

# Optimized data sharing in multicell MIMO with finite backhaul capacity

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**Abstract**—This paper addresses cooperation in a multicell environment where base stations (BSs) wish to jointly serve multiple users, under a constrained-capacity backhaul. We point out that for finite backhaul capacity a trade-off between sharing user data, which allows for full MIMO cooperation, and not doing so, which reduces the setup to an interference channel but also requires less overhead, emerges. We optimize this trade-off by formulating a rate splitting approach in which non-shared data (private to each transmitter) and shared data are superimposed. We derive the corresponding achievable rate region and obtain the optimal beamforming design for both shared and private symbols. We illustrate how the capacity of the backhaul determines how much of the user data is worth sharing across multiple BSs.

## I. INTRODUCTION

Interference is a major issue in several types of wireless networks. The related problem is especially acute in cellular networks with full spectrum reuse across all base stations (BSs) (see [1] and references therein). In traditional designs, each BS obtains from the backhaul the data intended for users in its coverage area alone, i.e., if one ignores cases of soft handover, data for users is not available at multiple BSs: this results in the so-called interference channel (IC) and was treated for the MISO case in [2] and [3] for example. Recent research rooted in MIMO theory has suggested the benefits of relaxing this constraint, allowing for user messages to be shared at multiple transmitters so that a giant broadcast MIMO channel ensues. In such a scenario, multicell processing in the form of joint precoding is realized, e.g. for the downlink: this scheme is referred to as network MIMO (a.k.a. multicell MIMO) (see [4], [5], [6] for example).

In this paper, we focus on the issue of data sharing in a multicell cooperation setup. Other issues, such as the complexity of a centralized implementation of network MIMO or CSI sharing are for example tackled in [7] and [8], respectively.

In fact, full data sharing subsumes high capacity backhaul links, which may not always be available, or even simply desirable. In fact, under limited backhaul rate constraints, data sharing consumes a precious fraction of the backhaul capacity which otherwise could be used to carry more data to the users: this overhead should thus be compensated by the capacity gain induced by the network MIMO channel over the classical IC.

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A number of recent interesting research efforts have considered networks with finite-capacity backhaul. For example, in [9], joint encoding for the downlink of a cellular system is studied under the assumption that the BSs are connected to a central unit via finite-capacity links. The authors investigate different transmission schemes and ways of using the backhaul capacity in the context of a modified version of Wyner's channel model. One of their main conclusions is that "central encoding with oblivious cells", whereby quantized versions of the signals to be transmitted from each BS, computed at the central unit, are sent over the backhaul links, is shown to be a very attractive option for both ease of implementation and performance, unless high data rate are required. If this is the case, the BSs need to be involved in the encoding, i.e., at least part of the backhaul link should be used for sending the messages themselves not the corresponding codewords. Another recent study which deals with a Wyner-like channel model is [10], which has taken an information-theoretic look at the problem of partial message exchange between neighboring BSs and derived the corresponding asymptotic multiplexing gain per-user as the number of users (and BSs) goes to infinity.

In [11], an optimization framework, for an adopted backhaul usage scheme, is proposed for the downlink of a large cellular system. A so-called joint transmission configuration matrix is defined: this specifies which antennas in the system serve which group of users. The backhaul to each BS is used to either carry quantized versions of the transmit signals computed centrally similarly to the scheme in [9], except that a more realistic system model is assumed; alternatively, the backhaul is used to carry uncoded binary user data.

In [12], a more information-theoretic approach is taken and a two-cell setup is considered in which, in addition to links between the network and each BS, the two multi-antenna BSs may be connected via a finite-capacity link: different usages of the backhaul are optimized and their rate regions compared under suboptimal maximum ratio transmission beamforming. [13] uses duality theory to optimize transmission for both quantized and unquantized message based cooperation schemes: for the unquantized case, intermediate schemes that time-share between no and full cooperation while meeting the backhaul constraints are proposed.

Imposing finite capacity constraints on the backhaul links brings with it a set of interesting research as well as practical questions, since more cooperation between BSs is expected in 4G cellular networks, in particular:

- Given the backhaul constraints, assuming that *not all* traffic is shared across transmitters, i.e assuming a certain part remains private to each transmitter, what rates can be achieved?

- How useful is data sharing when backhaul constraints are present? I.e., how do the rates achieved with a data sharing joint transmission enabling scheme compare to those achieved without data sharing, under limited backhaul?
- Is there a backhaul capacity lower bound below which it does not pay off to share user data?

In this work, we attempt to answer these questions by considering a setup in which a finite rate backhaul connects the network with each of the BSs, and focusing on how to use this given backhaul to serve the users in the system. To simplify exposition, we focus on the two-cell problem. A transmission scheme is specified whereby superposition coding is used to transmit signals to each user: each user's data is in fact split into two types, 'private' data sent by a single BS and 'shared' data transmitted via multiple bases. Thus for all non trivial traffic ratios between private and shared data, our system corresponds to a hybrid channel, which in an information theoretic sense may be considered as an intermediate between the MIMO broadcast (or "network MIMO") channel and the IC. Intuitively, such an approach should be useful as it allows to tune how much data is shared as a function of the backhaul constraints: if the backhaul is too constrictive, it may be better to simply have each user served by a single base station rather than have both messages routed to both base stations. This is because, although data sharing allows to convert the interference channel into a MIMO broadcast channel with higher capacity, data sharing occupies the resources that could otherwise be used to send fresh (non shared) data.

The corresponding rate region is expressed in terms of the backhaul constraints and the beamforming vectors used to carry the different signals: finding the boundary of the aforementioned region is reduced to solving a set of convex optimization problems. In doing so, we also solve the problem of optimal beamforming design for this hybrid IC/MIMO broadcast channel. We compare the rates achieved in such a rate splitting scheme to those obtained for network MIMO and the IC and illustrate the gains related to moderate sharing levels in certain realistic situations. An alternative approach to the use made here of the backhaul is the one proposed in [9] for example: as described earlier, such an approach assumes the BSs are oblivious to the encoding and uses the backhaul to forward quantized versions of the encoded symbols to be transmitted over the air. We also adapt this scheme to our setup and compare its performance to our rate splitting scheme.

**Notation:** Normal lower case letters, boldface lower case letters and boldface upper case letters are used to represent scalars, vectors and matrices, respectively. All vectors in the paper are column vectors.  $\mathbf{x}^T$ ,  $\mathbf{x}^*$  and  $\mathbf{x}^H$  denote the transpose, the conjugate and conjugate transpose or Hermitian of vector  $\mathbf{x}$ , respectively.  $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$  denotes a circularly symmetric complex Gaussian random vector of mean  $\boldsymbol{\mu}$  and covariance matrix  $\mathbf{C}$ . Finally  $\mathbf{S} \succeq 0$  means  $\mathbf{S}$  is a positive semidefinite (PSD) matrix.

## II. SYSTEM MODEL AND PROPOSED TRANSMISSION SCHEME

The system considered is shown in Figure 1. In this study, we focus on a two transmitter two receiver setup. As we emphasize the problem of precoding at the transmitter side, the receivers are assumed to have a single antenna whereas transmitters have  $N_t \geq 1$  antennas each:  $\mathbf{h}_{ij}$  is the  $N_t$ -dimensional complex vector corresponding to the channel between transmitter  $j$  and user  $i$ ;  $\mathbf{h}_i = [\mathbf{h}_{i1}^T, \mathbf{h}_{i2}^T]^T$  represents user  $i$ 's whole channel state vector. The signal received at user  $i$  will be given by

$$y_i = \sum_{j=1}^2 \mathbf{h}_{ij}^T \mathbf{x}_j + z_i, \quad (1)$$

where  $\mathbf{x}_j \in \mathbb{C}^{N_t}$  denotes BS  $j$ 's transmit signal, and  $z_i \sim \mathcal{CN}(0, \sigma^2)$  is the receiver noise.  $\mathbf{x}_j$  is subject to power constraint  $P_j$  so that

$$\mathbb{E} \|\mathbf{x}_j\|^2 \leq P_j, \quad j = 1, 2. \quad (2)$$

We assume a backhaul link of capacity  $C_j$  [bits/sec/Hz] between the central processor (CP) or the backbone network, and transmitter  $j$ , for  $j = 1, 2$ : the central processor collects all downlink traffic then routes it to individual (non shared traffic) or both (shared traffic) transmitters. In an attempt to bridge the IC situation (where the transmitters do not share user data) and the multi-cell MIMO scenario (where they do), we propose to split the user traffic content across two types of messages:

- *private messages* which are sent from the CP to only one of the transmitters, and
- *shared messages*, which are sent from the CP to both transmitters, and are consequently jointly transmitted.

Thus, the total information rate for user  $i$ ,  $r_i$ , will be split across  $r_{i1,p}$ ,  $r_{i2,p}$  and  $r_{i,c}$ , where  $r_{i,c}$  refers to the rate of the shared message for that user, and  $r_{ij,p}$  refer to the rate of the private message for user  $i$  reaching it from BS  $j$ :

$$r_i = \sum_{j=1}^2 r_{ij,p} + r_{i,c}. \quad (3)$$

*Assumptions* We assume each receiver does single user detection (SUD), in the sense that any source of interference is treated as noise. Note that this paper examines the costs and benefits of sharing user data, not that of sharing the channel state information (CSI), hence full global CSIT is assumed at each transmitter. A companion paper which focuses on the problem of CSIT sharing can be found in [14].

### A. Particular Cases

The transmission scheme introduced here covers the two particular cases of:

- An IC, obtained by forcing  $r_{ii,p} \equiv r_i$ ,  $i = 1, 2$ , and
- A network MIMO channel, obtained by forcing  $r_{ij,p} \equiv 0$ ,  $i = 1, 2$ ,  $j = 1, 2$ .

*Notation* In what follows,  $\bar{i} = \text{mod}(i, 2) + 1$ ,  $i = 1, 2$  and is used to denote the *other* transmitter/receiver depending on the context.

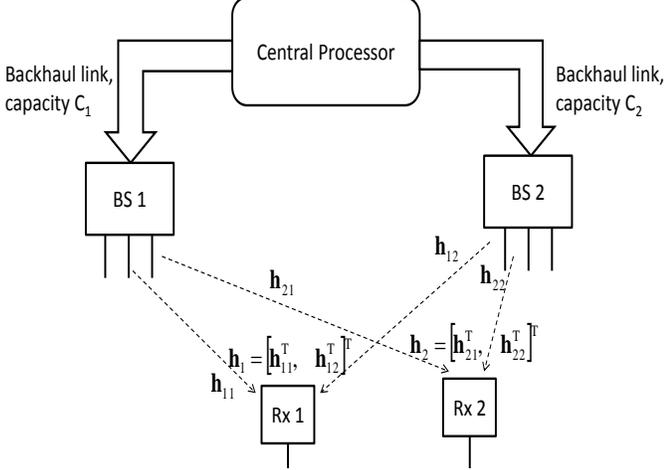


Fig. 1. Constrained backhaul setup. The rates of the messages carried by each backhaul link are represented. The central processor is assumed to collect all downlink traffic then route it to individual (non shared traffic) or both (shared traffic) transmitters.

### B. Backhaul usage

Here we introduce some fundamental inequalities imposed by the backhaul constraints which will be helpful in characterizing the achievable rate region for this hybrid IC/MIMO broadcast channel. Backhaul link  $j$  with finite capacity  $C_j$  serves to carry both private (from BS  $j$ ) and shared messages for both users, so that the following constraint applies:

$$C_j \geq \sum_{i=1}^2 r_{ij,p} + \sum_{i=1}^2 r_{i,c}, \quad j = 1, 2. \quad (4)$$

Using (3), this constraint can be rewritten as:

$$C_j \geq \sum_{i=1}^2 r_i - \sum_{i=1}^2 r_{i\bar{j},p}, \quad j = 1, 2. \quad (5)$$

Finally, the sum rate  $r = r_1 + r_2$  cannot exceed the total backhaul capacity, so that

$$r \leq C_1 + C_2. \quad (6)$$

### C. Over-the-air transmission

The channel between the two transmitters and user  $i$  can be viewed as a MAC with a common message [15]. The overall channel can be regarded as the superposition of two such channels, which interfere with each other so that the receiver noise at user  $i$  is enhanced by the interference due to the signals carrying user  $\bar{i}$ 's data; the total interference plus noise power at user  $i$  will be denoted by  $\sigma_i^2$ . We thus write the transmit signal of BS  $j$  as a superposition of two signals,  $\mathbf{x}_{ij}$ ,  $i = 1, 2$ , one intended for each user:

$$\mathbf{x}_j = \sum_{i=1}^2 \mathbf{x}_{ij}. \quad (7)$$

Restricting the transmission model to beamforming,  $\mathbf{x}_{ij}$  can be generated as:

$$\mathbf{x}_{ij} = \mathbf{w}_{ij,c} s_{i,c} + \mathbf{w}_{ij,p} s_{ij,p}, \quad (8)$$

where  $s_{i,c}$  and  $s_{ij,p}$  are independent  $\mathcal{CN}(0, 1)$  random variables;  $\mathbf{w}_{i,c} = [\mathbf{w}_{i1,c}^T \mathbf{w}_{i2,c}^T]^T \in \mathbb{C}^{2N_t}$ , is the beamforming vector carrying symbols  $s_{i,c}$ , and  $\mathbf{w}_{ij,p} \in \mathbb{C}^{N_t}$  is the beamforming vector carrying symbols  $s_{ij,p}$ .

The following proposition specifies a rate region corresponding to the over-the-air channel, achievable by transmit signals of the form given in (8). Details of the proof can be found in Appendix A.

**Proposition 1.** *The following rate region  $\mathcal{R}_{\text{air}}$  is achievable on the over-the-air segment:*

$$\begin{aligned} r_{ij,p} &\leq \log_2 \left( 1 + \frac{|\mathbf{h}_{ij}^T \mathbf{w}_{ij,p}|^2}{\sigma_i^2} \right), \quad j = 1, 2, i = 1, 2 \\ \sum_{j=1}^2 r_{ij,p} &\leq \log_2 \left( 1 + \frac{\sum_{j=1}^2 |\mathbf{h}_{ij}^T \mathbf{w}_{ij,p}|^2}{\sigma_i^2} \right), \quad i = 1, 2 \\ r_i &\leq \log_2 \left( 1 + \frac{|\mathbf{h}_i^T \mathbf{w}_{i,c}|^2 + \sum_{j=1}^2 |\mathbf{h}_{ij}^T \mathbf{w}_{ij,p}|^2}{\sigma_i^2} \right), \quad i = 1, 2 \end{aligned} \quad (9)$$

where

$$\sigma_i^2 = \sigma^2 + \sum_{j=1}^2 |\mathbf{h}_{ij}^T \mathbf{w}_{\bar{i},p}|^2 + |\mathbf{h}_i^T \mathbf{w}_{\bar{i},c}|^2, \quad (10)$$

and the beamforming vectors are subject to power constraint

$$\sum_{i=1}^2 (\|\mathbf{w}_{ij,c}\|^2 + \|\mathbf{w}_{ij,p}\|^2) \leq P_j, \quad j = 1, 2. \quad (11)$$

Note that since the rate region given by Proposition 1 corresponds to that of a MAC with a common message, thus for the rates to be achieved successive interference cancellation may be required at each receiver: however, this would involve the user's own signals, not those intended for the other user.

## III. ACHIEVABLE RATE REGION

The set of rate-tuples  $(r_1, r_{11,p}, r_{12,p}, r_2, r_{21,p}, r_{22,p})$  that belong to  $\mathcal{R}_{\text{air}}$  (see Proposition 1) and also satisfy the specified backhaul constraints defines an achievable rate region  $\mathcal{R}$ . We are particularly interested in its boundary and in the beamforming strategies to achieve points on this boundary. As the direct characterization of the rate region is a difficult task here, one may obtain the desired boundary by using the rate profile notion from [16]: a rate profile specifies how the total rate is split between the users. Points on the rate region boundary are thus obtained by solving the following problem for  $\alpha$  discretized over  $[0, 1]$ , where  $\alpha$  denotes the proportion of the total sum rate intended for user 1's data:

$$\begin{aligned}
& \max. r \\
& \text{s.t. } r_1 = \alpha r, \quad r_2 = (1 - \alpha)r \\
& \quad r_i \geq 0, r_{ij,p} \geq 0, i = 1, 2, j = 1, 2 \\
& \quad \sum_{j=1}^2 r_{ij,p} \leq r_i, i = 1, 2 \\
& \quad \sum_{i=1}^2 r_i - \sum_{i=1}^2 r_{i\bar{j},p} \leq C_j, \quad j = 1, 2 \\
& \quad (r_1, r_{11,p}, r_{12,p}, r_2, r_{21,p}, r_{22,p}) \in \mathcal{R}_{\text{air}}, \quad (12)
\end{aligned}$$

where  $\mathcal{R}_{\text{air}}$  was defined in Proposition 1 and the remaining constraints follow from (3) and (5).

This problem may be solved using a bisection method over  $r$ , which requires testing the feasibility of any chosen sum rate  $r$ : the latter is detailed below.

#### A. Establishing feasibility of a given rate pair $(r_1, r_2)$

Assume sum rate  $r$  and  $\alpha$  to be fixed. Thus,  $r_1 = \alpha r$ ,  $r_2 = (1 - \alpha)r$ . An important question toward characterizing the rate region boundary is how do we establish feasibility of this rate pair?

**Lemma 1.** *Rate pair  $(r_1, r_2)$  is achievable, if and only if a rate-tuple  $(r_1, r_{11,p}, r_{12,p}, r_2, r_{21,p}, r_{22,p})$  such that*

$$\sum_{i=1}^2 r_{ij,p} = \max(0, r_1 + r_2 - C_j) \equiv c_j, \quad j = 1, 2 \quad (13)$$

can be supported on the over-the-air segment.

*Proof:* What the lemma essentially means is that for a rate pair  $(r_1, r_2)$  to be achievable (in the context of our rate splitting approach) on the over-the-air segment it must be achievable with the private messages rate kept as low as possible. This is intuitive since having more data available at both terminals allows for more cooperation, both in terms of enhancing the desired signals at the terminals, and controlling the interference generated.

More formally, we show in Appendix B that if  $(r_1, r_2)$  cannot be achieved for rate-tuple  $(r_1, r_{11,p}, r_{12,p}, r_2, r_{21,p}, r_{22,p})$ , it cannot be achieved for any other rate-tuple with the same  $r_1$  and  $r_2$ , such that any of the private rates are higher. The last step of the proof consists in obtaining from the backhaul constraints, in particular Equation (5), a characterization of the minimum private rates boundary, which turns out to be the constraint given by (13). ■

Taking into consideration Lemma 1, one can verify that feasibility of a rate pair  $(r_1, r_2)$  may be checked by solving the following power minimization:

$$\mathcal{P}_{\min}: \quad \min. \quad \sum_{i=1}^2 \sum_{j=1}^2 (\|\mathbf{w}_{ij,c}\|^2 + \|\mathbf{w}_{ij,p}\|^2) \quad (14)$$

$$\text{s.t. } 0 \leq r_{1j,p} \leq c_j, \quad j = 1, 2 \quad (15)$$

$$c_1 + c_2 - r_2 \leq r_{11,p} + r_{12,p} \leq r_1 \quad (16)$$

$$(r_1, r_{11,p}, r_{12,p}, r_2, c_1 - r_{11,p}, c_2 - r_{12,p}) \in \mathcal{R}_{\text{air}}.$$

#### Solving $\mathcal{P}_{\min}$

As discussed above, we can establish feasibility of rate pair  $(r_1, r_2)$  by solving  $\mathcal{P}_{\min}$ , which is an optimization over both the private rates, and the beamforming vectors. Fixing the rates, the remaining power minimization problem can be shown to be equivalent to a convex optimization, and can thus be solved efficiently. Details can be found in Appendix C.

But how do we obtain the optimal private rates? Constraints (15) and (16) define a polyhedron. If a single  $c_i$  is zero,  $r_{1i,p} = 0$ , and the polyhedron collapses down to a line segment. If both  $c_i$ 's are zero, the line segment further collapses into a single point: in this case, there are no private messages and only shared messages are needed, as in the traditional network MIMO setup, and only the  $(r_{11,p}, r_{12,p})$  pair  $(0, 0)$  needs to be checked for feasibility. In the general case, we conjecture that to solve (14), it is enough to check feasibility at the corner points of the polytope defined in (15) and (16): in other words, to check feasibility we only need to solve the above convex optimization at a small number of points, which is what we do in our simulations.

Note that the above rate region could further be expanded by dirty-paper coding [17] the shared messages, thereby reducing the interference at one of the users: thus, if user 1's shared message is encoded first, dirty-paper coding of the other messages at user 2 ensures the corresponding signal does not cause interference to user 2.

#### B. Extension to $N > 2$ base stations

Throughout this paper, we focus on the two cell case. However, our approach can be extended to  $N > 2$  cells. However, the number of possible private and shared grows exponentially with  $N$ . In fact, for  $N$  cooperating BSs, messages for a certain user may be shared by  $k = 2, \dots, N$  BSs, and for each  $k$ , there will be  $\binom{N}{k}$  possible BS combinations. Thus, some simplification would be required. This may not be too restrictive since in general, a user in a cellular network is most sensitive to the signals reaching it from its 3 closest BSs, and would benefit most by receiving messages from these alone.

## IV. QUANTIZED BACKHAUL (OBLIVIOUS BASE STATIONS)

In our numerical results, we will be comparing the performance of the rate splitting approach, to another proposed in the literature, based on the notion of "oblivious base stations" [9], [11], [12]. Depending on the network configuration, it may also be possible to move the processing away from the BSs and assume these to be ignorant of the encoding scheme: this type of framework was recently proposed in [9], where the dirty paper encoded then quantized input signal in a Wyner-type channel model network is optimized. Owing to these differences in the setup, in this section, we adapt the scheme in [9] to our scenario, restricting the transmission model to linear beamforming.

#### A. Oblivious BS with Linear Beamforming

Before any quantization takes place and for a linear precoding scheme, the signal to be transmitted by base station  $i$ ,

$\mathbf{x}_i \in \mathbb{C}^{N_t}$  can be written as

$$\mathbf{x}_i = \mathbf{w}_{1i}s_1 + \mathbf{w}_{2i}s_2, \quad (17)$$

where  $s_k \sim \mathcal{CN}(0,1)$  are the symbols carrying user  $k$ 's message. Thus,

$$\mathbf{x}_j \sim \mathcal{CN}(\mathbf{0}_{N_t}, \mathbf{w}_{1j}\mathbf{w}_{1j}^H + \mathbf{w}_{2j}\mathbf{w}_{2j}^H). \quad (18)$$

Since the backhaul links have finite capacity, the designed  $\mathbf{x}_j$  may not be forwarded perfectly to the corresponding BS. Thus, quantization is resorted to. This may be modeled, as in [9], by a forward test channel of the form

$$\hat{\mathbf{x}}_j = \mathbf{x}_j + \mathbf{q}_j, \quad (19)$$

where  $\mathbf{x}_j$  is as specified above and  $\mathbf{q}_j$  is the quantization noise, independent of  $\mathbf{x}_j$  and such that  $\mathbf{q}_j \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_{\mathbf{q}_j})$ .  $\hat{\mathbf{x}}_j$  is what ends up being transmitted by BS  $j$ . Due to the backhaul constraint, the mutual information  $I(\hat{\mathbf{x}}_j, \mathbf{x}_j)$  must satisfy

$$I(\hat{\mathbf{x}}_j, \mathbf{x}_j) \leq C_j. \quad (20)$$

Moreover, given the power constraints, the covariance of  $\hat{\mathbf{x}}_j$ ,  $\mathbf{C}_{\hat{\mathbf{x}}_j}$  must be such that

$$\text{Tr}[\mathbf{C}_{\hat{\mathbf{x}}_j}] = \text{Tr}[\mathbf{C}_{\mathbf{x}_j} + \mathbf{C}_{\mathbf{q}_j}] \leq P_j. \quad (21)$$

The signal received at user  $k$  is thus<sup>1</sup>

$$y_k = \sum_{j=1}^2 \mathbf{h}_{kj}^T [\mathbf{x}_j + \mathbf{q}_j] + n_k = \sum_{j=1}^2 \mathbf{h}_{kj}^T \mathbf{w}_{kj} s_k + z_k \quad (22)$$

where

$$z_k = n_k + \sum_{j=1}^2 \mathbf{h}_{kj}^T [\mathbf{w}_{\bar{k}j} s_{\bar{k}} + \mathbf{q}_j]. \quad (23)$$

Let  $\mathbf{w}_k = [\mathbf{w}_{k1}; \mathbf{w}_{k2}]$  be the joint precoding vector carrying user  $k$ 's symbols.  $r_k$  will be upper bounded by

$$\log_2 \left( 1 + \frac{|\mathbf{h}_k^T \mathbf{w}_k|^2}{\sigma^2 + |\mathbf{h}_k^T \mathbf{w}_{\bar{k}}|^2 + \sum_{j=1}^2 \mathbf{h}_{kj}^T \mathbf{C}_{\mathbf{q}_j} \mathbf{h}_{kj}^*} \right) \quad (24)$$

To tackle the design of the precoding and the quantization, we distinguish between the case where  $N_t = 1$  and that when  $N_t \geq 2$ .

1)  $N_t = 1$ : In this case, the covariance matrix  $\mathbf{C}_{\mathbf{q}_j}$  boils down to a single parameter, the quantization noise variance  $\sigma_{\mathbf{q}_j}^2$ , so that (20) becomes

$$\log_2 \left( 1 + \frac{|\mathbf{w}_{1j}|^2 + |\mathbf{w}_{2j}|^2}{\sigma_{\mathbf{q}_j}^2} \right) \leq C_j. \quad (25)$$

Similarly, the power constraint at BS  $j$  reduces to  $|\mathbf{w}_{1j}|^2 + |\mathbf{w}_{2j}|^2 + \sigma_{\mathbf{q}_j}^2 \leq P_j$ .

It is clear that the best rates require  $\sigma_{\mathbf{q}_j}^2$  to be as small as possible. Thus, from the backhaul constraint,  $\sigma_{\mathbf{q}_j}^2 = \frac{|\mathbf{w}_{1j}|^2 + |\mathbf{w}_{2j}|^2}{2^{C_j - 1}}$ , and constraint (25) can be transformed into a convex second order cone constraint. This is however not the case if  $N_t \geq 2$ , which is now treated separately below.

2)  $N_t \geq 2$ :  $\mathbf{x}_j$ 's covariance matrix,  $\mathbf{w}_{1j}\mathbf{w}_{1j}^H + \mathbf{w}_{2j}\mathbf{w}_{2j}^H$ , is rank 2. Let  $\mathbf{U}_j \mathbf{\Lambda}_j \mathbf{U}_j^H$  be its eigenvalue decomposition, and let  $\mathbf{U}_j^{(1)}$  be the two columns corresponding to the nonzero eigenvalues, referred to as  $\lambda_{j,1}$  and  $\lambda_{j,2}$ . Premultiplying (19) by  $\mathbf{U}_j^{(1)H}$  yields an equivalent channel

$$\tilde{\mathbf{x}}_j = \tilde{\mathbf{x}}_j + \tilde{\mathbf{q}}_j, \quad (26)$$

such that  $\tilde{\mathbf{x}}_j = \mathbf{U}_j^{(1)H} \hat{\mathbf{x}}_j$ ,  $\tilde{\mathbf{x}}_j = \mathbf{U}_j^{(1)H} \mathbf{x}_j$ ,  $\tilde{\mathbf{q}}_j = \mathbf{U}_j^{(1)H} \mathbf{q}_j$ , all in  $\mathbb{C}^2$ .

$\mathbf{C}_{\tilde{\mathbf{x}}_j} = \text{diag}[\lambda_{j,1}, \lambda_{j,2}]$ . (20) becomes

$$\sum_{i=1}^2 \log_2 \left( 1 + \frac{\lambda_{j,i}}{\sigma_{\tilde{\mathbf{q}}_j,i}^2} \right) \leq C_j. \quad (27)$$

whereas (21) becomes  $\sum_{i=1}^2 (\lambda_{j,i} + \sigma_{\tilde{\mathbf{q}}_j,i}^2) \leq P_j$ . However, unlike the  $N_t = 1$  case, studying the above power and backhaul link constraints does not seem to offer a simple characterization of the quantization noise covariance matrix, and attempts to solve the problem only guarantee a local optimum.

When determining the corresponding achievable rate region in Section V, we use Matlab's *fmincon* function.

## V. NUMERICAL RESULTS

Throughout the simulations,  $C_1 = C_2$  and the common value is denoted  $C$ . Similarly,  $P_1 = P_2$  and the common value is denoted  $P$ . Since the rate region is established for one given channel instance, we illustrate the gains arising from finite shared messages over one example of a channel given by some arbitrary, yet fixed, coefficients. Later on we show Monte Carlo results obtained over fading channels.

Figures 2 and 3 show, for the following channel

$$\begin{aligned} \mathbf{h}_{11}^T &= [0.2939 - 1.1488\mathbf{i} \quad -1.5260 - 0.3861\mathbf{i}], \\ \mathbf{h}_{12}^T &= [0.3963 - 0.2679\mathbf{i} \quad 0.8306 + 0.6110\mathbf{i}], \\ \mathbf{h}_{21}^T &= [-0.7201 - 0.3025\mathbf{i} \quad -0.9658 - 0.1754\mathbf{i}], \\ \mathbf{h}_{22}^T &= [0.1952 - 0.0026\mathbf{i} \quad 1.7096 + 0.4040\mathbf{i}], \end{aligned}$$

and different values of the backhaul constraints, the rate regions achieved for an SNR ( $P/\sigma^2$ ) of 10 dB by the following different schemes:

- The proposed rate splitting scheme, which we label FRS (for Full Rate Splitting)
- The rate splitting scheme studied in [18], where private rates originate from only one of the two BSs ( $r_{i,j,p} = 0$ , for  $i \neq j$ ), which we label ARS (for Asymmetric Rate Splitting),
- Beamforming on the interference channel ( $r_{ii,p} = r_i$ ,  $i = 1, 2$ ), labeled IC,
- Network MIMO beamforming ( $r_{i,c} = r_i$ ,  $i = 1, 2$ ), labeled NM,
- The quantized backhaul network MIMO scheme, labeled QNM.

As can be seen, depending on  $C$ , the FRS scheme may achieve a total sum rate of up to  $2C$ , which is the maximum possible: this is the case in Figure 2 for example. One can also note

<sup>1</sup>Recall  $\bar{k}$  is used to denote the 'other' user/base station.

that if  $C$  is relatively low, one may be better off giving up on a network MIMO approach, especially if the backhaul is used to forward the messages themselves. As the backhaul capacity increases, the NM approach increases in appeal. The FRS and ARS approaches outperform it as  $C$  increases until the point where both achieve the same rate region: when this happens, the system is no longer backhaul-limited and becomes limited by the achievable rate region over the air interface.

Note that simulations not presented here have shown that QNM might provide slightly better results over a portion of the rate region: Of course, the applicability of either scheme may be limited by the network infrastructure itself and where the network ‘intelligence’ is located. Also, the rate region achieved by the FRS and ARS schemes are not always as smooth and can be nonconvex, similar to the QNM region, since we do not convexify the region by time sharing between different transmit strategies.

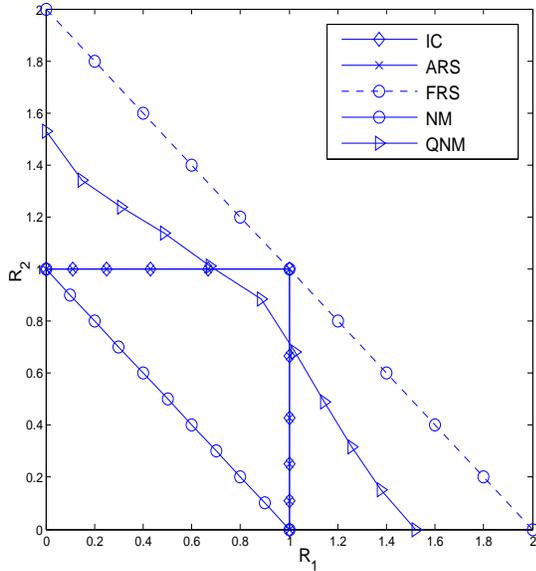


Fig. 2. Sample Rate Regions Comparison for 10dB SNR,  $C = 1$  bits/sec/Hz

Figure 4 shows the maximum sum rates achieved by the FRS scheme averaged over 100 channel samples, for  $N_t = 2$  and different values of the backhaul. The channels are assumed to be symmetric in the sense that

$$\mathbf{h}_{ii} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \quad i = 1, 2 \quad (28)$$

$$\mathbf{h}_{i\bar{i}} \sim \mathcal{CN}(\mathbf{0}, \epsilon \mathbf{I}), \quad i = 1, 2, \bar{i} = \text{mod}(i, 2) + 1 \quad (29)$$

where the parameter  $\epsilon$  quantifies the strength of the cross links (in an IC, this would be the strength of the interfering link). For low  $C$ , the maximum sum rate of  $2C$  is achievable for quite low SNR. As  $C$  increases, the saturation of the sum rate at  $2C$  occurs at higher SNR. Also shown in the figures is how much of the total data rate comes from private messages. Our feasibility check as detailed in Section III-A does not seek to maximize the total private messages: it simply checks the corner points of the feasible rate region defined by the backhaul constraints for feasibility over the air interface and exits at the first instance of a feasible set of rates. However, for  $C$  quite low, most of the data will be in the form of private

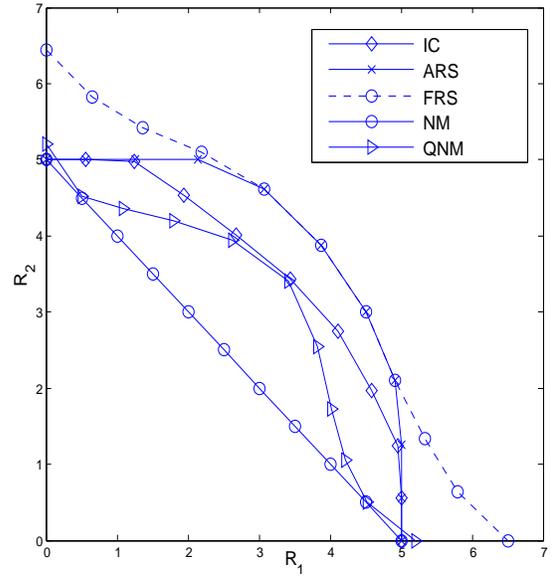


Fig. 3. Sample Rate Regions Comparison for 10dB SNR,  $C = 5$  bits/sec/Hz

messages, whereas as  $C$  increases, private messages will be required for higher values of the SNR only. Thus, for  $C = 10$ , for an SNR lower than 10 dB, the sum rate can almost always be maximized by a network MIMO approach.

Figure 5 shows the achieved maximum sum rate versus  $C$  for a 10 dB SNR: as noted earlier, for low  $C$ , the proposed scheme and the IC’s performance are quite close as further corroborated by the low shared rates, whereas as  $C$  increases, the shared rates increase and eventually NM and our scheme have similar performance. Another way of bridging the performance between the IC and NM under a constrained backhaul involves time-sharing between the non-cooperative (IC) and fully-cooperative (NM) scheme [13]: to obtain the best performance in this case, one needs to further allow for the possibility of bursty transmission, i.e. the BSs alternate between each serving its own user, both serving the users jointly and both being silent: such burstiness may not be desirable. The performance of such a scheme is also illustrated in Figure 5.

## VI. CONCLUSION

In this paper, we proposed to use the backhaul capacity to convey different types of messages: private messages transmitted from only one of the base stations, and shared messages jointly transmitted from several base stations. A corresponding achievable rate region for the two-cell setup was characterized and simulations have illustrated the benefit of the rate splitting approach adopted. The study shows how the portion of traffic that ought to be shared grows with the backhaul capacity limit. Our rate splitting approach was compared to one relying on quantization, which it normally outperforms.

## APPENDIX A MAC WITH A COMMON MESSAGE

For convenience, we reproduce the following result from [19], initially obtained by Slepian and Wolf [15], where  $I(\cdot; \cdot)$

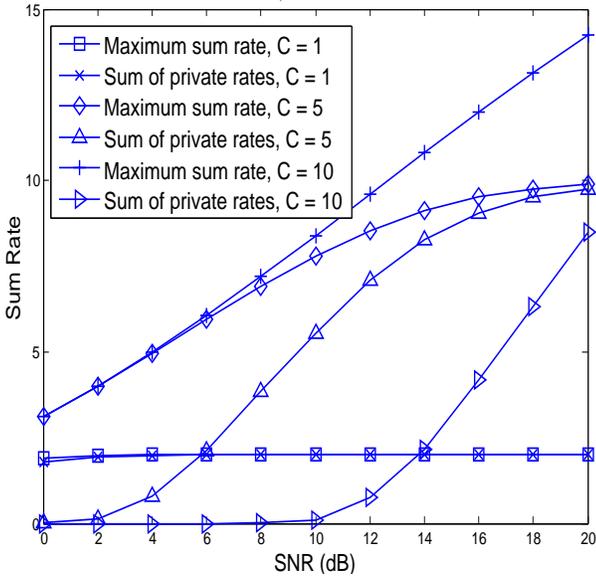


Fig. 4. Average maximum sum rate versus SNR for  $C = 1, 5$  and  $10$  bits/sec/Hz, and symmetric channels with cross channel variance  $.1$ . The figure also shows how much of the rates are in the form of private messages.

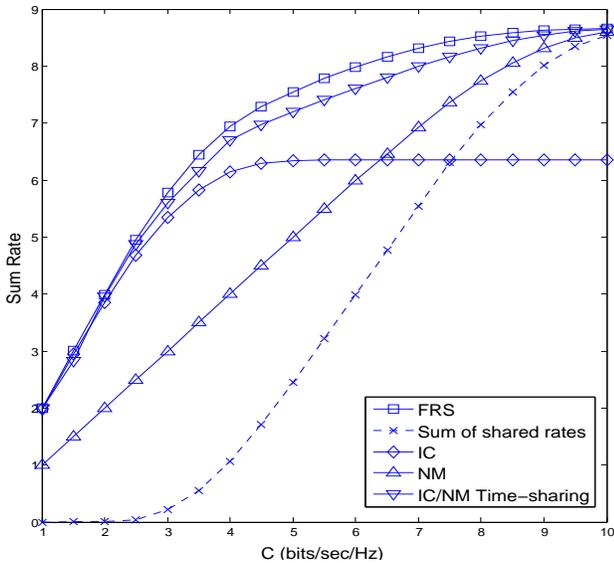


Fig. 5. Average maximum sum rate for FSR, IC, NM and a time-sharing between IC and NC scheme versus  $C$  for  $10$  dB SNR, and symmetric channels with cross channel variance  $.5$ . The figure also shows how much of the rates are in the form of shared messages in the FSR scheme.

denotes mutual information and  $I(\cdot; \cdot | \cdot)$  denotes conditional mutual information:

**Theorem 1** (MAC with a common message). *The sources put out statistically independent messages  $W_0, W_1, W_2$  with  $nR_0, nR_1, nR_2$  bits, respectively. The message  $W_0$  is seen by both encoders and is called the common message, whereas  $W_1$  and  $W_2$  appear only at the respective encoders 1 and 2, i.e., are private to those encoders. Encoder 1 maps  $(w_0, w_1)$  to a sequence  $x_1^n \in \mathcal{X}_1^n$ , encoder 2 maps  $(w_0, w_2)$  to a sequence  $x_2^n \in \mathcal{X}_2^n$ , and the channel  $P_{Y|X_1, X_2}(\cdot)$  puts out a sequence  $y^n \in \mathcal{Y}^n$ . Consider a distribution  $P_{U, X_1, X_2, Y}$  that factors as*

$P_U P_{X_1|U} P_{X_2|U} P_{Y|X_1, X_2}$ . The following rate region, denoted  $\mathcal{R}(P_U, P_{X_1|U}, P_{X_2|U})$ , is achievable:

$$\begin{aligned} R_1 &< I(X_1; Y | X_2, U), \\ R_2 &< I(X_2; Y | X_1, U), \\ R_1 + R_2 &< I(X_1, X_2; Y | U), \\ R_0 + R_1 + R_2 &< I(X_1, X_2; Y). \end{aligned} \quad (30)$$

One can further restrict attention to  $|\mathcal{U}| \leq \min(|\mathcal{Y}| + 3, |\mathcal{X}_1| |\mathcal{X}_2| + 2)$ . The capacity of the thus defined MAC is the union of such regions,

$$\mathcal{C}_{\text{MAC}} = \bigcup_{P_U, P_{X_1|U}, P_{X_2|U}} \mathcal{R}(P_U, P_{X_1|U}, P_{X_2|U}). \quad (31)$$

In the setup considered, let  $s_{i,c}, s_{i,j,p}$ ,  $j = 1, 2$ , be independent  $\mathcal{CN}(0, 1)$  random variables, and define the transmit signal of BS  $j$  to user  $i$ ,  $\mathbf{x}_{ij}$ , as in (8). The received signal at user  $i$ , as given by (1), becomes

$$y_i = \sum_{j=1}^2 \mathbf{h}_{ij}^T \mathbf{x}_{ij} + \tilde{z}_i, \quad (32)$$

$\tilde{z}_i \sim \mathcal{CN}(0, \sigma_i^2)$  is the receiver noise plus interference, which from (8), can be verified to be equal to

$$\begin{aligned} \sigma_i^2 &= \sigma^2 + \mathbb{E} \left| \sum_{j=1}^2 \mathbf{h}_{ij}^T \mathbf{x}_{ij} \right|^2 \\ &= \sigma^2 + \sum_{j=1}^2 |\mathbf{h}_{ij}^T \mathbf{w}_{ij,p}|^2 + |\mathbf{h}_i^T \mathbf{w}_{i,c}|^2. \end{aligned} \quad (33)$$

In the proposed transmission scheme,  $s_{i,c}$  is the equivalent of  $U$  in (30). Thus, the following rates are achievable, under perfect channel state information at the receiver (CSIR):

$$r_{ij,p} < I(\mathbf{x}_{ij}; y_i | \mathbf{x}_{i\bar{j}}, \mathbf{w}_{i,c} s_{i,c}), \quad j = 1, 2 \quad (34)$$

$$\sum_{j=1}^2 r_{ij,p} < I(\mathbf{x}_{i1}, \mathbf{x}_{i2}; y_i | \mathbf{w}_{i,c} s_{i,c}), \quad j = 1, 2 \quad (35)$$

$$r_i = \sum_{j=1}^2 r_{ij,p} + r_{i,c} < I(\mathbf{x}_{ii}, \mathbf{x}_{i\bar{i}}; y_i), \quad (36)$$

One can easily verify that the mutual information expressions are the ones in (9).

## APPENDIX B

Without loss of generality, we focus on the transmission to user 1, and hold that to user 2 (i.e.,  $\mathbf{w}_{21,p}, \mathbf{w}_{22,p}, \mathbf{w}_{2,c}$ ) fixed. The rate constraints at user 2 impose an interference constraint on user 1's transmission, so that

$$|\mathbf{h}_2^T \mathbf{w}_{1,c}|^2 + \sum_{j=1}^2 |\mathbf{h}_{2j}^T \mathbf{w}_{1j,p}|^2 \leq I_2. \quad (37)$$

Since we have fixed the transmission to user 2, we also have power constraints

$$\|\mathbf{w}_{1j,c}\|^2 + \|\mathbf{w}_{1j,p}\|^2 \leq p_{1j}, \quad j = 1, 2. \quad (38)$$

Fixing the transmission to user 2 also specifies  $\sigma_1^2$ . We now show that if we put lower bounds on  $|\mathbf{h}_{11}^T \mathbf{w}_{11,p}|^2$  and  $|\mathbf{h}_{12}^T \mathbf{w}_{12,p}|^2$ , and wish to maximize  $r_1$ , there is nothing to be gained by increasing any of these two quantities (which allows an increase of  $r_{11,p} + r_{12,p}$ ). The last constraint in (9) limits the total rate to user  $i$ ,  $i = 1, 2$ . Since  $\sigma_1^2$  is fixed, maximizing  $r_1$  is equivalent to maximizing, subject to (38) and (37),

$$|\mathbf{h}_1^T \mathbf{w}_{1,c}|^2 + \sum_{j=1}^2 |\mathbf{h}_{1j}^T \mathbf{w}_{1j,p}|^2. \quad (39)$$

Instead of solving the above problem, we consider its semidefinite programming (SDP) relaxation (see [20] for example)

$$\begin{aligned} \max. \quad & \mathbf{S}_{\geq 0}, \mathbf{S}_1 \geq 0, \mathbf{S}_2 \geq 0 \mathbf{h}_1^T \mathbf{S} \mathbf{h}_1^* + \sum_{j=1}^2 \mathbf{h}_{1j}^T \mathbf{S}_j \mathbf{h}_{1j}^* \\ \text{s.t.} \quad & \mathbf{h}_2^T \mathbf{S} \mathbf{h}_2^* + \sum_{j=1}^2 \mathbf{h}_{2j}^T \mathbf{S}_j \mathbf{h}_{2j}^* \leq I_2 \\ & \mathbf{h}_{1j}^T \mathbf{S}_j \mathbf{h}_{1j}^* \geq U_j, \quad j = 1, 2 \\ & \text{Tr} \mathbf{S}_j + \text{Tr} [\mathbf{S}_{jj}] \leq p_{1j}, \quad j = 1, 2, \end{aligned} \quad (40)$$

where we write  $\mathbf{S}$  as a  $2 \times 2$  block matrix, with block sizes  $N_t \times N_t$  as

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{12}^H & \mathbf{S}_{22} \end{bmatrix}. \quad (42)$$

We show that the optimal  $\mathbf{S}$ ,  $\mathbf{S}_1$  and  $\mathbf{S}_2$  will all have at most rank one. Moreover, we show that for a feasible problem, one can always let the constraints in (40) hold with equality at the optimum. This is done by following a similar approach to the proof of Lemma 1 in [21], the difference in our setup being that 1) we are dealing with multiple PSD matrices to optimize over simultaneously and 2) our problem includes per-BS power constraints.

The KKT conditions corresponding to problem (41) become

$$\mathbf{h}_{1j}^* \mathbf{h}_{1j}^T + \Phi_{jj} = \mu \mathbf{h}_{2j}^* \mathbf{h}_{2j}^T + \mu_j \mathbf{I}_{N_t}, \quad j = 1, 2 \quad (43)$$

$$\mathbf{h}_{11}^* \mathbf{h}_{12}^T + \Phi_{12} = \mu \mathbf{h}_{21}^* \mathbf{h}_{22}^T \quad (44)$$

$$(1 + \eta_j) \mathbf{h}_{1j}^* \mathbf{h}_{1j}^T + \Phi_j = \mu \mathbf{h}_{2j}^* \mathbf{h}_{2j}^T + \mu_j \mathbf{I}, \quad j = 1, 2 \quad (45)$$

$$\eta_j [\mathbf{h}_{1j}^T \mathbf{S}_j \mathbf{h}_{1j}^* - U_j] = 0, \quad j = 1, 2 \quad (46)$$

$$\mu \left[ 2\text{Re} [\mathbf{h}_{21}^T \mathbf{S}_{12} \mathbf{h}_{22}^*] + \sum_{j=1}^2 \mathbf{h}_{2j}^T (\mathbf{S}_{jj} + \mathbf{S}_j) \mathbf{h}_{2j}^* - I_2 \right] = 0 \quad (47)$$

$$\mu_j [\text{Tr} \mathbf{S}_j + \text{Tr} [\mathbf{S}_{jj}] - p_{1j}] = 0, \quad j = 1, 2 \quad (48)$$

$$\text{tr} [\mathbf{S} \Phi] = 0, \quad (49)$$

$$\text{tr} [\mathbf{S}_j \Phi_j] = 0, \quad j = 1, 2, \quad (50)$$

where  $\mu, \mu_1, \mu_2, \eta_1$  and  $\eta_2$  are Lagrange coefficients corresponding to the interference, transmit power at BS 1 and 2 and private useful received power constraints from BS 1 and 2, respectively;  $\Phi, \Phi_1$  and  $\Phi_2$  are PSD matrix corresponding to the PSD constraints on  $\mathbf{S}, \mathbf{S}_1$  and  $\mathbf{S}_2$ , respectively,

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^H & \Phi_{22} \end{bmatrix} \quad (51)$$

Combining (43) and (45), we get

$$\eta_j \mathbf{h}_{1j}^* \mathbf{h}_{1j}^T + \Phi_j = \Phi_{jj}. \quad (52)$$

- If the optimal  $\mu_j$  is strictly positive, i.e. the power constraint at BS  $j$  is met with equality at the optimum, then  $\Phi_j$  and  $\Phi_{jj}$  have rank at least  $N_t - 1$ , so that  $\mathbf{S}_j$  and  $\mathbf{S}_{jj}$  will have at most rank 1. Obviously,  $\mathbf{S}_j$  must have rank 1 to be able to satisfy the constraint (40), if  $U_j > 0$ .
- If the optimal  $\mu_j = 0$ , then since  $\Phi_j \succeq 0$  and  $\eta_j \geq 0$ , this is only possible if  $\mu > 0$  (the interference constraint is met with equality), and  $\mathbf{h}_{1j} = c_j \mathbf{h}_{2j}$ : in this case, beamforming will be optimal (no point in transmitting in any direction other than  $\mathbf{h}_{1j}^*$ ). As a result,  $\Phi_j = \mathbf{0}_{N_t \times N_t}$ . Moreover,  $\Phi_{jj} = \eta_j \mathbf{h}_{1j}^* \mathbf{h}_{1j}^T$ .

For both these cases, if  $\eta_j > 0$ , constraint (40) will be tight, and  $\mathbf{S}_{jj}$  will be zero: this would occur if either the power constraint or the interference constraint is met with equality concurrently with (40). To ensure that  $\mathbf{S}$  is PSD,  $\mathbf{S}_{12}$  must also be zero in this case. On the other hand, if  $\eta_j = 0$ , and it is possible to achieve  $\mathbf{h}_{1j}^T \mathbf{S}_j \mathbf{h}_{1j}^* > U_j$  while still meeting power and interference constraints, then it is obvious that better performance can be achieved by fixing  $\mathbf{h}_{1j}^T \mathbf{S}_j \mathbf{h}_{1j}^*$  at  $U_j$ , since as a worst-case, for given  $\mathbf{S}_j$  such that  $\mathbf{h}_{1j}^T \mathbf{S}_j \mathbf{h}_{1j}^* > U_j$ , one could let  $\mathbf{S}_{j,new} = \alpha_j \mathbf{S}_j$ , where  $\alpha_j$  is a scaling factor that ensures  $\mathbf{h}_{1j}^T \mathbf{S}_{j,new} \mathbf{h}_{1j}^* = U_j$  and  $\mathbf{S}_{jj,new} = (1 - \alpha_j) \mathbf{S}_j$ , then the power constraints are unaffected and one can improve performance by optimizing over  $\mathbf{S}_{12}$ .

We conclude the proof by showing that if  $\mathbf{S}_{jj}$  has rank one for  $j = 1, 2$ , then  $\mathbf{S}$  must be rank one. If  $\mu_1$  and  $\mu_2$  are strictly positive, this is obvious from KKT conditions (43) and (44) which together imply that the rank of  $\Phi$  is at least  $2N_t - 1$  so that the rank of  $\mathbf{S}$  is at most 1: since  $\mathbf{S}_{jj}$  has rank one, then  $\mathbf{S}$  cannot have rank zero, which completes the proof for this case. When,  $\mu_j = 0$ , i.e.,  $\mathbf{h}_{1j} = c_j \mathbf{h}_{2j}$ , then  $\mathbf{S}_{jj} = \bar{p}_j \frac{\mathbf{h}_{1j}^* \mathbf{h}_{1j}^T}{\|\mathbf{h}_{1j}\|^2}$ , for some power level  $\bar{p}_j$ . In this case,  $\Phi_{jj} = \mathbf{0}$ , and to ensure that  $\Phi$  is PSD,  $\Phi_{12}$  must be zero, and (44) becomes

$$\mathbf{h}_{11}^* \mathbf{h}_{12}^T = \mu \mathbf{h}_{21}^* \mathbf{h}_{22}^T. \quad (53)$$

Since  $\mathbf{h}_{1j} = c_j \mathbf{h}_{2j}$ , this implies that  $\mathbf{h}_{1\bar{j}} = c_{\bar{j}} \mathbf{h}_{2\bar{j}}$  for the other cell as well. Thus,  $\mathbf{S}_{\bar{j}\bar{j}} = \bar{p}_{\bar{j}} \frac{\mathbf{h}_{1\bar{j}}^* \mathbf{h}_{1\bar{j}}^T}{\|\mathbf{h}_{1\bar{j}}\|^2}$ , for some power level  $\bar{p}_{\bar{j}}$ .

Using results on Schur complements (see Appendix A.5.5 in [22]), to ensure  $\mathbf{S}$  is PSD, the following must hold

$$(I - \mathbf{S}_{11} \mathbf{S}_{11}^\dagger) \mathbf{S}_{12} = \mathbf{0} \quad (54)$$

$$\mathbf{S}_{22} - \mathbf{S}_{12}^H \mathbf{S}_{11}^\dagger \mathbf{S}_{12} \succeq 0 \quad (55)$$

$$(I - \mathbf{S}_{22} \mathbf{S}_{22}^\dagger) \mathbf{S}_{12}^H = \mathbf{0} \quad (56)$$

$$\mathbf{S}_{11} - \mathbf{S}_{12} \mathbf{S}_{22}^\dagger \mathbf{S}_{12}^H \succeq 0, \quad (57)$$

where  $\dagger$  denotes the pseudo-inverse. Letting  $\mathbf{S}_{jj} = \mathbf{v}_{jj} \mathbf{v}_{jj}^H$ , for

$j = 1, 2$ , for some vector  $\mathbf{v}_{jj}$ , these become

$$\left( I - \frac{\mathbf{v}_{11}\mathbf{v}_{11}^H}{\|\mathbf{v}_{11}\|^2} \right) \mathbf{S}_{12} = \mathbf{0} \quad (58)$$

$$\mathbf{v}_{22}\mathbf{v}_{22}^H - \frac{\mathbf{S}_{12}^H\mathbf{v}_{11}\mathbf{v}_{11}^H\mathbf{S}_{12}}{\|\mathbf{v}_{11}\|^4} \succeq \mathbf{0} \quad (59)$$

$$\left( I - \frac{\mathbf{v}_{22}\mathbf{v}_{22}^H}{\|\mathbf{v}_{22}\|^2} \right) \mathbf{S}_{12}^H = \mathbf{0} \quad (60)$$

$$\mathbf{v}_{11}\mathbf{v}_{11}^H - \frac{\mathbf{S}_{12}\mathbf{v}_{22}\mathbf{v}_{22}^H\mathbf{S}_{12}^H}{\|\mathbf{v}_{22}\|^4} \succeq \mathbf{0} \quad (61)$$

Let the singular value decomposition of  $\mathbf{S}_{12}$  be given by  $\mathbf{U}_{12}\mathbf{\Lambda}_{12}\mathbf{V}_{12}^H = \sum_{i=1}^{N_t} s_i \mathbf{u}_i \mathbf{v}_i^H$ . Premultiplying (58) by  $\mathbf{u}_l$  and post-multiplying it by  $\mathbf{v}_l^H$ , for  $l = 1, \dots, N_t$ , we get  $s_l = s_l \frac{|\mathbf{v}_{11}^H \mathbf{u}_l|^2}{\|\mathbf{v}_{11}\|^2}$ . This implies that either  $s_l = 0$ , or  $\mathbf{u}_l = e^{i\theta_l} \frac{\mathbf{v}_{11}}{\|\mathbf{v}_{11}\|}$ , for some angle  $\theta_l$ . Obviously, the latter case can occur for at most one  $l$ . On the other hand, pre- and post-multiplication by  $\mathbf{v}_l^H$  and  $\mathbf{u}_l$ , respectively, of (60) yields  $s_l = s_l \frac{|\mathbf{v}_{22}^H \mathbf{v}_l|^2}{\|\mathbf{v}_{22}\|^2}$ . This is satisfied if either  $s_l = 0$ , or if  $\mathbf{v}_l = e^{i\phi_l} \frac{\mathbf{v}_{22}}{\|\mathbf{v}_{22}\|}$ , for some angle  $\phi_l$ . Here too, the latter case can occur for at most one  $l$ . Assuming the singular values  $s_l$  are not all zeros, and letting the only strictly positive  $s_l$  be  $s_1$ , (59) and (61) become

$$\mathbf{v}_{22}\mathbf{v}_{22}^H \left( 1 - \frac{s_1^2}{\|\mathbf{v}_{22}\|^2 \|\mathbf{v}_{11}\|^2} \right) \succeq \mathbf{0} \quad (62)$$

$$\mathbf{v}_{11}\mathbf{v}_{11}^H \left( 1 - \frac{s_1^2}{\|\mathbf{v}_{22}\|^2 \|\mathbf{v}_{11}\|^2} \right) \succeq \mathbf{0}. \quad (63)$$

Thus,  $s_1 \leq \|\mathbf{v}_{22}\| \|\mathbf{v}_{11}\|$ . It is easy to verify that  $\mathbf{S}$  will be rank one, iff

- $\mathbf{v}_{jj}$  is zero for at least one of the BSs': in this case,  $s_1$  will be zero, as will  $\mathbf{S}_{12}$ .
- both  $\mathbf{v}_{jj}$  are not zero, and  $s_1 e^{i(\theta_1 - \phi_1)} = \|\mathbf{v}_{22}\| \|\mathbf{v}_{11}\|$ . For  $\mathbf{v}_{11} = \sqrt{\bar{p}_1} \frac{\mathbf{h}_{11}^*}{\|\mathbf{h}_{11}\|}$ ,  $\mathbf{v}_{22} = e^{i\tilde{\theta}} \sqrt{\bar{p}_2} \frac{\mathbf{h}_{12}^*}{\|\mathbf{h}_{12}\|}$  and letting  $\tilde{\phi} = \theta_1 - \phi_1$

$$\mathbf{S} = \begin{bmatrix} \bar{p}_1 \frac{\mathbf{h}_{11}^* \mathbf{h}_{11}^T}{\|\mathbf{h}_{11}\|^2} & s_1 e^{i(\tilde{\phi} - \tilde{\theta})} \frac{\mathbf{h}_{11}^* \mathbf{h}_{12}^T}{\|\mathbf{h}_{12}\| \|\mathbf{h}_{11}\|} \\ s_1 e^{i(\tilde{\theta} - \tilde{\phi})} \frac{\mathbf{h}_{12}^* \mathbf{h}_{11}^T}{\|\mathbf{h}_{12}\| \|\mathbf{h}_{11}\|} & \bar{p}_2 \mathbf{h}_{12}^* \mathbf{h}_{12}^T \end{bmatrix} \quad (64)$$

Without loss of generality, we can let  $\tilde{\phi} = 0$ .

$$\begin{aligned} & \mathbf{h}_1^T \mathbf{S} \mathbf{h}_1^* \\ &= \bar{p}_1 \|\mathbf{h}_{11}\|^2 + \bar{p}_2 \|\mathbf{h}_{12}\|^2 + 2s_1 \|\mathbf{h}_{12}\| \|\mathbf{h}_{11}\| \cos \theta \end{aligned} \quad (65)$$

$$\begin{aligned} & \mathbf{h}_2^T \mathbf{S} \mathbf{h}_2^* \\ &= \bar{p}_1 |c_1|^2 \|\mathbf{h}_{11}\|^2 + \bar{p}_2 |c_2|^2 \|\mathbf{h}_{12}\|^2 \\ &+ 2s_1 \|\mathbf{h}_{11}\| \|\mathbf{h}_{12}\| |c_1| |c_2| \cos(\theta + \theta_{c_2} - \theta_{c_1}) \end{aligned} \quad (66)$$

Now consider the problem

$$\max_{\theta, s_1} s_1 \cos \theta \quad (67)$$

$$\text{s.t. } s_1 \cos(\theta + \theta_{c_2} - \theta_{c_1}) \leq t \quad (68)$$

$$0 \leq s_1 \leq \sqrt{\bar{p}_1 \bar{p}_2} \quad (69)$$

Studying the KKT conditions of the above problem, one can show that the only case in which an optimal  $s_1 < \sqrt{\bar{p}_1 \bar{p}_2}$  is conceivable is if  $\theta_{c_2} - \theta_{c_1}$  (normalized to an

angle in  $[0, 2\pi)$  is equal to 0. However, if this is the case, then either  $\sqrt{\bar{p}_1 \bar{p}_2} < t$ , in which case it is trivial to see that the optimal  $s_1$  is indeed  $\sqrt{\bar{p}_1 \bar{p}_2}$ , or the optimum is equal to  $t$ , which can be achieved for  $s_1 = \sqrt{\bar{p}_1 \bar{p}_2}$  and an appropriate choice of  $\theta$ . We can thus restrict  $s_1$  to be equal to  $\sqrt{\bar{p}_1 \bar{p}_2}$ , which leads to a rank one  $\mathbf{S}$ . This completes the proof.

## APPENDIX C

The equivalence of (14) for fixed rates to a convex optimization is shown by

- Taking the SDP relaxation of the problem;
- Noting that the relaxed problem is convex and has zero duality gap;
- Noting that the optimal matrices will be rank one.

Such a strategy is followed in [23] for example, in the context of characterizing the Pareto boundary of the rate region of the MISO interference channel. SDP relaxations that are shown to have rank one optimal solutions appear elsewhere in the literature, e.g., in [24].

Let

$$\begin{aligned} \Gamma_{1j,p} &= 2^{r_{1j,p}} - 1, & \Gamma_{2j,p} &= 2^{c_j - r_{1j,p}} - 1, \\ \Gamma_{1,p} &= 2^{\sum_j r_{1j,p}} - 1, & \Gamma_{2,p} &= 2^{\sum_j (c_j - r_{1j,p})} - 1, \end{aligned} \quad (70)$$

$$\Gamma_i = 2^{r_i} - 1, \quad i = 1, 2, \quad (71)$$

and  $\mathbf{R}_i = \mathbf{h}_i^* \mathbf{h}_i^T$ ,  $\mathbf{R}_{ij} = \mathbf{h}_{ij}^* \mathbf{h}_{ij}^T$ , the SDP relaxation is

$$\min. \sum_{i=1}^2 \left( \text{Tr} \mathbf{S}_{i,c} + \sum_{j=1}^2 \text{Tr} \mathbf{S}_{ij,p} \right) \quad (72)$$

$$\text{s.t. } \Gamma_{ij,p} \left[ \sigma^2 + \sum_{j'} \text{Tr} [\mathbf{R}_{ij'} \mathbf{S}_{ij',p}] + \text{Tr} [\mathbf{R}_i \mathbf{S}_{i,c}] \right] \leq \text{Tr} [\mathbf{R}_{ij} \mathbf{V}_{ij,p}], \quad i, j = 1, 2$$

$$\Gamma_{i,p} \left[ \sigma^2 + \sum_{j'} \text{Tr} [\mathbf{R}_{ij'} \mathbf{R}_{ij',p}] + \text{Tr} [\mathbf{R}_i \mathbf{S}_{i,c}] \right]$$

$$\leq \sum_{j=1}^2 \text{Tr} [\mathbf{R}_{ij} \mathbf{S}_{ij,p}], \quad i = 1, 2$$

$$\Gamma_i \left[ \sigma^2 + \sum_{j'} \text{Tr} [\mathbf{R}_{ij'} \mathbf{S}_{ij',p}] + \text{Tr} [\mathbf{R}_i \mathbf{S}_{i,c}] \right]$$

$$\leq \text{Tr} [\mathbf{R}_i \mathbf{S}_{i,c}] + \sum_{j=1}^2 \text{Tr} [\mathbf{R}_{ij} \mathbf{S}_{ij,p}], \quad i = 1, 2$$

$$\sum_{i=1}^2 (\text{Tr} [\mathbf{D}_j \mathbf{S}_{i,c}] + \text{Tr} \mathbf{S}_{ij,p}) \leq P_j, \quad j = 1, 2$$

$$\mathbf{S}_{ij,p} \succeq \mathbf{0}, \quad i = 1, 2, j = 1, 2$$

$$\mathbf{S}_{i,c} \succeq \mathbf{0}, \quad i = 1, 2 \quad (73)$$

where  $\mathbf{D}_j$  is a diagonal matrix whose only non-zero entries are equal to 1 and occupy positions 1 to  $N_t$  along the diagonal, and positions  $N_t + 1$  to  $2N_t$  for  $j = 1$  and  $j = 2$ , respectively.

One can show that the optimal matrices will be rank one by considering individual useful and interfering terms corresponding to each term and noting that minimization of transmit power subject to useful and interference power constraints (and in the case of joint transmission terms, per-BS power constraints) will have a rank one optimal solution. As an example, consider

$$\begin{aligned} \min. \mathbf{s}_{i,c} \succeq 0 \quad & \text{Tr} \mathbf{S}_{i,c} \\ & \text{Tr} [\mathbf{R}_i \mathbf{S}_{i,c}] \geq U_i, \quad \text{Tr} [\mathbf{R}_i \mathbf{S}_{i,c}] \leq I_i \\ & \text{Tr} [\mathbf{D}_j \mathbf{S}_{i,c}] \leq p_j, \quad j = 1, 2. \end{aligned} \quad (74)$$

Letting  $\lambda_U, \lambda_I$  and  $\mu_j$  denote the Lagrangian coefficients corresponding to useful, interference and per-BS power constraints, respectively. Also let  $\Phi$  denote the Lagrangian PSD matrix corresponding to the PSD constraint on  $\mathbf{S}_{i,c}$ , the following KKT conditions must hold at the optimum:

$$\mathbf{I}_{2N_t} + \sum_{j=1}^2 \mu_j \mathbf{D}_j + \lambda_I \mathbf{R}_i = \mathbf{R}_i + \Phi \quad (75)$$

$$\text{tr} [\Phi \mathbf{S}_{i,c}] = 0. \quad (76)$$

Clearly, the right-hand side of (75) has rank  $2N_t$ . Since  $\mathbf{R}_i$  has rank 1,  $\Phi$  has rank at least  $2N_t - 1$ <sup>2</sup>. Using this fact when considering (76) implies  $\mathbf{S}_{i,c}$  has a most rank 1. Unless  $U_i = 0$ , it will necessarily have rank 1.

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**Randa Zakhour** Biography text here.

**David Gesbert** Biography text here.

<sup>2</sup>As noted earlier, a similar comment is used when considering the maximization of the useful term subject to power and interference constraints in the cognitive radio setup in [21]