The Effect of Delay Constraint and Causal Feedback on the Wideband Performance of Multiaccess Block-Fading Channels

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Abstract

We consider a multiaccess Gaussian block fading channel where transmitters have causal state information and must deliver their message within a finite delay. Variable-rate coding with input power constraint enforced on a per-codeword basis is examined. We characterize the average capacity region and average capacity region per unit energy, the limiting behavior for large delay and the wideband slope of the spectral efficiency vs. (E_b/N_0)*dB.*

1: Introduction and motivations

Fading multiaccess channels have attracted a lot of attention because of the huge increase in system performance potentially achieved by the combination of joint decoding and power control. In [3] the single user case with perfect channel state information at both transmitter and receiver was studied and the optimal ergodic waterfilling law was derived. It was shown that substantial improvement can be obtained in the low SNR regime, where it is possible to have reliable communication with arbitrarily low energy per bit if the fading distribution have unbounded support, while in the high SNR regime very little can be gain with respect to the case of no power control. Ergodic performance can be attained by constant-rate variable-power schemes. Things are dramatically different in a faded multiuser ergodic scenario. In fact, [4] shows that substantial improvement can be obtained in the whole SNR range due to the inherent diversity provided by fading that affects the user signals. In the power limited case, reliable communication takes place at the price of large decoding delay and high peak-to-average power ratio.

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Unfortunately, the real world is not as "ergodic" as one would like it to be. In general, codewords cannot be made long enough to reveal the whole fading statistics, because of delay constraints, and a certain minimum bit rate must be guaranteed in every fading state. The problem of incorporating delay limitations and quality of service requirements of practical systems in Shannon theoretic setting has been a perennial challenge. For example, in [6] the concept of *information outage probability* was introduced and in [5] the *delay limited capacity*, i.e., the set of rates achievable in all fading states, was characterized. All these approaches keep the transmission rate fixed and let the transmission power fluctuate.

In this paper we take a somewhat complementary point of view: we assume that codewords span a *finite* number of slots, with fading constant over each slot and varying independently from slot to slot, that the power constraint is enforced on a per-codeword basis and that transmitters have causal knowledge of the channel state [7]. We allow *variable rate coding* so that users can coordinate their rates in order to be always inside the fading-dependent capacity region. Here, the transmit power is fixed while the coding rates fluctuate. Consequently, we define the *long-term average* capacity region as the set of all achievable rates averaged over an arbitrarily long sequence of codewords. Moreover, in the energy-limited case investigated here, we also look for the largest achievable *long-term average capacity per unit energy* (bit/joule).

Since vanishing error probability is unattainable unless the number of degrees of freedom, (proportional to the product of time duration and bandwidth) grows without bound, we concentrate our analysis in the wideband regime where an asymptotic analysis is feasible even for codebooks of fixed duration. As shown recently in [8], information theoretic performance in the wideband regime is not only characterized by the capacity per unit energy, i.e., the *minimum energy per bit*, but also by the *wideband slope* of the spectral-efficiency as a function of E_b/N_0 (dB) (b/s/Hz/3 dB), which quantifies the bandwidth requirement for a given desired data rate. Our analysis will focus on both limits.

In this work we formally define a variable rate scheme that differs from standard channel driven codeword multiplexing [3]; we extend the analysis of delay constrained systems with causal feedback in [7] to the multiuser case and we give a coding theorem (achievability and converse part); we provide a limiting theorem for large delay; we formalize the "low SNR" analysis in the framework of capacity per unit-cost by showing that the power allocation policy that concentrates the whole transmit energy over one slot is optimal in terms of minimum energy per bit for any number of users; finally, we analyze the system performance in the wideband regime and we show that the "one-shot" policy is also optimal with respect to the wideband slope for the single user case and provide an slope region for the multiuser case achievable with TDMA in a completely decentralized way.

The paper is organized as follow: Section 2 gives a description of the system model and defines the variable rate coding scheme; Section 3 characterize the long-term average capacity region and Section 4 the long-term average capacity region per unit energy; Section 5 considers the performance in wideband regime; Section 6 reports the results for the Rayleigh fading case and in Section 7 we point some conclusions. The proofs of all the theorems reported in the following can be found in [9].

2: System model and definitions

We consider a block-fading Gaussian Multi-Access Channel (MAC) where K transmitters must deliver their message within a frame of N slots to the receiver by spending a fixed maximum energy. The number of complex dimensions per slot is $L = \lfloor WT \rfloor$, where T is the slot duration and W is the channel bandwidth. The baseband L-dimensional complex received vector in slot n is

$$\mathbf{y}_n = \sum_{k=1}^{K} c_{k,n}\mathbf{x}_{k,n} + \mathbf{z}_n \tag{1}$$

where $\mathbf{z}_n$ is a proper complex Gaussian random vector with i.i.d. components of zero mean and unit variance, $\mathbf{x}_{k,n}$ is the transmit signal of user k, $c_{k,n}$ is the fading coefficient for user k with power gain $\alpha_{k,n} = |c_{k,n}|^2$, assumed i.i.d. from slot to slot with *continuous* cdf $F_\alpha(x)$.

The receiver has perfect *non-causal* Channel State Information (CSI) while the transmitters have perfect *causal* CSI [7], i.e., in slot n the transmitters know the channel state up to time n, defined by

$$\mathcal{S}_n \triangleq \{c_{k,i} : k = 1, \cdots, K,\ i = 1, \cdots, n\} \tag{2}$$

Each transmitter k is subject to the per-codeword input constraint (referred to as "short-term" power constraint)

$$\frac{1}{NL}\sum_{n=1}^{N} |\mathbf{x}_{k,n}|^2 \le \gamma_k \tag{3}$$

where γ_k is the transmitted energy per symbol that, because of the normalized noise variance, has the meaning of *transmit* SNR. In the following we indicate with $\beta_{k,n} = 1/L\,|\mathbf{x}_{k,n}|^2$ the *instantaneous* SNR of user k in slot n.

For finite N and L no positive rate is achievable. However, we can consider a sequence of channels indexed by the slot length L and study the achievable rates in the limit for $L \to \infty$ and fixed N. This is a standard mathematical abstraction in the study of the limit performance of block-fading channels [6] and it is motivated by the fact that, in many practical applications, the product WT is large. Even in the limit of large L, the rate K-tuple at which reliable communication is possible over a frame of N slots is a random vector, because only a fixed number N of fading coefficients affect each user codeword. We allow *variable rate coding* so that users can coordinate their rates in order to be always inside the fading-dependent capacity region.

Variable-rate coding in our setting is essentially different from variable-rate coding in an ergodic setting, such as in [4, 3]. Here, we assume that each transmitter has an infinite "bit-reservoir" and, depending on the fading instantaneous realization, transmits a variable number of bits per frame. We model this setting by letting the message set size depend on the fading state. Consider user k, let $\mathcal{W}_{k,n} = \{W_{k,n}(\mathcal{S}_n) : \mathcal{S}_n \in CC^{nK}\}$ be a collection of message sets indexed by the channel state $\mathcal{S}_n$ and $|W_{k,n}(\mathcal{S}_n)| = M_{k,n}(\mathcal{S}_n)$ denote the cardinality of the message set $W_{k,n}(\mathcal{S}_n)$.

Definition 1. A variable-rate coding system is defined by:

a) An assignment of message sets to the fading states defined by $\mathcal{W}_{k,n}$ given above;

b) A sequence of encoding functions $\phi_{k,n} : W_{k,n}(\mathcal{S}_n) \times \mathbb{C}^{nK} \to \mathbb{C}^L$ such that $\phi_{k,n} : (w, \mathcal{S}_n) \mapsto \mathbf{x}_{k,n}$, where $w \in W_{k,n}(\mathcal{S}_n)$, and such that the resulting code word satisfy (3);

c) A decoding function $\psi : \mathbb{C}^{NL} \times \mathbb{C}^{NK} \to \bigotimes_{k=1}^{K}\bigotimes_{n=1}^{N}\{\cup\mathcal{W}_{k,n}\}$ such that $\psi : (\{\mathbf{y}_n : n = 1, \cdots, N\}, \mathcal{S}_N) \mapsto (\mathbf{w}_1, \cdots, \mathbf{w}_K)$, where $\{\cup\mathcal{W}_{k,n}\}$ is a shorthand notation to indicate the union of all message sets $W_{k,n}(\mathcal{S}_n) \in \mathcal{W}_{k,n}$, and where $\mathbf{w}_k = (w_{k,1}, \cdots, w_{k,N})$ is a sequence of messages such that $w_{k,n} \in W_{k,n}(\mathcal{S}_n)$. □

For given $\mathcal{S}_N$, the coding rate for user k of the above scheme is given by the K-tuple

$$R_k(\mathcal{S}_N) = \frac{1}{NL}\sum_{n=1}^{N} \log\left(M_{k,n}(\mathcal{S}_n)\right) \tag{4}$$

and the error probability is given by

$$P_e(\mathcal{S}_N) = \frac{1}{\prod_{k=1}^K \prod_{n=1}^N M_{k,n}(\mathcal{S}_n)} \cdot \sum_{\mathbf{w}_1,\cdots,\mathbf{w}_K} \Pr\left(\psi(\{\mathbf{y}_n\}, \mathcal{S}_N) \neq (\mathbf{w}_1, \cdots, \mathbf{w}_K) | (\mathbf{w}_1, \cdots, \mathbf{w}_K)\right) \quad (5)$$

Consider a sequence of frames, where coding and decoding are performed frame-by-frame according to a variable-rate coding scheme defined above, and where the channel state sequence $\mathcal{S}_N$ over each frame is generated according to some ergodic and stationary process. By the law of large numbers, the long-term average coding rate and error probability are given by $R_k = \mathrm{E}[R_k(\mathcal{S}_N)]$ and by $P_e = \mathrm{E}[P_e(\mathcal{S}_N)]$, where expectation is with respect to the joint statistics of the channel state $\mathcal{S}_N$. Since only average error probability over the messages selected uniformly is considered in the definition, our results do not hold for maximal error probability. In particular, there might exist a state subset for which the error probability is very large for some messages. However, the probability measure of such subset must vanish as $L \to \infty$. The operative definitions of long-term average capacity region and of long-term average capacity region per unit-energy mimic, respectively, the standard capacity region definition for input constrained channels [1] and definition of capacity region per unit cost given in [2]. For sake of brevity we omit them here.

Next, in analogy with [4, 2], we characterize the long-term average capacity region and the long-term average capacity per unit energy. We also give limiting theorems for large delay N.

3: The long-term average capacity region

Theorem 1. The long-term average capacity region is

$$C_{K,N}(\boldsymbol{\gamma}) = \bigcup_{\boldsymbol{\beta} \in \Gamma_{K,N}(\boldsymbol{\gamma})} \Big\{ \mathbf{R} \in \mathbb{R}_+^K \ : \ \forall \mathcal{A} \subseteq \{1, \cdots, K\} \sum_{k \in \mathcal{A}} R_k \leq \mathrm{E}\left[\frac{1}{N} \sum_{n=1}^N \log\left(1 + \sum_{k \in \mathcal{A}} \alpha_{k,n} \beta_{k,n}(\mathcal{S}_n)\right)\right]\Big\} \quad (6)$$

where the expectation is taken with respect to the channel state $\mathcal{S}_N$ and $\Gamma_{K,N}(\boldsymbol{\gamma})$ is the set of *feasible short-term causal power allocation policies* $\boldsymbol{\beta}$ defined as

$$\Gamma_{K,N}(\boldsymbol{\gamma}) \triangleq \left\{ \boldsymbol{\beta} \in \mathbb{R}_+^{KN} \ : \ \frac{1}{N} \sum_{n=1}^N \beta_{k,n}(\mathcal{S}_n) \leq \gamma_k \right\} \quad (7)$$

and where $\beta_{k,n}(\mathcal{S}_n)$ defines the causality constraint. $\diamond$

The explicit characterization of the boundary of $C_{K,N}(\boldsymbol{\gamma})$ can be done following the approach [4] and it is the closure of all points $\mathbf{R} \in \mathbb{R}_+^K$ solution of

$$\max_{\mathbf{R} \in C_{K,N}(\boldsymbol{\gamma})} \sum_{k=1}^K \mu_k R_k \quad (8)$$

for some $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_K) \in \mathbb{R}_+^K$. As in [4], the optimization in (8) can be turned into the optimization over the power policies $\boldsymbol{\beta} \in \Gamma_{K,N}(\boldsymbol{\gamma})$ of the functional

$$\sum_{k=1}^K \mu_{\pi_k} \mathrm{E}\left[\frac{1}{N} \sum_{n=1}^N \log\left(1 + \frac{\alpha_{\pi_k,n}\, \beta_{\pi_k,n}}{1 + \sum_{j<k} \alpha_{\pi_j,n}\, \beta_{\pi_j,n}}\right)\right] \quad (9)$$

where π is the permutation that orders $\boldsymbol{\mu}$ in decreasing order, i.e., $\mu_{\pi_1} > \cdots > \mu_{\pi_K}$. Problem (9) is solved by the Dynamic Programming algorithm:

Theorem 2. The solution of problem (9) is

$$\max_{\mathbf{R} \in C_{K,N}(\boldsymbol{\gamma})} \sum_{k=1}^K \mu_k R_k = \frac{1}{N} S_N^{(\mu)}(N\gamma_1, \cdots, N\gamma_K) \quad (10)$$

where the function $S_N^{(\mu)}(P_1, \cdots, P_K)$ is given by the Dynamic Programming recursion, for $n = 1, \cdots, N$

$$S_n^{(\mu)}(P_1, \cdots, P_K) = \mathrm{E}\left[\max_{p_k \in [0, P_k]} S_{n-1}^{(\mu)}(P_1 - p_1, \cdots, P_K - p_K) + \sum_{k=1}^K \mu_{\pi_k} \log\left(1 + \frac{\alpha_{\pi_k}\, p_{\pi_k}}{1 + \sum_{j<k} \alpha_{\pi_j}\, p_{\pi_j}}\right)\right] \quad (11)$$

with initial condition $S_0^{(\mu)}(P_1, \cdots, P_K) = 0$. $\diamond$

In [7], recursion (11) was computed numerically for $K = 1$. Although for finite N a closed form solution of (11) seems infeasible, for large N can prove that:

Theorem 3. In the limit for large N, the long-term average capacity region $C_{K,N}(\boldsymbol{\gamma})$ tends to the ergodic capacity region given in [4]. $\diamond$

4: The long-term average capacity region per unit energy

A byproduct of the proof of Theorem 1 is that the long-term average capacity region coincides with the standard "ergodic" capacity region of the N-slot extension channel, which is frame-wise memoryless. The following theorem is an immediate consequence of this fact and of the general theory of capacity per unit cost [2]:

Theorem 4. The long-term average capacity region per unit energy is given by

$$U_{K,N} = \bigcup_{\boldsymbol{\gamma} \in \mathbb{R}_+^K} \left\{ \mathbf{r} \in \mathbb{R}_+^K : (\gamma_1\, r_1, \cdots, \gamma_K\, r_K) \in C_{K,N}(\boldsymbol{\gamma}) \right\}$$

In analogy with [2], it is easy to show the following:

Theorem 5. The long-term average capacity region per unit energy is the hyper-cube

$$U_{K,N} = \left\{ \mathbf{r} \in \mathbb{R}_+^K : r_k \le U_{1,N} \right\} \quad (12)$$

where (with a slight abuse of notation)

$$U_{1,N} = \lim_{\gamma \to 0} \frac{1}{\gamma} \sup_{\boldsymbol{\beta} \in \Gamma_{1,N}(\gamma)} \mathrm{E}\left[\frac{1}{N} \sum_{n=1}^{N} \alpha_{k,n} \beta_{k,n}(\mathcal{S}_n) \right] \quad (13)$$

$\diamond$

The analytical solution of (13) was found in [7]. We report it here in our notation for later use:

Theorem 6. Define the recursion

$$s_n = \mathop{\mathrm{E}}_{\alpha}[\max\{s_{n-1}, \alpha\}] \quad (14)$$

for $n = 1, \ldots, N$, with initial condition $s_0 = 0$. The single-user long-term average capacity per unit energy is given by $U_{1,N} = s_N$ and it is achieved by the “one-shot” power allocation policy defined by

$$\beta_n^\star = \begin{cases} N\gamma & \text{if } n = n^\star(\boldsymbol{\alpha}) \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

where we define the “level-crossing” time $n^\star(\boldsymbol{\alpha}) = \min\{n \in \{1, \ldots, N\} : \alpha_n \ge s_{N-n}\}$ $\diamond$

In the limit for large N we have:

Theorem 7. For large N, $U_{1,N}$ tends to the ergodic capacity region per unit energy defined in [2] and given explicitly by

$$\lim_{N \to \infty} U_{1,N} = \sup\{\alpha\} \quad (16)$$

where $\sup\{\alpha\} = \inf\{x \ge 0 : F_\alpha(x) = 1\}$. $\diamond$

5: Performance in wideband regime

The optimality of a coding scheme in the wideband regime is defined and studied in [8]. Let $C(\gamma)$ be the capacity expressed in nat/dimension as a function of γ, and let $\mathsf{C}(E_b/N_0)$ denote the corresponding spectral efficiency in bit/s/Hz as a function of the energy per bit vs. noise power spectral density, E_b/N_0, given implicitly by

$$\begin{cases} \frac{E_b}{N_0} = \frac{\gamma \log 2}{C(\gamma)} \\ \mathsf{C}\left(\frac{E_b}{N_0}\right) = \frac{C(\gamma)}{\log 2} \end{cases} \quad (17)$$

The value $(E_b/N_0)_{\min}$ for which $\mathsf{C}(E_b/N_0) > 0 \Leftrightarrow E_b/N_0 > (E_b/N_0)_{\min}$ and the slope of the spectral efficiency in bit/s/Hz/(3 dB) are given by [8]

$$\left(\frac{E_b}{N_0}\right)_{\min} = \frac{\log 2}{\dot{C}(0)} \quad (18)$$

$$\mathcal{S}_0 = \frac{2\left(\dot{C}(0)\right)^2}{-\ddot{C}(0)} \quad (19)$$

where $\dot{C}(0)$ and $\ddot{C}(0)$ are the first and second derivative of the capacity function at $\gamma = 0$. From [2], we see immediately that the reciprocal of $(E_b/N_0)_{\min}$ is the capacity per unit energy (expressed in bit/joule) of the channel.

A signaling strategy is said to be *first-order optimal* if it achieves $(E_b/N_0)_{\min}$ and *second-order optimal* is it achieves $\mathcal{S}_0$ [8].

In the previous section we have shown that the one-shot power allocation $\boldsymbol{\beta}^\star$ (in conjunction with Gaussian variable-rate coding) achieves the capacity region per unit energy for all users for the block-fading MAC with causal transmitter CSI considered here. Then, we conclude that the one-shot policy is first-order optimal for any number of users K. Next, we study the wideband slope performance of $\boldsymbol{\beta}^\star$ in the single user case.

Theorem 8. $(E_b/N_0)_{\min}$ and $\mathcal{S}_0$ for the single-user block fading channel with causal transmitter CSI are given by

$$\left(\frac{E_b}{N_0}\right)_{\min} = \frac{\log 2}{s_N} \quad (20)$$

$$\mathcal{S}_0 = \frac{2\,(s_N)^2}{-N\,\ddot{S}_N(0)} \quad (21)$$

where s_N is given in (14), the function $S_N(P)$ is defined in (11) for $K = 1$ and $\mu_1 = 1$, $\ddot{S}_N(0)$ denotes the second derivative of $S_N(P)$ at $P = 0$ and is given by the recursion

$$\begin{aligned} -\ddot{S}_n(0) &= \Pr(\alpha \ge s_{n-1})\, \mathrm{E}[\alpha^2 | \alpha \ge s_{n-1}] \\ &\quad -\ddot{S}_{n-1}(0) \Pr(\alpha < s_{n-1}) \end{aligned} \quad (22)$$

with initial condition $\ddot{S}_0(0) = 0$. Furthermore, the one-shot power allocation policy $\boldsymbol{\beta}^\star$ achieves $(E_b/N_0)_{\min}$ and $\mathcal{S}_0$. $\diamond$

From the proof of Theorem 5 it follows that first-order optimality can be obtained either by using superposition coding or by using TDMA inside each slot. Since $\boldsymbol{\beta}^\star$ is also second-order optimal, the one-shot policy achieves the maximum possible slopes under TDMA:

Theorem 9. As the rates vanish, the slope region achieved by TDMA in conjunction with power policy $\beta^\star$ is

$$0 \le \sum_{k=1}^{K} \mathcal{S}_{0,\mathrm{tdma}}^{(k)} \le \mathcal{S}_0 \quad (23)$$

for every ratio R_j/R_k and every choice of the time sharing factors. $\diamond$

Note that slopes in (23) are achievable in a completely decentralized way: each slot is partition is K sub-slots and in each sub-slot the active user allots power according to $\boldsymbol{\beta}^\star$ and transmits an instantaneous rate of $\log(1 + \alpha_n N\gamma)$ if $n = n^\star(\boldsymbol{\alpha})$ without need of “rate coordination”.

6: Example: the Rayleigh fading channel

In order to illustrate the results of previous sections we consider the case of i.i.d. Rayleigh fading. The channels

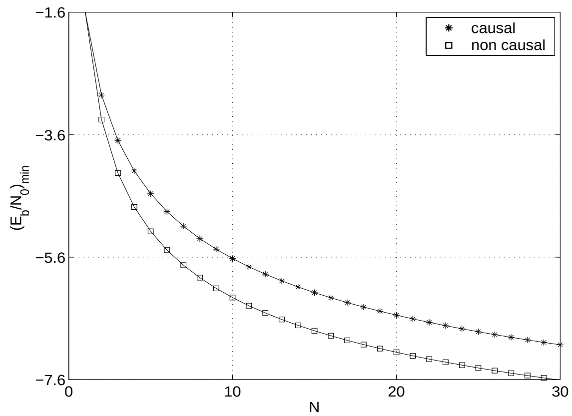

Figure 1. $(E_b/N_0)_{\min}$ **(dB) vs.** N**.**

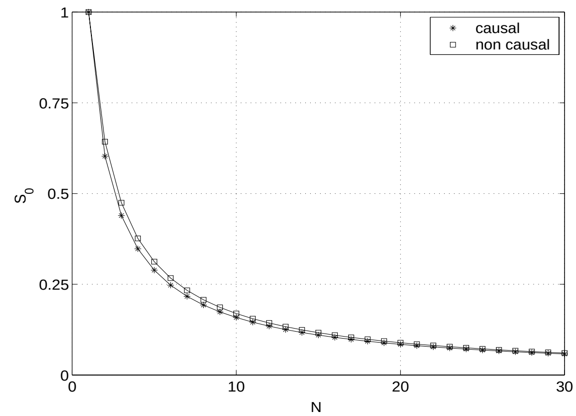

Figure 2. Single user slope vs. N**.**

gain law is $F_\alpha(x) = 1 - \mathrm{e}^{-x}$ for $x \geq 0$. The thresholds are

$$s_n = s_{n-1} + \mathrm{e}^{-s_{n-1}}$$

with $s_0 = 0$. The recursion for the second derivatives is

$$-\ddot{S}_n(0) = \mathrm{e}^{-s_{n-1}}(2 + 2s_{n-1} + s_{n-1}^2) - \ddot{S}_{n-1}(0)(1 - \mathrm{e}^{-s_{n-1}});$$

with $\ddot{S}_0(0) = 0$. If we allow the input to depend on the whole CSI $\mathcal{S}_N$, the first order optimal power policy (also second order optimal in the single user case) is

$$\beta_n^{\star\,\text{non-causal}} = \begin{cases} N\gamma & \text{if } \alpha_n = \max\{\alpha_1, \cdots, \alpha_N\} \\ 0 & \text{otherwise} \end{cases}$$

that gives

$$\left(\frac{E_b}{N_0}\right)_{\min}^{(\text{non-causal})} = \frac{\log 2}{\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\frac{1}{n}}$$

$$\mathcal{S}_0^{(\text{non-causal})} = \frac{2\left(\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\frac{1}{n}\right)^2}{N\sum_{n=1}^{N}\binom{N}{n}(-1)^{n+1}\frac{2!}{n^2}}$$

Figs. 1 and 2 show $(E_b/N_0)_{\min}$ and $\mathcal{S}_0$ vs. N and for both the causal and non-causal knowledge of the channel.

7: Conclusions

Theorem 9 has interesting consequences at protocol layer. In a system that requires aggressive power control management and transmission at low rate, like wireless sensor networks, the one-shot policy in conjunction with TDMA has an impact on the protocols stack. In fact, in order to optimize the average number of received bits per transmit joule, sequential polling of the all active users by the master, like in Bluetooth, that let them transmitting in TDMA, is suboptimal. Information theory suggests that the master station should send periodically a "reference signal" with the only purpose of enabling the users to measure their instantaneous channel; if a user has a packet to send, then it starts a timeout and measures the attenuation of the "reference signal" on every sub-slot of its time window; on the first slot where the channel gain is higher than the time varying threshold s it sends the packet with all the available energy, then it resets the timeout and waits for the next packet to send. Since transmission occurs at minimum E_b/N_0, the system uses in the most efficient way the available energy and reduces the multiaccess interference to the minimum. This protocol does not require frame synchronization among users nor does waste memory space to store past fading values.

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