Iterative Soft-SIC Joint Decoding and Parameter Estimation

Mari Kobayashi†, Joseph Boutros† and Giuseppe Caire⋆

† Ecole Nationale Supérieure des Télécommunications de Paris,
64 rue Barrault – 75634 Paris CEDEX, France.
Email: mari@wcs.sony.co.jp, boutros@com.enst.fr.
⋆ Institut Eurecom,
2229 Route des Crétes, B.P. 193
06904 Sophia-Antipolis CEDEX, France.
Email: caire@eurecom.fr

Abstract

We derive a low-complexity receiver scheme for joint multiuser decoding and parameter estimation of CDMA signals. The resulting receiver processes the users serially and iteratively, and makes use of *soft-in soft-out* single-user decoders, of soft interference cancellation and of *expectation-maximization* parameter estimation as the main building blocks. Computer simulations show that the proposed receiver achieves near single-user performance at very high channel load (number of users per chip) and outperforms conventional schemes with similar complexity.

1 Introduction

Among the several multiuser detection schemes proposed for CDMA [1], Serial and Parallel Interference Cancellation (SIC and PIC) are particularly attractive because they process directly the output of a bank of single-user matched filters (SUMF). The receiver front-end is identical to that of conventional detection. Therefore, these methods can be seen as an "add-on" post-processing to enhance the performance of a conventionale base-station receive when particularly high channel load is needed, and can be applied easily to either short and long spreading sequence formats [2].

The main preformance limitation of SIC/PIC schemes are: 1) error propagation caused by feeding back erroneous symbol decisions; 2) imperfect interference cancellation due to non-ideal knowledge of channel parameters (e.g., the complex amplitudes and delays of the users' multipath channels). In this work, we propose a receiver scheme which handles successfully both problems.

SIC is both simpler and more robust than PIC with respect to error propagation, since users can be ranked according to their signal-to-interference plus noise ratio (SINR) and decoded in sequence [3, 4]. Hence, we focus on SIC schemes. In early works, SIC is applied to uncoded transmission and hard decisions are used at each stage to remove the already detected users from the received signals. In order to prevent error propagation, the use of soft (or partial) interference cancellation and iterative SIC schemes has been proposed in different forms and by different authors (see for example [4, 5, 6]). More recently, the SIC approach has been combined with channel coding and Soft-In Soft-Out (SISO) decoding. The number of works in this direction is overwhelming. Without the ambition of being exhaustive, we refer to [7, 8, 9, 10, 11, 12]. A common feature of these algorithms is that single-user SISO decoders provide at each iteration an estimate of the a posteriori probabilities (APP) for the user code symbols, which are used to form a soft estimate of interference to be subtracted from the received signal. In this way, the contribution of a user is effectively subtracted from the signal only if its symbol decisions are sufficiently reliable.

A unified framework to iterative multiuser joint decoding based on factor-graphs and sum-product algorithm [13] is provided in [14]. In this framework, almost all algorithms previously proposed (notably, those of [7] and of [11, 12]) can be re-derived in a simple direct way. Moreover, as a consequence of the sum-product approach, it is found that extrinsic (EXT) probabilities [15] rather than APPs should be fed back to form the soft interference estimate. As confirmed experimentally by [16], APP-based soft interference cancellation yields a biased residual interference term which tends to cancel the useful signal, and the APP-based algorithms of [7, 11, 12] attain a worse overall spectral efficiency than their EXT-based counterparts derived and analyzed in [14].

In order to reduce parameter estimation errors, iterative SIC schemes can be naturally coupled with iterative parameter estimation in order to (hopefully) improve the estimates with the iterations, as long as the signal is "cleaned-up" from interference. We propose a low-complexity iterative soft-SIC algorithm for joint data detection and channel parameter estimation. The main building blocks of our receiver are SISO single-user decoders, soft interference cancellation stages and a channel parameter estimation updating step which is formally equivalent to one step of the Expectation-Maximization (EM) algorithm (see [17] and references therein). The key idea to achieve polynomial complexity in the number of users is to apply EM "locally", i.e., instead of using the true a posteriori distribution of the missing data given the observation and the current parameter estimate, we use the product distribution induced by the a posteriori marginal (symbol-by-symbol) probabilities output by the SISO decoders at each iteration. A longer version of this work can be found in [18].

The paper is organized as follows. In Section 2 the synchronous CDMA signal model is presented. In Section 3 we derive the proposed receiver structure. In Section 4 we present some numerical results and in Section 5 we summarize our conclusions.

Notation conventions: 1) Let $\mathbf{A}$ be a matrix, then $\mathbf{a}_n$, $\mathbf{a}^k$ and $a_{k,n}$ (or $[\mathbf{A}]_{k,n}$) denote the n-th column, the k-th row and the (k,n)-th element, respectively; 2) $\mathbf{z} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ indicates that the random vector $\mathbf{z}$ is complex circularly-symmetric jointly Gaussian with mean $E[\mathbf{z}] = \boldsymbol{\mu}$ and covariance $E[(\mathbf{z}-\boldsymbol{\mu})(\mathbf{z}-\boldsymbol{\mu})^H] = \boldsymbol{\Sigma}$; 3) $A \propto B$ means that A and B differ by a multiplicative term; 4) $A \doteq B$ means that A and B differ by an additive term; 5) Probability density functions (pdf) are denoted by $p(\cdot)$ and probability mass functions (pmf) are denoted by $\Pr(\cdot)$.

2 System model

We consider the uplink of a coded direct-sequence CDMA system with synchronous transmission over frequency-non-selective channels and Nyquist chip-shaping pulses. The system is frame-oriented, i.e., encoding and decoding is performed frame-by-frame and users are synchronous also at the frame level. In each frame, the complex baseband equivalent discrete-time signal originated by sampling at the chip rate the output of a chip-matched filter is given by [1]

$$\begin{cases} \mathbf{Y} = \mathbf{SWX} + \mathbf{N} & \text{Data transmission phase} \\ \mathbf{Y}^{(t)} = \mathbf{SWX}^{(t)} + \mathbf{N}^{(t)} & \text{Training phase} \end{cases} \tag{1}$$

where: 1) $\mathbf{Y} \in \mathbb{C}^{L \times N}$ and $\mathbf{Y}^{(t)} \in \mathbb{C}^{L \times T}$ are the arrays of received signal samples in the data and training phases, respectively; 2) $\mathbf{N} \in \mathbb{C}^{L \times N}$ and $\mathbf{N}^{(t)} \in \mathbb{C}^{L \times T}$ are the arrays of noise samples in the data and training phases, with i.i.d. components $\sim \mathcal{N}_{\mathbb{C}}(0, N_0)$; 3) $\mathbf{S} \in \mathbb{C}^{L \times K}$ contains the user spreading sequences by columns; 4) $\mathbf{W} = \text{diag}(w_1, \ldots, w_k)$ contans the user complex amplitudes w_k; 5) $\mathbf{X} \in \mathbb{C}^{K \times N}$ is the array of transmitted code symbols; 6) $\mathbf{X}^{(t)} \in \mathbb{C}^{K \times T}$ is the array of transmitted training symbols; 7) N, T, L and K denote the code block length and the training length (in symbols), the spreading factor (chips per symbol) and the number of users, respectively.

The total frame length in symbols is equal to $N + T$. Since the channel amplitudes remain constant over the whole frame and the system is synchronous, the position of training symbols in the frame is irrelevant and arbitrary. The user spreading sequences are normalized such that $\mathbf{s}_k|^2 = 1$. Hence, the signal-to-noise ratio (SNR) of user k is given by $\text{SNR}_k = |w_k|^2/N_0$.

At each frame, each user encodes a sequence of information bits into a code word $\mathbf{x}^k \in \mathcal{C}_k$, where $\mathcal{C}_k$ is the code book of user k, defined over a given complex signal set (e.g., a PSK or QAM constellation). In this paper we consider non-systematic non-recursive convolutional codes with trellis termination, mapped onto BPSK, so that $x_{k,n} \in \{-1, +1\}$. Each code word is independently interleaved before transmission.

3 Iterative joint data detection and parameter estimation

Without loss of generality, we assume that the user decoding order at each iteration is $k = 1, \dots, K$. Decoding of user k at iteration m in the soft-SIC receiver is based on the observed signal sequence

$$z_{k,n}^{(m)} = \underbrace{\frac{1}{\widehat{w}_k^{(m)}} \mathbf{s}_k^H \mathbf{y}_n}_{\text{SUMF output}} \underbrace{- \sum_{j=1}^{k-1} \mathbf{s}_k^H \mathbf{s}_j \frac{\widehat{w}_j^{(m)}}{\widehat{w}_j^{(m)}} \widehat{x}_{j,n}^{(m)}}_{\text{current iteration}} \underbrace{- \sum_{j=k+1}^{K} \mathbf{s}_k^H \mathbf{s}_j \frac{\widehat{w}_j^{(m)}}{\widehat{w}_j^{(m)}} \widehat{x}_{j,n}^{(m-1)}}_{\text{previous iteration}} \tag{2}$$

for $n = 1, \dots, N$, where $\{\widehat{w}_j^{(m)} : j = 1, \dots, K\}$ are estimates of the user amplitudes at iteration m, $\{\widehat{x}_{j,n}^{(m)} : j = 1, \dots, K\}$ are estimates of the user symbols provided by the decoders at iteration m.

Decoding is performed by a SISO decoder, which in the case of convolutional codes can be implemented efficiently by the forward-backward BCJR algorithm [19]. Let $p(z_{k,n}^{(m)} | x_{k,n} = a)$ be the conditional pdf of $z_{k,n}^{(m)}$. The SISO decoder for user k produces a marginal EXT pmf for $x_{k,n}$, given by

$$\text{EXT}_{k,n}^{(m)}(a) \propto \sum_{\mathbf{c} \in \mathcal{G}_k : c_n = a} \prod_{\ell \neq n} p(z_{k,\ell}^{(m)} | x_{k,\ell} = c_\ell) \tag{3}$$

where the normalization $\text{EXT}_{k,n}^{(m)}(+1) + \text{EXT}_{k,n}^{(m)}(-1) = 1$ is enforced. The corresponding APP is given by

$$\text{APP}_{k,n}^{(m)}(a) \propto p(z_{k,n}^{(m)} | x_{k,n} = a) \, \text{EXT}_{k,n}^{(m)}(a) \tag{4}$$

with again the normalization $\text{APP}_{k,n}^{(m)}(+1) + \text{APP}_{k,n}^{(m)}(-1) = 1$.

Assuming that $z_{k,n}^{(m)}$ is conditionally (marginally) circularly-symmetric complex Gaussian given $x_{k,n}$, its conditional pdf can be approximated as

$$p(z_{k,n}^{(m)} | x_{k,n} = a) \propto \exp\left(- \frac{|z_{k,n}^{(m)} - a|^2}{\nu_k^{(m)}} \right) \tag{5}$$

where $\nu_k^{(m)} = E[|z_{k,n}^{(m)} - x_{k,n}|^2]$ is the residual interference plus noise variance, which is independent of n under mild uniformity conditions on the user codes [14].

The SISO decoders output also APPs for the information bits, which will be used for final symbol-by-symbol decisions in the last iteration. For simplicity, we assume that the total number of iterations M is fixed for all users. In practice, M should be optimized according to the SNR and channel load K/L. Also, some dynamic stopping criterion might be used in order to minimize the number of iterations. We leave this interesting topic for future work.

Next, we address the estimation of the residual interference plus noise variance $\nu_k^{(m)}$, the estimation of the code symbols $x_{k,n}$ and the estimation of the user amplitudes w_k used in the soft-SIC (equation (2)). We also address the initialization of the receiver with training-based parameter estimation and some methods to combine training-based and EM-based estimation.

Estimation of the residual interference plus noise variance. The variance $\nu_k^{(m)}$ is unknown, and must be estimated on-line before each SISO decoding step. Let $\zeta_{k,n}^{(m)} = z_{k,n}^{(m)} - x_{k,n}$ denote the residual interference plus noise term in (2). A simple estimator for $\nu_k^{(m)}$ is given by

$$\widehat{\nu}_k^{(m)} = \frac{1}{N} \sum_{n=1}^{N} |z_{k,n}^{(m)}|^2 - 1 \tag{6}$$

Beside its simplicity, in [18] we provide a number of motivations justifying the use (6) to estimate $\nu_k^{(m)}$. In particular: 1) if $\zeta_{k,n}^{(m)}$ and $x_{k,n}$ are uncorrelated, then $\widehat{\nu}_k^{(m)}$ is an unbiased estimator; 2) if $x_{k,n}$ is i.i.d., uniformly distributed on $\{-1,+1\}$ (as in our case), $\zeta_{k,n}^{(m)}$ is i.i.d. $\sim \mathcal{N}_{\mathbb{C}}(0,\nu_k^{(m)})$, and $x_{k,n}, \zeta_{k,n}^{(m)}$ are uncorrelated, then the error variance of $\widehat{\nu}_k^{(m)}$, given by

$$E\left[\left|\nu_k^{(m)} - \widehat{\nu}_k^{(m)}\right|^2\right] = \frac{1}{N}\left(4\nu_k^{(m)} + (\nu_k^{(m)})^2\right) \tag{7}$$

is surprisingly close to the error variance of the Maximum-Likelihood (ML) estimator with known $x_{k,n}$, is given by

$$E\left[\left|\nu_k^{(m)} - \frac{1}{N}\sum_{n=1}^{N}|z_{k,n}^{(m)} - x_{k,n}|^2\right|^2\right] = \frac{1}{N}(\nu_k^{(m)})^2 \tag{8}$$

3) if the complex amplitude is estimated reliably, i.e., $\widehat{w}_k^{(m)} \approx w_k$, and if $x_{k,n}$ is uncorrelated with $\widehat{x}_{j,n}$ for $j \neq k$, then $\zeta_{k,n}^{(m)}$ and $x_{k,n}$ are practically uncorrelated. Moreover, under mild conditions on the user amplitudes, for large K the residual interference term $\zeta_{k,n}^{(m)}$ is asymptotically Gaussian in the large system limit. We conclude that for large N and K the estimator $\widehat{\nu}_k^{(m)}$ performs very close to the ML estimator for known coded symbols.

In the actual receiver implementation, the EXT and APP pmfs (3) and (4) are calculated by using (5) where $\nu_k^{(m)}$ is replaced by its estimate $\widehat{\nu}_k^{(m)}$ given by (6).

Soft estimation of the code symbols. The (non-linear) MMSE estimate of symbol $x_{k,n}$ given the observation $\mathbf{Y}$ is given by the conditional mean

$$x_{k,n}^{\text{mmse}} = E[x_{k,n}|\mathbf{Y}] = 2\Pr(x_{k,n} = +1|\mathbf{Y}) - 1 \tag{9}$$

where $\Pr(x_{k,n} = a|\mathbf{Y})$ is the a posteriori pmf of symbol $x_{k,n}$ given the whole observation $\mathbf{Y}$. We are tempted to replace $\Pr(x_{k,n} = a|\mathbf{Y})$ by $\text{APP}_{k,n}(a)$ given by the SISO output at iteration m, and claim that this choice minimizes the residual interference variance and it is therefore optimal. Unfortunately, this reasoning is incorrect (see [18, 14] for a proof by contradicion).

By using a rigorous derivation based on factor-graphs and on the application of the sum-product algorithm , it can shown that [14]: 1) even for perfectly know amplitudes and SISO input variances (i.e., $\widehat{w}_k^{(m)} = w_k$ and $\widehat{\nu}_k^{(m)}$), the residual interference term $\zeta_{k,n}^{(m)}$ is conditionally biased and the bias tends to cancel the useful signal, i.e.,

$$E[\zeta_{k,n}^{(m)}|x_{k,n} = a] = -\mu_{k,n}^{(m)} a$$

where $\mu_{k,n}^{(m)}$ is a non-negative quantity that may depend on k, n and on the iteration index m; 2) by using EXT-based instead of APP-based symbol estimates, the resulting residual interference term is conditionally unbiased, i.e., $E[\zeta_{k,n}^{(m)}|x_{k,n}] = 0$, and the overall soft-SIC algorithm attains better performance than its APP-based version for very high channel load. Remarkably, this effect is not visible for small channel load but, as K/L increases, the difference between APP-based and EXT-based soft-SIC schemes is more and more evident [16].

In passing, we notice also that a biased residual interference implies that $x_{k,n}$ and $\zeta_{k,n}^{(m)}$ are correlated (even for perfect amplitude estimation). Hence, the variance estimator (6) is asymptotically optimal for large N, K only when the symbol soft estimates are obtained from EXT pmfs.

Driven by the above considerations, we shall use the following soft symbol estimates

$$\widehat{x}_{k,n}^{(m)} = 2\text{EXT}_{k,n}^{(m)}(+1) - 1 \tag{10}$$

Which can be regarded as a "local " MMSE estimate of $x_{k,n}$ assuming that the a posteriori pmf of $x_{k,n}$ is $\text{EXT}_{k,n}^{(m)}(a)$.

Estimation of the user complex amplitudes. Let $\mathbf{w} = (w_1, \ldots, w_K)^T$, denote the vector of complex amplitudes to be estimated. The ML estimate of $\mathbf{w}$ given the observation $\mathbf{Y}$ is given by

$$\mathbf{w}^{\text{ML}} = \arg\max_{\mathbf{w}} \log p(\mathbf{Y}|\mathbf{w}) \tag{11}$$

where $p(\mathbf{Y}|\mathbf{w})$ is the conditional pdf of the observed signal given $\mathbf{w}$, given by

$$p(\mathbf{Y}|\mathbf{w}) \propto \sum_{\mathbf{x}^1 \in \mathcal{C}_1} \cdots \sum_{\mathbf{x}^K \in \mathcal{C}_K} \exp\left(-\frac{1}{N_0}\sum_{n=1}^{N} |\mathbf{y}_n - \mathbf{S}\mathcal{X}_n\mathbf{x}|^2\right) \tag{12}$$

where we have defined the diagonal matrix $\mathcal{X}_k = \text{diag}(x_{1,n}, \ldots, x_{K,n})$ and where we have used the fact that the channel input $\mathbf{X}$ is independent of the channel amplitudes, so that $\Pr(\mathbf{X}|\mathbf{w}) = \Pr(\mathbf{X}) =$ uniform on the Cartesian product of the code books $\mathcal{C}_1 \times \cdots \times \mathcal{C}_K$ and zero outside. From (12) it is clear that direct ML estimation of $\mathbf{w}$ is infeasible in any practical case, as it has complexity proportional to the total number of user code words.

Now, assume that the estimate $\widehat{\mathbf{w}}^{(m)}$ and the a posteriori probability $\Pr(\mathbf{X}|\mathbf{Y}, \widehat{\mathbf{w}}^{(m)})$ are available at iteration m. Then, we can produce an updated estimate $\widehat{\mathbf{w}}^{(m+1)}$ for next iteration by following the EM approach. In the language of the EM algorithm [17], $\mathbf{Y}$, $\mathbf{X}$ and $\{\mathbf{Y}, \mathbf{X}\}$ play the role of incomplete, missing and complete data. The EM update consists of computing the expected log-likelihood function of the complete data conditionally on the incomplete data and on the current parameter estimate (E-step), and maximizing the result with respect to the parameter (M-step) [31]. In our case, the complete data log-likelihood function is given by

$$\log p(\mathbf{Y}, \mathbf{X}|\mathbf{w}) \doteq \frac{2}{N_0}\text{Re}\{\mathbf{r}^H\mathbf{w}\} - \frac{1}{N_0}\mathbf{w}^H\mathbf{R}\mathbf{w} \tag{13}$$

where we define the vector $\mathbf{r} = \sum_{n=1}^{N} \mathcal{X}_n\mathbf{S}^H\mathbf{y}_n$ and the $K \times K$ matrix $\mathbf{R} = \sum_{n=1}^{N} \mathcal{X}_n\mathbf{S}^H\mathbf{S}\mathcal{X}_n$. By using (13) we obtain the E-step in the form

$$Q(\mathbf{w}, \widehat{\mathbf{w}}^{(m)}) \doteq \frac{2}{N_0}\text{Re}\{\bar{\mathbf{r}}^H\mathbf{w}\} - \frac{1}{N_0}\mathbf{w}^H\bar{\mathbf{R}}\mathbf{w} \tag{14}$$

where we let $\bar{\mathbf{r}} = E[\mathbf{r}|\mathbf{Y}, \widehat{\mathbf{w}}^{(m)}]$ and $E[\mathbf{R}|\mathbf{Y}, \widehat{\mathbf{w}}^{(m)}]$. These are given by $\bar{\mathbf{r}} = \sum_{n=1}^{N} \overline{\mathcal{X}}_n\mathbf{S}^H\mathbf{y}_n$ and by

$$[\bar{\mathbf{R}}]_{i,j} = \begin{cases} N & \text{for } i = j \\ \mathbf{s}_i^H\mathbf{s}_j \sum_{n=1}^{N} \overline{x_{i,n}x_{j,n}} & \text{for } i \neq j \end{cases}$$

where $\overline{\mathcal{X}}_n = \text{diag}(\bar{x}_{1,n}, \ldots, \bar{x}_{K,n})$ and where $\bar{x}_{k,n}$ and $\overline{x_{k,n}x_{j,\ell}}$ denote the first and second moments of the joint a posteriori pmf $\Pr(\mathbf{X}|\mathbf{Y}, \widehat{\mathbf{w}}^{(m)})$. By noticing that (14) is a quadratic form in $\mathbf{w}$ and that $\bar{\mathbf{R}}$ is non-negative definite, the M-step is readily obtained as

$$\widehat{\mathbf{w}}^{(m+1)} = \arg\max_{\mathbf{w}} Q(\mathbf{w}, \widehat{\mathbf{w}}^{(m)}) = \bar{\mathbf{R}}^{-1}\bar{\mathbf{r}} \tag{15}$$

The above procedure has still complexity exponential in K , since the computation of the first and second order moments of $\Pr(\mathbf{X}|\mathbf{Y}, \widehat{\mathbf{w}}^{(m)})$ is equivalent to the marginalization of the joint a posteriori pmf. Then, we shall apply the above EM step "locally", i.e., by replacing $\Pr(\mathbf{X}|\mathbf{Y}, \widehat{\mathbf{w}}^{(m)})$ by the product of the marginal APPs produced by the SISO decoders at the end of iteration m. Namely, we use the approximation $\Pr(\mathbf{X}|\mathbf{Y}, \widehat{\mathbf{w}}^{(m)}) \approx \prod_{k=1}^{K}\prod_{n=1}^{N} \text{APP}_{k,n}^{(m)}(x_{k,n}$ Thanks to the product form, the exponential complexity of the moment computation is reduced to linear. In fact, the moments of the product pmf are given by

$$\begin{aligned} \widetilde{x}_{k,n} &= 2\text{APP}_{k,n}^{(m)}(+1) - 1 \\ \widetilde{x_{k,n}x_{j,\ell}} &= \begin{cases} 1 & \text{for } (k,n) = (j,\ell) \\ \widetilde{x}_{k,n}\widetilde{x}_{j,\ell} & \text{otherwise} \end{cases} \end{aligned} \tag{16}$$

Finally, the proposed approximated EM updating step consists of computing (15) where $\bar{\mathbf{R}}$ and $\bar{\mathbf{r}}$ are approximated by replacing the true moments by their approximations (16).

Initialization and combining with the training phase. The overall iterative soft-SIC algorithm needs a sufficiently reliable initial estimate $\widehat{\mathbf{w}}^{(0)}$ of the complex user amplitudes. For the sake of initialization, a joint ML estimate of the complex amplitudes is obtained from the training phase. This is readily given by $\widehat{\mathbf{w}}^{(t)} = \left(\mathbf{R}^{(t)}\right)^{-1}\mathbf{r}^{(t)}$ where $\mathbf{r}^{(t)}$ and $\mathbf{R}^{(t)}$ have the same expression of $\mathbf{r}$ and $\mathbf{R}$, respectively, when replacing N by T and the sum is exptedned over the (known) training symbols. If the training sequences are mutually orthogonal, i.e., such that $\mathbf{X}^{(t)}(\mathbf{X}^{(t)})^H = T\mathbf{I}$, we obtain $\mathbf{R}^{(t)} = T\mathbf{I}$ and no matrix inverse is needed. It can be shown that this choice also minimizes the estimation error variance.

The receiver is initialized by letting $\widehat{\mathbf{w}}^{(0)} = \widehat{\mathbf{w}}^{(t)}$. Then, at iterations $m = 1, 2, \ldots$, the receiver exploits the updated estimate $\widehat{\mathbf{w}}^{(m)}$ provided by the EM step (15) by combining it in some way with the training-based estimate. We propose a method for combining the training-based and the EM-based parameter estimates which is much more effective than simply including the training symbols as completely known symbols into the EM computation (see [18] for details and comparison).

Assume for simplicity that the training sequences are mutually orthogonal (the optimal case). Then, $\widehat{\mathbf{w}}^{(t)} = \mathbf{w} + \boldsymbol{\eta}^{(t)}$ with $\boldsymbol{\eta}^{(t)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \frac{N_0}{T}\mathbf{I})$. In particular, the training-based estimator $\widehat{\mathbf{w}}^{(t)}$ is unbiased. From the definition of $\bar{\mathbf{r}}$ and (1) we obtain

$$\bar{\mathbf{r}} = \sum_{n=1}^{N} \widetilde{\mathcal{X}}_n \mathbf{S}^H(\mathbf{S}\mathcal{X}_n\mathbf{w} + \mathbf{n}_n) = \mathbf{R}'\mathbf{w} + \boldsymbol{\eta} \tag{17}$$

where $\mathbf{R}' = \sum_{n=1}^{N} \widetilde{\mathcal{X}}_n \mathbf{S}^H\mathbf{S}\mathcal{X}_n$ and $\boldsymbol{\eta} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{R}'')$ with $\mathbf{R}'' = \sum_{n=1}^{N} \widetilde{\mathcal{X}}_n \mathbf{S}^H\mathbf{S}\widetilde{\mathcal{X}}_n$. By using this into (15) we have $\widehat{\mathbf{w}}^{(m)} = \bar{\mathbf{R}}^{-1}\mathbf{R}'\mathbf{w} + \bar{\mathbf{R}}^{-1}\boldsymbol{\eta}$, and since $\bar{\mathbf{R}} \neq \mathbf{R}'$ (unless the code symbols are perfectly known), the result of EM is biased. For sufficiently large N the following approximations hold

$$\begin{aligned}
\bar{\mathbf{R}} &\approx N\mathbf{I} \\
\mathbf{R}' &\approx \operatorname{diag}\left(\sum_{n=1}^{N} x_{1,n}\widetilde{x}_{1,n}, \ldots, \sum_{n=1}^{N} x_{K,n}\widetilde{x}_{K,n}\right) \\
\mathbf{R}'' &\approx \operatorname{diag}\left(\sum_{n=1}^{N} |\widetilde{x}_{1,n}|^2, \ldots, \sum_{n=1}^{N} |\widetilde{x}_{K,n}|^2\right)
\end{aligned} \tag{18}$$

With these approximations we can express the biased EM estimate of user k amplitude as $\widehat{w}_k^{(m)} = \alpha_k w_k + \eta'_k$ where $\alpha_k = \frac{1}{N}\sum_{n=1}^{N} x_{k,n}\widetilde{x}_{k,n} \approx (1 - 2\epsilon_k^{(m)})\frac{1}{N}\sum_{n=1}^{N} |\widetilde{x}_{k,n}|$, $\eta'_k \sim \mathcal{N}_{\mathbb{C}}(0, \frac{N_0}{N}\beta_k^2)$ with $\beta_k^2 = \frac{1}{N}\sum_{n=1}^{N} |\widetilde{x}_{k,n}|^2$, and where $\epsilon_k^{(m)}$ is the symbol error probability at the SISO decoder output of user k and iteration m. Our goal is to obtain a combined estimator in the form $\widehat{\widehat{w}}_k^{(m)} = a_k\widehat{w}_k^{(m)} + b_k\widehat{w}_k^{(t)}$ where the coefficients a_k, b_k are chosen in order to minimize the error variance subject to the unbiased constraint, i.e., they are the solution of

$$\begin{cases} \text{minimize} & E[|a_k\eta'_k + b_k\eta_k^{(t)}|^2] \\ \text{subject to} & a_k\alpha_k + b_k = 1 \end{cases}$$

whose solution is readily obtained as $a_k\alpha_k/(\alpha_k^2 + (T/N)\beta_k^2)$ and $b_k = (T/N)\beta_k^2/(\alpha_k^2 + (T/N)\beta_k^2)$. This combining method provides an approximately unbiased estimate at each iteration. At the first iterations, when $\epsilon_k^{(1)} \approx 1/2$, only the result of training-based estimation is used. As the soft-SIC cleans-up the signal from interference and $\epsilon_k^{(m)}$ becomes small (converging the single-user performance), then $\alpha_k \approx \beta_k^2 \approx 1$ and $a_k \approx \frac{N}{T+N}$, $b_k \approx \frac{T}{T+N}$. These limiting values are precisely the maximal-ratio combining coefficient for estimating $\mathbf{w}$ from the unbiased noisy observations $\mathbf{w} + \boldsymbol{\eta}^{(t)}$ and $\mathbf{w} + \boldsymbol{\eta}$.

4 Results and conclusions

For simulation we considered a power controlled CDMA system with $K = 32$ users, $L = 16$, $N = 2000$ and training length $T = 4$ or $T = 32$. The SIC is limited to $M = 10$ iterations. Spreading sequences are randomly and independently generated with QPSK chips. All users make use of the 4-state rate 1/2 convolutional code with generators $(5, 7)_8$. Figs. 1 show the BER (worst user) of the system with $T = 32$ and Figs. 2 show analogous results for $T = 4$. Training-only estimation prevents the receiver to achieve the single-user BER, since interference cannot be canceled completely because of the estimation errors which do not vanish with iterations. Remarkably, the EM-based method converges to single user performance despite the large channel load and the small training available.

Even though similar algorithms can be found (with minor variations) in several other works, here we investigated in the details several new important aspects, namely: a simple and efficient way to estimate the residual

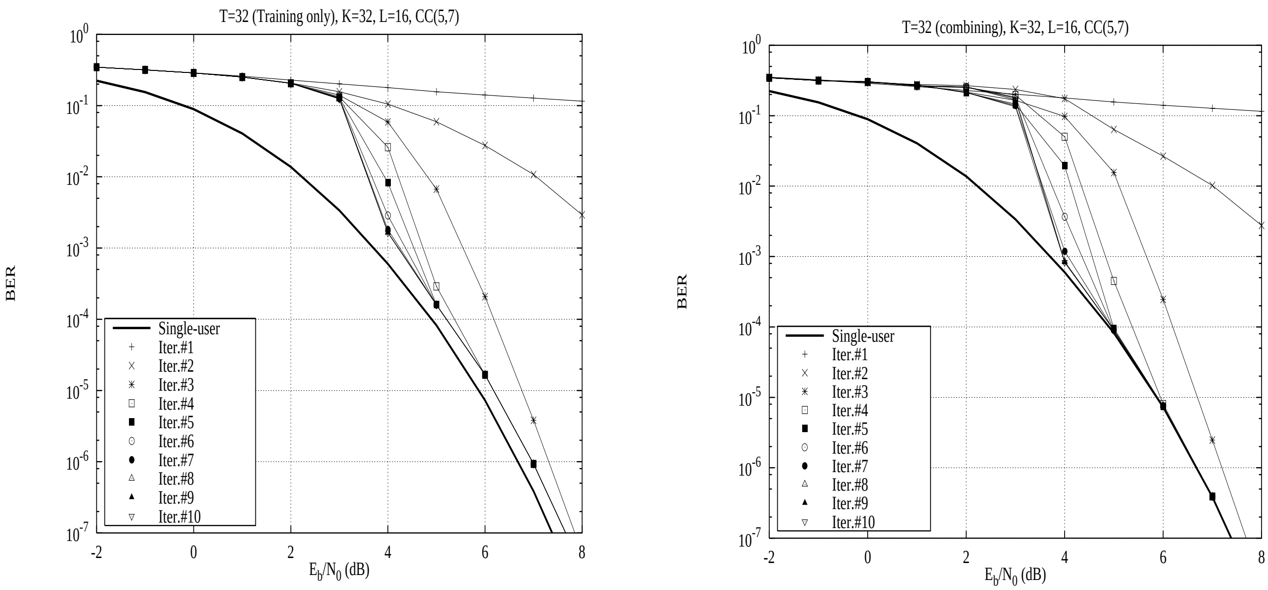

Figure 1: $K = 32$, $L = 16$, $T = 32$. Left: training estimation only. Right: EM-based joint parameter estimation.

interference plus noise variance at the SISO inputs; the issue of soft interference estimation based on EXT pmfs versus the conventional approach of using APPs; the correct formulation of EM estimation with channel coding, and the key approximation to bring complexity from exponential down to polynomial in the number of users; the use of training-based estimation together with EM updating. In particular, we provided a new method for combining the unbiased channel estimates provided by ML training-based estimation with the biased estimates provided by EM.

The full investigation of the optimal trade-off between training symbols fraction T/N and channel load K/L is out of the scope of this paper. However, from the simulation results shown here we can get some conclusions on the overall benefit of the proposed approach.

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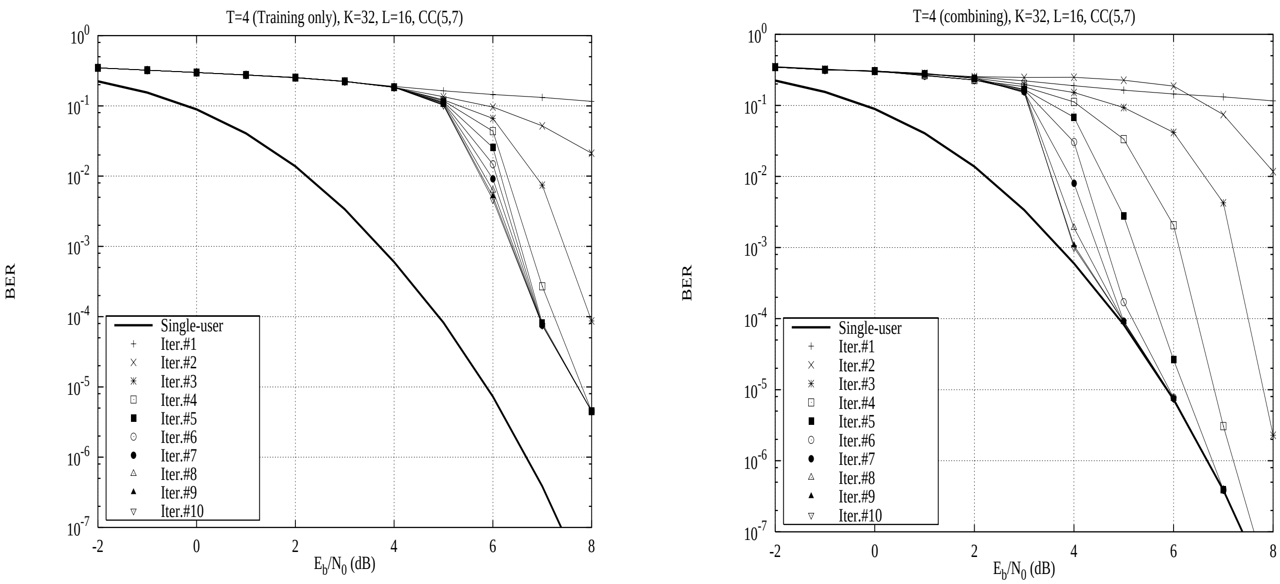

Figure 2: $K = 32$, $L = 16$, $T = 4$. Left: training estimation only. Right: EM-based joint parameter estimation.

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