Blind Space-time Receivers for CDMA Communications [1]

David Gesbert   Boon Chong Ng[†]   *Arogyaswami J. Paulraj*
Information Systems Laboratory,
Durand 102, Stanford University, Stanford, CA 94305-4055, USA.
[†] *DSO National Lab 20 Science Park Drive Singapore 118230, Singapore*
email: gesbert,paulraj@rascals.stanford.edu; nbooncho@starnet.gov.sg

Abstract

This chapter reviews space-time processing methods for CDMA mobile radio applications with emphasis on blind signal detection. We begin with a motivation for the use of blind space-time processing in CDMA. Next, we develop channel and signal models useful for blind processing. We follow this by considering first space-time single user receivers (ST-RAKE) and then review some basic theory of blind ST-RAKE algorithms. The important problem of multi-user detection (MUD) is considered next which leads to a novel technique allowing the estimation of the minimum-mean-square error linear MUD.

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1 Introduction

Direct sequence code-division-multiple-access (DS-CDMA) is a spread spectrum multiple access communication method that is expected to gain a significant share of the cellular market. CDMA is currently deployed in certain areas across the world under the IS-95 standard and has been selected as the main multiple access technology for third generation wireless systems. DS-CDMA has several attractive properties for personal wireless communications: its efficient use of bandwidth, its resistance to interferences and its flexibility for multimedia links.

In CDMA the users operate in the same frequency and time channel whereas in TDMA and FDMA the users are separated in time and frequency respectively. In DS-CDMA each user has a *unique* spreading code. This code operates at a chip rate P times greater than the information data rate. The DS-CDMA link therefore needs a large bandwidth channel which can be shared by multiple users. The spreading code can be viewed as a complex symbol waveform with a large time-bandwidth product (approximately P), whereas in TDMA the time-bandwidth product of the symbol waveform is small (approximately 1). A good general reference on DS-CDMA can be found in [1]. Other books on DS-CDMA include [2], [3] and [4]. The user codes can be designed to be orthogonal or quasi-orthogonal. With orthogonal codes, and in the absence of channel delay spread, the users do not interfere with each other and signal detection is noise limited. Single-user detection schemes (RAKE receiver) can then be used successfully. If non-orthogonal signaling is used or caused by the propagation environment (multipath, asynchronism amongst the users, etc..) the users interfere with each other and detection often becomes interference limited. In this case multi-user detection is favored.

As in other multiple access systems, the use of antenna arrays in CDMA base stations can improve system capacity, quality and coverage, making space-time signal processing for CDMA a promising technology. Space-time processing can be incorporated in single user receivers (space-time (ST) RAKE) and multi-user receivers.

Space-time CDMA receivers use knowledge of the received signature waveform (channel) across time and space for the user(s) of interest. In the coherent RAKE receiver, the channel information (which in its broadest description, includes the effect of the spreading sequence, multipath propagation and user asynchronism) is normally acquired through the use of training (pilot) symbols known both by the transmitter and receiver. Blind estimation methods have been recently proposed which circumvent the use of a periodic training and allow the receiver to recover the channel information from only the observation of the received signals and knowledge of the spreading sequence for the desired user. These methods can increase the spectral efficiency, facilitate system management and allow self-startup operation of the network when necessary. The incorporation of multiple antennas is especially beneficial in blind CDMA receivers because of the additional spatial structure conferred to the signal and the increased dimensionality of the observations. As a result blind space-time CDMA currently motivates much research, as illustrated by the increasing number of publications devoted to it.

The purpose of this chapter is not to provide an extensive literature review of blind CDMA receiver design, but rather to develop certain aspects of the topic. We first review the concept of blind processing in the context of single-user receivers. More specifically, blind processing can be incorporated to estimate the spatial (or space-time) beamformer used in the ST RAKE, as was first proposed in [5]. Next we investigate the use of blind algorithms in multiuser receivers. A novel algorithm to estimate a linear minimum mean square error (MMSE) MUD is presented. The interesting features of this latter algorithm include (i) its simplicity, (ii) its ability to bypass the channel estimation step, and (iii) its generality, as it requires only knowledge of the spreading sequence for the user of interest in order to develop the multiuser receiver. We begin with a description of the space-time signal model in CDMA networks.

2 Space-time signal models

We assume that $M > 1$ antenna elements are used only at the base station and that the mobile has a single omni antenna. The mobile transmits a possibly channel coded and modulated signal which does not incorporate any spatial (or indeed any special temporal) processing. The received continuous-time $M \times 1$ signal vector in a multiple antenna CDMA system has the following form

$$\mathbf{x}(t) \;=\; \sum_{q=1}^{Q} \mathbf{x}^q(t) \;+\; \mathbf{n}(t) \tag{1}$$

where Q is the number of users sharing the same channel, $\mathbf{n}(t)$ is the additive noise vector with element-wise variance σ^2, $\mathbf{x}^q(t)$ is the vector signal contribution from a single user and is given by

$$\mathbf{x}^q(t) \;=\; \sum_{n=-\infty}^{\infty} d^q(n)\mathbf{g}^q(t - nT_c) \tag{2}$$

where $\{d^q\}$ is the modulated chip stream, T_c is the chip period, and $\mathbf{g}^q(t)$ is the vector multipath channel

$$\mathbf{g}^q(t) \;=\; \sum_{l=0}^{D-1} \alpha_l^q \mathbf{a}_l^q a(t-\tau_l^q) \tag{3}$$

where $a(t)$ is the chip pulse, D is the maximum number of paths for any of the users, each with complex amplitude and delay α_l^q and τ_l^q, and $\mathbf{a}_l^q$ corresponds to the array response vector for the l-th path.

We consider here a fairly general situation in which arbitrary delays may result in large inter-chip-interference (ICI) and even possibly significant inter-symbol-interference (ISI). The lack of synchronism between the cell users is also accounted for in the channel model. The modulated chip stream is given by

$$d^q(n) = s^q(k)c^q(k, n-kP) \;\text{ with }\; k = \lfloor \frac{n}{P} \rfloor \tag{4}$$

where $s^q(k) \in \{-1;+1\}$ is the underlying bit[2] (or "symbol") stream for the q-th user, P is the spreading factor (length of symbol in chip durations) and $(c^q(k,0),...,c^q(k,P-1)$ is the spreading code used for the k-th symbol. The code may or may not be repeated across the symbols. Note that this fact will determine our ability to develop convenient multi-user receivers.

In the above model (3) we have assumed that the inverse signal bandwidth is large compared to the travel time across the array. Therefore the complex envelopes of the signals from a given path received by different antennas are identical except for phase and amplitude differences that depend on the path angle-of-arrival, array geometry and the element pattern. Thus, the use of multiple antennas at the receiver has merely converted a scalar channel $g^q(t)$ to a vector channel $\mathbf{g}^q(t)$.

Although blind CDMA can be defined in various ways, a common concern of it is the problem of estimating the symbols $s^q(k)$ transmitted by a single or multiple users given a minimal amount of signal/channel information available at the receiver. Existing approaches to blind CDMA differ mostly on

1. the degree of a priori signal/channel information and the generality of the channel model: τ_l^q known or unknown, synchronized or unsynchronized users, delay spread smaller or larger than a chipt period, a symbol period;

2. the receiver's structure: single-user or multiuser, linear or non-linear;

3. the estimation method of the receiver: use of finite alphabet information, second-order statistics or higher-order statistics.

[2] To differentiate from the binary code sequence, $s^q(k)$ is referred to as "symbol" in this chapter.

Throughout this paper, the channel responses $\{\mathbf{g}^q(t)\}$ are assumed to be unknown and completely arbitrary (with the exception of the pulse shape $a(t)$ in the blind RAKE receiver) and only knowledge of the spreading code for the user of interest is available. We focus on linear (single user or multi-user) receivers. Finally, we restrict our study to blind methods exploiting solely the second order statistics of $\mathbf{x}(t)$.

MIMO equivalent model Another interesting way to represent the CDMA multiple access model in (1) is to consider the stream of user's symbols $\{s^q(k)\}_{q=1..Q}$ as the signals driving a $Q \times M$ multiple inputs multiple outputs (MIMO) system, with vector output $\mathbf{x}(t)$ given by

$$\mathbf{x}(t) = \sum_{q=1}^{Q} \sum_{k=-\infty}^{\infty} s^q(k)\mathbf{h}^q(k, t - kT_s) + \mathbf{n}(t) \tag{5}$$

where $T_s = PT_c$ denotes the symbol period and $\mathbf{h}^q(k,t) = [h_1^q(k,t), ..., h_m^q(k,t)]^T$ is the $M \times 1$ impulse response of the channel from user q to the receive antenna array. Note that each symbol experiences a different channel in the case of aperiodic codes, hence the symbol index k in $\mathbf{h}^q(k,t)$. Using (2)-(4), the time-varying MIMO channel response is given by

$$\mathbf{h}^q(k,t) = \sum_{j=0}^{P-1} c^q(k,j)\mathbf{g}^q(t - jT_c) \quad \text{q=1,...,Q} \tag{6}$$

Time invariant MIMO In the particular case of periodic codes, the MIMO channel becomes *time invariant*

$$\mathbf{x}(t) = \sum_{q=1}^{Q} \sum_{k=-\infty}^{\infty} s^q(k)\mathbf{h}^q(t - kT_s) + \mathbf{n}(t) \tag{7}$$

$$\mathbf{h}^q(t) = \sum_{j=0}^{P-1} c^q(j)\mathbf{g}^q(t - jT_c) \quad \forall k \quad q = 1, ..., Q \tag{8}$$

and constitutes a useful model to develop blind multi-user receivers, as we will see in section 4.

3 Single-user receivers

3.1 Space-time RAKE

When the channel exhibits significant multipath with delay spread larger than one chip period, a receiver can be designed to match this channel. Note that in DS-CDMA multipath has both a positive and a negative effect. On one hand, the independently fading paths can be a valuable source of diversity. On the other hand, the multipath introduces inter-path interference similar to intersymbol interference in TDMA, and since exact orthogonalilty among the codes cannot be maintained at all lags, it also introduces multiple access interference (MAI).

A popular non-blind single-user receiver in the presence of multipath is the coherent RAKE combiner first proposed by Price and Green in 1958 [6]. The RAKE receiver uses multiple correlators, one for each path, and the outputs of the correlators (called fingers) are then combined into a single output to maximize the signal to noise ratio using a matched filter approach.

When the receiver is equipped with multiple antennas, additional spatial matched filtering can be performed, giving rise to the space-time RAKE, first proposed in [5]. We define the transmitted signature waveform of the q-th user by

$$p^q(k,t) = \sum_{j=0}^{P-1} c^q(k,j) a^q(t - jT_c)$$

Therefore, the output of the coherent ST-RAKE can be expressed as follows[3]:

$$\hat{s}^q(k) = \text{sign}\{\sum_l \alpha_l^{q^*} \mathbf{a}_l^{q^*} \int_{kT_s+\tau_l}^{(k+1)T_s+\tau_l} \mathbf{x}(t) p^q(k, t - kT_s - \tau_l) dt\} \tag{9}$$

3.2 Blind ST-RAKE - Principal Component Method

The implementation of the ST-RAKE requires knowledge of the delay, amplitude, and array response for each incoming path with its absolute phase information. Let us assume first that the path delays are known for the desired user. Recently, a principal component analysis technique was proposed to acquire blind estimates of $\alpha_l^q \mathbf{a}_l^q$ up to the path phase information[4], that can be exploited to implement a non coherent blind ST RAKE [7, 8].

[3]In (9) $\mathbf{a}_l^{q^*}$ plays the role of a spatial beamformer extracting the l-th path. More general beamforming solutions can also be used, e.g. max SINR beamformer.

[4]Phase information can be acquired from a one bit training.

The technique is based on the second-order statistics of both the pre-correlation data $\mathbf{x}(t)$ and post-correlation data $y_l^q(k)$ where $y_l^q(k)$ is defined for the l-th path from q-th user by

$$\mathbf{y}_l^q(k) = \int_{kT_s+\tau_l}^{(k+1)T_s+\tau_l} \mathbf{x}(t) p^q(k, t - kT_s - \tau_l) dt \qquad (10)$$

Let $\mathbf{R}_x = E(\mathbf{x}(t)\mathbf{x}(t)^*)$ and $\mathbf{R}_l^q = E(\mathbf{y}_l^q(k)\mathbf{y}_l^q(k)^*)$. These correlation matrices can be expressed as follows:

$$\begin{aligned} \mathbf{R}_x &= \mathcal{E}_l^q |\alpha_l^q|^2 \mathbf{a}_l^q \mathbf{a}_l^{q^*} + \mathbf{R}_{u,l}^q \\ \mathbf{R}_l^q &= P\mathcal{E}_l^q |\alpha_l^q|^2 \mathbf{a}_l^q \mathbf{a}_l^{q^*} + \mathbf{R}_{u,l}^q \end{aligned}$$

where $\mathcal{E}_l^q$ denotes the signal power in the desired path and $\mathbf{R}_{u,l}^q$ denotes the correlation matrix of all other paths and users' signals, developed in [8]. The trick here is that the covariance of the interfering paths $\mathbf{R}_{u,l}^q$ remains the same before and after despreading, whereas the processing gain P appears as a factor in the term related to the desired path after despreading. The path response $\alpha_l^q \mathbf{a}_l^q$ can be found up to a phase component as the dominant eigenvector of $\mathbf{R}_l^q - \mathbf{R}_x$.

A problem with this approach lies in the lost phase information for each path. More problematic is perhaps the estimation of the path delays τ_l^q. In fact these can be found by solving the problem above for a range of delays τ and selecting those offering the largest dominant eigenvalues of $\mathbf{R}_l^q - \mathbf{R}_x$. However this approach may be computationally intensive and is not necessarily robust given the possibly large number of both paths and users. A solution to this problem is to construct a receiver that does not use any constraint on the channel model in terms of the path parameters. This is shown below.

3.3 Space-time equalizer

An equivalent way to view the ST-RAKE shown in 3.1 is as a receiver forming a beam in the direction of each desired path and combining the beamformed signals coherently to generate an estimate of the chip sequence $d^q(n)$. The chip sequence is then despreaded with the user's code to produce symbols estimates. However, a more general linear single-user receiver consists of using an arbitrary space-time *chip equalizer* instead of a filter matched to the channel. This equalizer can generate a better estimate of the chip sequence, before despreading. As another advantage, a ST equalizer does not pose any constraint on the multipath structure. Let $(f_m^q(0), .., f_m^q(N-1))$ be the chip rate N-long impulse response of the equalizer on the m-th antenna. The equalizer's output and subsequent symbol estimates are given by:

$$\hat{d}^q(n) = \sum_{m=1}^{M} \sum_{i=0}^{N-1} f_m^q(i)^* x_m((n-i)T_c)$$

$$\hat{s}^q(k) \;=\; \sum_{j=0}^{P-1} c^q(k,j)\hat{d}^q(kP+j)$$

The MMSE chip equalizer, defined simply by[5] $\{f_m^q\} = \text{argmin} E|d^q(n) - \hat{d}^q|^2$ can be obtained from the well known Wiener equation

$$\mathbf{R}_{N,x}\mathbf{f} = \mathbf{r}_N$$

where $\mathbf{R}_{N,x}$ is the ST correlation matrix of $\{\mathbf{x}(nT_c), ..., \mathbf{x}((n-N+1)T_c)\}$, $\mathbf{f}$ is the unknown ST equalizer vector and $\mathbf{r}_M$ is the unknown crosscorrelation (channel) vector. In [9] it is shown that the principal component analysis of 3.2 can be generalized simply to allow the blind estimation of $\mathbf{r}_N$, up to a *single* phase ambiguity. Further improvements on this technique can be found in [10]. The reader is referred to these papers for more details due to lack of space.

4 Multi-user receivers

In practical wireless environments the orthogonality of the signaling scheme cannot be maintained due to multipath propagation and user asynchronism. The lack of orthogonality between the different users' received signature waveforms causes cross-correlation terms to arise at the output of the RAKE receiver. These terms cause significant MAI when the interfering users are received with higher power than the user of interest (near-far problem). In such cases, single user receivers are far from optimal and multi-user detectors (MUD) can be used which offer superior performance. Multi-user detectors are designed to eliminate MAI completely (in the absence of additive noise) despite the lack of orthogonality between the user's signature waveforms. MUD can be used to detect the signal of one single user of reference or possibly the signals of several cell sharing users (joint detection).

The optimum joint multiuser receiver in an AWGN channel is shown to consist of a bank of matched filters followed by a Viterbi algorithm [11]. The computational complexity of this receiver grows exponentially with the number of users. The informational complexity is also high, as the optimum MUD requires knowledge of all the users' channels. This makes the development of blind optimum MUD receivers a difficult task, although approximate and iterative maximum likelihood techniques are possible [12].

[5] A delay can also be introduced in the equalizer

4.1 Linear multiuser receivers

As an alternative to optimum receivers, linear multiuser (LMU) receivers for synchronous and asynchronous DS-CDMA have been proposed in [13] and [14] that exhibit the same degree of near-far resistance as the optimum multiuser receiver. They also have error rate performances comparable to the optimum multiuser receiver. When a repeated code scheme is used, LMU receivers can be designed to cancel ICI, ISI and MAI completely as we show below.

4.1.1 Notations and assumptions

The spatial vector signal (5) is first sampled at the chip rate[6]. A total of MP samples are collected per symbol period and stacked in a signal vector[7] defined by

$$\mathbf{x}(k) = [\mathbf{x}^T(kT_s), \mathbf{x}^T(kT_s + T_c), \ldots, \mathbf{x}^T(kT_s + (P-1)T_c)]^T \tag{11}$$

Since periodic codes are used, the time invariant MIMO model developed in (7) can be exploited. We define $(L+1)T_s$ to be the unrestricted maximum length of the channels for all users. Hence the degree of the vector channels is at most L. In fact, $L = 1$ is a typical value when considering practical CDMA data rates. A severe asynchronism among the users can result in $L = 2$.

In view of (7) we can write

$$\mathbf{x}(k) = \sum_{q=1}^{Q} \mathbf{H}^q \mathbf{s}^q(k) + \mathbf{n}(k) \tag{12}$$

where the $MP \times (L+1)$-dimensional matrix $\mathbf{H}^q$ is the channel matrix for user q and its $(pM+m)$-th row (with $1 \leq m \leq M$ and $0 \leq p \leq P-1$) is given by

$$\mathbf{H}^q(pM+m, :) = (h_m^q(pT_c), h_m^q(pT_c + T_s), \ldots, h_m^q(pT_c + LT_s)) \tag{13}$$

In (12), $\mathbf{s}^q(k) \stackrel{\triangle}{=} (s^q(k), s^q(k-1), \ldots, s^q(k-L))^T$ is the current symbol vector. Let N denote the length of the LMU filter. The following MNP-dimensional output vector is introduced:

$$\mathbf{X}(k) = (\mathbf{x}^T(k), \mathbf{x}^T(k-1), \ldots, \mathbf{x}^T(k-N+1))^T \tag{14}$$

[6]Oversampling can be also accommodated in this model

[7]A noise vector $\mathbf{n}(k)$ is defined similarly.

The vector $\mathbf{X}(k)$ can, similarly to $\mathbf{x}(k)$, be written in matrix form as

$$\mathbf{X}(k) = \sum_{q=1}^{Q} \begin{pmatrix} \boxed{\mathbf{H}^q} & & 0 \\ & \boxed{\mathbf{H}^q} & \\ & & \ddots & \\ 0 & & & \boxed{\mathbf{H}^q} \end{pmatrix} \begin{pmatrix} s^q(k) \\ s^q(k\text{-}1) \\ \vdots \end{pmatrix} + \begin{pmatrix} \mathbf{n}(k) \\ \mathbf{n}(k\text{-}1) \\ \vdots \end{pmatrix} \quad (15)$$

$$\triangleq \sum_{q=1}^{Q} \boldsymbol{\mathcal{H}}^q \mathbf{S}^q(k) + \mathbf{N}(k) \quad (16)$$

The global multiuser channel matrix $\boldsymbol{\mathcal{H}} \triangleq (\boldsymbol{\mathcal{H}}^1, \ldots, \boldsymbol{\mathcal{H}}^Q)$ has dimension $MNP \times Q(L+N)$. At this point it is important to mention that we will need

$$MP \geq Q$$

so that $\boldsymbol{\mathcal{H}}$ can be made tall (i.e. with more rows than columns) for a sufficiently large filter length N. Clearly, the use of multiple antennas ($M > 1$) allows us to support more users for a given spreading factor P. The tallness property of $\boldsymbol{\mathcal{H}}$ ensures the existence of exact *zero-forcing* LMU receivers. We further assume that the conditions under which $\boldsymbol{\mathcal{H}}$ has full column rank are satisfied [15]. These conditions are mild and are satisfied with probability one. In practice, the *condition number* of the matrix $\boldsymbol{\mathcal{H}}$ will determine the achievable performance of the blind estimation algorithm.

The problem of linear multi-user detection in the noise free case can be stated ideally as follows: given the channel matrices $\boldsymbol{\mathcal{H}}^u$ for all users $u = 1, ., Q$, find a vector $\mathbf{w}$ with MNP entries that satisfies:

$$\mathbf{w}^* \boldsymbol{\mathcal{H}}^q = (0, \ldots, 0, 1, 0, \ldots, 0), \quad (17)$$

$$\mathbf{w}^* \boldsymbol{\mathcal{H}}^u = (0, \ldots, 0), \ u \neq q \quad (18)$$

where q denotes the index of the desired user and where "*" denotes complex conjugate transpose. Note that while (17) corresponds to the elimination of ICI and ISI, (18) guarantees the suppression of MAI. The position of the "1" element in (17) is a free parameter that indicates the reconstruction delay of the receiver, to be taken between 0 and $L + N - 1$.

4.2 Blind receiver estimation

The estimation of a blind receiver as defined above can be closely related to the research over the last few years in blind MIMO estimation, see for instance [15, 16] and the references therein. In fact, the model shown in (15) is not CDMA dependent. For instance it is well known that the spatial separation of co-channel TDMA users leads to a similar model. However a significant advantage of CDMA in this context over the arbitrary MIMO problem is the availability of a rich temporal (code) structure embedded into the channel which can be exploited in addition to the spatial structure to separate the users.

To a great extent the existing literature on blind CDMA has focused on the problem of estimating the users' channels as a first step to obtaining the CDMA receiver. In the so-called subspace-based techniques, the signal subspace span($\boldsymbol{\mathcal{H}}$) is first obtained from the dominant eigenvectors of $\mathbf{R} = E(\mathbf{X}(k)\mathbf{X}^*(k))$. Next, the coefficients of $\boldsymbol{\mathcal{H}}^q$, $q = 1, .., Q$ are adjusted to match the signal subspace, by invoking a variant of the subspace identification theorem [17]. The knowledge of the spreading code is used to differentiate amomng the users. Instances of this approach can be found in [18, 19, 20, 21].

The problem of directly estimating a multi-user detector for CDMA signals in unknown channels has been also addressed. Often a MMSE receiver is derived in the restrictive case of no delay spread [22, 23]. Motivated by the work in [22], a minimum output energy technique is introduced in [24] that results in a blind linear receiver that has a performance close to the MMSE receiver at high SNR in the presence of delay spread.

However, it is possible to estimate blindly the MMSE receiver itself in the most general case as we show below. The technique presented here is based on exploitation of a set of linear conditions that take advantage of: (1) the subspace structure associated with the code of the desired user, and (2) the estimated signal subspace of the covariance matrix for the observed signals. The main contribution shows that these conditions, when brought together, fully characterize the MMSE receiver. This technique was first presented in [25]. We now review this algorithm which is based on a two-fold projection.

5 MMSE receiver estimation

In the presence of noise, the best (in the MMSE sense) linear receiver with delay δ, for user q, is given by:

$$\mathbf{w} = \arg\min E|\mathbf{w}^*\mathbf{X}(k) - s^q(k-\delta)|^2 \tag{19}$$

The vector $\mathbf{w}$ found from (19) satisfies the classical Wiener equation

$$\mathbf{R}\mathbf{w} = \mathbf{r}^q \tag{20}$$

where $\mathbf{R} = E(\mathbf{X}(k)\mathbf{X}^*(k))$ denotes the ST received covariance matrix and $\mathbf{r}^q = E(\mathbf{X}(k)s^{q*}(k-\delta))$ denotes the cross-correlation vector. The goal here is to identify $\mathbf{w}$ blindly without knowledge of $\mathbf{r}^q$.

5.1 Code subspace properties

Assuming i.i.d. symbols, it can be seen easily from (15) that $\mathbf{r}^q$ is given by

$$\mathbf{r}^q = \begin{pmatrix} \boxed{\mathbf{H}^q} & & 0 \\ & \boxed{\mathbf{H}^q} & \\ & \ddots & \\ 0 & & \boxed{\mathbf{H}^q} \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \tag{21}$$

Thus $\mathbf{r}^q$ coincides with the $(\delta+1)$th column of $\boldsymbol{\mathcal{H}}^q$. If the detector delay δ is chosen appropriately (namely $L \leq \delta \leq N-1$ assuming $L \leq N-1$), the vector $\mathbf{r}^q$ contains all the $(L+1)MP$ channel coefficients for user q, as well as $(N-L-1)MP$ zeros[8]. The point here is that for each antenna m there exists a simple selection-permutation matrix $\mathbf{T}_m$ that selects the $(L+1)P$ channel coefficients in $\mathbf{r}^q$ associated with this antenna, together with $(N-L-1)P$ zeros, and puts the selected entries in a chronologically ordered vector so that

$$\mathbf{T}_m\mathbf{r}^q = (h_m^q(0), \ldots, h_m^q((LP+P-1)T_c), 0, \ldots, 0)^T \tag{22}$$

For notational convenience, in what follows we will assume $\delta = L$, but the final result holds true for any admissible value of the delay. Using the relation shown in (8) for the m-th antenna, we can relate (22) to the known spreading sequence $\{c^q(n)\}$.

$$\begin{aligned} \mathbf{T}_m\mathbf{r}^q &= \begin{pmatrix} c^q(0) & & 0 \\ \vdots & \ddots & \\ c^q(P-1) & & c^q(0) \\ & \ddots & \vdots \\ 0 & & c^q(P-1) \\ \vdots & & \\ 0 & & 0 \end{pmatrix} \begin{pmatrix} g_m^q(0) \\ g_m^q(T_c) \\ \vdots \\ g_m^q(LPT_c) \end{pmatrix} \\ &\triangleq \mathbf{C}^q\mathbf{g}_m^q \end{aligned} \tag{23}$$

[8]This is due to the block Toeplitz structure of the matrix $\boldsymbol{\mathcal{H}}^q$.

The code matrix $\mathbf{C}^q$ of dimension $PN \times LP+1$ can be made tall by selecting $N > L$ and is known to the receiver. In contrast, $\mathbf{g}_m^q$ is unknown. Also, $\mathbf{g}_m^q$ contains a certain number of zero entries. Note however that the knowledge of this number is not necessary for the proposed method. Therefore the knowledge of the *actual channel length* in chip durations is not required here.

Introduce $\mathbf{U}^q$, an orthonormal basis for the orthogonal complement of $\mathbf{C}^q$. Hence, $\mathbf{U}^q$ is of size $PN \times (P(N-L)-1)$. Then, from (20) and (23), we obtain

$$\mathbf{U}^{q\,*}\mathbf{T}_m\mathbf{R}\mathbf{w} = \underbrace{\mathbf{U}^{q\,*}\mathbf{C}^q}_{0}\mathbf{g}_m^q = 0 \quad m = 1, .., M \tag{24}$$

Of course, (24) is valid for $m = 1, \ldots, M$. Thus (24) gives us a first set of $M(P(N-L)-1)$ equations.

5.2 Signal subspace properties

Let us assume that the additive noise is white[9] with variance σ^2. In view of (16), the correlation matrix of the received signal is then given by

$$\mathbf{R} = \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^* + \sigma^2\mathbf{I} \tag{25}$$

where $\mathbf{I}$ is the identity matrix of size MNP. The signal/noise subspace decomposition of $\mathbf{R}$ goes as follows:

$$\mathbf{R} = \mathbf{E}_s\boldsymbol{\Sigma}_s\mathbf{E}_s^* + \sigma^2\mathbf{E}_n\mathbf{E}_n^* \tag{26}$$

where the columns of $\mathbf{E}_s$ (resp. $\mathbf{E}_n$) are given by the $Q(N+L)$ dominant eigenvectors (resp. the $MNP - Q(N+L)$ least dominant eigenvectors) of $\mathbf{R}$.

A very useful result is that the desired MMSE detector always lies in the signal subspace defined by $\text{span}(\boldsymbol{\mathcal{H}}) = \text{span}(\mathbf{E}_s)$, as shown below. From (20) and (25) we have

$$(\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^* + \sigma^2\mathbf{I})\mathbf{w} = \mathbf{r}^q$$

which we can rewrite as

$$\mathbf{w} = (\mathbf{r}^q - \boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^*\mathbf{w})/\sigma^2 \tag{27}$$

In view of (21), $\mathbf{r}^q$ is by construction in $\text{span}(\boldsymbol{\mathcal{H}}^q)$, hence in $\text{span}(\boldsymbol{\mathcal{H}})$. From (27) $\mathbf{w}$ also belongs to $\text{span}(\boldsymbol{\mathcal{H}})$ and it follows that

$$\mathbf{E}_n^*\mathbf{w} = 0 \tag{28}$$

Equation (28) gives us an additional $MNP - Q(N+L)$ equations toward the determination of $\mathbf{w}$.

[9]This whiteness condition can be relaxed provided the noise covariance is known.

5.3 Algorithm

Based on the projection equations in (24) and (28), $\mathbf{w}$ can be written as the solution to

$$\boldsymbol{\mathcal{A}}\mathbf{w} = \begin{pmatrix} \boldsymbol{\mathcal{A}}_1 \\ \boldsymbol{\mathcal{A}}_2 \end{pmatrix} \mathbf{w} = 0 \tag{29}$$

where $\boldsymbol{\mathcal{A}}_2 = \mathbf{E}_n^*$ and $\boldsymbol{\mathcal{A}}_1 = \boldsymbol{\mathcal{U}}^{q*}\mathbf{T}\mathbf{R}$, with $\mathbf{T} = (\mathbf{T}_1^*, \cdots, \mathbf{T}_M^*)^*$ and

$$\boldsymbol{\mathcal{U}}^{q*} = \begin{pmatrix} \boxed{\mathbf{U}^q} & 0 & 0 \\ 0 & \boxed{\mathbf{U}^q} & \\ & & \ddots & \\ 0 & & & \boxed{\mathbf{U}^q} \end{pmatrix}$$

In order to determine $\mathbf{w}$ in a noisy situation, the projection equations (29) suggest optimizing the following simple quadratic cost function

$$J_{MMSE}(\mathbf{w}) = \mathbf{w}^*\{\boldsymbol{\mathcal{A}}_1^*\boldsymbol{\mathcal{A}}_1 + \alpha\boldsymbol{\mathcal{A}}_2^*\boldsymbol{\mathcal{A}}_2\}\mathbf{w} \tag{30}$$

where α is a tunable weight. Some additional constraint must be added in order to avoid trivial solutions. In the particular case of a unit-norm constraint, $\mathbf{w}$ is found as the minimum eigenvector of $\boldsymbol{\mathcal{A}}_1^*\boldsymbol{\mathcal{A}}_1 + \alpha\boldsymbol{\mathcal{A}}_2^*\boldsymbol{\mathcal{A}}_2$. Note that while $\mathbf{R}$ is estimated using sample averaging, $\mathbf{T}$ and $\boldsymbol{\mathcal{U}}^q$ can be pre-computed.

5.3.1 Analysis

We now show that (29) completely determines the MMSE equalizer. Let us first introduce some useful notation. Let $\boldsymbol{\mathcal{C}}^q$ be the tall modifed code matrix of size $MNP \times M(LP+1)$ defined as

$$\boldsymbol{\mathcal{C}}^q = \begin{pmatrix} \boxed{\mathbf{C}^q} & 0 & 0 \\ 0 & \boxed{\mathbf{C}^q} & \\ & & \ddots & \\ 0 & & & \boxed{\mathbf{C}^q} \end{pmatrix}$$

Let $\bar{\boldsymbol{\mathcal{H}}}$ be defined as

$$\bar{\boldsymbol{\mathcal{H}}} = \mathbf{T}\tilde{\boldsymbol{\mathcal{H}}}$$

where $\tilde{\boldsymbol{\mathcal{H}}}$ denotes the original channel matrix $\boldsymbol{\mathcal{H}}$ from which the $(\delta+1)$th column of $\boldsymbol{\mathcal{H}}^q$ has been removed. The rows of $\tilde{\boldsymbol{\mathcal{H}}}$ are then organized antenna-wise using $\mathbf{T}$ to form $\bar{\boldsymbol{\mathcal{H}}}$. The main result can be summarized as follows:

Proposition 1:

Assuming that (i) $Q(L+N) \leq M(P(N-L)-1)$, and (ii) the matrix $(\bar{\boldsymbol{\mathcal{H}}}, \boldsymbol{\mathcal{C}}^q)$ has full column rank, then the matrix $\boldsymbol{\mathcal{A}}$ has rank $MNP-1$; hence the only vector in null$(\boldsymbol{\mathcal{A}})$ *determines the MMSE equalizer completely.*

Proof:

Using (26), $\boldsymbol{\mathcal{A}}_1$ admits the following orthogonal decomposition:

$$\boldsymbol{\mathcal{A}}_1 = \boldsymbol{\mathcal{A}}_{11} + \boldsymbol{\mathcal{A}}_{12}$$

where

$$\begin{aligned} \boldsymbol{\mathcal{A}}_{11} &= \boldsymbol{\mathcal{U}}^{q*}\mathbf{T}\mathbf{E}_s\boldsymbol{\Sigma}_s\mathbf{E}_s^* \\ \boldsymbol{\mathcal{A}}_{12} &= \sigma^2\boldsymbol{\mathcal{U}}^{q*}\mathbf{T}\mathbf{E}_n\mathbf{E}_n^* \end{aligned}$$

Note that the row span of $\boldsymbol{\mathcal{A}}_{12}$ is included in that of $\boldsymbol{\mathcal{A}}_2$ and therefore that the rank of $\boldsymbol{\mathcal{A}}$ is also given, due to the orthogonality property between $\boldsymbol{\mathcal{A}}_{11}$ and $\boldsymbol{\mathcal{A}}_2$, by:

$$\text{rank}\begin{pmatrix} \boldsymbol{\mathcal{A}}_{11} \\ \boldsymbol{\mathcal{A}}_2 \end{pmatrix} = \text{rank}(\boldsymbol{\mathcal{A}}_{11}) + \text{rank}(\boldsymbol{\mathcal{A}}_2)$$

The rank of $\boldsymbol{\mathcal{A}}_2$ is clearly $MNP - Q(L+N)$. In order to obtain the total rank of $\boldsymbol{\mathcal{A}}$, we shall show that rank$(\boldsymbol{\mathcal{A}}_{11}) = Q(L+N)-1$. It is clear that there exists a square invertible matrix, denoted $\mathbf{F}$, such that $\mathbf{E}_s = \boldsymbol{\mathcal{H}}\mathbf{F}$. Thus $\boldsymbol{\mathcal{A}}_{11}$ is given by:

$$\boldsymbol{\mathcal{A}}_{11} = \boldsymbol{\mathcal{U}}^{q^*}\mathbf{T}\boldsymbol{\mathcal{H}}\mathbf{F}\boldsymbol{\Sigma}_s\mathbf{E}_s^*$$

Hence rank$(\boldsymbol{\mathcal{A}}_{11}) = $ rank$(\boldsymbol{\mathcal{U}}^{q^*}\mathbf{T}\boldsymbol{\mathcal{H}})$. But, by construction, $\mathbf{T}\boldsymbol{\mathcal{H}} = [\bar{\boldsymbol{\mathcal{H}}}, \mathbf{Tr}^q]\mathbf{J}$ where $\mathbf{J}$ is a simple permutation matrix that shifts the last column to the $(\delta+1)$th position. Thus $\boldsymbol{\mathcal{U}}^{q^*}\mathbf{T}\boldsymbol{\mathcal{H}} = [\boldsymbol{\mathcal{U}}^{q^*}\bar{\boldsymbol{\mathcal{H}}}, \mathbf{0}]$ and it follows that rank$(\boldsymbol{\mathcal{A}}_{11}) = $ rank$(\boldsymbol{\mathcal{U}}^{q^*}\bar{\boldsymbol{\mathcal{H}}})$. Making use of the fact that $\boldsymbol{\mathcal{U}}^q$ is the orthogonal complement of $\boldsymbol{\mathcal{C}}^q$, we easily check that $\boldsymbol{\mathcal{U}}^{q^*}\bar{\boldsymbol{\mathcal{H}}}$ has full column rank $Q(L+N)-1$ under (i) and (ii). It thus follows that, the total rank of $\boldsymbol{\mathcal{A}}$ is $Q(L+N) - 1 + MNP - Q(L+N) = MNP - 1$. ■

Note that in the noise free case $\mathbf{w}$ given by (29) is not unique since there exist several admissible solutions to (20) as $\mathbf{R}$ has a non-empty null space. However, all solutions generate a desired MUD.

5.4 Numerical examples

We validate the proposed algorithm by a number of numerical experiments. In the simulations we use arbitrary (random) channels and code sequences for all the users, illustrating that orthogonality among the codes is not necessary in MUD. The channel order is taken to be $L = 1$ corresponding to an ISI of two symbols and we chose the memory of the equalizer to be $N = 2 > L$. In all of the experiments we use the weight $\alpha = 1$. All the channels for the users are normalized so we define the transmitted and received signal-to-noise ratio as

$$\mathrm{SNR} = 10 \log 1/\sigma^2 \tag{31}$$

Since the goal is the blind estimation of the MMSE receiver, we can quantify the performance of our algorithm in terms of the mean-squared error (MSE) at the output of the receiver: $\mathrm{MSE} = E|\hat{s}^q(k) - s^q(k)|^2$.

In Figure 1 we compare the proposed algorithm with the minimum output energy (MOE) method presented in [24]. MOE is formulated for the single antenna case only, so we first choose $M = 1$. As in [24], we assume a scenario with $Q = 10$ users and codes of length $P = 31$. We use 256 and 512 symbols in the estimation and assume perfect power control. We see that the proposed algorithm shows a significant performance improvement when compared to the MOE method, especially for low to medium-high SNRs. As the SNR increases, the performances of the two methods become similar, a result that is consistent with the fact that the MOE receiver approaches the MMSE solution for high SNR [26].

In Figure 2 we study the robustness of the proposed method to the lack of power control (the near-far problem). We assume a scenario where the power (in dB) decreases linearly with user index, and the power of the strongest and weakest of the 10 users differs by 10 dB (in this case the SNR is defined with respect to the strongest user). We plot the MSE as a function of user index, and choose $M = 4$ antennas and a spreading factor $P = 9$ to have a value of the loading ratio MP/Q comparable to the one in the previous example (512 symbols are used in the estimation). We see a small increase in the MSE as the power of the users decreases, indicating the desired near-far robustness of the proposed receiver.

In Figure 3 we verify that the performance of the proposed method approaches that of the asymptotic MMSE solution as the number of data points increases, hence supporting the claims of Proposition 1. We use the same multiple antenna scenario as above with perfect power control and plot the MSE of one of the users as a function of SNR for different numbers of symbols together with the asymptotic curve.

6 Summary

In this chapter, we first introduced the basic concepts of blind estimation in single user space-time CDMA receivers. Space-time RAKE and its generalization were briefly reviewed. Linear multiuser detectors based on the MIMO model of CDMA were then developed. Finally, a technique that exploits the code subspace structure of CDMA signals is used to obtain a blind MMSE multiuser receiver in the context of very general channels. The new blind MMSE receiver is shown to converge to the optimal MMSE solution asymptotically in numerical experiments as well as analytically, regardless of SNR.

Further research to extend space-time blind multiuser algorithms to CDMA embedded with long codes are currently under investigation.

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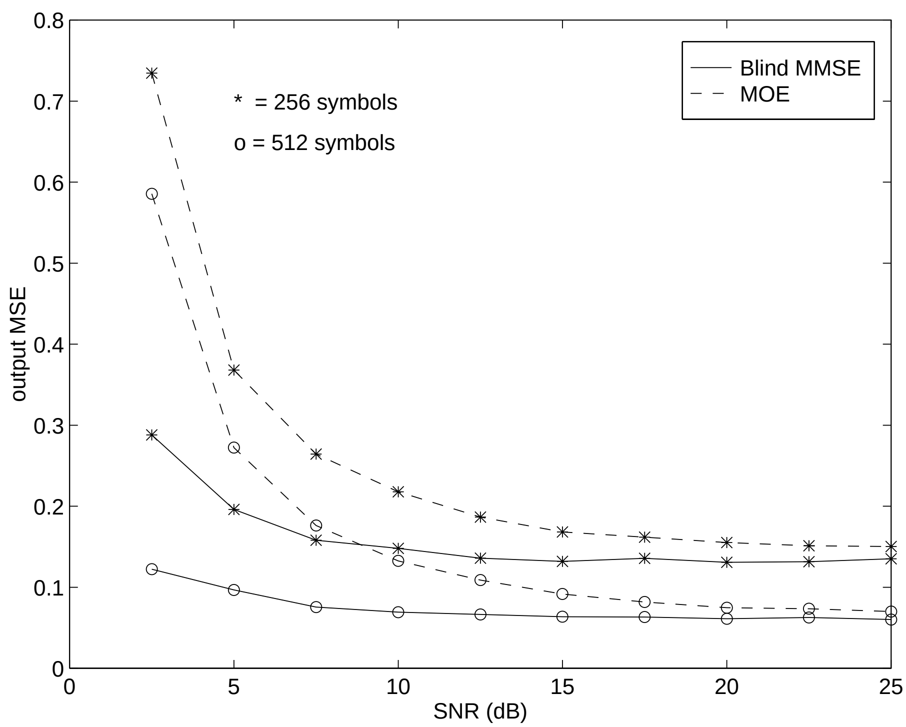

Figure 1: MSE as a function of SNR, for the proposed blind MMSE and the minimum output energy (MOE) methods ($Q = 10,\ P = 31,\ M = 1$). Perfect power control, 256 ("*") and 512 ("o") symbols are used in the estimation.

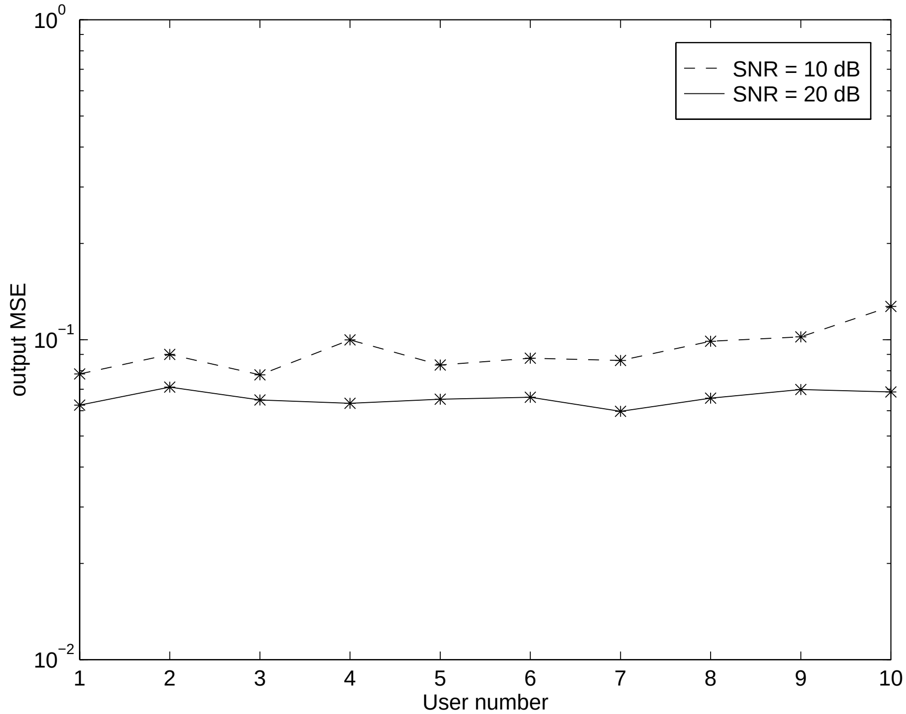

Figure 2: MSE as a function of user index for imperfect power control (user no. 1 has power 10dB above user no. 10). The SNR refers to the strongest user and 512 symbols are used in the estimation ($Q = 10,\ P = 9, M = 4$).

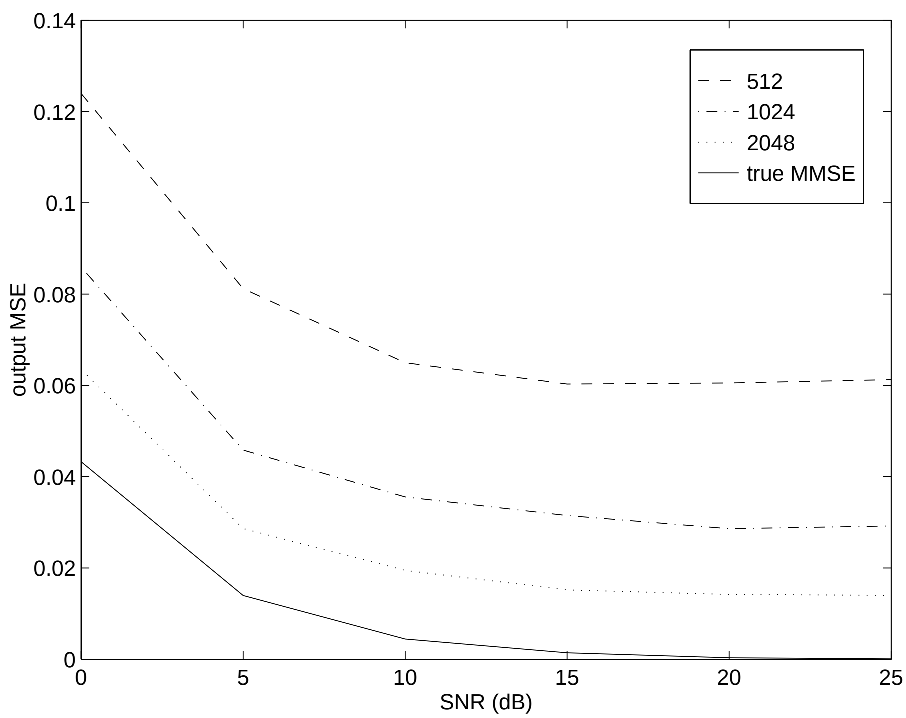

Figure 3: MSE as a function of SNR for different number of symbols (512, 1024 and 2048) used in the estimation, together with the MSE obtained using the true (asymptotic) covariance matrix. Perfect power control ($Q = 10,\ P = 9, M = 4$).