Space-Time Coding with Bit Interleaved Coded Modulations

Nicolas GRESSETT

31, March 2006
Motivations: High data rates and large frame sizes,

1. Achieve near capacity performance over ergodic MIMO channels

2. Achieve near outage capacity performance over block-fading MIMO channels

Selected solution:

- BICM is a flexible and an efficient technique for space-time coding
- Iterative decoding to achieve near ML performance with tractable complexity
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We consider frequency non-selective $n_t \times n_r$ MIMO channels represented by the complex matrix $H$.

One transmitted codeword observes $n_c$ channel states:
- $n_c = 1 \Rightarrow$ quasi-static channel,
- $n_c = \infty \Rightarrow$ ergodic channel.

Bit Interleaved Coded modulation (BICM):

The channel output is $y = zSH + \eta$, where $z \in (\text{M-QAM})^{sn_t}$. 
Channel model and Transmitter scheme

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- Bit Interleaved Coded modulation (BICM):

  The channel output is $y = zSH + \eta \quad z \in (M-QAM)^{sn_t}$
Extrinsic information is given by the classical formula:

\[
\xi(c_\ell) \propto \sum_{z' \in \Omega(c_\ell = 1)} \left[ \left( e^{-\frac{\|y - z' S H \|^2}{2N_0}} \right) \prod_{r \neq \ell} \pi(c_r) \right]
\]

⇒ List sphere decoder for near optimum reduced complexity detection
Part I

BICM optimization for ergodic channels
Large interleaver size ⇒ no interference between the erroneous bits in the pairwise error probability

Exact pairwise error probability has been derived, the asymptotic performance is

\[ \text{BER} \propto E \left[ \prod_{k=1}^{w_{\text{Hmin}}} \left( \frac{2N_0}{d_k^2} \right)^{n_r} \right] \propto E \left[ \left( \frac{2N_0}{d_k^2} \right)^{n_r} \right]^{w_{\text{Hmin}}} \]

The coding gain depends on the error correcting code and on the binary mapping (via the equivalent BSKs)
Large interleaver size ⇒ no interference between the erroneous bits in the pairwise error probability

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$$\text{BER} \xrightarrow{N_0 \to 0} E \left[ \prod_{k=1}^{w_{H_{\text{min}}}} \left( \frac{2N_0}{d_k^2} \right)^{n_r} \right] \xrightarrow{N_0 \to 0} E \left[ \left( \frac{2N_0}{d_k^2} \right)^{n_r} \right]^{w_{H_{\text{min}}}}$$

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Performance over ergodic channels

- Large interleaver size $\Rightarrow$ no interference between the erroneous bits in the pairwise error probability

- Exact pairwise error probability has been derived, the asymptotic performance is

$$\text{BER} \overset{N_0 \to 0}{\propto} E \left[ \prod_{k=1}^{w_{H_{\text{min}}}} \left( \frac{2N_0}{d_k^2} \right)^{n_T} \right] \propto E \left[ \left( \frac{2N_0}{d_k^2} \right)^{n_T} \right]^{w_{H_{\text{min}}}}$$

- The coding gain depends on the error correcting code and on the binary mapping (via the equivalent BSKs)
Mapping optimization

Mapping figure of merit

Maximizing the figure of merit $\mathcal{F}_\Omega = E_D [d^{-2n_r}]^{-1}$ of the mapping $\Omega$ maximizes the coding gain.

Gray Mapping

Optimized mapping
Mapping optimization

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Maximizing the figure of merit \( \mathcal{F}_\Omega = E_D \left[ d^{-2n_r} \right]^{-1} \) of the mapping \( \Omega \) maximizes the coding gain

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Multi-dimensional mappings

- \( n_{map} \) complex dimensions among \( n_t \) are used to increase the BSK distances

<table>
<thead>
<tr>
<th>( n_r = 4 ), ( n_{map} = 1 )</th>
<th>BPSK</th>
<th>QPSK</th>
<th>16-QAM</th>
<th>64-QAM</th>
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<tr>
<td>0.00</td>
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<tr>
<td>( n_r = 4 ), ( n_{map} = 2 )</td>
<td>0.69</td>
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<td>10.98</td>
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<td>( n_r = 4 ), ( n_{map} = 3 )</td>
<td>3.35</td>
<td>6.16</td>
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<td>( n_r = 4 ), ( n_{map} = 4 )</td>
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**Table:** Asymptotic gains (in dB) with respect to Gray mapping

- BSA algorithm is used for the mapping optimization

- Simple error correcting codes are used to allow a good convergence of the iterative receiver
Ergodic 4 × 4 MIMO channel, interleaver size is 8192 bits, rate 1/2 NRNSC and Turbo-codes, QPSK modulation, 20 decoding iterations
Part II

BICM optimization for block-fading MIMO channels
Ideal interleaving condition

**Proposition**

For any pair of codewords, an ideal interleaver places different bits between the two codewords in QAM symbols that will be transmitted on different time periods, and equiprobably distribute them over all the channel states.

This condition may not be achieved with a practical interleaver.
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Singleton bound for single antenna block fading channels

- Originally proposed by Knopp & Humblet (1997)

- Given an error correcting code $\mathcal{C}$ of rate $R_c$, the diversity observed at the output of the decoder of a BICM transmitted over a single antenna block fading channel with $n_c$ independent blocks is upperbounded by

$$SB = \left\lfloor n_c(1 - R_c) + 1 \right\rfloor$$

- Full diversity $n_c$ is achieved only if $R_c \leq 1/n_c$

- Example of a parity code with $R_c = 2/3$ over a block fading channel with $n_c = 2$ realizations

\[
\begin{array}{|c|c|c|}
\hline
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\hline
\end{array}
\quad \Rightarrow \quad
\begin{array}{|c|c|c|}
\hline
& 1 & \text{2} \\
1 & 1 & \text{2} \\
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\hline
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Transmitted codeword

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000 \implies 111 \implies 101 \implies 110
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Linear Precoder

- Assume a linear precoder $S$ of size $sn_t \times sn_t$

- The spreading matrix $S$ multiplies the block diagonal channel matrix corresponding to $s$ time periods

$$H = \text{diag}\{H_1, \ldots, H_1, H_2, \ldots, H_2, \ldots, H_{n_s}, \ldots, H_{n_s}\}$$

$$s' = s/n_s$$

where $n_s$ is the number of independent channel realizations in $H$

- $n_t \times n_r$ MIMO block fading channel with $n_c$ blocks $\Rightarrow sn_t \times sn_r$ correlated MIMO block fading channel with $n_c/n_s$ blocks
Singleton bound with linear precoding

**Assumption on the Precoder/detector**

Under ideal interleaving, we assume that the detector perfectly converts the $sn_t \times sn_r$ correlated MIMO $n_c/n_s$-block-fading channel $SH_k$ with QAM input into a $1 \times sn_r$ SIMO $n_t n_c / s$-block-fading channel with BSK input, assuming that $s$ is a divisor of $n_t n_c$. (only satisfied if the Precoder is well designed)

- We established the Singleton bound on the maximum diversity order after decoding of the BICM

$$SB = sn_r \left[ \frac{n_c n_t}{s} (1 - R_c) + 1 \right]$$

- An optimized interleaver should be used to achieve Singleton bound diversity

- The diversity $\delta$ is upper-bounded by

$$\delta \leq \min(n_c n_t n_r, sn_r d_{H_{min}}, SB)$$
Singleton bound with linear precoding

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Full diversity with linear precoding

Choose the minimal $s$ dividing $n_c n_t$ that gives full diversity order:

$$n_c n_t n_r \leq s n_r \left\lfloor \frac{n_c n_t}{s} \left(1 - R_c\right) + 1 \right\rfloor \Rightarrow s \geq R_c n_c n_t$$

Linear precoding + ideal interleaving $\Rightarrow$ full diversity

Modified Singleton bound on the diversity order, coding rate $R_c = 1/2$, $n_r = 1$, and $n_c = 1$ channel state
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**Linear precoding + ideal interleaving \( \Rightarrow \) full diversity**

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<th>( n_t ) ( s )</th>
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Modified Singleton bound on the diversity order, coding rate \( R_c = 1/2 \), \( n_r = 1 \), and \( n_c = 1 \) channel state
Coding gain considerations: Non precoded scheme

- Consider a pair of codewords with Hamming distance $w \geq n_t n_c$

- Define $D = \{d_1, \ldots, d_w\}$ the Euclidean distance set of the $w$ equivalent Binary Shift Keying modulations (BSKs)

- Define $\gamma_k^2 = \sum_{i=1}^{\kappa_k} d_{k,i}^2$ the equivalent factorized distances corresponding to the $k$-th channel state

- The coding gain on the non precoded block fading channel is given by
  
  $$G_{bf}(D) = \left( \prod_{k=1}^{n_t n_c} \gamma_k^2 \right)^{1/(n_t n_c)}$$

- The ideal coding gain is given by
  
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- As $w$ increases, the ratio $G_{ideal}/G_{bf}$ decreases

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Saturation of the additional coding gain obtained by increasing the number of terms in the averaging.
Linear precoder influence on the coding gain

Consider a $2 \times 1$ quasi-static MIMO channel

Without linear precoding, the coding gain is

$$G_{bf} = \sqrt{\gamma_1 \gamma_2}$$

With ideal linear precoding $s = 2$, the coding gain is

$$G_{2,1} = \frac{\gamma_1 + \gamma_2}{2}$$
Linear precoder influence on the coding gain

Consider a $2 \times 1$ quasi-static MIMO channel

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  $$\gamma_1^2 / 2 \gamma_2^2 / 2 \xrightarrow{\text{with ideal linear precoding}} G_{2,1} = \frac{\gamma_1^2 + \gamma_2^2}{2}$$
Optimal coding gain with linear precoding

- We derive the exact pairwise error probability of the BICM with linear precoding and ideal interleaving.

- Full diversity is observed at the decoder output $\Rightarrow$ the optimal coding gain with $S_{s,n_s}$ is

$$G_{s,n_s,\text{opt}} = \frac{n_c/n_s}{\prod_{k=1}^{n_c/n_s} \left( \sum_{l=1}^{s n_t} \frac{\gamma_{k,l}^2}{n_t n_s} \right)^{n_s/n_c}}$$

- The optimal coding gain is not necessarily achievable.

- We have the relation

$$G_{\text{ideal}} \geq G_{s,n_s,\text{opt}} \geq G_{s,n_s} \geq G_{bf}$$
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- The optimal coding gain is not necessarily achievable.

- We have the relation

$$G_{\text{ideal}} \geq G_{s,n_s,\text{opt}} \geq G_{s,n_s} \geq G_{bf}$$
Assume that $n_t = 4$, $n_r = 1$, $n_s = 2$ and $s = 8$

$$H = \text{diag}\{H_1, H_1, H_1, H_1, H_2, H_2, H_2, H_2\}$$
Full spatial spreading optimal linear precoding

**Full spatial spreading precoder design criterion**

A linear precoder with \( s = n_t n_s \) achieving a diversity order \( sn_r \) and providing maximum coding gain at the decoder output has to satisfy the following conditions:

1. The \( n_s \) subparts of the rows in the \( sn_t \times sn_t \) precoding matrix have the same Euclidean norm.
2. In each of the \( n_s \) subparts, the \( n_t \) subparts are orthogonal and have the same Euclidean norm.

New modified cyclotomic rotations were presented to both

1. Satisfy the optimality conditions for coded performance
2. Provide good uncoded performance \( \Leftrightarrow \) good performance at the first iteration of the iterative decoding
DNA: A class of quasi-optimal linear precoders

**DNA conditions for non-full spreading**

Independent precoding the $\frac{n_t}{s'}$ groups of $s' = \frac{s}{n_s}$ antennas ($s$ is given by the Singleton bound) provides full diversity and quasi-optimal coding gain:

$$G_{s,n_s} = \prod_{k=1}^{n_c/n_s} \prod_{l_2=1}^{n_t/s'} \left( \sum_{l_1=1}^{ss'} \frac{\gamma_{2,\{l_2,l_1\}}}{s} \right)^{s/(n_cn_t)}$$

- The independent precoding of the $s'$ antennas is equivalent to a full spreading decoding $\Rightarrow$ direct utilization of the modified cyclotomic matrices
- Existence for all values of $n_t$, $n_c$, $n_s$ and $s$
- The decoder sees a $1 \times sn_r$ SIMO channel with BPSK input and $n_tn_c/s$ independent blocks
Example: $n_t = 4$, $n_c = 1$, BPSK input

Pseudo random Interleaver with block separation constraint
Interleaver design

Example: \( n_t = 4, n_c = 1 \), BPSK input

Pseudo random Interleaver with block separation constraint
Modified turbo-codes: Invariant FER with respect to the frame length

- Boutros & al have recently presented near outage capacity turbo-codes (Allerton 2004)
- The modified parallel turbo-code structure is the following

The coded bits are in phase with the information bits, the optimized interleaver for convolutional codes can be applied without any modification.
Assumption:
Ideal interleaving

DNA precoders
parameters $s, n_s$ and $n_t$

Conversion of the $n_t \times n_r$ MIMO $n_c$-block fading channel into a $1 \times s n_r$ SIMO $n_c n_t / s$ block fading channel at the detector output

Singleton bound,
choice of $s$ from $n_t n_c$ and the error-correcting code rate

Channel interleaver optimization
parameters $s, n_c$ and $n_t$

Full diversity $n_c n_t n_r$

Quasi-optimal coding gain

Minimum $s$ providing full diversity

Achieves the assumption with a good probability

Singleton bound,
the error-correcting code rate
Quasi-optimal coding gain
Achieves the assumption with a good probability

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Minimum $s$ providing full diversity
Rate-$1/2$ NRNSC codes, QPSK modulation, $2 \times 1$ MIMO channel, $n_c = 1$, 3 iterations. $L_c N_c = 1024$. 

Outage probability, gaussian input
Outage probability, QPSK input
PR Interleaver, NRNSC 7,5
Optimized Interleaver, NRNSC 7,5
Rate-1/2 NRNSC 7,5 code, QPSK modulation, 2 × 2 MIMO channel, $n_c = 2$, $n_s = 1$, 3 iterations. $L_c N_c = 256$. 

Outage Proba., Gaussian input
Outage Proba., QPSK input
PR Interleaver, No precoding
Optim. Interleaver, No precoding
Optim. Interleaver, DNA $s=2$
Optim. Interleaver, Golden code
Rate-1/2 Turbo code, $(7, 5)_8$ constituent, QPSK modulation, $2 \times 2$ MIMO channel, $n_c = 1$, 10 iterations
$R_c = 1/2$, $2 \times 1$ MIMO channel, SNR=15 dB
Conclusions

How to optimize a BICM for MIMO channels, with iterative joint detection based on soft output sphere detector and a probabilistic SISO channel decoder?

- The channel is ergodic or block fading with $n_t n_c \gg w$
  - Turbo-codes + Gray mapping
  - Multi-dimensional mappings + elementary codes

- The channel is block-fading
  - Singleton bound: $s \geq R c n_c n_t$
  - If $s \neq 1$: DNA modified cyclotomic linear precoder
  - Optimized interleaver
  - Modified turbo-code

$\Rightarrow$ Near (outage) capacity performance
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$\implies$ Near (outage) capacity performance
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