

Burst Mode Non-Causal Decision-Feedback Equalizer based on Soft Decisions

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Abstract: The Non-Causal Decision-Feedback Equalizer (NCDFE) is a decision-aided equalizer that uses not only past decisions, like DFEs, but also future decisions, which usually come from another, classical equalizer. When there are no errors on the decisions, the NCDFE attains the Matched Filter bound (MFB). In practice, it suffers from the propagation of errors. We propose an implementation of the NCDFE based on soft decisions where only the most reliable decisions are fed back: this decreases error propagation and allows performance closer to the matched filter bound.

I Introduction

The principle of the Non-Causal Decision-Feedback Equalizer (NCDFE) was first proposed by Proakis [1]: this equalizer uses past and future decisions in order to cancel all the ISI present in the signal. Gersho and Lim [2] introduced its MMSE design: the forward filter is proportional to the matched filter and the feedback filter applied to the past and future symbol decisions w.r.t. the symbol to be detected, is the cascade of the channel and the forward filter, without the central coefficient. These past and future symbol decisions come from another classical equalizer, linear or DFE (note that the past decisions may come from the NCDFE itself). A burst mode unbiased MMSE version based on MLSE was also proposed in [3].

When no errors on the decisions are made, this equalizer attains the Matched Filter Bound (MFB) and is then potentially more powerful than the other equalizers, linear or DFEs. However the error propagation phenomenon can cause some degradations, like for the classical DFE, and the NCDFE may bring only a marginal improvement. Our purpose is to build a non-causal decision-feedback equalizer where only the most reliable symbols are fed back. Symbols are estimated through several iterations: each iteration finds the linear MMSE estimate of the unknown symbols and hard decisions are made only on the most reliable symbols which are then considered as known. The estimation of the remaining unknown symbols is done taking into account the new known symbols, which increase their estimation performance. We define a measure of reliability related to

the MMSE estimation error variance: it depends on the experimental conditions, on the position of the symbol in the burst and on the presence of known symbols.

II Problem Formulation

We consider a single-user multichannel model: this model results from the oversampling of the received signal and/or from reception by multiple antennas. Consider a sequence of symbols $a(k)$ received through m channels of length N and coefficients $\mathbf{h}(i)$:

$$\mathbf{y}(k) = \sum_{i=0}^{N-1} \mathbf{h}(i)a(k-i) + \mathbf{v}(k), \quad (1)$$

$\mathbf{v}(k)$ is an additive independent white Gaussian noise with $r\mathbf{v}\mathbf{v}^H(k-i) = E\mathbf{v}(k)\mathbf{v}^H(i) = \sigma_v^2 I_m \delta_{ki}$.

The symbol constellation is assumed known. When the input symbols are real, it will be advantageous to consider separately the real and imaginary parts of the channel and received signal as:

$$\begin{bmatrix} \text{Re}(\mathbf{y}(k)) \\ \text{Im}(\mathbf{y}(k)) \end{bmatrix} = \sum_{i=0}^{N-1} \begin{bmatrix} \text{Re}(\mathbf{h}(i)) \\ \text{Im}(\mathbf{h}(i)) \end{bmatrix} a(k-i) + \begin{bmatrix} \text{Re}(\mathbf{v}(k)) \\ \text{Im}(\mathbf{v}(k)) \end{bmatrix} \quad (2)$$

Let's rename $\mathbf{y}(k) = [\text{Re}^H(\mathbf{y}(k)) \text{Im}^H(\mathbf{y}(k))]^H$, and idem for $\mathbf{h}(i)$ and $\mathbf{v}(k)$, we get again (1), but this time, all the quantities are real. The number of channels gets doubled, which has for advantage to increase diversity. Note that the monochannel case does not exist for real input constellations.

Assume we receive M samples, concatenated in the vector $\mathbf{Y}_M(k)$:

$$\mathbf{Y}_M(k) = \mathcal{T}_M(\mathbf{h}) A_{M+N-1}(k) + \mathbf{V}_M(k) \quad (3)$$

$\mathbf{Y}_M(k) = [\mathbf{y}^H(k-M+1) \cdots \mathbf{y}^H(k)]^H$, similarly for $\mathbf{V}_M(k)$, and $A_M(k) = [a^H(k-M-N+2) \cdots a^H(k)]^H$, where $(\cdot)^H$ denotes Hermitian transpose. $\mathcal{T}_M(\mathbf{h})$ is a block Toeplitz matrix filled out with the channel coefficients grouped in the vector \mathbf{h} . We assume that some symbols are known: A_k contains the M_k known symbols and A_u , the M_u unknown symbols. We shall simplify the notation in (2) with $k = M-1$ to:

$$\mathbf{Y} = \mathcal{T}(\mathbf{h}) A + \mathbf{V} = \mathcal{T}_k(\mathbf{h}) A_k + \mathcal{T}_u(\mathbf{h}) A_u + \mathbf{V} \quad (4)$$

[†] The work of Elisabeth de Carvalho was supported by Laboratoires d'Electronique PHILIPS under contract Cifre 297/95. The work of Dirk Slock was supported by Laboratoires d'Electronique PHILIPS under contract LEP 95FAR008 and by the EURECOM Institute

A Influence of the known symbols

The structure of the burst-mode multichannel classical equalizers has been established in [4]. We derived the linear and decision-feedback equalizers in their Minimum-Mean-Squared-Error (MMSE) Zero-Forcing (ZF), MMSE and Unbiased MMSE (UMMSE) versions when some symbols in the burst are known, as well as expressions for the output SNRs.

As an example, in figure 1, we show the SNR at the output of the MMSE Linear Equalizer (LE). In the following we will use two important properties of the burst mode equalization:

- The SNR depends on the position of the symbol on the burst.
- For a given symbol, the SNR is higher when there are known symbols in the burst and especially when the symbol is surrounded by known symbols.

As seen in figure 1, when no symbols are known, performance at the edges degrades: the middle symbols appear in N outputs whereas the symbols at the edges appear in strictly less than N outputs and thus there is less information about them. When $N-1$ known symbols are present at each end of the burst, performance is better for the symbols located at the edges: after eliminating the contributions of the known symbols, the outputs at the edges contain strictly less than N symbols, so there is more information on those symbols, which are then better estimated. We also show the case of 10 and 50 known symbols dispersed all over the burst: and can see the advantage of taking into account the presence of known symbols in the burst to estimate the unknown symbols.

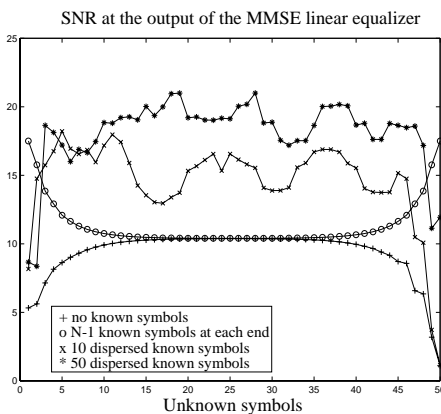


Figure 1: SNR at the output of a MMSE LE as a function of unknown symbol position in the burst: influence of the presence of known symbols on the estimation of the unknown symbols

B Burst Mode NCDFE

The burst mode structure of the NCDFE was derived in [3] based on MLSE. Its structure is given in figure 2. The forward filter is the multichannel matched filter $\mathcal{T}^H(h)$ followed by a

ing to the nature of the equalizer, MMSE or UMMSE: for the UMMSE NCDFE [3], $D = (\text{diag}(\mathcal{T}^H(h)\mathcal{T}(h)))^{-1}$ ($\text{diag}(\cdot)$ is a diagonal matrix containing the diagonal of its argument). The non-causal feedback filter consists in the forward filter without the central coefficient. \hat{A} may be the output of another equalizer or the output of the burst mode NCDFE at a previous iteration. If \hat{A} contains no errors, the performance of the NCDFE attains the MFB.

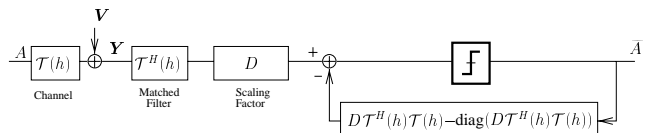


Figure 2: Burst Mode Non Causal DFE

IV NCDFE based on Soft Decisions

A MMSE LE gives as estimates of the unknown symbols A_u based on the observations \mathbf{Y} and the known symbols A_k :

$$\hat{A}_u = C_{A_u Y_u} C_{Y_u Y_u}^{-1} (\mathbf{Y} - \mathcal{T}_k(h) A_k) \quad (5)$$

$$= \left(\mathcal{T}_u^H(h) \mathcal{T}_u(h) + \frac{\sigma_v^2}{\sigma_a^2} I \right)^{-1} \mathcal{T}_u^H(h) (\mathbf{Y} - \mathcal{T}_k(h) A_k) \quad (6)$$

We recognize a non-causal decision-feedback structure where only the known symbols are fed back. Assume you want to detect one symbol in the burst and you know all the other symbols, the solution in (6) would give the output of the MMSE NCDFE corresponding to this symbol.

We will consider the UMMSE. The MMSE equalizer is indeed biased: at each output $\hat{a}_{MMSE}(k)$ of the MMSE equalizer, the term of interest is $\beta(k)a(k)$, with $\beta(k) < 1$. For convenience, we prefer to work with unscaled quantities. The UMMSE equalizer is simply a rescaled version of the MMSE equalizer giving for each output: $(\beta(k))^{-1} \hat{a}_{MMSE}(k)$. It has a lower output SNR than the MMSE but offers the advantage to give a lower probability of error, except for constant modulus constellations for which the probability of error remains the same.

A Soft Decisions

We will consider here only a BPSK; the principle of soft decisions could be extended to other constellations though. For each output of the UMMSE, $\hat{a}(k) = a(k) + \tilde{a}(k)$ where $\tilde{a}(k)$ contains intersymbol interference and noise, the sum of which can be approximated by a centered Gaussian variable. The variance of the error $\tilde{a}(k)$ is [4]:

$$\sigma_{\tilde{a}}^2(k) = \frac{1}{\sigma_v^2 \left(\left(\mathcal{T}_u^H(h) \mathcal{T}_u(h) + \frac{\sigma_v^2}{\sigma_a^2} I \right)^{-1} \right)_{k,k}} - 1 \quad (7)$$

We will not consider a hard decision scheme as shown in figure 3, but a soft decision scheme that will give hard decisions

ten based on the \tanh curve (see figure 3). Indeed, the MMSE estimate of $\hat{a} = a + \tilde{a}$, with a taking with equal probability the values $+1$ and -1 and \tilde{a} a centered Gaussian random variable independent of a , hypotheses verified (with the Gaussian approximation) in our problem, is:

$$\hat{a} = \tanh\left(\frac{\hat{a}}{\sigma_{\tilde{a}}^2}\right) \quad (8)$$

We envisage an iterative scheme with each iteration composed of two steps. In the first step we perform linear estimation of the symbols based on the received data and the symbol estimates from the previous iteration. The first step would correspond to the NCDFE if the symbol estimates were perfect. The second step performs element-wise nonlinear estimation. The optimal nonlinearity to be used in the second step is the $\tanh(\cdot)$. However, with such nonlinear symbol estimates, the design of the linear estimator for step one in the next iteration becomes non-trivial. Therefore, we propose the following simplified nonlinearity:

$$\hat{a} = f_{\alpha}(\hat{a}) = \begin{cases} \hat{a} & |\hat{a}| < \alpha \\ \text{sign}(\hat{a}) & |\hat{a}| \geq \alpha \end{cases} \quad (9)$$

$\alpha(k)$ gives the reliability of the symbol estimate and depends on $\sigma_{\tilde{a}}^2$: it is determined by searching the best MMSE estimate of \hat{a} of the form $f_{\alpha}(\hat{a})$ shown in figure 3:

$$\min_{\alpha} E (a - f_{\alpha}(\hat{a}))^2 \quad (10)$$

A closed form expression for α could not be found. However a

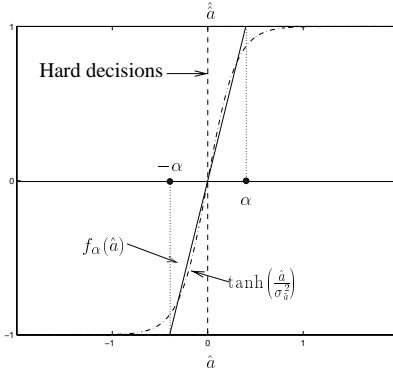


Figure 3: Soft Decision Curves

linear approximation w.r.t. $\sigma_{\tilde{a}}^2$ seemed to match well, especially for low $\sigma_{\tilde{a}}^2$: $\alpha = 1.33\sigma_{\tilde{a}}^2$ (see figure 4).

The complete iterative scheme is depicted in figure 5. $\hat{A}_{soft,i}$ denotes the \hat{a} for which $|\hat{a}| < 1$, whereas $\hat{A}_{hard,i}$ denotes the \hat{a} for which $|\hat{a}| = 1$. $\hat{A}_{hard}(i)$ denotes the accumulation of $\hat{A}_{hard,1}, \dots, \hat{A}_{hard,i}$. $\hat{A}_{lin,i}$ is a linear combination of $\hat{A}_i = \{\hat{A}_{soft,i}, \hat{A}_{hard}(i)\}$ and \mathbf{Y} , i.e., $\hat{A}_{lin,i}$ is a linear estimate of the remaining undecided symbols in terms of the received data and soft decisions for all symbols. One can observe that $\hat{A}_{lin,i}$ is in fact also a linear combination of only $\hat{A}_{hard}(i)$ and \mathbf{Y} and since the $\hat{A}_{hard}(i)$ are assumed to be error-free, the MMSE design of $\hat{A}_{lin,i}$ becomes tractable.

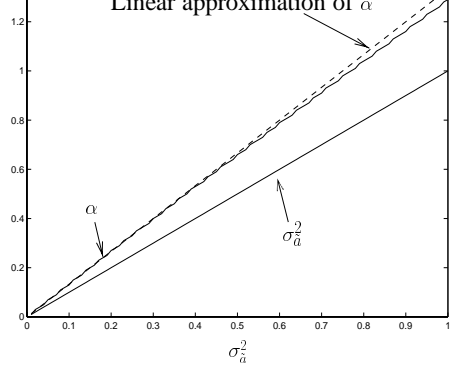


Figure 4: Linear approximation of α

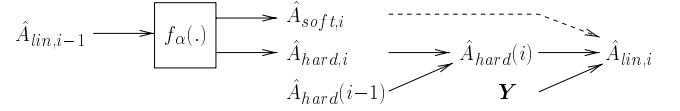


Figure 5: Iterative Soft Decision Scheme

B NCDFE based on soft decisions

The implementation of the NCDFE based on the soft decisions is now as follows:

1. Linear MMSE estimates of the unknown symbols based on \mathbf{Y} and A_k are computed by (6).
2. For each estimate $\hat{a}(k)$, the reliability measure $\alpha(k)$ is computed and the soft decision strategy (9) is applied. The hard decisions are treated as known symbols.

Steps 1-2 are reiterated until $\hat{A}_{hard,i}$ is empty.

Feeding back only the most reliable symbols allows to avoid the phenomenon of error propagation. Furthermore, as the presence of known symbols allows to increase the estimation quality of the unknown symbols, the feedback will help the detection of symbols on which errors could have been made by using a simple linear equalizer. At the end of this process, the symbols that remain non reliable even when using the feedback from known symbols are decided upon. Few iterations of the algorithm are necessary in general as will be seen in the simulations.

This strategy allows to automatically adapt the reliability intervals to the experimental conditions:

- The noise level: α is all the larger as the noise level is large.
- The presence of known symbols will be reflected in the value of $\alpha(k)$. Figure 6 shows the evolution of the reliability intervals from one iteration to another (for a randomly chosen channel at 5 dB): most of the symbols that remain unknown at the second iteration are located at the edges where indeed performance is lower. At the second iteration, the reliability on those symbols increases due to the feedback of the known symbols.

The reliability intervals have however for disadvantage to be based on mean quantities, and it may happen that hard decisions on certain realizations of $\hat{a}(k)$ considered as reliable are in

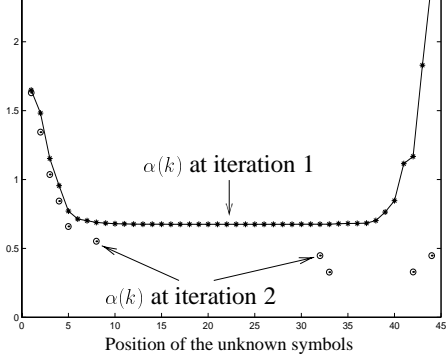


Figure 6: Evolution of the reliability intervals

fact false: this problem may arise especially at the first iteration for bad experimental conditions like low SNR or ill-conditioned channels. To alleviate this problem, α should be taken larger than the previously given value that we will denote α_o . In our simulations, we took $\alpha = \alpha_1 = 2\sigma_a^2$: this choice gave indeed better results for bad experimental conditions.

In our simulations, we will compare the performance of this NCDFE with the following more optimal but also more complex scheme:

1. We compute the output of the NCDFE with feedback A_k as in (6).
2. We choose the output with highest magnitude, *i.e.* the most reliable symbol estimate, make a decision on it and add it to the list of known symbols in A_k . Reiterate 1-2 until A_u is empty.

V Perspectives

A Changes in correlations

The incorporation of the soft decisions creates some problems. Indeed, it introduces correlations between $a(k)$ and $\tilde{a}(k)$, and then, as $\tilde{a}(k)$ combines noise and input burst components, between the noise \mathbf{V} and the symbols A , originally independent, as well as between the elements of A . Figure 8 shows the joint distribution of $a(k)$ and $\tilde{a}(k)$, for the reliable and non-reliable $\tilde{a}(k)$ for a case where $\alpha(k) = 1$ (see figure 7). $a(k)$ and $\tilde{a}(k)$ are correlated in both cases, and $\tilde{a}(k)|a(k)$ is not Gaussian anymore, only marginal distribution of $\tilde{a}(k)$ remains unchanged.

The formulation in (6), valid only when the noise and the symbols, as well the different symbols between themselves are independent, did not seem to be very sensitive to the correlation changes. And we kept this expression in its original form when using the soft decisions. The incorporation of these changes are the subject of on-going research.

B Complexity

The matrix $\mathcal{T}^H(h)\mathcal{T}(h) + \frac{\sigma_v^2}{\sigma_a^2}I$ being quasi-Toeplitz and banded, fast algorithms (based on the Schur algorithm) allow

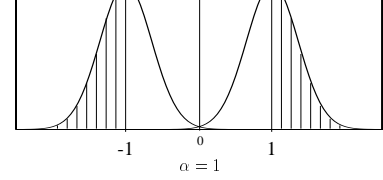


Figure 7: Distribution of the symbol estimates at the output of the UMMSE equalizer: reliable decisions such that $|\hat{a}(k)| \geq 1$ (example for $\alpha = 1$)

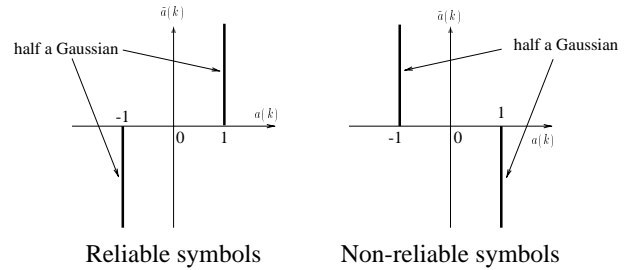


Figure 8: Joint distribution of $\tilde{a}(k)$ and $a(k)$ for the reliable (left) and non reliable decisions (right)

the computation of the equalizer output with a complexity of order MN . However $\mathcal{T}_u^H(h)\mathcal{T}_u(h) + \frac{\sigma_v^2}{\sigma_a^2}I$ loses the quasi-Toeplitz property but is still banded: a complexity of order $M_u N^2$ can then be achieved. On-going studies are trying to reduce this complexity to an order of $M_u N$. But it has to be noted that M_u is in general small (see the simulation part) the complexity $M_u N^2$ should not be an obstacle.

C Use of a Decision-Feedback Equalizer

At each step of the algorithm, the unknown symbols are estimated by a MMSE linear equalizer. This equalizer could be replaced by a DFE which gives in general better results: this DFE should however use among its past decisions only the most reliable ones.

VI Simulations

We tested the NCDFE based on soft decisions on two channels with real coefficients. The first channel \mathbf{H}_1 ($m = 2$, $N = 5$) was randomly chosen, the second channel $\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1.01 \end{bmatrix}$ has a nearly common zero. We plot the total number of errors for a input burst of length $M+N-1$, with $M = 100$ for a given input burst averaged over the burst length and 1000 Monte-Carlo runs of the noise. The SNR is at 7 dB and 10 dB. No known symbols are initially present in the burst.

We show the different measures in the following order (from left to right):

1. The optimal scheme previously described where only the most reliable decision is fed back at each iteration. The

hard decisions; two iterations of the hard NCDFE are done.

2. The proposed soft decision strategy with α_o followed by 2 iterations of the hard decision NCDFE.
3. The proposed soft decision strategy with α_1 followed by 2 iterations of the hard decision NCDFE.
4. The MMSE LE on which hard decisions are made followed by 2 iterations of the hard decision NCDFE.
5. The MMSE DFE on which hard decisions are made followed by 2 iterations of the hard decision NCDFE.

The MFB is also shown as reference: it is computed by averaging the number of errors at the output of a NCDFE with feedback of the exact symbols.

For both channels, it first has to be noticed that the hard NCDFE improves performance significantly w.r.t. the MMSE LE or DFE, but this is not always the case.

For channel H_1 (figure 9), we see that our soft decision strategy improves performance w.r.t. to the classical linear or DFE equalizer and w.r.t. to these same equalizers followed by the hard NCDFE. The soft decision strategy using α_1 attains the MFB at 10dB. The use of α_o or α_1 is approximately equivalent. Few steps of the algorithm were required: 2.5 for α_o and 2.9 for α_1 at 7 dB and 1.5 for α_o and 1.8 for α_1 at 10 dB. The experimental conditions being good, most of the symbol estimates are considered as reliable: only approximately 5% of the symbols remains unknown after the first iteration at 7 dB, and only 2% at 10dB. Our procedure remains however useful.

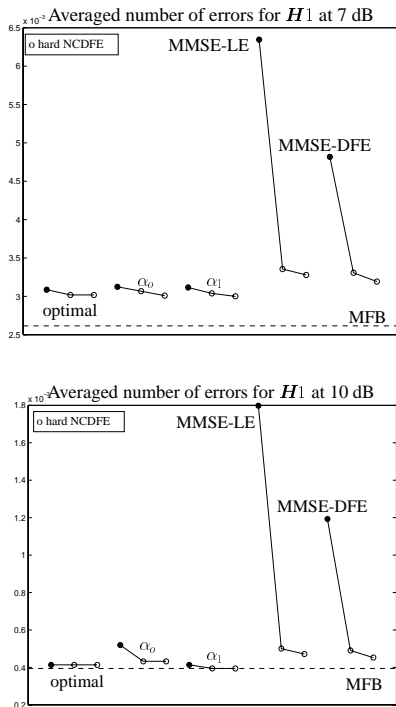


Figure 9: Average number of errors for channel H_1 at 7 dB and 10 dB.

mental decisions; two iterations of the hard NCDFE are done. In this example, we see more significantly the advantage of the soft decision strategy, and the necessity of choosing an α higher than α_o : indeed with α_o , false decisions were taken as reliable even at the first iteration. More iterations are done than in the previous example: 3 for α_o , 3.75 for α_1 at 7 dB and 2.1 for α_o , 2.2 for α_1 at 10 dB. Furthermore, at 5 dB, 12% for α_o , 26% for α_1 of the symbols remains unknown after the first iteration, and 7% for α_o , 12% for α_1 at 10 dB. At 10 dB, we also see that a classical DFE performs better than our soft decision procedure: this suggest the use of a DFE estimation of the unknown symbols instead of a linear one as already mentioned in previous section.

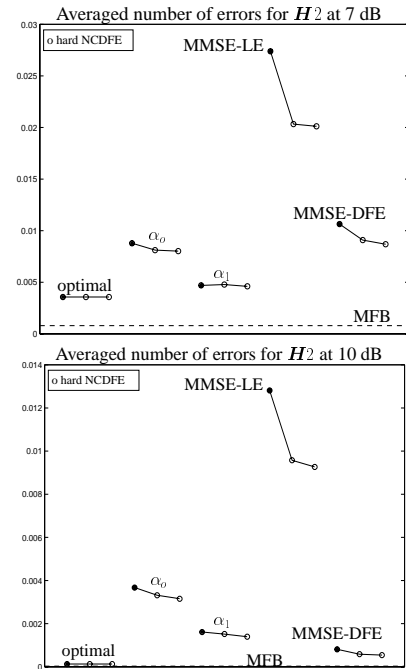


Figure 10: Average number of errors for channel H_2 at 7 dB and 10 dB.

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