Risk-Aware Resource Allocation for Semantic Communications: A Cumulative Prospect Theory Approach

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1. INTRODUCTION

- Goal-oriented Semantic Network Optimization: need for multi-objective/criteria optimization with semantics-based utilities and for handling multi-modal information.
- In this framework, it is crucial to capture through a metric
  - ➢ the importance (semantic) of the information {subjective evaluation}
  - ➢ the subjective occurrence of the events
- These features can be handled by
  - ➢ risk, which can be thought conceptually as a metric of importance
  - ➢ non-linear transformation of the rational probabilities of event occurrence
- We are developing a risk-sensitive decision-making framework for goal-oriented semantic communication.
- In literature, there are to major theories about risk-sensitive based analysis:
  - ➢ Expected Utility Theory (EUT)
  - ➢ (Cumulative) Prospect Theory ((C)PT)

Risk-neutral analysis

$$\mathbb{E}[f(x)] = \sum p(x) \cdot f(x)$$

EUT analysis

$$\mathbb{E}[u(x)] = \sum p(x) \cdot u(x)$$

(C)PT analysis

$$\mathbb{E}_w[u(x)] = \sum w\big(p(x)\big) \cdot u(x)$$

- To go beyond risk neutrality and to incorporate the subjective valuation of data/information, we leverage **(Cumulative) Prospect Theory**

2. CPT KEY FEATURES

Reference Dependence: need for a reference point that separates the domain into loss and gain subdomain. The quantities are perceived through changes instead of states.

$$u(x_0) = 0,\quad u(x) > 0\ \forall\, x > x_0,\quad u(x) < 0\ \forall\, x < x_0,\quad u'(x) > 0\ \forall x \in \mathbb{R}$$

Diminishing Marginal Utility: there is diminishing sensitivity toward the scale of changes.

$u(x)$ is a non-linear function in both subdomains

Asymmetric Risk Attitudes: loss subdomain is characterized by risk-seeking and on the other hand gain subdomain is characterized by risk aversion.

$u''(x) \leq 0, x > x_0$, risk aversion over gains
$u''(x) \geq 0, x < x_0$, risk seeking over losses

Loss Aversion: losses are more important than the equivalent gains.

$$\frac{u'(0^-)}{u'(0^+)} \equiv \lambda > 1$$

and loss aversion definitions

Probability Distortion: overweight of small probabilities and underweight of moderate and high probabilities.

non linear probability weighting function, inverse S-shape

In prospect theory, the weighting of the probabilities occurs on the probability mass function or probability density function. This violates the first-order stochastic dominance. Hence, in cumulative prospect theory, the weighting of the probabilities occurs on the cumulative probabilities or the cumulative density function.

3. CONTRIBUTIONS IN THEORY

- Risk aversion metric for a decision making between a binary gamble and a stationary point:

$$\mathcal{R}_{stationary}(x, \delta_1, \delta_2) = \frac{u(x) - u(x-\delta_1)}{u(x+\delta_2) - u(x)}$$
$$0 < \delta_1, \delta_2$$

- Risk aversion metric for a decision making between two nested binary gambles:

$$\mathcal{R}_{gamble}^{equal}(x, \delta_1, \delta_2, \delta_3, \delta_4) = \frac{u(x-\delta_3) - u(x-\delta_1)}{u(x+\delta_4) - u(x+\delta_2)},$$
$$0 \leq \delta_3 \leq \delta_1, 0 \leq \delta_2 \leq \delta_4$$

- Increasing non-symmetric bet aversion:, the rejection of all non-symmetric fair gambles is an increasing function of the scaling if and only if
$$u'(x_0 + w\cdot\delta_2) < u'(x_0 - w\cdot\delta_1)\ \forall\ w, \delta_1, \delta_2 \in \mathbb{R}_+$$
- Equivalent definition for Neilson's weak loss aversion:
$$u'(x_0^-)\cdot(x - x_0) < u(x) < u'(x_0^+)\cdot(x - x_0), \forall\ x < x_0$$
- Equivalent definition for Neilson's strong loss aversion:
$$\frac{u(x_0+\delta_4) - u(x_0+\delta_2)}{\delta_4 - \delta_2} < \frac{u(x_0-\delta_3) - u(x_0-\delta_1)}{(-\delta_3) - (-\delta_1)}$$
$$\forall\ \delta_1, \delta_2, \delta_3, \delta_4 > 0$$
- Interpretation on loss aversion definitions based on binary gambles analysis:

Symmetric bet aversion ⇔ $\mathcal{R}_{stationary}(x_0, \delta, \delta) > 1$

Increasing symmetric bet aversion ⇔
$$\frac{\partial}{\partial\delta}\Big(\mathcal{R}_{stationary}(x_0, \delta, \delta)\Big) > 0$$

Neilson's weak loss aversion ⇔
$$\mathcal{R}_{stationary}(x_0, \delta_1, \delta_2) > \frac{\delta_1}{\delta_2}$$

Increasing non-symmetric bet aversion ⇔
$$\frac{\partial}{\partial w}\Big(\mathcal{R}_{stationary}(x_0, w\cdot\delta_1, w\cdot\delta_2)\Big) > 0$$

Neilson's strong loss aversion ⇔ In the case of nested gambles, the region of acceptance the nested gamble instead of the non- nested increases with respect to the risk neutral case.

4. GOAL-ORIENTED SEMANTIC RESOURCE ALLOCATION WITH RISK-AVERSE AGENTS

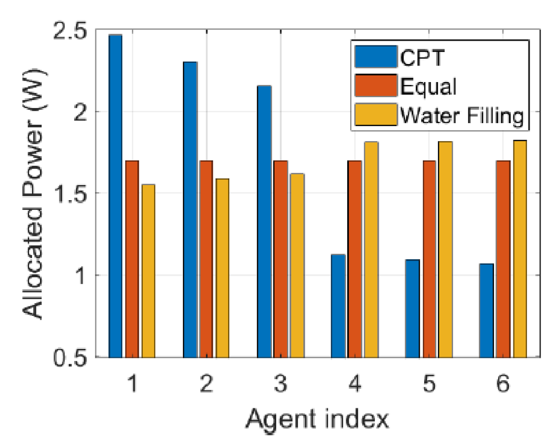
- We consider the downlink of a wireless system with N orthogonal channels and N agents.
- Objective: determining the optimal power allocation under a total power budget constraint
- Metric: signal-to-noise ratio ($SNR = \frac{P(i)\cdot|h(i)|^2}{N_0}$), which is subjectively evaluated by each extended-CPT agent.
- Utility function: $u(x) = \begin{cases} \lambda_1 \cdot \dfrac{\mu_1 - e^{\frac{\alpha}{\gamma_1}(x-x_0)}}{\alpha}, & x \geq x_0 \\ \lambda_2 \cdot \dfrac{\mu_2 - e^{\frac{\beta}{\gamma_2}(x-x_0)}}{\beta}, & x < x_0 \end{cases}$
- The utility function can be thought as a first-step expansion of well studied Expected Utility (EUT)
  - ➢ Introduction of reference point
  - ➢ Introduction of loss aversion
  - ➢ Risk averse behavior remains in both but each one subdomain has different parametrization $\left(\frac{\alpha}{\gamma_1}, \frac{\lambda_1}{\gamma_1}, \frac{\beta}{\gamma_2}, \frac{\lambda_2}{\gamma_2} < 0\right)$.
- The summation is weighted by subjective assessment of the probability p_i for the i-th agent, $w(p_i)$, which reflect aspects such as the channel activation or availability of information flow.
- Any j-th agent within the impact region of i-th agent falls under the influence of it. Consequently, the subdomain of the j-th agent is determined by the subdomain of the i-th agent.
- Thus, as loss aversion increases or the j-th agent is less active than the i-th agent, the impact region expands.
- Additionally, the inverse S-shaped PWF amplifies the influence of agents with lower p_i, while reducing the impact of more active agents.

$$\min_{\boldsymbol{P}} \sum_{i=1}^{N} w(p(i)) \cdot u\left(\frac{P(i)\cdot|h(i)|^2}{N_0}\right)$$
s.t. $0 \leq P(i)\ \forall i$
$$\sum_{i=1}^{N} P(i) \leq P_{total}$$

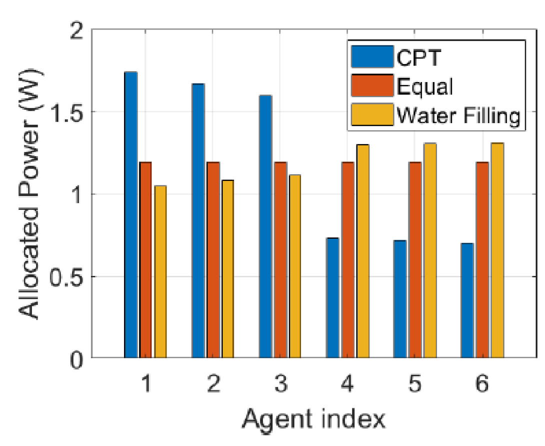
→ Lagrangian dual problem $\max_{\boldsymbol{k},\mu} \min_{\boldsymbol{P}} \mathcal{L}(\boldsymbol{k}, \mu, \boldsymbol{P})$ Slater's conditions, Stationarity, Complementary slackness

→ Given the total power constraint → bisection search within the appropriate interval in order to find the exact value of μ.

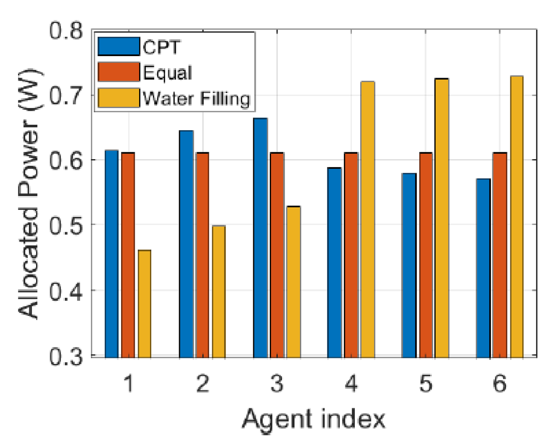
Region of influence for the i-th agent: $\left(-\left(\frac{\gamma_1}{\lambda_1}\cdot\frac{1}{w(p_i)}\cdot\frac{N_0}{|h(i)|^2}\right)^{-1}, -\left(\frac{\gamma_2}{\lambda_2}\cdot\frac{1}{w(p_i)}\cdot\frac{N_0}{|h(i)|^2}\right)^{-1}\right]$



All agents in gain subdomain

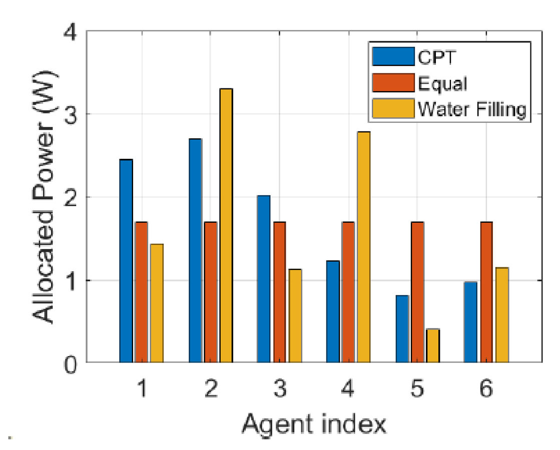


Intermediate region

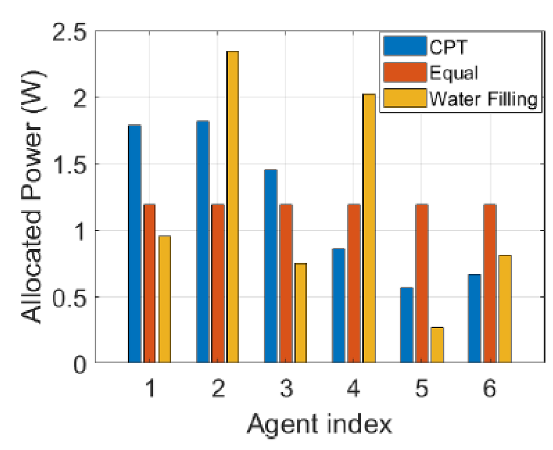


All agents in loss subdomain

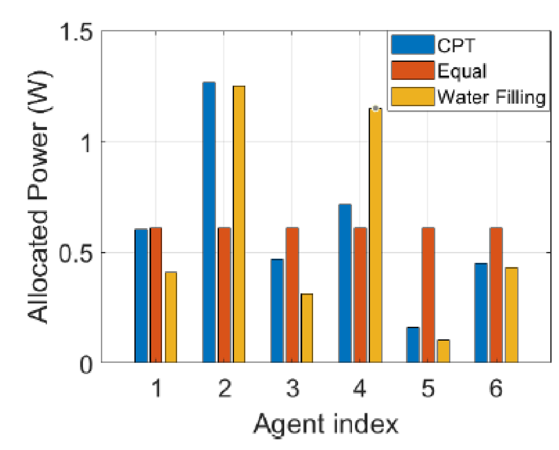
Optimal power allocation with equal weights $w(p_i) = 1, \forall\, i$



All agents in gain subdomain



Intermediate region



All agents in loss subdomain

Optimal power allocation with unequal weights $w(p_i)$