Pathwise Polynomial Expansion Interference Cancellation for DS-CDMA

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Abstract

In DS-CDMA systems with time-varying multipath propagation, one of the main obstacles to linear multiuser detection is the large number of parameters which have to be estimated from scarce training data. Pathwise Interference cancellation is an approach that allows to separate the parameters into fastly varying and slowly varying parameters, thereby allowing the scarce training data to be used in the estimation of the fastly varying parameters with a short time constant while the slowly varying parameters can be estimated over a much larger time interval. We will investigate the application of polynomial expansion (PE) to pathwise processing and propose the use of a weighting factor per signal component. We show that these weighting coefficients not only achieve significant improvements in the presence of power imbalances between users and paths w.r.t. scalar weighting, but also achieve further improvement due to the better estimation of the fastly varying parameters.

1 Introduction

One of the main problems in linear multiuser detection is the amount of parameters that have to be estimated from relatively few training data. In particular, the fastly varying parameters of the mobile channel in a multipath, fading environment can pose serious difficulties to interference cancellation and data detection. Pathwise Interference Cancellation (PWIC) is an approach that allows to separate the parameters into fastly varying and slowly varying parameters, thereby allowing the scarce training data to be used in the estimation of the fastly varying parameters while the whole of the received signal can be used to estimate the slowly varying parameters over a much larger time interval. Since the interference cancellation takes place between individual multipath components before spatial-temporal recombination, the signal thus obtained contains the desired parameters at an improved SINR compared to the received signal and hence allows improved channel estimation [1][2][3].

Polynomial expansion (PE) is an approximation technique for LMMSE receivers and is particularly well suited for CDMA due to the presence of a large number of small correlations. The fundamental principle of PE is to avoid the relatively costly correlation matrix inverse required by an LMMSE/Decorrelator receiver by considering the correlation matrix to be a small perturbation of an identity matrix and approximating the inverse of the correlation matrix by a polynomial expansion in the perturbation matrix or, equivalently, in the correlation matrix itself. However, for PE to work, adapted weighting factors have to be introduced. By appropriately choosing the weighting coefficients, every additional term in the PE can be guaranteed to improve performance and hence divergence concerns get eliminated.

PE has, in various forms, received a fair amount of attention recently in the literature [5] [6] [7] [8] etc. Some works on PE have analysed the choice of scalar weighting factors on the basis of asymptotic system analysis, leading to weight values that can be determined a priori.In this paper, we propose to introduce diagonal weighting matrices which corresponds to one weighting factor per signal component. We shall see that such multiple coefficients not only improve performance substantially in the presence of power imbalances between users and paths, but also further improvement due to the fast adaptation of these weights is possible since the instantaneous channel states will reflect the power imbalances very strongly.

Moshavi, who first introduced PE [4], applied the polynomial expansion to the joint set of RAKE outputs for the various users. In this way, the polynomial expansion receiver involves only (de)spreading and channel (matched) filtering operations and hence is mostly parameterized in terms of the channel parameters (as opposed to the general coefficients of a general linear receiver). Honig and coworkers apply the PE principle to the received signal directly and were able to show [9] that PE is equivalent to the *Multistage Wiener Filter* [10] in this case. We propose to introduce polynomial expansion at the level of the pathwise RAKE outputs. As compared to Moshavi's approach, the PE is situated before the maximum ratio recombination

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of the path contributions and leads to pathwise interference cancellation which will allow to estimate the path parameters (amplitudes, or even angles in the spatio-temporal case) with improved SINR and hence with reduced estimation error. The diagonal weighting factors we introduce will hence provide a weighting per path (or even possibly per antenna element per path in the spatio-temporal case). Maximum ratio combining after pathwise PE corresponds then to a version of the G-RAKE (the path amplitudes multiplied by arbitrary eighting factors become arbitrary recombination coefficients).

2 Data Model

For the received DS-CDMA signal model, we assume the K users to be transmitting linearly modulated signals over a linear, specular multipath channel with additive gaussian noise in an asynchronous fashion. Furthermore, we assume that the basestation receiver utilizes an antenna array with Q elements. The channel impulse response is characterised for users $k \in [1 \dots K]$ by

$$\mathbf{h}_k(t) = \sum_{m=1}^{M} A_{k,m}\mathbf{h}(\theta_{k,m})\delta(t-\tau_{k,m})$$

where $\mathbf{h}_k$ and $\mathbf{h}_{k,m} = \mathbf{h}(\theta_{k,m})$ are column vectors of dimension Q, the number of sensors employed at the receiver. $\mathbf{h}_{k,m}$ defines the response of the antenna array and is a function of the Direction of Arrival (DoA), $\theta_{k,m}$, of the signal. For identifiability reasons, we chose the anntenna response vector to have unity power, $\mathbf{h}_{k,m}^H\mathbf{h}_{k,m} = 1$. Further, the specular channel is characterised by $A_{k,m}$ and $\tau_{k,i}$, the complex amplitude and the path delays, respectively. M is the number of specular paths. The channel parameters can be divided into two classes: fastly and slowly varying parameters. The slowly varying parameters are the delays, $\tau_{k,m}$, the DoA, $\theta_{k,m}$, and the short-term path power, $E|A_{k,m}|^2$. Hence, the fast varying parameters are the complex phases and amplitudes, $A_{k,m}$. At the receiver front-end, the received signal before sampling is written as

$$\begin{aligned}\mathbf{y}(t) &= \sum_{k=1}^{K}\Big\{\sum_{n=-\infty}^{\infty}\sum_{m=1}^{M}(A_{k,m}a_k[n]) \\ &\quad \times \sum_{l=0}^{L-1} s_k[l]\mathbf{h}_{k,m}p(t-\tau_{k,m}-lT_c-nT)+\mathbf{n}(t)\Big\}\end{aligned} \tag{1}$$

$\mathbf{y}(t)$ and the Additive White Gaussian Noise (AWGN), $\mathbf{n}(t)$, are vector signals due to the use of multiple sensors and are of dimensions $Q \times 1$. $a_k[n], p(t)$ are the transmitted symbols for user k and the pulse-shaping filter, respectively. At the receiver front-end, the received signal given in equation (1) is lowpass-filtered and sampled at $1/T_s$. The spreading codes, $s_k(.)$ are assumed to be periodic of length $LT_c = T$ here. We obtain therefore

$$\mathbf{y}[n] = \sum_{i=-\infty}^{\infty}\mathbf{P}[n-i]\mathbf{SHAa}[i]+\mathbf{v}[n] \tag{2}$$

where $\mathbf{y}[n] = [\mathbf{y}[n+0\cdot T_c/J]\dots\mathbf{y}[n+(LJ-1)\cdot T_c/J]^T$, i.e. we stacked all samples of the received signal for the duration of a symbol period T into $\mathbf{y}[n]$. $\mathbf{v}[n]$ is the sampled and low-pass filtered contribution of the noise, $\mathbf{n}(t)$. $\mathbf{a}[n] = [a_1(n)a_2(n)\dots a_K(n)]^T$ contains the data symbols of all K users for a given n, ^{T} indicating the matrix transpose, $\mathbf{A} = diag(\mathbf{A}_1 \dots \mathbf{A}_K)$ is the block diagonal matrix containing the complex amplitude coefficients for each user such that $\mathbf{A}_k = [A_{k,1}^H \dots A_{k,M}^H]^H$, $\mathbf{H} = diag(\mathbf{H}_1 \dots \mathbf{H}_K)$ where $\mathbf{H}_k = diag(\mathbf{h}_{k,1}\dots\mathbf{h}_{k,M})$ where both $\mathbf{H}_k$ and $\mathbf{H}$ are block diagonal matrices and $\mathbf{h}_{k,m}$ is a column vector. $\mathbf{S} = diag(\mathbf{S}_1\dots\mathbf{S}_K)$ where $\mathbf{S}_k = [\mathbf{I}_M \otimes (\mathbf{s}_k \otimes \mathbf{I}_Q)]$; $\mathbf{s}_k = [s_k[0]\dots s_k[L-1]]^T$ represents the spreading code vector, $\mathbf{I}_M$ and $\mathbf{I}_Q$ denote identity matrices of dimensions $M \times M$ and $Q \times Q$, respectively. $\otimes$ signifies the Kronecker product. $\mathbf{P} = [\mathbf{p}_{n,1}\dots\mathbf{p}_{n,K}]$; $\mathbf{p}_{n,k} = [\mathbf{p}_{n,k,1}\dots\mathbf{p}_{n,k,M}]$ and

$$\mathbf{p}_{n,k,m} = \begin{bmatrix} \mathbf{p}_{n,k,m,0,0} & \cdots & \mathbf{p}_{n,k,m,0,L-1} \\ \vdots & \ddots & \vdots \\ \mathbf{p}_{n,k,m,LJ-1,1} & \cdots & \mathbf{p}_{n,k,m,LJ-1,L-1}\end{bmatrix}$$

where $\mathbf{p}_{n,k,m,r,l} = [p\,(nT+(r/J-l)T_c-\tau_{k,m})\otimes\mathbf{I}_Q]$. Let us define the received signal in the q-domain where q is the advance operator, i.e. $qy_n = y_{n+1}$ w.r.t the symbol period. To this end, let us reformulate the received signal as given in (2) in the q-domain.

$$\begin{aligned}\mathbf{y}[n] &= \mathbf{P}(q)\mathbf{SHAa}[n]+\mathbf{v}[n] \\ &= \mathbf{E}(q)\mathbf{a}[n]+\mathbf{v}[n] \\ &= \mathbf{E}_k(q)a_k[n]+\sum_{i=1;i\neq k}^{K}\mathbf{E}_i(q)a_i[n]+\mathbf{v}[n]\end{aligned} \tag{3}$$

where $\mathbf{P}(q) = \sum_i \mathbf{P}[i]q^{-i}$ and we split up the signal into user k's contribution and interference terms.

$$\begin{aligned}\underline{\underline{\mathbf{E}}}_k(q)\mathbf{H}_k\mathbf{A}_k &= \sum_{m=1}^{M}\underline{\mathbf{E}}_{k,m}(q)\mathbf{h}_{k,m}A_{k,m} \\ &= \sum_{m=1}^{M}\mathbf{E}_{k,m}(q)A_{k,m} = \underline{\mathbf{E}}_k(q)\mathbf{A}_k = \mathbf{E}_k(q)\end{aligned} \tag{4}$$

Furthermore, we can define

$$\begin{aligned}\mathbf{x}[n] &= \mathbf{H}^H\underline{\underline{\mathbf{E}}}^\dagger(q)\mathbf{y}[n] \\ &= \underbrace{\mathbf{H}^H\underline{\underline{\mathbf{E}}}^\dagger(q)\underline{\underline{\mathbf{E}}}(q)\mathbf{H}}_{\mathbf{R}(q)=\mathbf{I}+\overline{\mathbf{R}}(q)}\mathbf{Aa}[n]+\mathbf{H}^H\underline{\underline{\mathbf{E}}}^\dagger(q)\mathbf{v}[n] \\ &= \underline{\mathbf{E}}^\dagger(q)\underline{\mathbf{E}}(q)\mathbf{Aa}[n]+\underline{\mathbf{E}}(q)^\dagger\mathbf{v}[n]\end{aligned} \tag{5}$$

where $\underline{\mathbf{E}}^\dagger(q) = \underline{\mathbf{E}}^H(1/q^*)$ is the *paraconjugate* and

$$\sum_i \mathbf{E}^\dagger_{k,m}[i]\mathbf{E}_{k,m}[-i] = 1, \forall\, k \in \{1\ldots K\}, m \in \{1\ldots M\}$$

$\mathbf{x}[n] = [x_{1,1}[n]\ldots x_{K,M}[n]]^T$ are the matched filter or RAKE outputs, spatially but not temporally recombined. Assuming normalised spreading codes, $\mathbf{R}(q) = \underline{\mathbf{E}}^\dagger(q)\underline{\mathbf{E}}(q) = \sum_i \mathbf{R}[i]q^{-1}$ and $diag(\mathbf{R}[0]) = \mathbf{I}$ due to the normalisation of $\mathbf{h}_{k,m} : ||\mathbf{h}_{k,m}|| = 1$.

3 Polynomial Expansion in Pathwise Interference Cancellation

To illustrate the principle of pathwise polynomial expansion, it is beneficial to briefly consider a simplified, synchronous signal model with a single path per user. In this case, we can write

$$\begin{aligned} \mathbf{y}[n] &= \mathbf{S}_e\mathbf{A}\mathbf{a}[n] + \mathbf{v}[n] \\ \mathbf{x}[n] &= \mathbf{S}_e^H\mathbf{S}_e\mathbf{A}\mathbf{a}[n] + \mathbf{S}_e^H\mathbf{v}[n] \qquad (6) \\ \mathbf{R} &= \mathbf{S}_e^H\mathbf{S}_e \qquad (7) \end{aligned}$$

where $\mathbf{S}_e = [\mathbf{s}_1\ldots\mathbf{s}_K]$ and $\mathbf{A}$ as well as $\mathbf{a}[n]$ as defined previously then the matched filter becomes simply $\mathbf{S}_e^H$ and it is clear that $\mathbf{R}$ is simply the matrix of spreading code cross-correlations with unit elements on the diagonal. Therefore, $\mathbf{R} = \mathbf{I} + \overline{\mathbf{R}}$, where $\overline{\mathbf{R}}$ contains the off-diagonal elements of $\mathbf{R}$, $[\overline{\mathbf{R}}]_{ij} \leq 1 \forall\{i,j\}$. From (6) it can be seen immediately that the pathwise decorrelating receiver for $\mathbf{x}[n]$ is given by $\mathbf{R}^{-1}$. We can now expand $\mathbf{R}^{-1}$ as a Polynomial in $\overline{\mathbf{R}}$ such that $\mathbf{R}^{-1} = \left(\mathbf{I}+\overline{\mathbf{R}}\right)^{-1} = \sum_{b=0}^{\infty}\left(-\overline{\mathbf{R}}\right)^b$, provided that there is a matrix norm $|||\overline{\mathbf{R}}||| < 1$ to ensure convergence. We can approximate the inverse of the correlation matrix by

$$\mathbf{R}^{-1} \approx \tilde{\mathbf{R}}^{-1} = \sum_{b=0}^{B}(-\overline{\mathbf{R}})^b \qquad (8)$$

where we have truncated the infinite summation to $B+1$ terms. However, as would be expected, such a truncation is suboptimal and can only improve the SINR over the RAKE when the off-diagonal elements in $\mathbf{R}$ are few and small, i.e. for low system loading factors. In the noiseless case, a first-order expansion $(B = 1)$ in (8) leads to an amplitude-data product estimate given by

$$\begin{aligned} \widehat{\mathbf{A}\mathbf{a}}[n] &= \left(\mathbf{I} - \overline{\mathbf{R}}\right)\mathbf{x}[n] \\ &= (2\mathbf{I} - \mathbf{S}_e^H\mathbf{S}_e)\mathbf{S}_e^H\mathbf{S}_e\mathbf{A}\mathbf{a}[n] \qquad (9) \end{aligned}$$

Note that the complexity introduced by PE is essentially twice the complexity of the RAKE for every stage. In particular, every stage introduces an additional spreading followed by a despreading operation ($\mathbf{S}_e^H\mathbf{S}_e$). From this, we can write, without loss of generality, the expression for the signal-to-interference ratio (SIR) for user one from $\left[\widehat{\mathbf{A}\mathbf{a}}[n]\right]_1$, where $[.]_1$ denotes the first element of the vector.

$$SIR_{PE} = \frac{|A_1|^2(1-\mathbf{s}_1^H\overline{\mathbf{S}}_e\overline{\mathbf{S}}_e^H\mathbf{s}_1)(1-\mathbf{s}_1^H\overline{\mathbf{S}}_e\overline{\mathbf{S}}_e^H\mathbf{s}_1)}{\mathbf{s}_1^H(\mathbf{I}-\overline{\mathbf{S}}_e\overline{\mathbf{S}}_e^H)\overline{\mathbf{S}}_e\mathcal{P}\overline{\mathbf{S}}_e^H(\mathbf{I}-\overline{\mathbf{S}}_e\overline{\mathbf{S}}_e^H)\mathbf{s}_1} \qquad (10)$$

where we have taken the expectancy w.r.t. the data, i.e. $E\{\mathbf{a}[n]\mathbf{a}[n]^H = \sigma^2\mathbf{I}\}$ and $\mathbf{S}_e = [\mathbf{s}1\ \overline{\mathbf{S}}_e]$; $\overline{\mathbf{A}} = diag[A_2\ldots A_K]$; $\mathcal{P} = \overline{\mathbf{A}}^H\overline{\mathbf{A}}$. In the large-system limit, with the number of users K and the spreading gain N going to infinity while keeping the loading factor $\alpha = K/N$ constant, the SIR is given by

$$SIR_{PE;K,N\to\infty,\alpha=const.} = \frac{(\alpha-1)^2}{\alpha^2(\alpha+1)} \qquad (11)$$

The derivation of (11) cannot be shown here due to the lack of space. In comparison, an equivalent analysis of the RAKE receiver is well known to give

$$SIR_{RAKE;K,N\to\infty,\alpha=const.} = \frac{1}{\alpha} \qquad (12)$$

For the simple PE to perform better than the RAKE we can hence see from (11) and (12) that we require $\alpha < 1/3$ in the noiseless, synchronous case. A similar analysis leading to the same conconclusion has recently been presented in [11].

The performance of PE can be much improved by introducing scalar polynomial coefficients d_b according to some design criterion in (8) as has been documented in various publications e.g. [4][5][12][6] [9] and we will hence not treat this case here. Instead, we propose to increase the degrees of freedom available to us by introducing a scalar coefficient *per path*. Returning to the more general case of asynchronous transmission, let us define $\mathbf{D}_b = diag[d_{b,1}\ldots d_{b,KM}]$ and write the approximated inverse of $\mathbf{R}(q)$ as a polynomial in $\overline{\mathbf{R}}(q)$ or equivalently in $\mathbf{R}(q)$ since there is a one-to-one relationship between the expansions in $\mathbf{R}(q)$ and $\overline{\mathbf{R}}(q)$. Hence,

$$\tilde{\mathbf{R}}^{-1}(q) = \sum_{b=0}^{B}\mathbf{D}_b\overline{\mathbf{R}}^b(q) \qquad (13)$$

analogous to the synchronous case in (8). Typically, we would only be interested in $B \in \{1,2\}$ stages after the RAKE in order to keep complexity at a reasonable level. Choosing $\mathbf{D}_0$ to be an identity matrix, and defining $\mathbf{z}[n] = \mathbf{R}(q)\mathbf{x}[n]$ we can write (13) for $B = 1$ as

$$\tilde{\mathbf{R}}^{-1}(q) = \mathbf{I} + (\mathbf{I} - \mathbf{D}\mathbf{R}(q))$$

which allows us to determine $\mathbf{D}$ blindly by minimising the following variance criterion.

$$\begin{aligned} \mathbf{D}^o &= \arg\min_{\mathbf{D}} E||(\mathbf{I}-\mathbf{D}\mathbf{R}(q))\mathbf{x}[n]||^2 \qquad (14) \\ &= diag\{\mathbf{R}_{xz}\}\left(diag\{\mathbf{R}_{zz}\}\right)^{-1} \end{aligned}$$

The resulting performance will be evaluated by simulation. Note that no matrix inversions are required to compute the $d_{i,j}$'s in the above approach.

An alternative is to extend the approach in (14) to a pilot-assisted scenario. In that case, we can formulate a minimisation for $\mathbf{D}$ using the following LMMSE criterion.

$$\begin{aligned}\mathbf{D}^o &= \arg\min_{\mathbf{D}} E\|\mathbf{A}\mathbf{a}[n] - (\mathbf{x}[n] - \mathbf{D}(\mathbf{R}(q) - \mathbf{I})\mathbf{x}[n])\|^2 \\ &= diag\{\mathbf{A}(\mathbf{R}_{ax} - \mathbf{R}_{az}) - \mathbf{R}_{xx} + \mathbf{R}_{xz}\} \ldots \\ &\quad \times [diag\{\mathbf{R}_{xx} - 2Re\{\mathbf{R}_{xz}\} + \mathbf{R}_{zz}\}]^{-1} \qquad (15)\end{aligned}$$

In the above examples, we have so far assumed $\mathbf{D}_0 = \mathbf{I}$ based on the polynomial expansion of $\mathbf{R}(q)$. This is, however, not optimal in general. We can therefore generalise equation (15) to an arbitrary number of stages with a matrix polynomial coefficient per stage as

$$\mathbf{D}_i^o = \arg\min_{\mathbf{D}_i : i \in 0\ldots B} E\|\mathbf{A}\mathbf{a}[n] - \sum_{b=0}^{B} \mathbf{D}_b \mathbf{R}^b(q)\mathbf{x}[n]\|^2 \qquad (16)$$

which can be solved through a set of linear equations. Looking at any row j in equation (16), we can equivalently write

$$\begin{aligned}\mathbf{d}_j^o &= \arg\min_{\mathbf{d}_j} E|A_j a_l[n] - \mathbf{d}_j \zeta_j[n]|^2 \\ &= A_j E(a_l[n]\zeta_j^H[n])(E\zeta_j[n]\zeta_j^H[n])^{-1} \qquad (17)\end{aligned}$$

where $j \in \{1 \ldots KM\}$ is the path index, $l = \lceil \frac{j}{M} \rceil$ the corresponding datasymbol, $\mathbf{d}_j = [d_{0,j} \ldots d_{B,j}]$ and $\zeta_j = [z_{0,j} \ldots z_{B,j}]^T$, $\mathbf{z}_b[n] = \mathbf{R}^b(q)\mathbf{x}[n] = [z_{b,1}[n] \ldots z_{b,KM}[n]]^T$ and hence the problem decouples nicely into a path-by-path solvable problem. It worth noting that this is not the case when the polynomial coefficient matrix D_b is replaced by a scalar as the solution for the coefficients involves the summation over the paths j and hence there is no decoupling between paths nor users, i.e.

$$\begin{aligned}d_i^o &= \arg\min_{d_i : i \in 0\ldots B} E\|\mathbf{A}\mathbf{a}[n] - \sum_{b=0}^{B} d_b \mathbf{R}^b(q)\mathbf{x}[n]\|^2 \\ &= \sum_j E(A_j a_l[n]\zeta_j^H[n])(\sum_j E\zeta_j[n]\zeta_j^H[n])^{-1}\end{aligned}$$

Path recombining after the pathwise PE interference cancellation will give the symbol estimates:

$$\hat{\mathbf{a}}[n] = \mathbf{K}^H \mathbf{F}(q)\left[\mathbf{R}(q)\mathbf{A}\mathbf{a}[n] + \underline{\mathbf{E}}^\dagger(q)\mathbf{v}[n]\right]$$

where $\mathbf{K}$ is a general recombination matrix of the same block diagonal structure as $\mathbf{A}$, namely $\mathbf{K} = diag(\mathbf{K}_1, \ldots, \mathbf{K}_K)$. Maximum ratio combining is $\mathbf{K} = \mathbf{A}$. $\mathbf{F}(q)$ defines the linear filter corresponding to one of the PE approaches above in (14),(15) or (16). For maximum ratio combining note also, since

$$\hat{\mathbf{a}}[n] = \mathbf{A}^H \sum_{b=0}^{B} \mathbf{D}_b \mathbf{R}^b(q)\mathbf{x}[n] = \sum_{b=0}^{B} \mathbf{W}_b \mathbf{R}^b(q)\mathbf{x}[n]$$

where $\mathbf{W}_b = \mathbf{A}^H \mathbf{D}_b$ is another block diagonal matrix, stage $b = 0$ hence corresponds to a G-RAKE. Note however, that the direct application of the G-RAKE approach above would no longer provide the pathwise, SINR enhanced, outputs. For the symbol estimate of user one, we have

$$\hat{a}_1[n] = \mathbf{K}_1^H \left[\mathbf{Z}_1(q)\mathbf{A}_1 a_1[n] + \overline{\mathbf{Z}}_1(q)\overline{\mathbf{A}}_1\overline{\mathbf{a}}_1[n] + \mathbf{X}(q)\mathbf{v}[n]\right]$$

where

$$\begin{aligned}\mathbf{K} &= diag(\mathbf{K}_1, \overline{\mathbf{K}}_1) &(18)\\ [\mathbf{I}_M \mathbf{0}]\,\mathbf{F}(q)\mathbf{R}(q) &= [\mathbf{Z}_1(q)\overline{\mathbf{Z}}_1(q)] &(19)\\ \mathbf{X}(q) &= [\mathbf{I}_M \mathbf{0}]\,\mathbf{F}(q)\underline{\mathbf{E}}^\dagger(q) &(20)\\ \mathbf{a}[n] &= [a_1[n]\overline{\mathbf{a}}_1^T[n]]^T &(21)\\ \mathbf{A} &= diag(\mathbf{A}_1, \overline{\mathbf{A}}_1) &(22)\end{aligned}$$

and $(.)_1$ is a signal model component acting on the useful signal contribution of user one whereas $\overline{(.)}_1$ defines the interfering terms. Hence, the output SINR of user one can be written as

$$\begin{aligned}SINR &= \frac{\sigma_a^2|\mathbf{K}_1^H\mathbf{Z}_1[0]\mathbf{A}_1|^2}{\mathbf{K}_1^H\mathbf{R}_1\mathbf{K}_1} \qquad (23)\\ \mathbf{R}_1 &= \sigma_a^2 \sum_{i\neq 0} \mathbf{Z}_1[i]\mathbf{A}_1\mathbf{A}_1^H\mathbf{Z}_1^H[i] \\ &\quad + \sigma_a^2 \sum_i \overline{\mathbf{Z}}_1[i]\overline{\mathbf{A}}_1\overline{\mathbf{A}}_1^H\overline{\mathbf{Z}}_1^H[i] + \sigma_v^2 \sum_i \mathbf{X}[i]\mathbf{X}^H[i]\end{aligned}$$

Maximum ratio combining is, however, not optimal and performance can be further improved by maximising the output SINR for the symbol estimate with respect to the recombining vector, $\mathbf{K}_1$. It can be shown that

$$SINR_{max} = \sigma_a^2 \mathbf{A}_1^H \mathbf{Z}_1^H[0]\mathbf{R}_1\mathbf{Z}_1[0]\mathbf{A}_1$$

when the optimised recombination is given by $\mathbf{K}_1^o = \mathbf{R}_1^{-1}\mathbf{Z}_1[0]\mathbf{A}_1$ Numerical results are shown in the next section for a number of scenarios.

4 Simulations

The simulations show the output SINR as a function of the input SNR and are obtained for user 1 using the expression for the SINR given in (23). The SNR is computed w.r.t. the power of user 1. The spreading codes are periodic, and made up of iid random variables $s_{k,l} \in \frac{1}{L}\{+1, -1\}$. Delay spread is half a symbol period and the user delays are uniformly distributed for asynchronous channels. PE denotes the basic receiver in (8), PE-D corresponds to (14), PE-DD to (15) and PE-DDD to (16). PE-DDs and PE-DDDs have

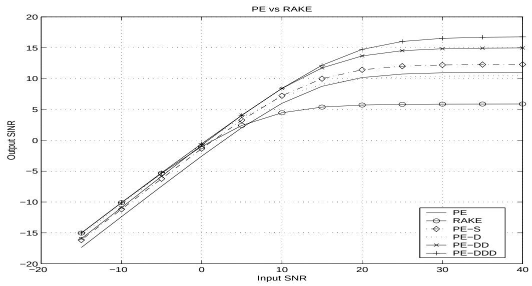

Figure 1: **L=16,K=4,SIR= -10dB,synchronous,MRC**

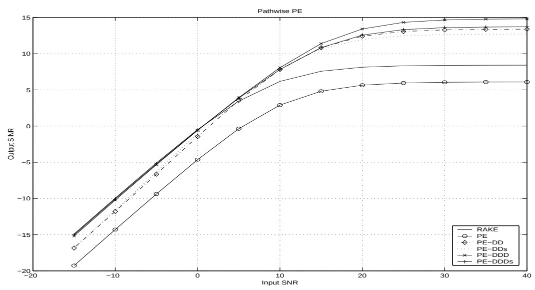

Figure 2: **L=16,K=3,M=3,asynchronous,MRC**

the diagonal coefficient matrices replaced by scalar coefficients. In figure 1, the single path, synchronous user case is shown, averaged over channel realisations for low ($K/L = 0.25$) system load. The SIR w.r.t. user 1 is -10dB and individual user powers are unequal. It can clearly be seen that the proposed approaches largely outperform both the RAKE and the PE. The PE-DDD receiver performs best, although not very much better than the PE-DD. In figure 2 performances are compared for the scalar and the diagonal matrix weighting in an asynchronous system, respectively. The PE-DD and PE-DDD approaches perform both better than their respective scalar equivalent (PE-DDs and PE-DDDs). Note also that due to the additional interference introduced from the multipaths, the PE approach now performs significantly worse than the RAKE despite the same loading, $K/L = 0.25$, as in the synchronous case in figure 1. Figure 3 shows the comparison of the the diagonal weighting approaches with maximum ratio combining and SINR optimised path recombining. The approaches using max. SINR recombining (PE-DD/MSINR and PE-DDD/MSINR), significantly outperform their equivalent using maximum ratio combining (PE-DD and PE-DDD).

5 Conclusions

Polynomial expansion (PE) is an approximation technique for LMMSE receivers and is particularly well suited for CDMA, due to the presence of a large number of small correlations. However, for PE to work, adjustment factors have to be introduced. We have shown that giving each signal component a separate scaling factor allows for improved performance at a small cost. Also, we have introduced PE

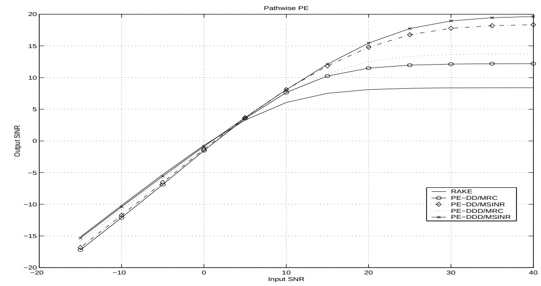

Figure 3: **L=12,K=4,M=3,asynchronous**

at the path level, which allows for interference cancellation and hence improved parameter estimation at the path level.

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