COMPARISON BETWEEN UNITARY AND CAUSAL APPROACHES TO BACKWARD ADAPTIVE TRANSFORM CODING OF VECTORIAL SIGNALS

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ABSTRACT

In a transform coding framework, we compare the optimal causal approach (LDU, Lower-Diagonal-Upper) to the optimal unitary approach (Karhunen-Loeve Transform, KLT). The criterion of merit used for this comparison is the coding gain, defined for a transformation T as the ratio of the average distortion obtained with the identity transformation over the average distortion obtained with T. Both transforms are known to yield the same gain when they are computed on the signal covariance matrix R. The purpose of this paper is to compare the behavior of these two transformations when the ideal transform coding scheme gets perturbed, that is, when only an estimate $R+\Delta R$ of R is known. In this case, not only the transformation itself will be perturbated, but also the bit allocation mechanism. We compare the two approaches in two cases. Firstly, ΔR is caused by a quantization noise : the coding scheme is based on the statistics of the quantized data. We find that the coding gain in the unitary case is higher than in the causal case. In a second case, ΔR corresponds to an estimation noise : the coding scheme is based on an estimate of R based on a finite amount of available data. In this case, both causal and unitary approaches are strictly equivalent, because of the unimodularity and decorrelating properties of the transformations. Simulations results confirming the predicted behavior of the coding gains with perturbations are reported.

1. INTRODUCTION

Consider a stationary Gaussian vectorial source $\{X\}$. This source may be composed of any scalar sources $\{x_i\}$, for example audio signals. In the classical transform coding framework, a linear transformation T is applied to each N-vector X to produce an N-vector $Y=TX$ whose components y_i are independently quantized using a scalar quantizer Q_i. A number of bits r_i is attributed to each Q_i under the constraint $\sum_i r_i = Nr$. For an entropy constrained scalar quantizer of a Gaussian source, the high resolution distortion is $\mathrm{E}(y_i^q(k)-y_i(k))^2 = \sigma^2_{q_i} = c2^{-2r}\sigma^2_{y_i}$, where $c=\frac{\pi e}{6}$.

An important property of commonly used transformations is that, if a noise (for example quantization noise) is added to the signal Y, then its power will be the same in the transform and in the signal domains. This property is sometimes referred to as "unity noise gain" property [3]. The coding gain for T is then defined as

$$G_T = \frac{E\|\tilde{X}\|^2_{(I)}}{E\|\tilde{X}\|^2_{(T)}} = \frac{E\|\tilde{X}\|^2_{(I)}}{E\|\tilde{Y}\|^2_{(T)}}, \tag{1}$$

where I is the identity matrix, and the notation $\|\tilde{X}\|^2_{(T)}$ denotes the variance of the quantization error on the vector X, obtained for a transformation T. The optimal bit allocation yields the well known distortion for the vectorial signal $\{Y\}$: $\mathrm{E}||\tilde{Y}||^2_T = \frac{1}{N}\sum_{i=1}^N \sigma^2_{q_i} = Nc2^{-2r}\left(\prod_{i=1}^N \sigma^2_{y_i}\right)^{\frac{1}{N}} = N\sigma^2_q$. $\sigma^2_{q_i}$ is independent of i, and the number of bits assigned to the ith component is $r+\frac{1}{2}log_2\frac{\sigma^2_{y_i}}{\left(\prod_{i=1}^N \sigma^2_{y_i}\right)^{\frac{1}{N}}}$.

In the next section, we recall the main characteristics of the optimal causal approach (LDU) when optimized on R, and summarize the reasons why its performance is the same as the best unitary approach (KLT).

However, a backward adaptive coding scheme requires that neither the transformation nor the parameters of the bit allocation are transmitted to the decoder. So suppose now that the coding scheme is based on $\widehat{R}_{XX} = R+\Delta R$ instead of R, where $\widehat{R}_{XX}$ is available at both encoder and decoder. Then the computed transformation will be $\hat{T} = T+\Delta T$, and the distortion will be proportional to the variances of the signals transformed by means of $\hat{T}$ instead of T, say $\sigma^{2'}_{y_i}$. Moreover, the bits r_i should be attributed on the basis of estimates of the variances available at both encoder and decoder also, that is, $(\hat{T}\widehat{R}_{XX}\hat{T})_{ii}$, where $(.)_{ii}$ denotes the ith diagonal element of $(.)$. Hence, we get the following measure of distortion for a transformation $\hat{T}$ based on $\widehat{R}_{XX}$:

$$E\|\tilde{Y}\|^2_{(\hat{T})} = E\sum_{i=1}^N c2^{-2[r+\frac{1}{2}log_2\frac{(\hat{T}\hat{R}_{XX}\hat{T}^T)_{ii}}{(\Pi_{i=1}^N(\hat{T}\hat{R}_{XX}\hat{T}^T)_{ii})^{\frac{1}{N}}}]}\sigma^{2'}_{y_i}. \tag{2}$$

where the expectation is w.r.t ΔR in case it is non-deterministic. In the third section,we compare this distortion when ΔR is caused by a quantization noise : the coding scheme is based on the statistics of the quantized data, under high resolution assumption. In the fourth part, ΔR corresponds to an estimation noise : the coding scheme is based on an estimate of R due to a finite amount of K vectors : $\widehat{R}_{XX} = \frac{1}{K}\sum_{i=1}^K X_iX_i^T$. The fifth part is dedicated to simulation results.

2. OPTIMAL CAUSAL AND UNITARY APPROACHES WITHOUT PERTURBATION

In the causal case, $Y=LX=X-\overline{L}X$, where $\overline{L}X$ is the reference vector. The output X^q is $Y^q+\overline{L}X$. Note that the recon-

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struction error $\tilde{X}$ equals the quantization error $\tilde{Y}$:

$$\tilde{X} = X - X^q = X - \overline{L}X - Y^q = Y - Y^q = \tilde{Y}, \qquad (3)$$

as in the unitary case. As detailed in [2], the optimal L in terms of coding gain is such that $LR_{XX}L^T = diag\{\sigma^2_{y_1}, ...\sigma^2_{y_N}\}$, where $diag\{...\}$ represents a diagonal matrix whose elements are $\sigma^2_{y_i}$. In other words, the components y_i are the prediction errors of x_i with respect to the past values of X, the $X_{1:i-1}$, and the optimal coefficients $-L_{i,1:i-1}$ are the optimal prediction coefficients. Since each prediction error y_i is orthogonal to the subspaces generated by the $X_{1:i-1}$, the y_i are orthogonal. It follows that $R_{XX} = L^{-1}R_{YY}L^{-T}$, which represents the LDU factorization of R_{XX}. Referring to (1), the coding gain without perturbation for the optimal causal transform can be written as

$$G_L^{(0)} = \frac{E||\tilde{Y}||^2_I}{E||\tilde{X}||^2_L} = \frac{E||\tilde{X}||^2_I}{E||\tilde{Y}||^2_L} = \left(\frac{\det\left[diag(R_{XX})\right]}{\det\left[diag(LR_{XX}L^T)\right]}\right)^{\frac{1}{N}}, \qquad (4)$$

where $diag(R)$ denotes here the diagonal matrix that corresponds to the diagonal of the matrix R. Now, using the unimodulariry of L, $\det(diag(R_{YY})) = \det(R_{XX}) = \det\Lambda$, where Λ is the eigenvalue matrix of R_{XX}. The coding gain is

$$G_L^{(0)} = \left(\frac{\det\left[diag(R_{XX})\right]}{\det\left[diag(LR_{XX}L^T)\right]}\right)^{\frac{1}{N}} = \left(\frac{\det\left[diag(R_{XX})\right]}{\det\Lambda}\right)^{\frac{1}{N}} = G_V^{(0)}, \qquad (5)$$

where V denotes a KLT of R_{XX}. Thus, for an optimal bit allocation, the coding gains of the KLT and the LDU are the same without perturbation for three reasons : both transformations ensure that the power of the quantization error is the same in the transform and in the signal domains, they are totally decorrelating transforms, and finally they are unimodular.

3. QUANTIZATION EFFECTS ON THE CODING GAINS

Suppose we compute the transformation on the basis of quantized data. The statistics of the quantized data are assumed to be perfectly known in this section. In other words, we assume that the decoder disposes of an infinite number of quantized vectors X_i^q, and hence of $R_{X^qX^q}$. Under the assumptions of high resolution (uncorrelated white noise), optimal bit assignment and unity noise gain property of the transformation, $\Delta R = E\tilde{X}\tilde{X}^T = \sigma_q^2 I$, where $\sigma_q^2 = c2^{-2r}\left(\prod_{i=1}^N \sigma^2_{y_i}\right)^{\frac{1}{N}}$. Thus, for $\hat{T} = I, \hat{V}$ and $\hat{L}$, we shall compute (the subscript q refers to quantization)

$$E\|\tilde{Y}\|^2_{(\hat{T},q)} = \sum_{i=1}^N c2^{-2[r+\frac{1}{2}log_2\frac{(\hat{T}R_{X^qX^q}\hat{T}^T)_{ii}}{(\Pi_{i=1}^N(\hat{T}R_{X^qX^q}\hat{T}^T)_{ii})^{\frac{1}{N}}}]}\sigma^{2'}_{y_i}, \qquad (6)$$

3.1. Identity Transformation

In this case, the number of bits attributed to the quantizer Q_i is $r + \frac{1}{2}log_2\frac{(R_{X^qX^q})_{ii}}{(\prod_{i=1}^N(R_{X^qX^q})_{ii})^{\frac{1}{N}}}$, and the variance $\sigma^{2'}_{y_i}$ are indeed $(R_{XX})_{ii}$. Thus

$$E\|\tilde{Y}\|^2_{(I,q)} = \sum_{i=1}^N c2^{-2[r+\frac{1}{2}log_2\frac{(R_{X^qX^q})_{ii}}{(\Pi_{i=1}^N(R_{X^qX^q})_{ii})^{\frac{1}{N}}}]}(R_{XX})_{ii} \qquad (7)$$

$$= \sum_{i=1}^N c2^{-2r}\left(\det diag\{R_{X^qX^q}\}\right)^{\frac{1}{N}}\frac{(R_{XX})_{ii}}{(R_{X^qX^q})_{ii}}. \qquad (8)$$

One shows that

$$\sum_{i=1}^N \frac{(R_{XX})_{ii}}{(R_{X^qX^q})_{ii}} = tr\{\left(I + \sigma_q^2(diagR_{XX})^{-1}\right)^{-1}\},$$

where tr denotes the trace operator, and $\det(diagR_{X^qX^q}) = \det(diagR_{XX})\det(I + \sigma_q^2(diagR_{XX})^{-1})$ and we find

$$\begin{aligned} E\|\tilde{Y}\|^2_{(I,q)} &= E\|\tilde{Y}\|^2_{(I)}\tfrac{1}{N}(\det(I+\sigma_q^2(diagR_{XX})^{-1}))^{\frac{1}{N}} \\ &\quad \times tr\{\left(I+\sigma_q^2(diagR_{XX})^{-1}\right)^{-1}\}. \end{aligned} \qquad (9)$$

The distortion is slightly increased because the bits allocated on the basis of variances of quantized signals are not the optimal ones (the variance of the quantization noise is not equal in each branch i). An approximation of (9) up to the second order of the perturbation gives

$$\begin{aligned} &E\|\tilde{Y}\|^2_{(I,q)} = c2^{-2r}(\det diag\{R_{XX}\})^{1/N} \\ &\times(\Pi_{i=1}^N(\tfrac{\sigma_q^2}{(R_{XX})_{ii}}))^{1/N}\textstyle\sum_{i=1}^N(1+\frac{1}{(R_{XX})_{ii}})^{-1} \\ &\approx E\|\tilde{Y}\|^2_{(I)}(1+\tfrac{\sigma_q^4}{N^2}(\tfrac{N-1}{2}\textstyle\sum_{i=1}^N\frac{1}{(R_{XX})^2_{ii}} \\ &-\textstyle\sum_{i=1}^N\sum_{j>i}\frac{1}{(R_{XX})_{ii}(R_{XX})_{jj}})) \end{aligned} \qquad (10)$$

3.2. KLT

As observed in [4] also, if V denotes a KLT of R_{XX}, then $V(R_{XX} + \sigma_q^2 I)V^T = \Lambda + \sigma_q^2 I = \Lambda^q$, and V is also a KLT of $R_{XX} + \sigma_q^2 I$. Thus, the perturbation term $\sigma_q^2 I$ on R_{XX} does not change the backward adapted transformation, and the variances of the transformed signals remain unchanged : $\sigma^{2'}_{y_i} = \lambda_i$. However, the decoder estimates the variances $(VR_{X^qX^q}V^T)_{ii} = \lambda_i + \sigma_q^2$, on the basis of which the coder assigns the bits r_i. Thus, the actual distortion is

$$E\|\tilde{Y}\|^2_{(V,q)} = \sum_{i=1}^N c2^{-2[r+\frac{1}{2}log_2\frac{(VR_{X^qX^q}V^T)_{ii}}{(\Pi_{i=1}^N(VR_{X^qX^q}V^T)_{ii})^{\frac{1}{N}}}]}(VR_{XX}V^T)_{ii}, \qquad (11)$$

wich can be computed in a similar way as in 3.1. We find

$$E\|\tilde{Y}\|^2_{(K,q)} = E\|\tilde{Y}\|^2_{(K)}\frac{1}{N}(\det(I+\sigma_q^2(\Lambda^{-1})))^{\frac{1}{N}}tr\{\left(I+\sigma_q^2(\Lambda^{-1})\right)^{-1}\}. \qquad (12)$$

Again, the increase in distortion comes from the perturbation occuring on the bit allocation mechanism. Up to the second order of perturbation, an expression similar to (10) is

$$\begin{aligned} &E\|\tilde{Y}\|^2_{(K,q)} \\ &= c2^{-2r}(\det diag\{R_{XX}\})^{1/N}(\Pi_{i=1}^N(\tfrac{\sigma_q^2}{\lambda_i}))^{1/N}\textstyle\sum_{i=1}^N(1+\frac{1}{\lambda_i})^{-1} \\ &\approx E\|\tilde{Y}\|^2_{(K)}\left[1+\tfrac{\sigma_q^4}{N^2}(\tfrac{N-1}{2}\textstyle\sum_{i=1}^N\frac{1}{(\lambda_i)^2}-\sum_{i=1}^N\sum_{j>i}\frac{1}{\lambda_i\lambda_j})\right] \end{aligned} \qquad (13)$$

The corresponding expression for the coding gain is

$$G_{K,q} = G^0\frac{(\det(I+\sigma_q^2(diagR_{XX})^{-1}))^{\frac{1}{N}}tr\{\left(I+\sigma_q^2(diagR_{XX})^{-1}\right)^{-1}\}}{(\det(I+\sigma_q^2(\Lambda^{-1})))^{\frac{1}{N}}tr\{\left(I+\sigma_q^2(\Lambda^{-1})\right)^{-1}\}}, \qquad (14)$$

whose second order approximation is

$$\begin{aligned} G_{K,q} \approx G^0[1+\tfrac{\sigma_q^4}{N^2}(\tfrac{N-1}{2}\textstyle\sum_{i=1}^N(\frac{1}{(R_{XX})^2_{ii}}-\frac{1}{(\lambda_i)^2}) \\ -\textstyle\sum_{i=1}^N\sum_{j>i}(\frac{1}{(R_{XX})_{ii}(R_{XX})_{jj}}-\frac{1}{\lambda_i\lambda_j}))]. \end{aligned} \qquad (15)$$

3.3. LDU

In the causal case, the coder uses a transformation L' such that $L'R_{X^qX^q}L'^T = R'_{YY}$. R'_{YY} is the diagonal matrix of the estimated variances involved in the bit allocation (L' and R'_{YY} are both available to the decoder). In this case, the difference vector $Y = X - \overline{L'}X^q$, the quantization noise is filtered by the rows of $\overline{L'}$. Note that $E\|\tilde{X}\|^2_{L',q}$ still equals $E\|\tilde{Y}\|^2_{L',q}$, since $\tilde{X} = X^q - X = Y^q + \overline{L'}X^q - X = Y^q - (X - \overline{L'}X^q) = Y^q - Y = \tilde{Y}$. One shows that the actual variances of the signals y_i obtained with L' are $(L'R_{X^qX^q}L'^T - \sigma_q^2 I)_{ii}$ [2]. In this case, one finds

$$\begin{aligned} E\|\tilde{Y}\|^2_{(L',q)} &= \sum_{i=1}^N c2^{-2[r+\frac{1}{2}log_2\frac{(L'R_{X^qX^q}L'^T)_{ii}}{(\Pi_{i=1}^N(L'R_{X^qX^q}L'^T)_{ii})^{\frac{1}{N}}}]} \\ &\quad\times(L'R_{X^qX^q}L'^T - \sigma_q^2 I)_{ii} \\ &= E\|\tilde{Y}\|^2_{(L)}\frac{1}{N}(\det(I+\sigma_q^2(\Lambda^{-1})))^{\frac{1}{N}}tr\{\left(I+\sigma_q^2(R'^{-1}_{YY})\right)\}. \end{aligned} \tag{16}$$

The increase in distortion comes not only from the perturbation occuring on the bit allocation mechanism but also from the filtering of the quantization noise. Up to the first order of perturbation,we obtain

$$E\|\tilde{Y}\|^2_{(L',q)} \approx E\|\tilde{Y}\|^2_{(K)}\left[1+\frac{\sigma_q^2}{N}(\sum_{i=1}^N\frac{1}{\lambda_i}-\frac{1}{\sigma_{y_i}^2})\right], \tag{17}$$

where $_N$

$$\begin{aligned} &\approx E\|\tilde{Y}\|^2_{(I)}[1+\ E\ \frac{N-1}{2N^2}\sum_{i=1}^N(\frac{(\Delta R)_{ii}}{(R_{XX})_{ii}})^2 \quad] \ (R_{XX})_{ii} \\ &\qquad - E\ \frac{1}{N^2}\sum_i\sum_{j>i}\frac{(\Delta R)_{ii}}{(R_{XX})_{ii}}\frac{(\Delta R)_{jj}}{(R_{XX})_{jj}}] \end{aligned} \tag{21}$$

The expectation of the first term in (21) is

$$E\ \frac{N-1}{2N^2}\sum_{i=1}^N\left(\frac{(\Delta R)_{ii}}{(R_{XX})_{ii}}\right)^2 = \frac{N-1}{2N^2}\sum_{i=1}^N\frac{2(R_{XX})^2_{ii}}{K(R_{XX})^2_{ii}} = \frac{N-1}{NK}. \tag{22}$$

The expectation of the second term is

$$\begin{aligned} &E\ \frac{1}{N^2}\sum_i\sum_{j>i}\frac{(\Delta R)_{ii}}{(R_{XX})_{ii}}\frac{(\Delta R)_{jj}}{(R_{XX})_{jj}} \\ &\approx \frac{2}{k}\sum_i\sum_{j>i}\frac{(R_{XX})^2_{ij}}{(R_{XX})_{ii}(R_{XX})_{jj}} \\ &\approx \frac{2}{k}\|\triangleright\left((diag\{R_{XX}\})^{1/2}R_{XX}(diag\{R_{XX}\})^{1/2}\right)\|^2_F \end{aligned} \tag{23}$$

where $\triangleright(A)$ denotes the strictly lower triangular matrix made with the strictly lower trianguler part of A, and $\|A\|^2_F$ the squared Frobenius norm of A. If $D = diag\{R_{XX}\}$, we obtain

$$\begin{aligned} &E\ \frac{1}{N^2}\sum_i\sum_{j>i}\frac{(\Delta R)_{ii}}{(R_{XX})_{ii}}\frac{(\Delta R)_{jj}}{(R_{XX})_{jj}} \approx \frac{1}{K}(\|D^{-\frac{1}{2}}R_{XX}D^{-\frac{1}{2}}\|^2_F \\ &-\|diag\{D^{-\frac{1}{2}}R_{XX}D^{-\frac{1}{2}}\|^2_F) \\ &= \frac{1}{K}(tr\{R_{XX}D^{-1}R_{XX}D^{-1}\}). \end{aligned} \tag{24}$$

Finally, the expected distortion for Identity with estimation noise is, for sufficiently high K,

$$E\|\tilde{Y}\|^2_{(I,K)} \approx E\|\tilde{Y}\|^2_{(I)}\left(1+\frac{1}{K}(1-\frac{tr\{R_{XX}D^{-1}R_{XX}D^{-1}}{N^2}\})\right). \tag{25}$$

4.2. KLT

In the unitary case, the expected distortion is

$$E\|\tilde{Y}\|^2_{(\hat{V},k)} = E\sum_{i=1}^N c2^{-2[r+\frac{1}{2}log_2\frac{(\hat{V}\hat{R}_{XX}\hat{V}^K)_{ii}}{(\Pi_{i=1}^N(\hat{V}\hat{R}_{XX}\hat{V}^T)_{ii})^{\frac{1}{N}}}]}(\hat{V}R_{XX}\hat{V}^T)_{ii}. \tag{26}$$

Using the fact that $\hat{V}\widehat{R}_{XX}\hat{V}^T$ is diagonal, we can write (26) as

$$E\|\tilde{Y}\|^2_{(\hat{V},K)} = Ec2^{-2r}\left(\det\hat{V}\widehat{R}_{XX}\hat{V}\right)^{\frac{1}{N}}\sum_{i=1}^N\frac{(\hat{V}R_{XX}\hat{V}^T)_{ii}}{(\hat{V}\widehat{R}_{XX}\hat{V}^T)_{ii}} \tag{27}$$

Because of the unimodularity of $\hat{V}$, the determinant in (27) may be written as

$$\begin{aligned} \det \hat{V}\widehat{R}_{XX}\hat{V}^T &= \det(R_{XX}+\Delta R) \\ &= (\det R_{XX})\det(I+R_{XX}^{-1}\Delta R) \end{aligned} \quad (28)$$

The sum in (27) may be written as

$$\sum_{i=1}^{N}\frac{(\hat{V}R_{XX}\hat{V}^T)_{ii}}{(\hat{V}\widehat{R}_{XX}\hat{V}^T)_{ii}} = tr\{(\hat{V}\widehat{R}_{XX}\hat{V}^T)^{-\frac{1}{2}}\hat{V}R_{XX}\hat{V}^T(\hat{V}\widehat{R}_{XX}\hat{V}^T)^{-\frac{1}{2}}\} = tr\{(I+R_{XX}^{-1}\Delta R)^{-1}\} \quad (29)$$

Thus, (27) is equivalent to

$$E\|\tilde{Y}\|^2_{(\hat{V},K)} = E\|\tilde{Y}\|^2_{(K)}\frac{1}{N}E(\det(I+R_{XX}^{-1}\Delta R))^{\frac{1}{N}}tr\{\left(I+R_{XX}^{-1}\Delta R\right)^{-1}\} \quad (30)$$

Using similar developments as in the previous section, the expected distortion for the KLT when the transformation is based on k vectors is finally, under high resolution assumption

$$E\|\tilde{Y}\|^2_{(\hat{V},k)} \approx E\|\tilde{Y}\|^2_{(K)}\left(1+\frac{N-1}{2K}+\frac{N-1}{NK}\right). \quad (31)$$

The associated coding gain is,where $D = diag\{R_{XX}\}$,

$$G_{\hat{V},K} = \frac{E\|\tilde{Y}\|^2_{(I,K)}}{E\|\tilde{Y}\|^2_{(\hat{V},K)}} \approx G^0\left(1-\frac{1}{KN^2}tr\{RD^{-1}RD^{-1}\}-\frac{N-1}{2K}+\frac{1}{NK}\right). \quad (32)$$

4.3. LDU

As stated in the introduction of this section, the expected distortion with $\hat{L}$ computed on $\widehat{R}_{XX}$ is

$$\begin{aligned} E\|\tilde{Y}\|^2_{(\hat{L},K)} &= E\sum_{i=1}^{N}c2^{-2\left[r+\frac{1}{2}log_2\frac{(\hat{L}\widehat{R}_{XX}\hat{L}^T)_{ii}}{(\Pi_{i=1}^{N}(\hat{L}\widehat{R}_{XX}\hat{L}^T)_{ii})^{\frac{1}{N}}}\right]} \\ &\quad \times(\hat{L}R_{XX}\hat{L}^T)_{ii} \\ &= Ec2^{-2r}\underbrace{\left(\det\hat{V}\widehat{R}_{XX}\hat{V}\right)^{\frac{1}{N}}}_{=\det\widehat{R}_{XX}=\det(R_{XX})\det(I+R_{XX}^{-1}\Delta R)}\underbrace{\sum_{i=1}^{N}\frac{(\hat{L}R_{XX}\hat{L}^T)_{ii}}{(\hat{L}\widehat{R}_{XX}\hat{L}^T)_{ii}}}_{=tr\{(I+R_{XX}^{-1}\Delta R)^{-1}\}} \\ &= E\|\tilde{Y}\|^2_{(\hat{V},K)}, \end{aligned} \quad (33)$$

where the equality concerning the determinants comes from the unimodularity of the transformations $\hat{L}$ and $\hat{V}$. The equality concerning the trace comes from their decorrelating property. Thus, the distortion and coding gain with estimation noise are the same in the causal and the unitary cases, and are given up to the first order in K by (30) and (32).

5. SIMULATIONS

For the simulations, we used entropy constrained scalar quantizers Q_i and real Gaussian i.i.d. vectors with covariance matrix $R_{XX} = HR_{AR1}H^T$. R_{AR1} is the covariance matrix of a first order autoregressive process with normalized correlation coefficient ρ. H is a diagonal matrix whose ith entry is $(N-i+1)^{1/3}$ (decreasing variances).

In Figure 1, the coding gain with quantization noise is plotted for KLT (upper curves) and LDU (lower curves) with $\rho = 0.9$, $N = 4$. The theoretical exact expressions are given by (14) and (18), and the approximated expressions by (15) and (19).

The coding gains in presence of estimation noise are compared for LDU and KLT in Figure 2, for $N = 4$ and $\rho = 0.9$ (mean over 100 realizations). The observed behaviors of the transformation corresponds quite well to the theoretically predicted ones for $K \approx$ a few tens.

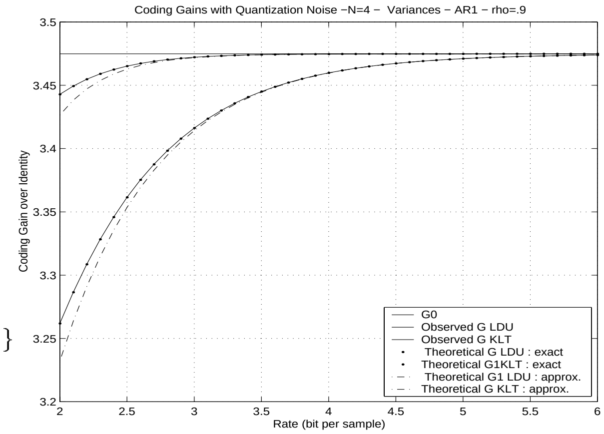

Fig. 1. Coding Gains vs rate in bit/sample.

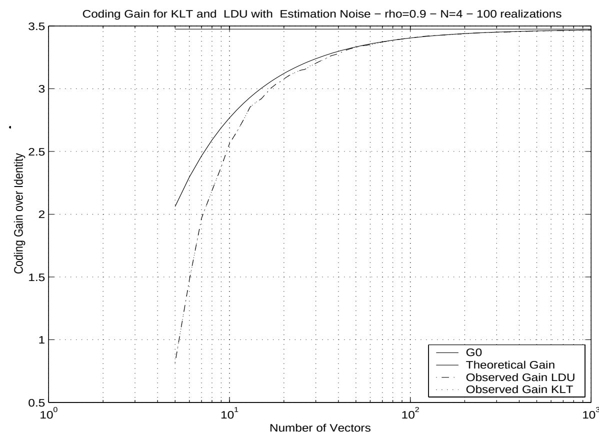

Fig. 2. Gains for KLT and LDU with estimation noise

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