Design and Performance of a Low-Complexity Iterative Multiuser Joint Decoder Based on Viterbi Decoding and Parallel Interference Cancellation

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***Abstract*—We propose a low complexity multiuser joint Parallel Interference Cancellation (PIC) decoder for Direct-Sequence CDMA. An estimate of the Multiple-Access Interference (MAI) signal is formed by weighting the hard decisions produced by hard-output Viterbi decoders. Such MAI interference estimate is subtracted from the received signal in order to improve decoding in the next iteration. By using asymptotic performance analysis of random-spreading CDMA, we optimize the feedback weights at each iteration. Then, we consider two performance limitation factors: the bias of residual interference and the *ping-pong* effect. We propose then a modification of the basic PIC algorithm, which allows higher channel load and/or faster convergence to the single-user performance. The main conclusion of this work is that, in most practical cases, SISO decoders are not needed to attain very high spectral efficiency, and simple conventional Viterbi decoding suffices for most practical settings.**

I. Introduction and motivation

The recently proposed UMTS standard adopted W-CDMA and a combination of TDMA-CDMA as the multiple access schemes for FDD and TDD modes, respectively. In both FDD and TDD modes, the UMTS basic receiver scheme contemplates the use of conventional Single-User Matched Filtering (SUMF) or *linear* Multiuser Detection (such as MMSE filtering or Decorrelation) followed by single-user decoding. Since Multiple-Access Interference (MAI) is treated as additional background noise, powerful and high-complexity channel coding such as 256-states convolutional codes and turbo codes [1] are envisaged in order to attain low Bit Error Rates (BER) at low decoder input signal-to-interference plus noise ratio (SINR). In any case, a channel load (number of users per chip) larger than 1 is difficult to be achieved by a *linear* front-end and single-user decoding [2]. On the other hand, Information Theory shows that much larger channel load can be achieved provided that a *non-linear* multiuser joint decoder is employed [2], [3]. This may range from the impractically complex optimal joint decoder to practically appealing successive interference cancellation approaches [4]. In practice, successive interference cancellation must cope with decision errors which prevent perfect cancellation of already decoded users. Then, several *iterative* schemes have been proposed, which limit the deleterious effect of decision errors by feeding back soft-estimates of the detected symbols (see for example [5], [6]). These schemes require Soft-Input Soft-Output (SISO) decoders, implemented by the forward-backward BCJR algorithm [7] which represent a non-negligible factor in the complexity of whole receiver. In real CDMA applications, the maximum achievable channel load is often limited by synchronization and channel estimation issues, rather than by the ultimate capability of the decoder itself. Hence, it makes sense to investigate simpler joint decoding schemes, which outperform the conventional linear front-end approach and yield performance similar to the SISO-based schemes and lower decoding complexity. Driven by this consideration, this paper proposes and analyzes the performance of a low complexity iterative multiuser receiver scheme where SISO blocks are replaced by simpler Viterbi decoders. This paper also discusses the bias problem in the residual interference, and the *ping-pong* effect [8] proposing modifications of the basic receiver.

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II. System model

We consider the uplink of a DS-CDMA system where U users send *encoded* information to a common receiver. We restrict our model to synchronous CDMA (where synchronism is at the chip, symbol, code word, and/or block level), and we assume that the propagation channels of all users are slowly time-varying and frequency flat. Although unrealistic, these assumptions are roughly applicable to the UMTS-TDD uplink system where the TDMA component ensures block synchronism between users and where indoor and pico-cells have usually a very short channel delay spread. The received discrete-time baseband signal corresponding to the n^{th} transmitted symbol, after chip matched filtering and sampling at chip-rate, is given by

$$\boldsymbol{y}[n] = \boldsymbol{S}[n]\boldsymbol{W}\boldsymbol{a}[n] + \boldsymbol{\nu}[n], \tag{1}$$

for $n = 0, 1, \dots, N-1$ where:

- $\boldsymbol{S}[n] = [\boldsymbol{s}_0[n], \dots, \boldsymbol{s}_{U-1}[n]]$ is a $L \times U$ matrix whose columns contain the user spreading sequences for the n^{th} symbol. In particular $\boldsymbol{s}_u[n] = [s_{0,u}[n], s_{1,u}[n], \dots, s_{L-1,u}[n]]^{\text{T}}$ is the complex spreading sequence of the u^{th} user in the n^{th} symbol, whose elements $s_{l,u}[n]$ are QPSK symbols in the set $\{(\pm 1 \pm j)/\sqrt{2L}\}$, so that $\boldsymbol{s}_u^{\text{H}}[n]\boldsymbol{s}_u[n] = 1$, $\forall u = 0, \dots, U-1$ and $\forall n$. L is the *spreading factor* (number of chip per symbol), assumed common to all users.
- $\boldsymbol{W}$ is a $U \times U$ diagonal matrix containing the channel complex amplitudes, such that, $\text{diag}\{\boldsymbol{W}\} = [w_0, \dots, w_{U-1}]$, where w_u is the u^{th} user channel coefficient.

- $\boldsymbol{a}[n] = [a_0[n], \ldots, a_{U-1}[n]]^{\mathrm{T}}$ contains the user coded symbols transmitted at time n.
- $\boldsymbol{\nu}[n] = [\nu_0[n], \ldots, \nu_{L-1}[n]]^{\mathrm{T}}$ contains the background noise samples, where $\nu_l[n]$ is an i.i.d. circularly symmetric Gaussian random variable with distribution $\mathcal{N}_C(0, N_0)$.

We assume that all users make use of convolutional coding and BPSK modulation, so that $a_u[n] \in \{\pm 1\}$, and interleave their code word before transmission. Hence, the sequence of symbols $\{a_u[n] \ \forall n\}$ represents the code word of user u *after interleaving*. With the above assumptions, and considering perfectly power-controlled system with equal power users, the received energy per symbol for user u is given by $E_{s\,u} = |w_u|^2 = E_s \ \forall u$. The output of a the SUMFs bank is

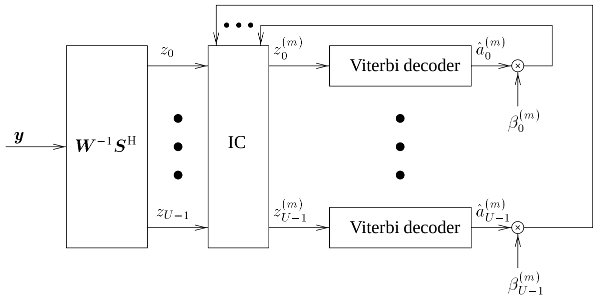


Fig. 1. Block diagram of the proposed receiver.

given by

$$\boldsymbol{z}[n] = \mathrm{Re}\left\{\boldsymbol{W}^{-1}\boldsymbol{S}^{\mathrm{H}}[n]\boldsymbol{y}[n]\right\} = \boldsymbol{T}[n]\boldsymbol{a}[n] + \boldsymbol{v}[n] \qquad (2)$$

where, $\boldsymbol{T}[n] = \mathrm{Re}\{\frac{1}{E_s}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{S}^{\mathrm{H}}[n]\boldsymbol{S}[n]\boldsymbol{W}\}$, and the noise term, $\boldsymbol{v}[n]$ is given by $\boldsymbol{v}[n] = \mathrm{Re}\{\frac{1}{E_s}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{S}^{\mathrm{H}}[n]\boldsymbol{\nu}[n]\}$ with distribution $\boldsymbol{v}[n] \sim \mathcal{N}(\boldsymbol{0}, \frac{N_0}{2E_s}\boldsymbol{T}[n])$. In the following, we omit the time index n when it is not necessary. From (2), the SUMF output for the u^{th} user can be written as

$$z_u = a_u + \sum_{k=0, k\neq u}^{U-1} \boldsymbol{T}_{u,k} a_k + v_u \qquad (3)$$

where $\boldsymbol{T}_{u,k}$ is the $(u,k)^{\mathrm{th}}$ element of $\mathbf{T}$ and where the first term is the desired user symbol, the second term is the MAI and v_u is the additive Gaussian noise. Figure 1 shows the proposed iterative receiver, where the signal z_u passes through an IC stage that uses the estimates $\hat{a}_u^{(m)}$ of the symbols a_u to remove the MAI. The superscript $^{(m)}$ denotes m^{th} iteration. The hard decisions $\hat{a}_u^{(m)} \in \{\pm 1\}$ provided by a bank of Viterbi decoders, are weighted by the factors $\beta_u^{(m)} \in [0,1]$, so that the signal at the output of the IC stage is given by

$$z_u^{(m)} = a_u + \sum_{k=0, k\neq u}^{U-1} \boldsymbol{T}_{u,k}\left(a_k - \beta_u^{(m)}\hat{a}_k^{(m)}\right) + v_u \qquad (4)$$

At the first iteration, the initial estimated symbols are set to zero, $\hat{a}_u^{(0)} = 0$, $\forall u$ so that $z_u^{(0)} = z_u$. In the case of perfect symbol estimates and $\beta_u^{(m)} = 1$, (4) reduces to $z_u^{(m)} = a_u + v_u$ where the MAI is completely removed and single user performance is attained. The weighting factors $\beta_u^{(m)}$ are intended as an index of reliability on the estimated symbols $\hat{a}_u^{(m)}$, such that $\beta_u^{(m)} = 1$ in the case of completely reliable symbol estimates, and $\beta_u^{(m)} = 0$ if they are not reliable at all.

III. Feedback weight optimization

Rigorous asymptotic analysis of such iterative schemes in the large system limits (i.e., for $N, U, L \to \infty$ with $U/L = \alpha$ fixed and $U/N \to 0$) is derived in [9] from the general approach of density evolution over graphs, currently used to analyze the limit performance of message-passing iterative decoders, and by using the result from the theory of large random matrices developed in [10] for the asymptotic analysis of linear CDMA receivers. In [9] it is also shown that this analysis holds only if the symbol estimates $\hat{a}_u^{(m)}$ are functions of the decoder *extrinsic information*. The decoder extrinsic information is defined for a SISO decoder based on the sum-product algorithm, such as the BCJR algorithm, but it is not defined for a sequence-wise ML decoder such as the Viterbi algorithm. Hence, we shall optimize the weights $\beta_u^{(m)}$ for a "fictitious" receiver where the Viterbi decoders are replaced by BCJR decoders and where the hard decisions $\hat{a}_u^{(m)}$ are obtained by one-bit quantization of the extrinsic likelihood ratios produced by the latters. We hasten to say that the fictitious receiver has no practical relevance, for the obvious reason that if BCJR decoders are used, then much more efficient soft-estimation of the interfering symbols (see for example [5], [6]) could be used. However, as it will be clear from the rest of this section, the weight optimization based on asymptotic analysis of the fictitious receiver allows us to derive a very simple expression for the optimal weights, independent of the user sequences and their mutual correlations, and a very simple practical algorithm for calculating these weights on-line.

Under the assumption that the spreading sequences are random with i.i.d. chips, uniformly distributed over the QPSK constellation, and that the phases of the channel coefficients are i.i.d., uniformly distributed over $[0, 2\pi)$, in the limit for $U, L \to \infty$ with $U/L = \alpha$ (where α is the *channel load*), the signal to noise ratio at the decoders input in the m^{th} iteration for the fictitious system can be written as [9]

$$\mathrm{SINR}^{(m)} = \frac{2E_s/N_0}{1 + \alpha E_s/N_0 \mu^{(m)}} \qquad (5)$$

where $\mu^{(m)} = \mathrm{E}[|a - \beta^{(m)}\hat{a}^{(m)}|^2]$ is the variance of the residual interference given by any one of the interfering users. Since the system is perfectly symmetric and all the users are equivalent, we have dropped the dependence on the user index u, meaning that (5) holds for every user. The above expectation is taken with respect to the symbols in a codeword and it can be expanded as

$$\mu^{(m)} = 1 + \left(\beta^{(m)}\right)^2 - 2\beta^{(m)}\left(1 - 2\epsilon^{(m)}\right) \qquad (6)$$

where $\epsilon^{(m)} = \Pr(a \neq \hat{a}^{(m)})$ is the Symbol Error Rate (SER) of a decoder with an input signal-to-noise ratio $\mathrm{SINR}^{(m-1)}$. Assuming that the residual interference plus noise at iteration m is Gaussian [1] the SER is a known function of the decoder input SINR. In other words, we can write $\epsilon^{(m)} = f(\mathrm{SINR}^{(m-1)})$ where $f(\cdot)$ is a function depending only on the employed convolutional code. We shall refer to this function as the "SER characteristic" of the user code. The SINR at iteration m can be rewritten as $\mathrm{SINR}^{(m)} = 2\frac{E_s}{N_0}\eta^{(m)}$ where $\eta^{(m)} = 1/(1 + \alpha\frac{E_s}{N_0}\mu^{(m)})$ and $\mu^{(m)} = 1 + (\beta^{(m)})^2 - 2\beta^{(m)}(1 - 2f(\mathrm{SINR}^{(m-1)}))$. The term $\eta^{(m)}$ is the degradation factor of the SINR at iteration m with respect to the single-user SNR, that is, $2E_s/N_0$. By definition $\eta^{(m)}$ it is Multiuser Efficiency (ME) at iteration m. Single user performance is achieved when $\eta^{(m)} = 1$. In order to maximize the SINR, $\beta^{(m)}$ is chosen in order to minimize the residual interference variance. From (6), we find that the optimal weighting factor is given by $\beta^{(m)} = 1 - 2\epsilon^{(m)}$ yielding the residual interference variance

$$\mu^{(m)} = 4\epsilon^{(m)}(1 - \epsilon^{(m)}) \tag{7}$$

Since $\epsilon^{(m)} \in [0, 1/2]$ is a non-increasing function of $\mathrm{SINR}^{(m-1)}$ and $\mu^{(m)} = 4\epsilon^{(m)}(1 - \epsilon^{(m)})$ is a non-increasing function of $\epsilon^{(m)}$ in the range $[0, 1/2]$, then $\{\eta^{(m)}\}$ is a non-decreasing sequence, upper-bounded by 1. Hence, if $\lim_{m\to\infty} \eta^{(m)} = 1$, the single-user performance is achieved by all users in the system. The evolution of the ME with the iterations is described by the one-dimensional non-linear dynamical system $\eta^{(m)} = \Psi(\eta^{(m-1)})$ with initial condition $\eta^{(0)} = (2E_s/N_0)/(1 + \alpha E_s/N_0)$, where the mapping function $\Psi(\cdot)$ is defined by

$$\Psi(\eta) = \left(1 + 4\alpha\frac{E_s}{N_0} f(2\eta E_s/N_0)(1 - f(2\eta E_s/N_0))\right)^{-1} \tag{8}$$

Figure 2 shows the function $\Psi(\eta)$ for $E_b/N_0 = 5$dB, the 4-states convolutional code of rate $1/2$ and octal generators $\{5, 7\}$ (denoted in the following by $\mathrm{CC}(5, 7)$) and for channel loads $\alpha = 2$ and $\alpha = 2.35$. For the sake of comparison, we show also the evolution of the same system when the BCJR decoder provides soft extrinsic estimates as proposed by [5]. The $\Psi(\eta)$ function in this case is derived in [9]. When the channel load is increased, the Ψ curves are modified so that for a certain threshold load α, the curve corresponding to the BCJR decoder with weighted hard decisions is tangent to the diagonal (see Figure 2 right). This means that this system has reached its maximum load and is not able to converge to single user performance. On the contrary, the system using soft decisions still converges to single user performance. This shows that using SISO decoding and soft feedback also provides a higher threshold load, i.e., an overall maximum achievable spectral efficiency of the system.

[1] In the large-system limit and under mild technical conditions this assumption is valid

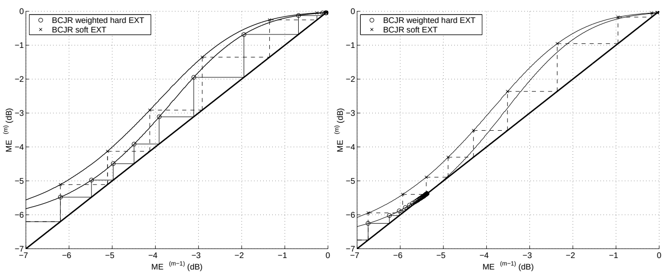


Fig. 2. Evolution of the ME given by the asymptotic analysis for $\mathrm{CC}(5, 7)$, $E_b/N_0 = 5$dB, and for channels loads $\alpha = 2$ (left), and $\alpha = 2.35$(right).

A. Implementation of the basic weighting algorithm

In a non-asymptotic situation, the ME is not identical for all users and, as a consequence, the optimal weighting factors $\beta_u^{(m)}$ do depend on the user index and not only on the iteration index. Moreover, the SINR at the decoder input for each iteration must be reliably estimated in order to compute the weighting factors for the next iteration. Driven by the above asymptotic analysis, we propose to compute the weighting factor for the u^{th} user at iteration m as

$$\beta_u^{(m)} = 1 - 2\, f\left(\mathrm{SINR}_{u_{\mathrm{est}}}^{(m-1)}\right) \tag{9}$$

where $f(\cdot)$ is the (known) SER code characteristics, and $\mathrm{SINR}_{u_{\mathrm{est}}}^{(m)}$ is the estimated SINR at the u^{th}decoder input of the m^{th} iteration. In order to estimate the input SINR, we can use the estimator given by

$$\mathrm{SINR}_{u_{\mathrm{est}}}^{(m)} = \left(\frac{1}{N}\sum_{n=0}^{N}\left|z_u^{(m)}[n]\right|^2 - 1\right)^{-1} \tag{10}$$

and proposed in [11], where it is shown that $\nu_u^{(m)} = \frac{1}{N}\sum_{n=0}^{N}\left|z_u^{(m)}[n]\right|^2 - 1$ is an unbiased estimator of the residual MAI plus noise variance at the decoder input if the residual MAI

$$\zeta_u^{(m)} = \sum_{k=0, k\neq u}^{U-1} \boldsymbol{T}_{u,k}\left(a_k - \beta_u^{(m)}\hat{a}_k^{(m)}\right) \tag{11}$$

is uncorrelated with the desired variable a_u.

IV. Compensation of the bias

It can be shown [9], [12] that when the symbol estimates $\hat{a}_u^{(m)}$ are provided by decision statistics "containing" the current observation interval, such as in a Viterbi decoder or by a symbol-by-symbol MAP decoder, then the residual interference term given by (11) is conditionally biased given a_u, i.e. $\mathrm{E}[\zeta_u^{(m)}|a_u] = \delta_u^{(m)}a_u$, where the bias coefficient $\delta_u^{(m)}$ is non-positive and depends on the system parameters and on the user and iteration index. On the contrary, if the symbol estimates $\hat{a}_u^{(m)}$ are provided by decision statistics "not containing" the current observation interval, i.e., they are based on the decoder *extrinsic information* [9], then in the limit for large block length (i.e., $N \to \infty$) and random interleaving the residual interference is conditionally unbiased, i.e., $\mathrm{E}[\zeta_u^{(m)}|a_u] = 0$. We can re-write the input of the u^{th} decoder at iteration m given in (3) as

$$z_u^{(m)} = (1 + \delta_u^{(m)})a_u + \widetilde{\zeta}_u^{(m)} + v_u \quad (12)$$

where $\widetilde{\zeta}_u^{(m)}$ is uncorrelated with a_u. The true SINR in the presence of bias is given by

$$\mathrm{SINR}_{\mathrm{u}_{\mathrm{true}}}^{(m)} = \frac{(1 + \delta_u^{(m)})^2}{\mathrm{E}[|\widetilde{\zeta}_u^{(m)} + v_u|^2]} \quad (13)$$

The SINR estimator (10) in the presence of bias, for large N, converges in probability to

$$\mathrm{SINR}_{\mathrm{u}_{\mathrm{est}}}^{(m)} \to \frac{1}{\mathrm{E}[|\widetilde{\zeta}_u^{(m)} + v_u|^2] + (1 + \delta_u^{(m)})^2 - 1}$$

Since $\delta_u^{(m)} \leq 0$ (i.e., the bias tends to decrease the useful signal term), we conclude that the estimator (10) tends to overestimate the true SINR at the decoder input with high probability. As a consequence, the weights $\beta_u^{(m)}$ computed according to (9) are mismatched in the presence of bias. This effect is shown in Figure 3(a) that refers to a system with $U = 38$, $L = 16$ (corresponding to $\alpha = 2.375$) and $E_b/N_0 = 5$dB.

In order to overcome this problem, we may think of compensating the SINR estimate in the first iteration. In a large random system, the *average* bias term $\bar{\delta}^{(m)}$ (where averaging is over all users) is a function of the load α, of the users SNRs and of the convolutional codes used. Hence, it can be numerically pre-computed and stored in a look-up table. Let $\hat{\delta}^{(m)}$ denote the pre-computed average bias at iteration m. Then, the following SINR estimator, taking into account the bias correction, can be used in (9) in order to compute the feedback weights

$$\mathrm{SINR}_{\mathrm{u}_{\mathrm{comp}}}^{(m)} = \frac{(1 + \hat{\delta}^{(m)})^2}{\frac{1}{N}\sum_{n=0}^{N-1} \left|z_u^{(m)}[n]\right|^2 - (1 + \hat{\delta}^{(m)})^2} \quad (14)$$

where "comp" stands for "compensated". In Figure 3(b), the performance of the proposed receivers for $U = 38$ are shown in terms of BER plotted versus the number of iterations. The bold curve represents the single-user performance, that for $E_b/N_0 = 5$dB and $\mathrm{CC}(5,7)$ is approximately BER$= 8 \cdot 10^{-5}$. The BER of the Viterbi-based receiver that uses $\mathrm{SINR}_{\mathrm{u}_{\mathrm{est}}}^{(m)}$ converges to the single-user performances in 12 iterations. The curve labeled "Weighted Viterbi (true SINR)" converging in 7 iterations represents the ideal receiver that has perfect knowledge of the bias for each user in every iteration, so that it can compute the feedback weights using $\mathrm{SINR}_{\mathrm{true}}^{(m)}$. The curve labeled "Weighted Viterbi (comp. SINR)" corresponds to the receiver that computes the feedback weights using $\mathrm{SINR}_{\mathrm{comp}}^{(m)}$ for the first iteration and converges in 9 iterations. Finally, the receiver based on the BCJR hard weighted decisions does not converge. Notice that the "Weighted Viterbi (comp. SINR)" receiver by providing a faster convergence to single user performance it also decreases the receiver overall complexity.

V. PING-PONG EFFECT AND ITS COMPENSATION

The proposed receivers allow system loads up to a certain threshold above which the system cannot achieve single user performance. In such high load situations, the system parameters as BER, SER, ME, and bias tend to oscillate between two convergence patterns [8]. This phenomenon is called *ping-pong* and it is related to the bias in the residual interference term. In fact, it does not appear when feedback is obtained from a SISO decoder extrinsic information [9]. [2] A further investigation reported in [13] showed that such a bistable situation is due to a fixed subset of the estimated symbols that flip when passing from one iteration to another, while the other estimated symbols in the complementary subset do not change. A countermeasure to this problem proposed in [13] consists of introducing a perturbation into the bistable situation, by feeding back to the IC stage the average of the estimates obtained from the two previous iterations. Thus, the contribution of the flipping symbols (considered as not reliable) is nullified. By considering this idea, the variance of the residual interference, $\mu^{(m)}$, can be expressed by

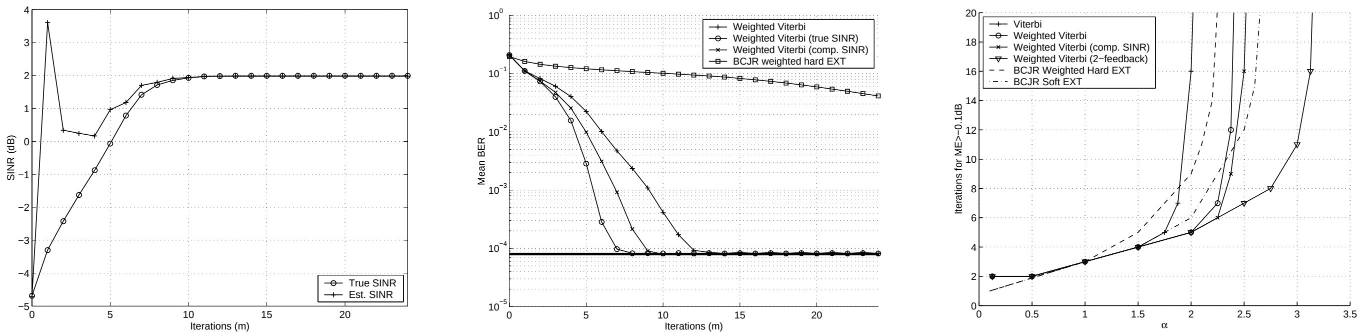
$$\mu^{(m)} = \mathrm{E}\left[\left|a - \beta_1{}^{(m)}\hat{a}^{(m)} - \beta_2{}^{(m)}\hat{a}^{(m-1)}\right|^2\right] \quad (15)$$

where the two previous estimates are now weighted with the coefficients $\beta_1^{(m)}$ and $\beta_2^{(m)}$. Furthermore (15) can be expanded as

$$\begin{aligned}\mu^{(m)} &= 1 + \left(\beta_1^{(m)}\right)^2 + \left(\beta_2^{(m)}\right)^2 - 2\beta_1^{(m)}\mathrm{E}\left[a\hat{a}^{(m)}\right] \\ &\quad -2\beta_2^{(m)}\mathrm{E}\left[a\hat{a}^{(m-1)}\right] + 2\beta_1^{(m)}\beta_2^{(m)}\mathrm{E}\left[\hat{a}^{(m)}\hat{a}^{(m-1)}\right] \quad (16)\end{aligned}$$

By minimizing the above equation with respect to both $\beta_1{}^{(m)}$ and $\beta_2{}^{(m)}$ we obtain $\beta_1{}^{(m)} = \frac{A-BC}{1-C^2}$, $\beta_2{}^{(m)} = \frac{B-AC}{1-C^2}$ where $A = \mathrm{E}\left[a\hat{a}^{(m)}\right] = 1 - 2\epsilon^{(m)}$, $B = \mathrm{E}\left[a\hat{a}^{(m-1)}\right] = 1 - 2\epsilon^{(m-1)}$ and $C = \mathrm{E}\left[\hat{a}^{(m)}\hat{a}^{(m-1)}\right]$. Notice that A and B are calculated using the SINR estimates at iteration m and $m-1$ respectively, while $C = \frac{1}{N}\sum_{i=0}^{N-1}\hat{a}_u^{(m)}[i]\hat{a}_u^{(m-1)}[i]$. It can be experimentally observed that, if the decoder converges to the single-user performance, $\beta_1^{(m)}$ and $\beta_2^{(m)}$ tend rapidly to $\beta_1{}^{(m)} \approx \frac{1}{2}(1 - 2\epsilon^{(m)})$, $\beta_2{}^{(m)} \approx \frac{1}{2}(1 - 2\epsilon^{(m-1)})$ as m increases. For $m = 1$, $\beta_2^{(1)}$ is irrelevant (since $\hat{a}_u^{(0)} = 0$) and $\beta_1^{(1)}$ computed according to (9). For $m = 2$, $\beta_2^{(2)} = \frac{1}{2}\beta_1^{(1)}$ and $\beta_1^{(2)} = \frac{1}{2}(1 - 2\epsilon^{(2)})$ is computed according to (9) by replacing $\mathrm{SINR}_{\mathrm{est}}^{(1)}$ by $\mathrm{SINR}_{\mathrm{comp}}^{(1)}$. For $m = 3, 4, \ldots$, $\beta_1^{(m)} = \frac{1}{2}(1 - 2\epsilon^{(m)})$ is computed according to (9), without need for compensation.

[2] For the extrinsic-based schemes, there exist obviously a threshold load above which single-user performance cannot be achieved, but no oscillatory behavior appears.



(a) ”Weighted Viterbi” receiver, $U = 38$

(b) BER of several receivers, $U = 38$

(c) Iterations required for ME> -0.1dB

Fig. 3. Performance of the proposed receivers using the CC$(5, 7)$ channel code, $L = 16$, and $E_b/N_0 = 5$dB

VI. Comparisons and conclusions

Figure 3(c) presents the system performance in terms of the number of iterations required to reach a $ME \geq -0.1$dB as a function of the channel load α, for $E_b/N_0 = 5$dB and CC$(5, 7)$. The curve labeled “BCJR weighted hard EXT” corresponds to the fictitious system with weighted extrinsic-based hard decisions. This curve shows a vertical asymptote for $\alpha = 2.3$ (threshold load). The curve labeled “BCJR soft EXT” corresponds to the system of [5], using SISO decoding and soft estimates in the feedback. In this case, the vertical asymptote is for $\alpha = 2.7$. The curve labeled “Viterbi” corresponds to a conventional hard-output PIC decoder where all feedback weights are set to 1, achieving a threshold load $\alpha = 2$. The curve labeled by “Weighted Viterbi” corresponds to the receiver proposed in this paper, based on weighted hard decisions where the weights are given by $\beta^{(m)} = 1 - 2\epsilon^{(m)}$ but hard decisions are provided by Viterbi decoders. In this case, the threshold load is $\alpha = 2.4$. The curve labeled “Weighted Viterbi (comp. SINR)” refers to the receiver that computes the feedback weights by compensating the SINR for the first iterations as described by (14). This decoder yields a small increase of the threshold load with respect to “Weighted Viterbi”. Finally the curve labeled “Weighted Viterbi (2-feedback)” correspond to the modified decoder that weights the hard output of the two previous iterations. The increase of performance given by this solution allows a threshold load $\alpha = 3.15$, outperforming “BCJR sof EXT” decoder. The two BCJR curves are obtained by the asymptotic formulas. The other curves are obtained by simulating finite-dimensional systems with spreading factor $L = 16$ and increasing number of users U. Remarkably, for $\alpha < 2.3$ the SISO-based and the Viterbi systems need around the same number of iterations to reach single user performance. Considering that the complexity of the Viterbi decoder is (at least) half of the BCJR one, the “Weighted Viterbi” receiver is convenient in the range of load $\alpha \leq 2.3$. Since this range is usually much larger than today’s conventional CDMA systems, and the channel load is often limited by the ability of the system to synchronize and perform channel estimation, we argue that for most applications the proposed receiver is just enough, and more complicated SISO decoders are basically not needed.

References

[1] 3GPP, “ETSI TS 125 222, Multiplexing and channel coding TDD 3GPP TS 25.222 Version 4.0.0 Release 4,” tech. rep., March 2001.

[2] S. Verdu and S. Shamai, “Spectral efficiency of CDMA with random spreading,” *IEEE Trans. on Inform. Theory*, vol. 45, pp. 622–640, March 1999.

[3] S. Shamai and S. Verdu, “The impact of frequency-flat fading on the spectral efficiency of CDMA,” *IEEE Trans. on Inform. Theory*, vol. 47, pp. 1302–1327, May 2001.

[4] M. Varanasi and T. Guess, “Optimum decision feedback multiuser equalization with successive decoding achieves the total capacity of the Gaussian multiple access channel,” in *Proc. Asilomar Conference*, (Pacific Groove, CA), November 1997.

[5] P. Alexander, A. Grant, and M. Reed, “Iterative detection in code-division multiple-access with error control coding,” *European Trans. on Telecomm.*, vol. 9, pp. 419–425, September 1999.

[6] X. Wang and V. Poor, “Iterative (Turbo) soft interference cancellation and decoding for coded CDMA,” *IEEE Trans. on Commun.*, vol. 47, pp. 1047–1061, July 1999.

[7] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,” *IEEE Trans. on Inform. Theory*, vol. 20, pp. 284–287, March 1974.

[8] L. K. Rasmussen, “On ping-pong effects in linear interference cancellation for CDMA,” *IEEE 6th int. Symp. on Spread-Spectrum Tech. & Appli.*, September 2000.

[9] J. Boutros and G. Caire, “Iterative multiuser decoding: unified framework and asymptotic performance analysis.” submitted to IEEE Trans. on Inform. Theory, August 2000.

[10] D. Tse and S. Hanly, “Linear multiuser receivers: Effective interference, effective bandwidth and capacity,” *IEEE Trans. on Inform. Theory*, vol. 45, pp. 641–675, March 1999.

[11] M. Kobayashi, J. Boutros, and G. Caire, “Successive interference cancellation with SISO decoding and EM channel estimation.” to appear on JSAC (special issue on multiuser detection), 2001.

[12] S. Marinkovic, B. Vucetic, and J. Evans, “Improved iterative parallel interference cancellation,” in *Proc. ISIT 2001*, (Washington DC, USA), p. 34, June 2001.

[13] E. G. Ström and S. L. Miller, “Iterative demodulation of othogonal signaling formats in asynchronous DS-CDMA systems,” *Proceedings of the ISSSE*, pp. 184–187, July 2001.