Design approaches for LDPC-encoded CDMA

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Abstract

We investigate the spectral efficiency achievable by random synchronous CDMA with QPSK modulation and binary error-control codes, in the large system limit where the number of users, the spreading factor and the code block length go to infinity. For given LDPC code ensembles, we provide methods to optimize the spectral efficiency assuming a successive stripping decoder, for the cases of equal rate and equal power users. Finally, we show that the proposed methods apply to practical (fixed block length) codes and yield in fact a simple and effective tool for designing encoders and decoders for CDMA.

1 Introduction

All points in the capacity region of the Gaussian multiple-access channel are achievable by successive single-user decoding and interference cancellation (stripping) [1, 2]. This generalizes to the CDMA channel as shown in [3], where the optimal stripping decoder incorporates MMSE filters against yet undecoded users at each successive cancellation stage. Key to the optimality of stripping is the use of Gaussian codes of rate arbitrarily close (but not larger) than the capacity of the channel obtained by removing the already decoded users. In this way, optimal spectral efficiency is achieved by simple single-user coding and decoding, with linear complexity in the number of users. One drawback of successive stripping is that, in general, every user must transmit at a different rate, or must be received at a different SNR level, (unless the spreading waveforms are appropriately chosen, which is impractical under frequency selective fading).

Existing nonorthogonal CDMA systems [4, 5] are largely based on pseudo-random waveforms. The maximum spectral efficiency of randomly spread (synchronous) CDMA, in the large system limit, where the number of users and the spreading factor grow without bound while their ratio tends to a constant β, was found in [6]. Capacity is achieved by Gaussian user code ensembles. However, practical systems make use of discrete small-size modulation alphabets, such as QPSK. Therefore it is of interest to investigate the penalty in achievable spectral efficiency incurred by the use of QPSK.

We consider a pragmatic approach to QPSK-modulated CDMA based on applying single-user binary coding and the same stripping decoding approach which would be optimal for Gaussian user codes. Moreover, we constrain our "practical" CDMA system to have only a finite number of coding rates and/or of received SNR levels. For this setting, we compute the achievable spectral efficiency in the large system regime with optimal (i.e., single-user capacity achieving) binary codes and with the best known LDPC code ensembles [7], in the limit for large code block length, in the cases of equal received SNRs and equal rate users. Remarkably, the proposed equal power and equal rate design approaches can be effectively applied to non-asymptotic code block length, and provide a simple tool to dimension CDMA systems for given target BER, user codes, and desired spectral efficiency. In order to illustrate the effectiveness of the approach, we give an explicit example of equal power system based on a family of Irregular Repeat-Accumulate (IRA) codes [8] specially designed to cover a large range of user rates.

2 Synchronous CDMA canonical model

We consider the complex baseband discrete-time channel model

$$\mathbf{y}_i = \mathbf{S}\mathbf{A}\mathbf{x}_i + \mathbf{n}_i, \qquad i = 1, \dots, n \tag{1}$$

originated by sampling at the chip-rate a synchronous CDMA system [9], where: 1) $\mathbf{y}_i, \mathbf{n}_i \in \mathbb{C}^N$, are the vector of received chip-rate samples and the corresponding AWGN samples $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$ received at time i; 2) $\mathbf{S} \in \mathbb{C}^{N \times K}$ contains the user spreading sequences by columns; 3) $\mathbf{A} = \mathrm{diag}(A_1, \dots, A_K)$ contains the user complex amplitudes; 4) $\mathbf{x}_i \in \mathbb{C}^K$ is the vector of user modulation symbols transmitted at time i, taking on values in a unit-energy signal set; 5) N, K and n denote the spreading factor, the number of users and the code block length, respectively.

Spreading sequences are random with i.i.d. chips with zero mean, variance $1/N$ and finite fourth order moment. The k-th user per-symbol SNR is given by $|A_k|^2$. Users are partitioned into J classes. The size of class j is K_j, and we denote by $\beta_j = K_j/N$ the "class load" of class j. The total channel load is $\beta = \sum_{j=1}^{J} \beta_j$. Users in class j have the same received SNR, denoted by γ_j, where, without loss of generality, $\gamma_1 \leq \cdot \leq \gamma_J$. Let user k in class j have rate $R_{j,k}$. The total system spectral efficiency is given by $\rho = \sum_{j=1}^{J} \beta_j R_j$ where we define the average user rate in class j as $R_j = \frac{1}{K_j} \sum_{k=1}^{K_j} R_{k,j}$. The users individual E_b/N_0's are in general different. Nevertheless, for the sake of comparison with a reference equal-rate equal-power system, we define a "system" E_b/N_0 by

$$\left(\frac{E_b}{N_0}\right)_{\mathrm{sys}} \triangleq \frac{\sum_{j=1}^{J} \beta_j \gamma_j}{\sum_{j=1}^{J} \beta_j R_j} \tag{2}$$

with the meaning that $(E_b/N_0)_{\mathrm{sys}}$ coincides with the individual E_b/N_0's in the case where users are dynamically assigned to the classes so that each user belongs to class j for a fraction β_j/β of the time.

3 Existing Results on Fundamental limits

In [6] the spectral efficiency (in bit/s/Hz) of random CDMA in the large system limit ($K, N \to \infty$ with $K/N = \beta$) subject to an input power constraint is found to be

$$\mathsf{C}(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \mathsf{C}^{\mathrm{mmse}}(\boldsymbol{\beta}, \boldsymbol{\gamma}) + \log_2 \frac{1}{\eta} + (\eta - 1)\log_2 e \tag{3}$$

where $\boldsymbol{\beta} \triangleq (\beta_1, \dots, \beta_J)$ and $\boldsymbol{\gamma} \triangleq (\gamma_1, \dots, \gamma_J)$, η is the solution to the Tse-Hanly equation [10], which for later use we write as

$$\eta = f_J(\eta, \beta_J) \tag{4}$$

where we define

$$f_j(\eta, z) \triangleq \left(1 + z\frac{\gamma_j}{1 + \gamma_j \eta} + \sum_{i=1}^{j-1} \beta_i \frac{\gamma_i}{1 + \gamma_i \eta}\right)^{-1} \tag{5}$$

and where $\mathsf{C}^{\mathrm{mmse}}(\boldsymbol{\beta}, \boldsymbol{\gamma})$ is the achievable spectral efficiency of a system based on linear MMSE filtering followed by single-user decoding, given by

$$\mathsf{C}^{\mathrm{mmse}}(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \sum_{j=1}^{J} \beta_j \log_2(1 + \gamma_j \eta) \tag{6}$$

The spectral efficiencies in (3) and in (6) are achieved with codes whose empirical distributions are Gaussian.

The supremum of (3) over all possible $J, \boldsymbol{\beta}, \boldsymbol{\gamma}$ (for a fixed E_b/N_0 and β) is achieved by $J = 1$ (one class only). The supremum over β is achieved for $\beta \to \infty$, and coincides with the AWGN single-user capacity $\mathsf{C}^\star$, implicitly given by

$$\frac{2^{\mathsf{C}^\star} - 1}{\mathsf{C}^\star} = E_b/N_0 \tag{7}$$

Spectral efficiency $\mathsf{C}(\boldsymbol{\beta}, \boldsymbol{\gamma})$ can be achieved by single-user decoding with successive stripping and MMSE filtering against undemodulated users. Suppose that users are decoded one by one, starting from users in class J, then class $J-1$ and so on. Then, $\mathsf{C}(\boldsymbol{\beta}, \boldsymbol{\gamma})$ can be written as

$$\mathsf{C}(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \sum_{j=1}^{J} \int_0^{\beta_j} \log_2(1 + \gamma_j \eta_j(z)) dz \tag{8}$$

where $\eta_j(z)$ is the solution to $\eta = f_j(\eta, z)$.

4 Approaching the optimal spectral efficiency

Substantial progress has been made in the last few years in designing binary codes and decoders whose rate comes fairly close to single-user capacity at vanishing BER. Among those modern codes are Turbo codes, Repeat-Accumulate (RA) codes, and Low-Density Parity-Check codes (LDPC), all of which are decoded by efficient iterative techniques (see the special issue [11] and references therein). These code ensembles are characterized by rate-threshold pairs (R, g), such that for $\text{SNR} \geq g$ the BER can be made arbitrarily small as $n \to \infty$. The threshold g is normally fairly close (see Figure 1) to the SNR solution of $R = C_{\text{qpsk}}(\text{SNR})$, where

$$C_{\text{qpsk}}(\text{SNR}) = 2\left(1 - \int_{-\infty}^{\infty} \log_2\left(1 + e^{-2\text{SNR} - 2\sqrt{\text{SNR}}v}\right) \frac{e^{-v^2/2}}{\sqrt{2\pi}} dv\right) \tag{9}$$

is the QPSK-input AWGN channel capacity, as a function of SNR.

Then, it makes sense to design CDMA systems assuming that decoding is error-free when the decoder operates above its threshold SNR. Our goal is to find the vectors $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ so that, at each stripping decoder stage, the threshold requirement of each single-user decoder is satisfied. We shall consider two alternative design problems: (1) equal-rate, non-uniform SNR, and (2) equal-power, non-uniform rate systems.

Loss of stripping decoding with QPSK. In the large system limit, under our system assumptions, it is well-known that the residual interference at the output of the MMSE filter at any cancellation stage is complex Gaussian with circular symmetry. Assuming optimal QPSK codes characterized by the rate-threshold pairs $(R, C_{\text{qpsk}}^{-1}(R))$, for $R \in [0, 2]$, (see Fig. 1), the spectral efficiency achieved by a stripping decoder is given by

$$\mathsf{C}_{\text{qpsk}}(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \sum_{j=1}^{J} \int_0^{\beta_j} C_{\text{qpsk}}(\gamma_j \eta_j(z)) dz \tag{10}$$

Fig. 2 shows $\mathsf{C}_{\text{qpsk}}(\beta, \gamma)$ and $\mathsf{C}(\beta, \gamma)$ (for a single-class system, i.e., $J = 1$) vs. β, for $E_b/N_0 = 3$ and 10 dB. The corresponding AWGN capacity $\mathsf{C}^\star$ is shown for comparison. We notice that the QPSK loss gets more pronounced as E_b/N_0 increases but that for any fixed E_b/N_0 and sufficiently large β, the loss vanishes.

Equal-rate CDMA system design. Assume that users in all classes make use of codes drawn randomly and independently from the same family with rate-threshold pair (R, g), and that users in each class i are decoded in parallel, while classes are stripped off from J to 1, i.e., in decreasing SNR order. The SINR at the output of the MMSE filter for class i, assuming that classes $i+1, \ldots, J$ have been perfectly canceled, is given by $\gamma_i \eta_i(\beta_i)$. Hence, the condition for successive decodability of all users is $\eta_i(\beta_i) \geq g/\gamma_i$, for all $i = 1, \ldots, J$.

We fix the received power levels $\boldsymbol{\gamma}$, and consider the optimization of the class loads $\boldsymbol{\beta}$. Without loss of generality, we assume $\gamma_1 \geq g$, since for all j such that $\gamma_j < g$, we would have trivially $\beta_j = 0$. This problem can be formulated as a linear programming problem as follows. Because of the monotonicity of the function in (5), if $\eta_j(z)$ satisfies $\eta_j(z) = f_j(\eta_j(z), z)$, then

$$\forall \; x \in [0, \infty[\qquad x \leq \eta_j(z)^* \Leftrightarrow x \leq f_j(x, z) \tag{11}$$

Accordingly, the successive decodability condition is equivalent to

$$\left(1 + \sum_{j=1}^{i} \beta_j \frac{\gamma_j}{1 + \gamma_j \frac{g}{\gamma_i}}\right)^{-1} \geq \frac{g}{\gamma_i}, \quad \forall \; i = 1, \ldots, J \tag{12}$$

which can be written in compact form as $\mathbf{A}\boldsymbol{\beta} \leq \mathbf{b}$, where $\mathbf{A}$ is a $J \times J$ lower triangular matrix with non-zero elements

$$a_{i,j} = \frac{(1+g)\gamma_j}{\gamma_i + \gamma_j g} \in]0, 1] \tag{13}$$

and $\mathbf{b}$ is a positive vector with elements

$$b_i = \frac{(1+g)(\gamma_i - g)}{\gamma_i g} \tag{14}$$

Notice that $a_{i,i} = 1$, $a_{i,j}$ (for $1 \leq j \leq i$) is increasing with j and decreasing with i and b_i is increasing with i.

For a desired spectral efficiency $\rho = \beta R$, the optimal vector $\boldsymbol{\beta}$ which achieves (if possible) arbitrarily small BER with minimal $(E_b/N_0)_{\text{sys}}$ is the solution of the linear program:

$$\left\{ \begin{array}{ll} \text{minimize} & \sum_{i=1}^{J} \beta_i \gamma_i \\ \text{subject to} & \mathbf{A}\boldsymbol{\beta} \leq \mathbf{b} \\ & \sum_{i=1}^{J} \beta_i \geq \beta, \\ & \boldsymbol{\beta} \geq \mathbf{0} \end{array} \right. \tag{15}$$

We have the following result:

Proposition 1. Let $\hat{\mathbf{x}}$ be the solution of the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$. The feasible set in (15) is nonempty if and only if $\beta \leq \sum_{j=1}^{J} \hat{x}_j$. Denote by $\hat{J}$ the minimum i for which $\beta \leq \sum_{j=1}^{i} \hat{x}_j$, then the solution of (15) is given explicitly by

$$\beta_i^\star = \left\{ \begin{array}{ll} \tau_i - \tau_{i+1}, & i = 1, \ldots, \hat{J} \\ 0 & i > \hat{J} \end{array} \right. \tag{16}$$

where $\tau_{\hat{J}+1} = 0$ and $\tau_i = \beta - \sum_{j=1}^{i-1} \hat{x}_j$ for $i = 1, \ldots, \hat{J}$.

The proof follows by finding an equivalent problem whose dual has a polymatroid structure [12], and by showing that the objective function computed for solution (16) and that of the dual problem yield the same value (the details are given in [13]). □

Equal-power CDMA system design. We consider a system with the same class-by-class successive decoding scheme from class J to class 1, as before. Classes have all the same SNR γ, but each class j makes use of a different code ensemble characterized by the rate-threshold pair (R_j, g_j). We assume $R_1 \geq \cdots \geq R_J$ and $g_1 \geq \cdots \geq g_J$. For example, the pairs (R_j, g_j) can be obtained by sampling the curve of Fig. 1 (assuming optimal binary random codes) or by taking the points corresponding to good existing LDPC codes [7] (see again Fig. 1). Without loss of generality, we assume $\gamma \geq g_1$, since for all j such that $\gamma < g_j$, we would have trivially $\beta_j = 0$.

The successive decodability condition is now given by $\eta_i(\beta_i) \geq \frac{g_i}{\gamma}$ for all $i = 1, \ldots, J$, which translates into

$$\sum_{j=1}^{i} \beta_j \leq b_i, \qquad i = 1, \ldots, J \tag{17}$$

with

$$b_i = \frac{(1 + g_i)(\gamma - g_i)}{\gamma g_i} \tag{18}$$

using again property (11). Hence, for given rate-threshold pairs (R_j, g_j), the spectral efficiency $\rho = \sum_{i=1}^{J} \beta_i R_i$ maximized over the class loads is obtained as the solution of the following linear program:

$$\left\{ \begin{array}{ll} \text{maximize} & \sum_{i=1}^{J} \beta_i R_i \\ \text{subject to} & \mathbf{L}\boldsymbol{\beta} \leq \mathbf{b} \\ & \sum_{i=1}^{J} \beta_i \leq \beta, \\ & \boldsymbol{\beta} \geq \mathbf{0} \end{array} \right. \tag{19}$$

where $\mathbf{L}$ is a lower triangular $J \times J$ matrix with non-zero elements all equal to 1 and where $\mathbf{b} = (b_1, \ldots, b_J)^T$ with b_i given in (18). We have the following result:

Proposition 2. The problem (19) is always feasible, and its solution satisfies $\sum_{i=1}^{J} \beta_i = \beta$ if and only if $\beta \leq b_J$. Denote by $\hat{J}$ the minimum i for which $\beta \leq b_i$, then the solution to (19) is given explicitly by

$$\beta_i^{\star} = \left\{ \begin{array}{ll} b_i - b_{i-1}, & i = 1, \ldots, \hat{J} - 1 \\ \beta - b_{\hat{J}-1}, & i = \hat{J} \\ 0 & i > \hat{J} \end{array} \right. \tag{20}$$

where $b_0 \stackrel{\Delta}{=} 0$.

The proof follows again by finding an equivalent problem with the polymatroid structure (the details are given in [13]). □

5 Numerical examples

In this section we give some examples of the equal-rate and the equal-power system designs. First, we consider good (infinite length) LDPC codes found in [7]. Then, we apply the optimization approach to finite-dimensional systems, based on explicitly designed IRA codes, with finite block length.

Spectral efficiency with LDPCs. In Fig. 3, the curves denoted by "LDPC" and "discr.QPSK" are the spectral efficiencies achieved by the equal-power design with the LDPC code family found in [7] with rate-threshold pairs corresponding to the marks in Fig. 1, and rate-threshold pairs obtained by sampling the QPSK capacity curve from $R = 0.05$ to 1.95 with step 0.1. The curves denoted by "QPSK, $\beta = 1, 5, 10, 20$" are the spectral efficiencies given by formula (10), achievable with optimal QPSK codes using infinitely fine sampling of the rate-threshold capacity curve. $\mathsf{C}^{\star}$ is shown for comparison. We observe that in order to approach $\mathsf{C}^{\star}$, it is necessary to have many different classes. Even a relatively finely discretized distribution of rates (such as curve "discr.QPSK") is quite far from $\mathsf{C}^{\star}$. Moreover, in order to approach $\mathsf{C}^{\star}$ at high $(E_b/N_0)_{\rm sys}$ a very high value of β is required. As expected, the low $(E_b/N_0)_{\rm sys}$ behavior of spectral efficiency is dominated by the class with lowest coding rate (and SNR threshold). In fact, the value at which spectral efficiency becomes zero is given by g_J/R_J, which is the minimum E_b/N_0 to have a vanishing fraction of users at non-zero rate.

It is worthwhile to mention that the equal-power spectral efficiency curves are obtained as the upper envelope of the solution of (19), over all $\gamma \geq g_J$ and $\beta \in [0, b_J]$, i.e., for all pairs (γ, β) for which the solution (20) holds.

In Fig. 4, the curves denoted by "LDPC, R=0.2, 1.0, 1.8" are the spectral efficiencies achieved by the equal-rate design with the LDPC codes of rate 0.2, 1.0 and 1.8 bit per QPSK symbol (corresponding to binary rate 0.1, 0.5 and 0.9), in the family of Fig. 1. The equal-rate spectral efficiency curves were obtained considering increasing values of β, and, for each β, a vector $\boldsymbol{\gamma}$ obtained by discretizing the interval $[g, \bar{\gamma}(\beta)]$ with step of 0.01 dB, where $\bar{\gamma}(\beta)$ is the minimum γ_J for which the feasible set of (15) is non-empty. The variable-rate design is able to approach quite closely $\mathsf{C}^\star$ for low user coding rate, at the price of a very large load β and a large number of power levels.

5.1 Practical finite-dimensional systems based on IRAs

In order to validate the equal-rate and equal-power CDMA system optimization in the case of practical finite-length codes and finite spreading factor, we consider the family of IRA codes of block length $n = 2000$ (binary symbols) given in Table 1, explicitly designed to cover a large range of rates (see [13] for the details). The SNR thresholds were determined by simulation, for a target BER of 10^{-4}. IRA codes are decoded by the standard "belief-propagation" iterative message-passing decoder [8, 13]. We plug the rate-threshold pairs given in Table 1 in our asymptotic equal-rate and equal-power system optimization. The actual number of users per each class is obtained by fixing a desired spreading factor N and by letting $K_j = \lfloor \beta_j N \rfloor$, where β_j is the partial load for class j, resulting from optimization. Table 2 presents two design examples for the codes of Table 1 and $\rho \approx 2$ bit/s/Hz.

In a finite-dimensional system users have not all the same SNR degradation η (or Asymptotic Multiuser Efficiency [9]) with respect to their single-user SNR, at every stripping decoder step, because of the asymmetry due to finite-length random sequences. Fig. 5 shows a snapshot simulation of the evolution of the AME vs. the stripping decoder iterations for the systems of Table 2. At each stripping decoder step, all users of the corresponding class is decoded in parallel, with 15 IRA decoder iterations per user. The minimum and maximum AME (over the user population) for the two systems are indicated by the vertical intervals. The equal rate system (squares) converges to single-user performance for all users in exactly 9 steps, i.e., one step per class is sufficient despite the residual randomness of the finite-dimensional system. On the contrary, the equal power system (diamonds) needs 14 stripping decoder steps (twice the requirement of the infinite-dimensional system).

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Rate (QPSK)	g (dB), 12 iterations	g (dB), 20 iterations	Code name
1.8030	7.1103	7.1103	code 1
1.6030	5.4103	5.3103	code 2
1.4020	4.3603	4.3603	code 3
1.2050	3.1003	3.1003	code 4
1.0020	2.2603	2.2603	code 5
0.7970	1.5303	1.5303	code 6
0.5880	-0.7997	-0.8997	code 7
0.4580	-1.2697	-1.2697	code 8
0.1930	-4.4897	-4.4897	code 9

Table 1: Table of IRA codes and thresholds for BER $\leq 10^{-4}$, QPSK modulation and block length 2000 (binary symbols).

System	γ (dB)	Code	Users per class K_j
Equal power	8.2497	1	8
$\rho = 2.0651$		2	10
$E_b/N_0 = 7.6831$ dB		3	6
$\beta = 1.8125$		4	7
		5	5
		6	4
		7	18
Equal rate	3.3035	5	10
$\rho = 2.0666$	4.3035		9
$E_b/N_0 = 7.5836$ dB	5.3035		8
$\beta = 2.0625$	6.3035		7
	7.3035		7
	8.3035		7
	9.3035		7
	10.3035		6
	11.3035		5

Table 2: Equal power and equal rate finite-dimensional system design examples, for $N = 32$.

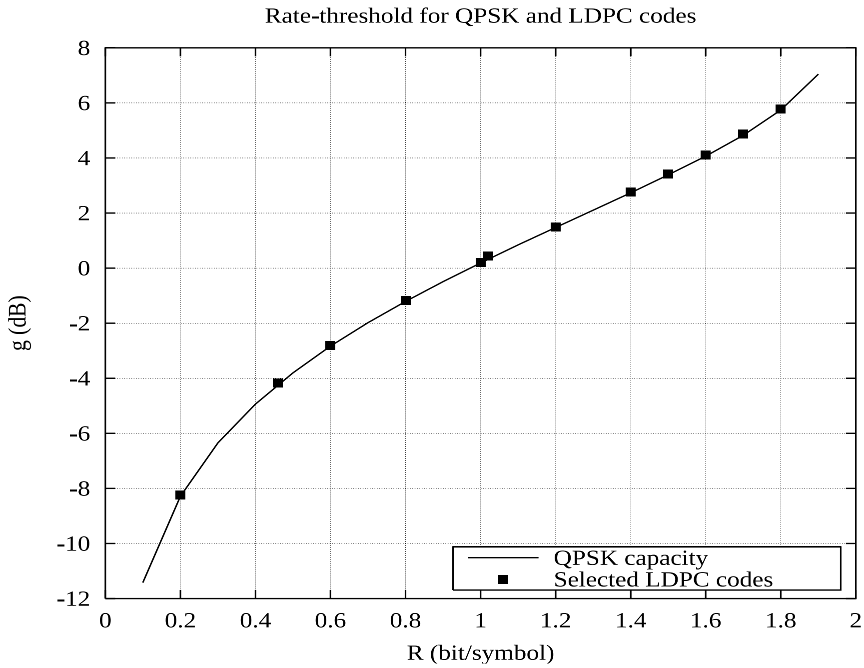

Figure 1: Rate-threshold pairs corresponding to QPSK capacity and for some LDPC codes from [7].

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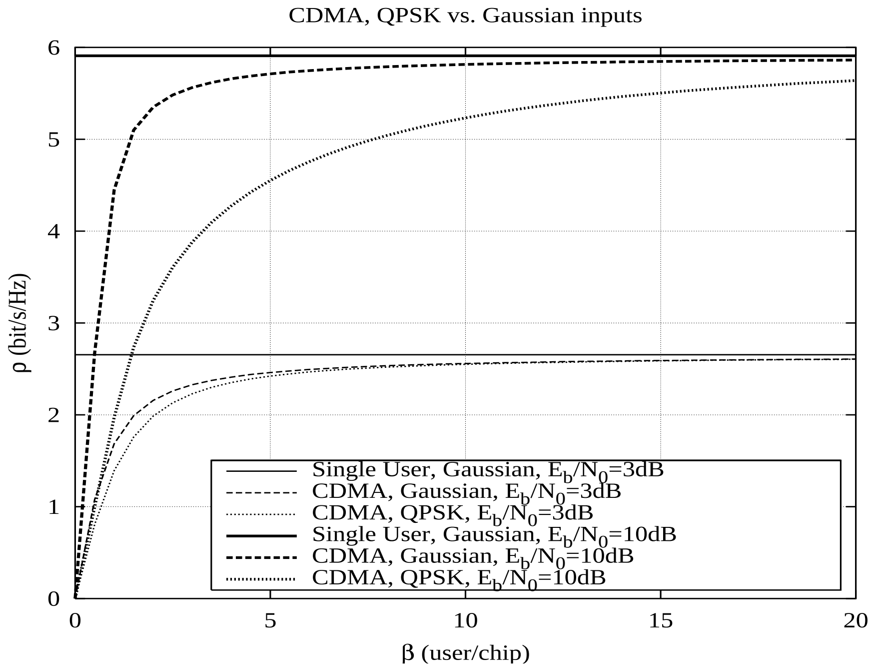

Figure 2: Spectral efficiency vs. β for random CDMA with Gaussian and QPSK inputs (with stripping decoder).

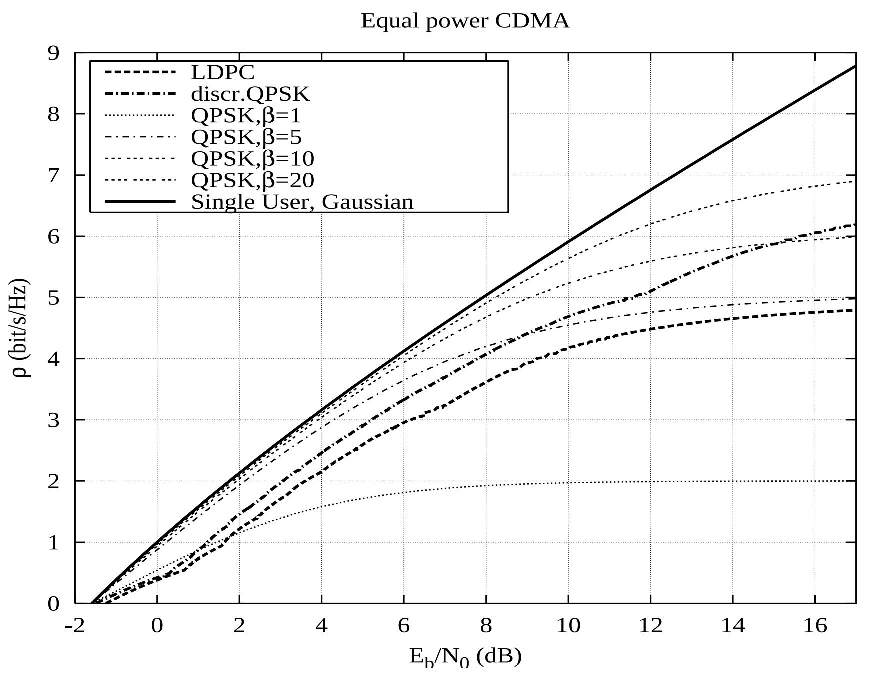

Figure 3: Spectral efficiency of LDPC and optimal QPSK codes with equal-power design.

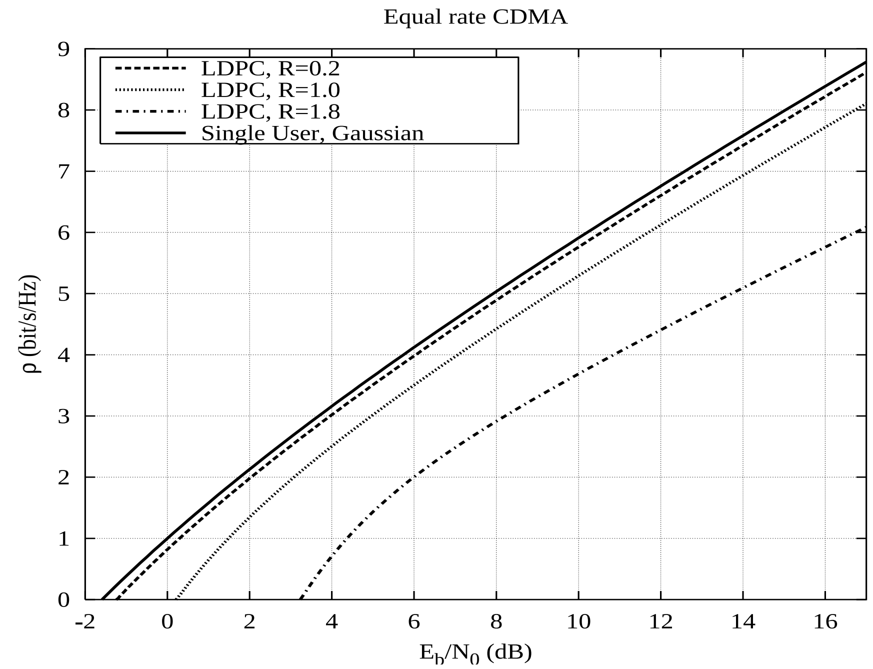

Figure 4: Spectral efficiency of some LDPC codes with equal-rate design.

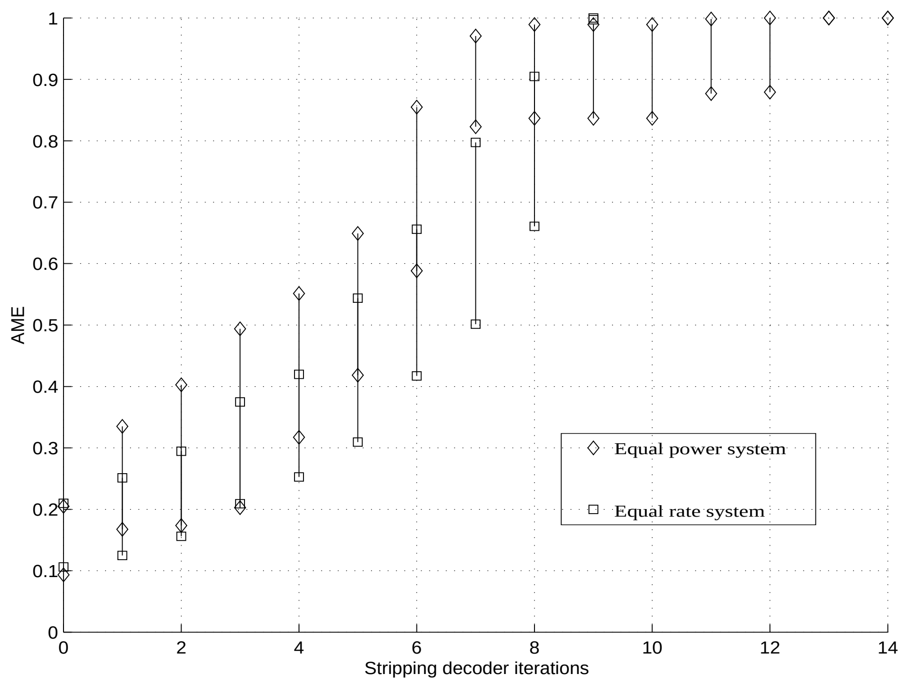

Figure 5: AME vs. stripping decoder iterations (snapshot simulation) for the systems of Table 2.