6 Transmitting over Ill-conditioned MIMO channels: from spatial to constellation multiplexing

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This chapter addresses the problem of multiplexing multiple data streams in a multiple input multiple output (MIMO) system in the presence of channel matrix ill-conditioning brought by fading correlation and/or a Rice component. Conventional multiplexing schemes based on the separation of the stream's spatial signatures (commonly referred to as spatial multiplexing—SM, V-BLAST architecture) rely on the *linear* independence between the channel responses corresponding to each transmit antenna. Consequently, such schemes suffer considerably from effects bringing ill conditioning in the MIMO channel matrix, such as fading correlation and Rice components. In an attempt to robustify SM schemes for deployment in a wide range or propagation terrains, we investigate the use of so-called *constellation* multiplexing (CM) whereby distinct M-QAM streams are superposed to form a higher-order QAM constellation with rate equivalent to the sum of rates of all original streams. CM schemes do not rely on MIMO channel full rankness to function properly. We thus seek an approach that allows bridging SM and CM schemes. We show that this can be realized in the form of a linear diagonal precoder. This in turn yields an adaptive rate-preserving MIMO multiplexing algorithm that can operate smoothly at any degree of correlation or Ricean factor. Conventional SM and CM schemes are shown to be particular cases of the presented family of schemes.

6.1. Introduction

Multiple input and multiple output (MIMO) systems, are capable of providing a large increase in capacity compared to traditional single antenna systems [1, 2] (see also [3] for an overview of the area). This increase in capacity is however dependent upon the fact that the channels from a certain transmitter to the array of receivers, seen as vectors, are linearly separable. In other words, the conditioning of the equivalent MIMO channel matrix should be as good as possible. The capacity of MIMO systems can be shown to degrade if there are, for example, severe correlations present at the transmitter and/or receiver side [4, 5]. Similarly, [6] demonstrated that line of sight (LOS) components, while having a positive effect on the outage behavior of the channel, are also capable of reducing the ergodic (i.e., average) capacity of MIMO systems. That is because the matrix representing the LOS component of wireless MIMO channels is typically extremely ill conditioned [5], at least if the base station antennas are located on a single array, and thus does not lend itself to matrix inversion. In fact with either strong transmit correlations or a high Ricean factor, the capacity behavior of the MIMO channel will ultimately become similar to that of a SIMO/MISO, with a possible additional array gain depending on the partial channel knowledge at the transmitter. If the LOS channel is very dominating, then the capacity falls back to that of an AWGN SISO system with additional array gain at the receiver.

Although the negative impact of correlation and the Rice component on average capacity behavior of MIMO systems is significant, the effect it has on the BER behavior of actual spatial multiplexing schemes [1, 7] is much more dramatic. That is, because conventional SM schemes (such as in linear MIMO detectors) rely explicitly or implicitly (such as in maximum likelihood MIMO detectors) on linear separability of the input spatial signatures to detect the data, unless a form of joint encoding is applied across the streams to differentiate them. In fact, any ill-conditioned components present in the channel effectively increase the linear dependence of the input streams and makes stream separation and decoding a difficult task. For example, current schemes like SM (V-BLAST) literally break down in the presence of correlation levels close to one or high Ricean factors. As a result, algorithms such as V-BLAST simply fail to adapt themselves and extract the nonzero capacity that is present in highly correlated or strongly Ricean channels.

Designing appropriate transmission techniques that can adjust to various kinds of channel and terrain scenarios is therefore an important and practical issue for the successful deployment of MIMO systems. To tackle this problem, the correlation and LOS component structure can be assumed to remain static over a "long-enough" period of time so that the corresponding channel parameters, unlike the fast fading coefficients, can be sent to the transmitter at regular intervals using a low rate feedback logical channel. This is consistent with upcoming third generation wireless standards and beyond.

To robustify the transmission of independent streams we build on the concept of constellation multiplexing (CM) whereby distinct streams (e.g., M-QAM) are superposed to form a higher-order constellation (say N-QAM $N > M$) with rate equivalent to the sum of rates of all original streams. For instance, a 16-QAM signal is the superposition of two 4-QAM signals. By construction CM schemes do not rely on MIMO channel full rankness to function properly. In contrast to SM schemes, the substreams in CM schemes are differentiated through *power scaling* rather than through spatial signatures. Indeed, two 4-QAM signals with appropriate scaling, forming a single 16-QAM, require only one transmit and receive antenna to be sent and detected. Hence, such schemes are clearly robust with respect to fading correlation and the Ricean factor. In this paper, we build upon the following two simple observations.

(i) CM schemes can be cast as spatial multiplexing schemes over a low-rank MIMO channel cascaded with a linear diagonal precoder.

(ii) In the case of constellations with regular lattice, the precoder is such that the probability of error for each stream is identical.

This suggests that simple linear precoding can be used for spatial multiplexing schemes as an attempt to circumvent effect of channel ill conditioning. This also suggests an optimization metric based on equating probabilities of error among the substreams. Precoding for correlated/Ricean MIMO channels has been considered among others in the case of (low-rate) space-time (ST) block coding schemes (including but not limited to [8]). In the case of spatial mutiplexing schemes (which can be seen as high-rate ST codes) however, the effect of propagation-related ill conditioning is much more dramatic because the transmitter design cannot guarantee channel orthogonality.

Although precoding for such correlated scenarios have previously been considered [9, 10, 11], the focus has mainly been on transmit correlation and quite often on capacity issues rather than on designing robust practical algorithms. To minimize the BER in the presence of transmit correlation and LOS channel, a transmit precoding scheme based on per-antenna phase shifting was proposed in [12] to improve the system performance. The main downside of this approach is that a numerical search is required to find the optimal phases.

Here, instead, the focus is on low-complexity, closed-form solutions for SM precoders that tackle both transmit correlation and Ricean channels. The precoder is found as the solution to a linear equation parametrized as a function of the transmit correlation coefficient and the line-of-sight MIMO channel matrix.

The precoder is derived under the BER balancing criterion (BBC) which aims at giving the same error performance over all spatially multiplexed streams, a criterion that is proved to determine our precoder fully. Clearly the BBC criterion can be further weighted in order to accommodate unequal quality of service (QoS) targets.[1] The precoder is obtained under the hypothesis of a hybrid linear receiver (mixing concepts of maximum ratio and minimum mean square error receivers), but the resulting precoding coefficients are applicable over a wider range of receivers as shown in our simulations. In fact, the precoder can be interpretated as a linear data transform, bridging between SM and CM schemes. Under the case of zero correlation, Rayleigh channels, it matches the conventional SM approach. To the other extreme, it simply falls back to a CM scheme in the case of rank deficiency of the channel matrix. In that case, the scheme behaves equivalent to sending a single higher-order modulation whose independent components are mapped to the different antennas. In between these cases, the proposed algorithm will operate smoothly at all levels of correlation and Ricean factors.

The paper is organized as follows. First we present the mathematical models for signals and channels under the MIMO spatial multiplexing framework (Section 6.2). Then we introduce constellation multiplexing and its interpretation as a precoded form of spatial multiplexing for single transmit antenna systems (Section 6.3). In Section 6.4 we derive the precoder for the general case of MIMO with correlation and Rice component, first for two transmitters and then for any

[1] This issue is not addressed here however.

number of transmitters. We give numerical results in Section 6.6. Conclusions are given in Section 6.7.

Notations. The following notations are adopted throughout this paper. Vectors and matrices are denoted using bold-face lower-case letters and capitals, respectively. The symbol $(\cdot)^{\dagger}$ is used to refer to the Moore-Penrose pseudoinverse, while $\mathbf{H}_{:,i}$ points to the i'th column of the matrix $\mathbf{H}$ in a Matlab fashion. Similarly, $\mathbf{H}_{i,:}$ denotes the i'th row. $E\{\}$ is the expectation operator. The symbol $*$ refers to complex transpose of a vector/matrix.

6.2. Signal and channel models

6.2.1. Signal model

We consider the multiplexed transmission of N simultaneous data symbols $s_1, s_2, \dots, s_N$ over a memoryless channel. The symbols are *treated as* independent, but an outer-code may be applied across the streams prior to multiplexing.

The symbols are selected from a normalized constellation such that $E\{|s_i|^2\}=1$. The minimum distance between two symbols for the given modulation is denoted by $d_{\min}$, while $d_{\max}$ ($\geq d_{\min}$) is the minimum distance between two constellation points with highest amplitude. In the 4-QAM case, $d_{\min} = d_{\max}$ as all symbols are transmitted with equal power. For 16-QAM, we have $d_{\min} = 3d_{\max}$, and so forth.

6.2.2. Channel model

We consider a Ricean MIMO flat fading channel consisting of N transmit antennas and M ($\geq N$) receive antennas. For the sake of tractability, we will deal with correlation present at the transmitter only. A generalization of the precoding problem to deal in an elegant way with the receive correlation remains an open problem, although another version of this problem for precoding of space-time block codes was addressed in several papers, including, for example, [8, 13].

With transmit correlation and LOS component the channel matrix can be described by

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0\mathbf{R}_t^{1/2} + \sqrt{\frac{K}{K+1}}\mathbf{H}_{\text{los}}. \tag{6.1}$$

The $M \times N$ channel matrix $\mathbf{H}_0$ consists of complex Gaussian zero mean unit-variance independent and identically distributed (i.i.d) elements, while $\mathbf{R}_t$ is the $N \times N$ transmit correlation matrix. $\mathbf{H}_{\text{los}}$, also of dimensions $M \times N$, is the LOS channel matrix, possibly being ill conditioned, and K defines the Ricean factor. The choice of $K = 0$ leads to a standard Rayleigh fading channel.

The baseband equivalent of the N-dimensional signal vector observed at the receiver can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{6.2}$$

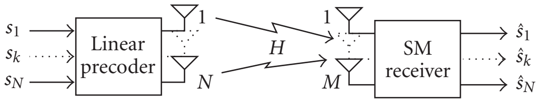

Figure 6.1. Spatial multiplexing with precoding.

where **n** is the M-dimensional noise vector whose entries are i.i.d complex Gaussian with zero mean and a variance of σ_n^2.

6.2.3. Linear precoding

We consider a memory less linear precoding of the N symbols according to the illustration in Figure 6.1. Although the most general precoder would take the form of an N by N matrix, we limit ourselves to a *diagonal* precoding of the symbols for the sake of closed-form derivability of the precoding coefficients. Furthermore, we argue (and confirm later) that robustness with respect to ill conditioning is brought about by the diagonal elements of the precoder first and foremost, as is also hinted at in Section 6.3.

The diagonally precoded transmitted vector, denoted by **s**, is given by

$$\mathbf{s} = \left[\sqrt{P}_1 s_1 \sqrt{P}_2 e^{j\phi_2} s_2 \cdots \sqrt{P}_N e^{j\phi_N} s_N\right]^T. \tag{6.3}$$

$P_1, \ldots, P_N$ represent power levels allocated, respectively, to input symbols $s_1, \ldots, s_N$, and are selected to satisfy $\sum_{i=1}^{N} P_i = 1$. $\phi_2, \ldots, \phi_N$ correspond to phase shifts on each transmit antenna. Notice that the first symbol does not undergo a phase change and can be regarded as a reference point for all other phase components. We therefore define $\phi_1 = 0$. Standard SM (e.g., V-BLAST) assigns equal weights $P_i = 1/N$ and $\phi_i = 0$ for $1 \leq i \leq N$.

6.3. Constellation multiplexing

The concept of constellation multiplexing lies in the remark that higher (say QAM) constellations can be designed from the superposition of lower-order constellations. For instance, a single 16-QAM constellation achieving a rate of 4 bits per symbol can be realized by superposing two 4-QAM constellations, each achieving a rate of 2 bits per symbol, with scaling of 1/4 between the two 4-QAM signals, as shown in Figure 6.2 [14]. In the same manner, two superposed 16-QAM signals can be used from a 256-QAM signal if the second one is scaled in power by a factor 1/16. Or, again, three superposed 4-QAM signals will form a single 64-QAM signals if a power scaling of, respectively, 1/4 and 1/16 is used for the second and third 4-QAM signals. In general, the superposition of N 2^m-QAM constellations, achieving each m bits persymbol for a total rate of mN bits persymbol, can be represented by a single 2^{mN}-QAM constellation with the same total rate. To obtain a regular grid for the 2^{mN}-QAM constellation, it suffices to apply a geometric scaling with factor r_m for every additional 2^m-QAM constellation. The factor r_m is a function of the ratio $d_{\max}/d_{\min}$ corresponding to the constellation used.

In other words, let s belong to a regular 2^{mN}-QAM constellation. Then there exist $r_m \in [0, 1]$ and $s_1, s_2, \ldots, s_N$ symbols, each belonging to a 2^m-QAM constellation, such that s can be written in the form of

$$s = \sum_{k=1}^{N} r_m^k s_k, \tag{6.4}$$

where r_m is such that $\sum_{k=1}^{N} r_m^{2k} = 1$ so as to preserve the unit power of the signal.

6.3.1. Precoding for rank one channels

Clearly, the signal shown in (6.4) can be reinterpretated as the signal being transmitted in a MIMO channel with rank-one with a simple precoder. Taking (6.2) with a rank-one MIMO channel in the generic form of $\mathbf{H} = [\mathbf{h}, \mathbf{h}, \ldots, \mathbf{h}]$, where $\mathbf{h}$ is an $M \times 1$ complex vector, and using (6.3) we find

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \mathbf{h} \sum_{k=1}^{N} \sqrt{P_k} e^{j\phi_k} s_k + \mathbf{n} = \mathbf{h}s + \mathbf{n}, \tag{6.5}$$

where the precoding coefficients are selected as

$$\begin{aligned} P_k &= r_m^{2k}, \quad k = 1 \cdots N, \\ \phi_k &= 0, \quad k = 1 \cdots N. \end{aligned} \tag{6.6}$$

Therefore, transmitting over a rank-one MIMO channel with an appropriate diagonal precoder is equivalent to transmitting over a higher-order constellation signal over a SIMO channel. This suggests that linear precoding can be used to deal effectively with rank deficient MIMO channels. In what follows, we explore this idea further and obtain closed-form precoding coefficients to handle all in-between scenarios with intermediate correlation and Ricean factors. We assume that the transmitter, in order to derive the precoder, has only knowledge of the slow-varying parameters of the channel (TDD system or FDD system with a low-rate feedback) such as the correlation structure, the line-of-sight MIMO component, and the Ricean factor.

For the sake of exposition, in the next section we start by describing the optimization procedure for the 2×2 case. The derivation is later extended to the case of arbitrary number of transmitter and receiver antennas.

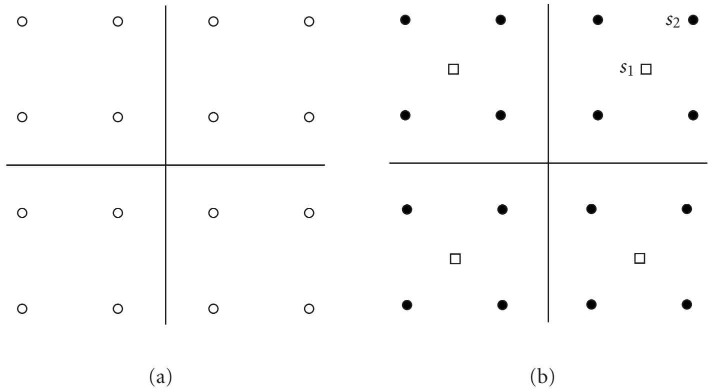

Figure 6.2. (a) Resulting equivalent 16QAM constellation. (b) Illustration of two superimposed 4-QAM signals s_1 and s_2. The latter is scaled down in power by 1/4.

6.4. Precoder optimization for 2×2 MIMO system

6.4.1. Receiver structure

As an aid toward finding a simple closed-form expression for the precoding weights, our calculation assumes a particular receiver combining structure based on maximum ratio combining (MRC). The principle behind the decoding structure is to successively estimate the substreams symbols in an iterative fashion, similar to V-BLAST [1], where the zero-forcing procedure is replaced with an MRC.

Although no optimality of this decoding method is claimed here, we draw the reader's attention on the fact that this approach allows us to derive the precoder expression in a compact fashion. It also yields results which make good intuitive sense and can easily be interpretated. Note that the use of MRC is consistent with bringing robustness against an ill conditioned channel, since a matrix inversion-based detector will generate a large noise enhancement in that case. Nevertheless, we show through simulations that the precoder behaves well when matched at the receiver with other more practical decoding methods such as maximum likelihood (ML) decoding. Simulation results are also presented with under MMSE decoding.

6.4.2. Detected signals

Writing out in full, the Ricean, transmit correlated, channel matrix in (6.1) in the 2×2 situation:

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0 \begin{bmatrix} \alpha & \beta e^{j\psi} \\ \beta e^{-j\psi} & \alpha \end{bmatrix} + \sqrt{\frac{K}{K+1}} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}, \tag{6.7}$$

where by construction $\alpha^2 + \beta^2 = 1$, and $\rho = 2\alpha\beta$ is the modulus of the antenna correlation coefficient, and channel coefficients $h_{i,j}$ describe the components of the LOS matrix.

Without loss of generality in the decoding procedure we assume $P_1 \geq P_2$ and as the first part of the decoding, the receiver implements an MRC with the first row of $\mathbf{H}^*$. We find the following from (6.2), (6.3), and (6.7):

$$z_1 = (\mathbf{H}^*)_{1,:}\mathbf{y} = \tau_1\sqrt{P_1}s_1 + \tau_2\sqrt{P_2}e^{j\phi_2}s_2 + (\mathbf{H}^*)_{1,:}\mathbf{n}, \tag{6.8}$$

τ_1 denotes the total gain (normalized by $\sqrt{P_1}$) for s_1 as a result of the MRC, while τ_2 represents the effects of the interference. Further normalizing z_1 in (6.8), or alternatively from

$$\frac{1}{\tau_1}z_1 = \sqrt{P_1}s_1 + \frac{\tau_2}{\tau_1}\sqrt{P_2}e^{j\phi_2}s_2 + \frac{1}{\tau_1}(\mathbf{H}^*)_{1,:}\mathbf{n}, \tag{6.9}$$

where

$$\tau_1 = (\mathbf{H}^*)_{1,:}\mathbf{H}_{:,1}, \qquad \tau_2 = (\mathbf{H}^*)_{1,:}\mathbf{H}_{:,2}, \tag{6.10}$$

is the channel-related interference factor.

Equations (6.8) and (6.9) show that symbol s_2 will be superimposed upon s_1 as a function of the channel matrix, whose long-term behavior depends on the transmit correlation, the K factor, and the LOS channel matrix. Notice that this superimposition effectively reduces the minimum distance for detection of symbol s_1.

At this point we make the assumption that the interference's magnitude is small enough not to "move" the symbol s_1 out of its decision boundary, that is,

$$\left|\frac{\tau_2}{\tau_1}\right|\sqrt{P_2}d_{\max} \leq \frac{1}{2}\sqrt{P_1}d_{\min}, \tag{6.11}$$

a symbol decision can be made on z_1 to obtain an estimate for s_1. For the sake of deriving the precoder, we assume that the correct decision is made on s_1, hence the symbol is subtracted from $\mathbf{y}$:

$$\hat{\mathbf{y}} = \mathbf{y} - \mathbf{H}_{:,1}\sqrt{P_1}s_1. \tag{6.12}$$

The robustness of the precoder with respect to the assumption made in (6.11) is demonstrated implicitly in the simulations. An estimate for the second symbol can now be obtained through a second MRC,

$$z_2 = (\mathbf{H}^*)_{2,:}\hat{\mathbf{y}} = \tau_3\sqrt{P_2}e^{j\phi_2}s_2 + (\mathbf{H}^*)_{2,:}\mathbf{n}, \tag{6.13}$$

where $\tau_3 = (\mathbf{H}^*)_{2,:}\mathbf{H}_{:,2}$.

6.4.3. Average channel behavior

We wish to design the precoder exclusively based upon knowledge of long-term parameters $h_{i,j}$, K, α, β, ψ, with no dependence on short-term varying parameter $\mathbf{H}_0$.

The performance of detection of s_1 depends on the instantaneous minimum distance in $\hat{z}_1$, however for the optimization of the weights P_1, P_2, and phases to be independent of $\mathbf{H}_0$, we base ourselves upon an "average" channel behavior. To this end, we introduce the following quantity modified from (6.8) by averaging over the instantaneous channel realizations:

$$\hat{z}_1 = E\{\tau_1\}\sqrt{P_1}s_1 + E\{\tau_2\}\sqrt{P_2}e^{j\phi_2}s_2. \tag{6.14}$$

A rather straightforward calculation can then be used to show that (see Section 6.5)

$$\begin{aligned} E\{\tau_1\} &= \frac{1}{K+1}\left(2 + K\left(h_{1,1}^* h_{1,1} + h_{2,1}^* h_{2,1}\right)\right), \\ E\{\tau_2\} &= \frac{1}{K+1}\left(2\rho e^{j\psi} + K\left(h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}\right)\right). \end{aligned} \tag{6.15}$$

Clearly, with $\rho = 0$ and $K = 0$ we find $E\{\tau_1\} = 2$ and $E\{\tau_2\} = 0$. This shows the MRC returning an average array gain factor of 2 (3dB).

6.4.4. Evaluation of minimum distances under average channel behavior

Next we evaluate the minimum distances, which dictate the error performance of the symbols, under average channel behavior, by considering the absolute average value of each individual gain factor in $E\{\tau_1\}$ and $E\{\tau_2\}$. For s_1, the minimum distance is found from (6.14) for, for example, $s_2 = -s_1$,

$$\delta_1 = E\{\tau_1\}\sqrt{P_1}d_{\min} - \overline{E\{\tau_2\}}\sqrt{P_2}d_{\max}, \tag{6.16}$$

where

$$\overline{E\{\tau_2\}} = \frac{1}{K+1}\left(2\rho + K\left|h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}\right|\right), \tag{6.17}$$

$\overline{E\{\tau_2\}}$ represents the average absolute gain coming from the correlation and the LOS channel components.

Similarly, the gain for s_2 under average channel behavior in (6.13) can be described by

$$\hat{z}_2 = E\{\tau_3\}\sqrt{P_2}e^{j\phi_2}s_2, \tag{6.18}$$

where one can show that (see Section 6.5)

$$E\{\tau_3\} = \frac{1}{K+1}\left(2 + K\left(h_{1,2}^* h_{1,2} + h_{2,2}^* h_{2,2}\right)\right). \tag{6.19}$$

This leads to the following minimum distance for s_2:

$$\delta_2 = E\{\tau_3\}\sqrt{P}_2 d_{\min}. \tag{6.20}$$

6.4.5. Precoding coefficients with the BER balancing criterion (BBC)

6.4.5.1. Phase optimization

If the average gain coming from $E\{\tau_2\}$ is nonzero, then by selecting the phase ϕ_2 accordingly, the distance from the decision boundary can be maximized for s_1. For an arbitrary QAM modulation, this is done by selecting ϕ_2 at the emitter such that

$$\phi_2 = -\angle E\{\tau_2\}, \tag{6.21}$$

which aligns up the symbols in a coherent fashion.

6.4.5.2. Power optimization

The noise entries of $\mathbf{H}_{1,:}^*\mathbf{n}$ and $\mathbf{H}_{2,:}^*\mathbf{n}$ all follow the same distribution, similarly all components in $\mathbf{H}^*$ also have an identical statistical structure. Thus, the noise factors have identical variance when averaged over $\mathbf{H}_0$. We can therefore equate the average probability of error for s_1 and s_2 simply by equating the minimum distances found in (6.16) and (6.20),

$$E\{\tau_1\}\sqrt{P}_1 d_{\min} - \overline{E\{\tau_2\}}\sqrt{P}_2 d_{\max} = E\{\tau_3\}\sqrt{P}_2 d_{\min}, \tag{6.22}$$

under constraint

$$P_1 + P_2 = 1. \tag{6.23}$$

For clarity we rewrite (6.22) as

$$\mu_1\sqrt{P}_1 - \mu_2\sqrt{P}_2 = \mu_3\sqrt{P}_2, \tag{6.24}$$

where we have defined $\mu_1 = E\{\tau_1\}d_{\min}$, $\mu_2 = \overline{E\{\tau_2\}}d_{\max}$, and $\mu_3 = E\{\tau_3\}d_{\min}$. The weights for this 2×2 system can easily be computed as functions of μ to be

$$P_1 = \frac{(\mu_2 + \mu_3)^2}{\mu_1^2 + (\mu_2 + \mu_3)^2}, \qquad P_2 = \frac{\mu_1^2}{\mu_1^2 + (\mu_2 + \mu_3)^2}, \tag{6.25}$$

or written out in full:

$$P_1 = \frac{\left[d_{\max}(2\rho + K|\alpha|) + d_{\min}(2 + K(|h_{1,2}|^2 + |h_{2,2}|^2))\right]^2}{d_{\min}^2(2+K(|h_{1,1}|^2+|h_{2,1}|^2))^2+[d_{\max}(2\rho+K|\alpha|)+d_{\min}(2+K(|h_{1,1}|^2+|h_{2,1}|^2))]^2},$$

$$P_2 = \frac{d_{\min}^2(2+K(|h_{1,1}|^2+|h_{2,1}|^2))^2}{d_{\min}^2(2+K(|h_{1,1}|^2+|h_{2,1}|^2))^2+[d_{\max}(2\rho+K|\alpha|)+d_{\min}(2+K(|h_{1,1}|^2+|h_{2,1}|^2))]^2}, \tag{6.26}$$

where $\alpha = h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}$.

6.4.6. Interpretations

Observe that $\alpha = h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}$ in the expression for $\overline{E\{\tau_2\}}$ directly measures the ill conditioning of $\mathbf{H}_{\text{los}}$. For instance, in the (unrealistic) case that the LOS component is orthogonal, then $\alpha = 0$. This is intuitively appealing because one expects the precoder to depend on whether the LOS component is easily invertible or not.

Special cases.

(i) **No LOS:** with a small K, the expressions give more attention to the effects of transmit correlation. For instance, $K = 0$ gives $\phi_2 = -\psi$, while $\mu_1 = 2$, $\mu_2 = 2\rho$, and $\mu_3 = 2$, which results in the following solution:

$$P_1 = \frac{(1 + (d_{\max}/d_{\min})\rho)^2}{1 + (1 + (d_{\max}/d_{\min})\rho)^2}, \qquad P_2 = \frac{1}{1 + (1 + (d_{\max}/d_{\min})\rho)^2}. \tag{6.27}$$

Notice that $d_{\max}/d_{\min} = 1$ for 4-QAM and $d_{\max}/d_{\min} = 3$ for 16-QAM, for example. In the 4-QAM case, the precoder is thus

$$P_1 = \frac{(1+\rho)^2}{1+(1+\rho)^2}, \qquad P_2 = \frac{1}{1+(1+\rho)^2}. \tag{6.28}$$

(a) **Uncorrelated:** with no correlation $\rho = 0$ which yields equal power transmission, justifying the standard V-BLAST design corresponding to spatial multiplexing.

(b) **Fully correlated:** with full correlation $\rho = 1$ we find $P_1 = 0.8$ and $P_2 = 0.2$. Interestingly, this corresponds to the power allocation for a regular 2D constellation. For instance, a unit power 16-QAM constellation can be seen as the superposition of two 4-QAM constellations with respective powers 0.8 and 0.2 (see Figure 6.2). Hence, the spatial multiplexing is here replaced by constellation multiplexing.

(ii) **Strong LOS:** with $K \to \infty$ and a strongly ill conditioned $\mathbf{H}_{\text{los}}$, we find $\mu_1 \approx \mu_2 \approx \mu_3$, giving $P_1 = 0.8$ and $P_2 = 0.2$ (4-QAM). If the LOS component is better conditioned, the scheme performs a mix of spatial and constellation multiplexing.

6.5. Optimization for an arbitrary MIMO system

For a general MIMO setup, the MRC precoder may easily be extended as follows. We first assume that the power weights satisfy

$$P_1 \geq P_2 \geq \cdots \geq P_N. \tag{6.29}$$

Thus, in an iterative detection procedure, s_1 becomes the first symbol to be decoded, followed by s_2, and so forth, in a chronological order.

To derive the appropriate values of $P_1, \ldots, P_N$ and phases, the average gain and interference factors need to be calculated. The average gain coming from cross interference of the LOS channel and the remaining channel $\mathbf{H}_0\mathbf{R}_t^{1/2}$ is clearly zero, that is, $E\{(\mathbf{H}_0\mathbf{R}_t^{1/2})^*\mathbf{H}_{\text{los}}\} = E\{\mathbf{H}_{\text{los}}^*(\mathbf{H}_0\mathbf{R}_t^{1/2})\} = \mathbf{0}$.

We therefore obtain

$$\begin{aligned} E\{\mathbf{H}^*\mathbf{H}\} &= \frac{1}{K+1}E\{(\mathbf{H}_0\mathbf{R}_t^{1/2})^*(\mathbf{H}_0\mathbf{R}_t^{1/2})\} + \frac{K}{K+1}\mathbf{H}_{\text{los}}^*\mathbf{H}_{\text{los}} \\ &= \frac{M}{K+1}\mathbf{R}_t + \frac{K}{K+1}\mathbf{H}_{\text{los}}^*\mathbf{H}_{\text{los}}. \end{aligned} \tag{6.30}$$

Element k, l $(1 \leq k, l \leq N)$ can then be written out explicitly as

$$E\{\mathbf{H}^*\mathbf{H}\}_{k,l} = \frac{1}{K+1}\left(M\rho_{k,l} + K\sum_{i=1}^{M} h_{i,l}^* h_{i,k}\right). \tag{6.31}$$

Taking into account the absolute gain coming from both factors, we define

$$\gamma_{k,l} = \frac{1}{K+1}\left(M\rho_{k,l} + K\left|\sum_{i=1}^{M} h_{i,l}^* h_{i,k}\right|\right). \tag{6.32}$$

As previously, channel coefficients $h_{i,j}$ represent elements of the LOS matrix, while $\rho_{k,l}$ describes the nonnegative coefficients of the correlation matrix $\mathbf{R}_t$, where $\rho_{k,k} = 1$.

Assuming an iterative MRC receiver, the average minimum distance for s_1 becomes

$$\delta_1 = \gamma_{1,1}\sqrt{P_1}d_{\min} - \gamma_{1,2}\sqrt{P_2}d_{\max} - \cdots - \gamma_{1,N}\sqrt{P_N}d_{\max}. \tag{6.33}$$

After a symbol estimation/subtraction, the minimum distance for s_2 can be found:

$$\delta_2 = \gamma_{2,2}\sqrt{P_2}d_{\min} - \gamma_{2,3}\sqrt{P_3}d_{\max} - \cdots - \gamma_{2,N}\sqrt{P_N}d_{\max}. \tag{6.34}$$

By repeating this N times, we obtain expressions for N minimum distances,

$$\delta_N = \gamma_{N,N}\sqrt{P_N}d_{\min}. \tag{6.35}$$

6.5.1. Phase optimization

To cancel out the phase shifts introduced for s_1, the most significant symbol, we set

$$\phi_i = -\angle E\{\mathbf{H}^*\mathbf{H}\}_{1,i} = -\angle\left(M\rho_{1,i} + K\sum_{k=1}^{M} h_{k,i}^* h_{k,1}\right), \tag{6.36}$$

for $i = 2, \dots, N$.

If the transmitter and receivers are positioned far from each other, and the arrays are placed broadside to each other, which is a practical situation in many applications, then the channel model can be approximated as [6]

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0\mathbf{R}_t^{1/2} + \sqrt{\frac{K}{K+1}}e^{j\theta}\mathbf{1}. \tag{6.37}$$

If, in addition, a constant phase shift, or an exponential correlation model, is applied for the transmit correlation matrix, then the solution provided by (6.36) will hold for all symbols [15] as long as K is small. For larger values of K, $\mathbf{H}_{\text{los}}$ will dominate and the single phase shift it introduces can be canceled out for all symbols by selecting $\phi_i = -\theta$, $i = 2, \dots, N$.

Nevertheless, for this particular MRC receiver, the exact phase rotation is also dependent upon $\mathbf{H}_0$, which the transmitter is unaware of, and the selection of (6.36) will in practice therefore only have minor effect. In contrast, the decoder of [16] eliminates $\mathbf{H}_0$ before further processing and the phase change thus plays a more important role. The essential information destined to differentiate the signals is though determined by the choice of power weights.

6.5.2. Weight optimization

To guarantee all symbols an equal error rate, it is sufficient that values for $\sqrt{P_1}, \sqrt{P_2}, \dots, \sqrt{P_N}$ be selected so that on average the minimum symbol distance observed for each symbol is identical,

$$\delta_1 = \delta_N, \qquad \delta_2 = \delta_N, \dots, \delta_{N-1} = \delta_N. \tag{6.38}$$

Based on (6.38), the following linear system can then be set up as part of the problem to find the appropriate power levels:

$$\mathbf{\Delta p} = \mathbf{0}, \tag{6.39}$$

where

$$\boldsymbol{\Delta} = \begin{bmatrix} \gamma_{1,1}\check{d} & -\gamma_{1,2}\hat{d} & -\gamma_{1,3}\hat{d} & \cdots & -\gamma_{N,N}\check{d} - \gamma_{1,N}\hat{d} \\ 0 & \gamma_{2,2}\check{d} & -\gamma_{2,3}\hat{d} & \cdots & -\gamma_{N,N}\check{d} - \gamma_{2,N}\hat{d} \\ & & \cdots & & \\ 0 & 0 & 0 & \gamma_{N-1,N-1}\check{d} & -\gamma_{N,N}\check{d} - \gamma_{N-1,N}\hat{d} \end{bmatrix}, \tag{6.40}$$

$$\mathbf{p} = \left[\sqrt{P_1}\sqrt{P_2}\cdots\sqrt{P_N}\right]^T \tag{6.41}$$

and $\mathbf{0}$ is a vector with N zero elements. To obtain a compact notation we have used $\check{d} = d_{\min}$ and $\hat{d} = d_{\max}$.

The upper triangular system (6.40) only contains $N-1$ equations for N unknowns, however, any solution must also satisfy $\sum_{i=1}^{N} P_i = 1$. Therefore, $\mathbf{p}$ can be found as the only unit-norm all-positive vector in the null space of $\boldsymbol{\Delta}$. For a proof we refer to the appendix. Therefore, the BBC criterion determines the precoder in a unique way (up to trivial ambiguities like a rotation, etc.).

Extreme LOS cases.

(i) Ideal Rayleigh case: if $K = 0$ and $\rho = 0$, we find $\gamma_{k,l} = 0$, $(1 \leq k,\ l \leq N,\ k \neq l)$ and from (6.40) one can easily see that this gives $P_i = 1/N$, that is, equal power distribution across all streams.

(ii) On the other hand, with a strong K factor and high level of ill conditionality (e.g., (6.37)), we can assume all $\gamma_{k,l}$ to be of roughly equal value. For instance, in the 4QAM case (i.e., with $d_{\min} = d_{\max}$), this gives rise to the following (scale corrected) matrix:

$$\boldsymbol{\Delta} = \begin{bmatrix} 1 & -1 & -1 & \cdots & -2 \\ 0 & 1 & -1 & \cdots & -2 \\ & & \cdots & & \\ 0 & 0 & & 1 & -2 \end{bmatrix}. \tag{6.42}$$

This linear system can easily be solved through backsubstitution and under the energy constrain one arrives to

$$P_i = \frac{3 \cdot 4^N}{4^i\left(4^N - 1\right)}, \quad i = 1, \ldots, N. \tag{6.43}$$

The energy for this setup decreases by one quarter from symbol s_i to s_{i+1}. The final form of the received signal $\hat{z}_1$ will conclusively simply correspond to a standard 4^N-QAM modulation.

6.6. Simulations

This section demonstrates the effectiveness of the precoder through Monte Carlo simulations. We use the following receiver structures and compare the results with and without precoding

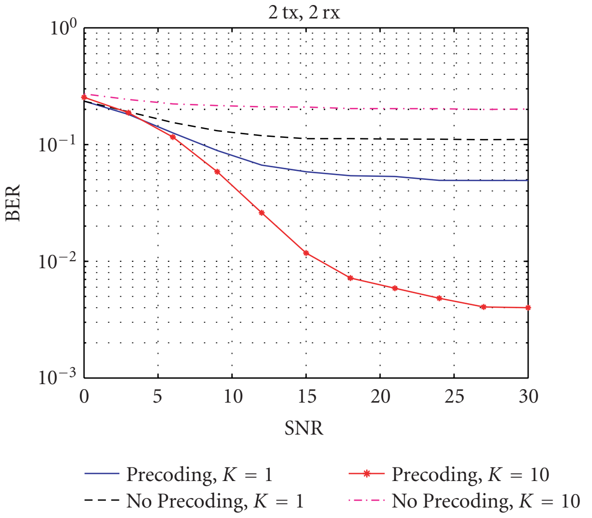

Figure 6.3. $K = 1$, $K = 10$, MRC with/without precoding.

(i) MRC SIC (successive interference cancelling) decoder, used to derive the precoding weights.

(ii) MMSE SIC decoder. The receiver is similar to the one above, but rather implements an MMSE matrix inversion to estimate the symbols in each iteration.

(iii) ML, an exhaustive maximum likelihood search is carried out.

Illustrating the first simulation, Figure 6.3 shows the simulation results for the MRC decoder assuming $K = 1$, $K = 10$ under the channel model of (6.37) and no transmit correlation. The use of MRC introduces residual symbol interference showing up as a flooring effect, however, the precoder nevertheless manages to bring in a noticeable improvement.

Illustrating the second simulation plot, Figure 6.4 displays the use of MMSE SIC receiver structure with/without precoding under the same channel conditions as previously. In Figure 6.5, the same simulations have been extended with transmit correlation being set at $\rho = 0.8$. Even at low K-factors, having a precoder clearly becomes beneficial.

Figure 6.6 uses ML as the decoder, the K-factors being 10 and 15 and with no transmit correlation assumption. A high K-factor with precoding makes the slope of the curve steeper as the fading is virtually nonexisting.

Finally, in Figure 6.7 we demonstrate the use of ML decoding at SNR of 15 dB, $K = 0$, for a 2×2 setup with transmitter correlation ranging from $\rho = 0$ to $\rho = 1$. The difference between ML with or without precoding is relatively small at low correlation levels but becomes very substantial with higher degrees of transmit correlation.

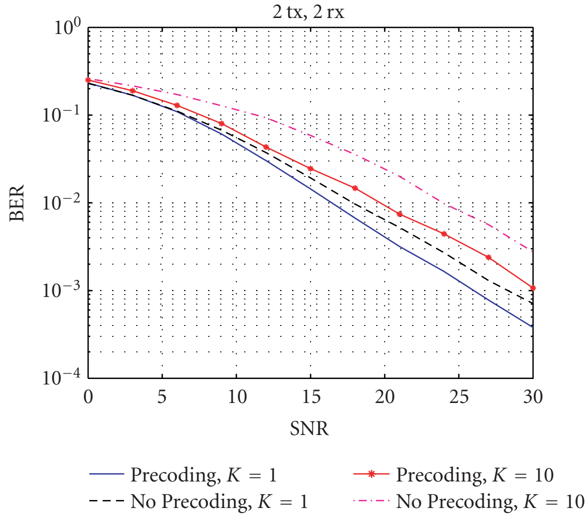

Figure 6.4. $K = 1$ and $K = 10$, MMSE SIC with/without precoding.

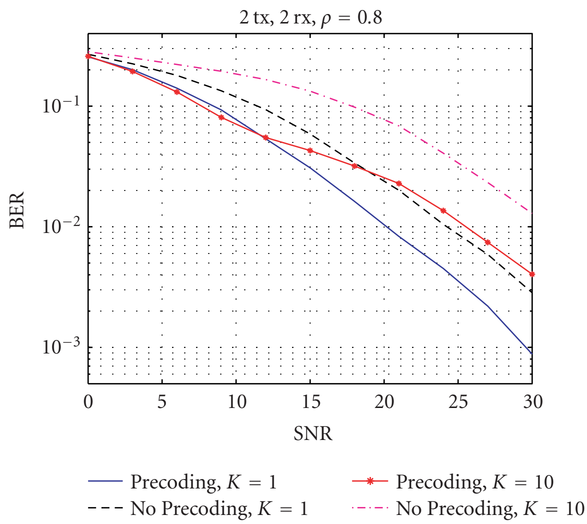

Figure 6.5. $K = 1$ and $K = 10$, $\rho = 0.8$, MMSE SIC with/without precoding.

6.7. Conclusions

In this chapter, we proposed a simple closed-form power weighting approach making use of the average channel knowledge to adapt the transmitted constellation.

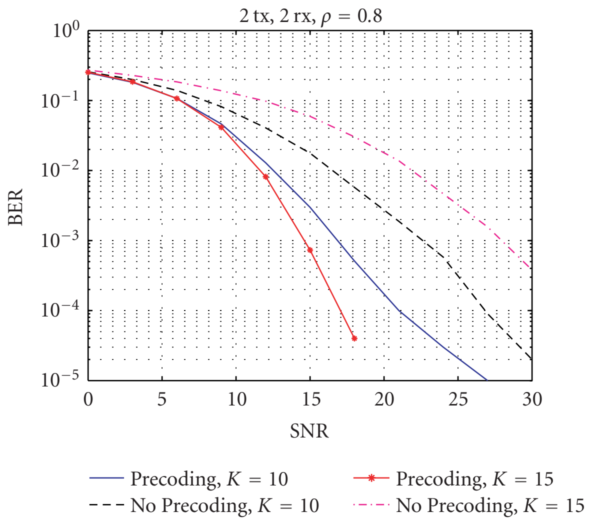

Figure 6.6. $K = 10$ and $K = 15$, ML with/without precoding.

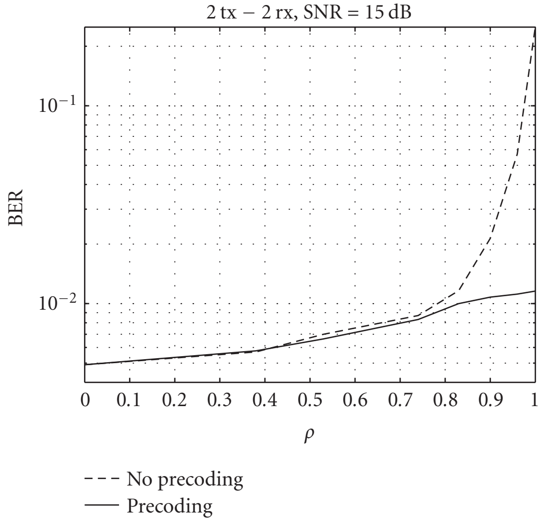

Figure 6.7. 2×2 case. Improvement due to the proposed precoder with the ML receiver as function of correlation.

The derivation assumes an MRC-based decoder however the weights may be applied on a wider range of receiver structures. This offers a way to preserve a constant data rate for any correlation level and for well- or ill-behaved LOS components.

Appendix

Due to the specific structure of (6.40) there exists a solution to (6.39), where all entries of $\mathbf{p}$ are nonnegative. Observe that the left $N - 1 \times N - 1$ submatrix of $\boldsymbol{\Delta}$ is upper triangular and contains unit entries on the diagonal, while all other elements are nonpositive. The last column of $\boldsymbol{\Delta}$ however consists of all strictly negative entries.

From the format of (6.40), it is clear that $\sqrt{P_N} \neq 0$ otherwise all elements in $\mathbf{p}$ would become zero. Without loss of generality, we can therefore set $\sqrt{P_N} = 1$. Moving the last column to the right-hand side gives a strictly positive vector and the upper unit triangular system can be solved by backsubstitution to find $\sqrt{P_{N-1}}, \dots, \sqrt{P_1}$. A suitable scaling can then assure proper normalization and the result follows.

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Abbreviations

AWGN	Additive white Gaussian noise
BER	Bit error rate
BBC	BER balancing criterion
CM	Constellation multiplexing
LOS	Line of sight
MIMO	Multiple input multiple output
MISO	Multiple input single output
M-QAM	M - quadrature amplitude modulation
QoS	Quality of service
SIMO	Single input multiple output
SM	Spatial multiplexing
V-BLAST	Vertical Bell-labs layered space-time

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