

Achieving Multiuser Diversity under Hard Fairness Constraints

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I. INTRODUCTION

The focus of this study is an extension of [1] where the concept of *multiuser diversity* was introduced in the context of a single-cell multiuser channel. The primary result of this work is that using proper dynamic resource allocation strategies on Rayleigh fading channels can yield a non-negligible power boost which can be as high as 5 or 6 dB, for a moderate number of users employing a joint resource allocation policy. This gain grows logarithmically with the number of users. The results were extended in [2] where the complete fading multiuser channel capacity region was found. It was shown in [3] and [4] that similar results hold for the fading broadcast channel.

The main practical issue arising from these results is fairness. Users must wait (or the basestation in the case of the broadcast channel) until their channel conditions are favourable enough to transmit. We show in this work that multiuser diversity can still be achieved in a multi-carrier system using simple orthogonal multiplexing even under the constraint that each user receives the same constant bandwidth resource at any time instant.

II. PROBLEM FORMULATION

Consider the N -user system with N parallel channels. The parallel channels would typically model a multi-carrier system, where each sub-channel is an independent sub-carrier. Each sub-channel is an AWGN channel with noise variance N_0 and the signal on a particular sub-channel is

$$r_i = \sum_{n=0}^{N-1} \sqrt{P_{n,i}(\mathbf{H})h_{n,i}}x_{n,i} + z_i, i = 0, 1, \dots, N-1 \quad (1)$$

where $x_{n,i}$ and $h_{n,i}$ are the signal and channel gain for user i on sub-channel n and z_i is the noise on sub-channel i . $P_n(\mathbf{H})$ is the transmit power for user i on sub-channel n , and $\mathbf{H} = \{h_{n,i}\}$ is the $N \times N$ matrix of channel gains. Let us now assume that $\mathbb{E}x_{n,i}^2 = 1$ and $\mathbb{E}P_{n,i}(\mathbf{H}) \leq P/N$, so that the average transmit power of user i across all channels is P . If we consider the multiuser case, we have that the achievable sum rate is upper-bounded as

$$\sum_{n=0}^{N-1} R_n \leq \mathbb{E}_{\mathbf{H}} \sum_{i=0}^{N-1} \frac{1}{2} \log_2 \left(1 + \frac{1}{N_0} \sum_{n=0}^{N-1} P_{n,i}(\mathbf{H})h_{n,i} \right) \text{ bits/dim} \quad (2)$$

The optimal power control law for user n on sub-channel i is given in [1, 2]

Let us now impose a hard fairness constraint on the problem, namely that each user is granted one sub-channel at any given time instant and transmits with constant power P . This policy grants each user an equal opportunity to transmit across the channel. The power allocation problem now amounts to choosing a permutation vector π , mapping users to sub-carriers. The maximum achievable sum rate under this hard fairness constraint is upper-bounded by

$$\sum_{n=0}^{N-1} R_n \leq \mathbb{E}_{\mathbf{H}} \max_{l=0, \dots, N-1} \sum_{n=0}^{N-1} \frac{1}{2} \log_2 \left(1 + \frac{P}{N_0} h_{\pi_n, l} \right), \quad (3)$$

when the users (guided by the receiver) jointly select the optimal allocation vector given the instantaneous channel gains. We also consider a simpler allocation strategy which does not involve computation of mutual information functionals and will lead to tractable analytical expressions. We choose the permutation π_{l^*} where

$$l^* = \arg \max_{l=0, \dots, N-1} \min_{n=0, \dots, N-1} h_{\pi_n, l} \quad (4)$$

Here we guarantee that at any given time instant the channel with the minimum allocated gain is the best possible among all allocation permutations. This is somewhat similar to the high-SNR behaviour of the optimal allocation policy based on mutual information.

In Figure 1 we show the achievable rates (over i.i.d. Rayleigh fading channels) for the optimal “fair” allocation policy and the sub-optimal policy π_{l^*} for $N = 2, 3, 4$. We see that multiuser diversity can still be successfully achieved with a hard fairness constraint and provides a non-negligible gain with respect even to a non-fading channel (lowest curve).

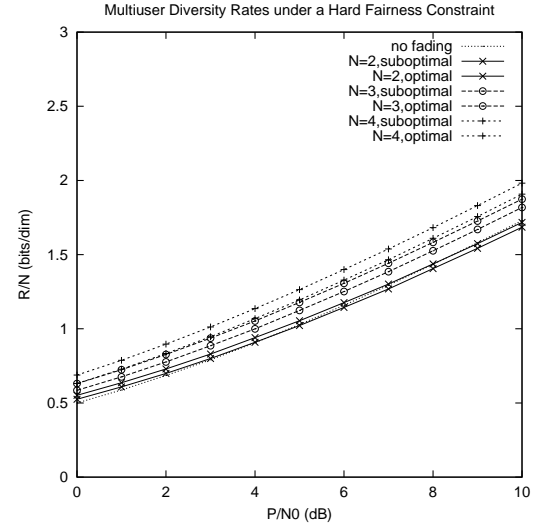


Figure 1: Multiuser Diversity Rates Under Optimal “Fair” Allocation and Sub-optimal π_{l^*}

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