Importance-aware Sampling of a Two-State Markov Source for Real-time Tracking

Mehrdad Salimnejad*, Marios Kountouris†, and Nikolaos Pappas*
*Department of Computer and Information Science, Linköping University, Linköping, Sweden
†Communication Systems Department, EURECOM, Sophia-Antipolis, France
{mehrdad.salimnejad, nikolaos.pappas}@liu.se, marios.kountouris@eurecom.fr

***Abstract*—This work considers the problem of real-time remote tracking and reconstruction of a two-state Markov process for actuation. The transmitter sends samples from an observed information source to a remote monitor over an unreliable wireless channel. We propose a state-aware randomized stationary sampling and transmission policy, which considers the importance of different states and their impact on the communication objective. We then analyze the performance of the proposed policy and compare it with existing goal-oriented joint sampling and transmission policies using relevant metrics. Specifically, we assess the real-time reconstruction error, the cost of actuation error, and the consecutive error metrics. Furthermore, a constrained optimization problem is formulated and solved so as to minimize the average cost of actuation error by determining optimal sampling probabilities. Our results show that the optimal state-aware randomized stationary policy outperforms other policies in scenarios with constrained sampling for fast-evolving sources. In addition, when the source changes slowly, although the semantics-aware policy tends to be more effective, the optimal state-aware randomized stationary policy excels under certain conditions.**

I. Introduction

An increasing number of applications and services today rely on cyber-physical and real-time communication systems for remote monitoring and control. These systems require reliable, effective, and timely acquisition, processing, and transport of large amounts of data sources. In this regard, a new communication paradigm has been proposed, which takes into account the *semantics of information* [1]. This paradigm considers the significance, goal-oriented usefulness, and contextual importance of information as a means to generate, transmit, and reconstruct data in time-sensitive and data-intensive communication systems. A highly relevant yet challenging problem in this context is the design of techniques that incorporate joint source sampling, transmission, and reconstruction, which consider the dynamics of the information source, enabling real-time remote tracking for actuation. Most prior works on remote tracking have mainly proposed sampling/scheduling policies to minimize the estimation error, disregarding the significance and utility of the information generated and transmitted for specific goals and contextual requirements. The works [2]–[4] consider the scheduling problem in event-triggered estimation. Optimal estimation and transmission policies for remote estimation over time-varying packet drop channels are analyzed in [5]. The works [6] and [7] have studied optimal sampling strategies for real-time remote monitoring and estimation of stochastic processes. Various approaches to remote state estimation under communication constraints for linear time-invariant (LTI) systems are investigated in [8] and [9]. The main objective of the aforementioned studies is to propose sampling and transmission strategies that minimize estimation errors, without considering the utilization of information. Recent works [1], [10]–[15] have introduced metrics that capture the semantics and effectiveness of information. These metrics aim to leverage the synergies between data processing, information transmission, and signal reconstruction.

This paper considers the problem of real-time remote tracking of an information source in a time-slotted communication system, where a two-state Markov process is sampled and transmitted as packets to a remote receiver via an unreliable wireless channel. The receiver then takes a specific action based on the estimated state of the information source. This work extends the results of [10] and [14], which have investigated real-time tracking and reconstruction of an information source for actuation purposes. We propose a sampling and transmission policy named *state-aware randomized stationary*, which accounts for varying sampling and success probabilities depending on the information source's states. This is important when the states signify actuation commands or other tasks of varying significance, emphasizing the need for distinct sampling frequencies. Furthermore, we analyze the performance of our proposed strategy in terms of time-averaged reconstruction error, cost of actuation error, and consecutive error metrics. Then, we compare our approach with previously proposed joint sampling and transmission policies [10] and [14]. We also solve an optimization problem with the objective to minimize the average cost of actuation error subject to a time-averaged sampling cost constraint. Our analysis establishes the regimes and the conditions under which the proposed policy outperforms state-of-the-art counterparts.

II. System Model

In this paper, we study a time-slotted communication system that involves sampling an information source $X(t)$ at each time slot t. The sampled data is then transmitted to the receiver over a wireless channel. The receiver, operating as

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an actuator, performs actions based on the reconstructed state of the information source. To model the information source, we use a discrete-time Markov chain (DTMC) with two states, represented as $X(t), t \in \mathbb{N}$. Therein, the self-transition probability and the probability of transition to another state at time slot $t+1$ are defined as $\Pr\big[X(t+1)=i\big|X(t)=j\big] = \mathbb{1}(i=0,j=0)(1-p)+\mathbb{1}(i=0,j=1)q+\mathbb{1}(i=1,j=0)p+\mathbb{1}(i=1,j=1)(1-q)$.

In this paper, we consider various sampling and transmission strategies for the states of the information source. We designate the act of sampling at time slot t when the information source is in state i $(i=0,1)$ as $\alpha_i^{\mathrm{s}}(t)$. If $\alpha_i^{\mathrm{s}}(t)$ is equal to 1, it indicates that the source in state i is sampled; otherwise, it remains at 0. Furthermore, the transmission action associated with $\alpha_i^{\mathrm{s}}(t)=1$ is denoted as $\alpha_i^{\mathrm{tx}}(t)$. When $\alpha_i^{\mathrm{tx}}(t)=1$, it signifies that the sampled data is transmitted, while $\alpha_i^{\mathrm{tx}}(t)=0$ implies that the transmitter remains idle. At each time slot t, the receiver constructs an estimate $\hat{X}(t)$ of the process $X(t)$ based on successfully received samples. The channel state $h_i(t)$ assumes the value 1 if the information source in state i is both sampled and decoded successfully by the receiver; otherwise, it is 0. To quantify the success probability of sampling and transmission when the information source is in state i, we define $p_{\mathrm{s}_i}=\Pr\big[h_i(t)=1\big]$. Note that incorporating different success probabilities opens up intriguing possibilities for implementing simple state-aware power control. The transmitter receives immediate and error-free feedback in the form of acknowledgment (ACK) or negative-ACK (NACK) packets, enabling it to have perfect knowledge of the reconstructed source state at time slot t $\big(\hat{X}(t)\big)$[1]. Additionally, we assume that a sample is discarded if its transmission fails.

III. Sampling and Transmission Policies

We introduce a novel sampling and transmission policy called the *state-aware randomized stationary* policy. This policy involves triggering sample generation in a probabilistic manner at each time slot. Specifically, we propose a scheme that enables assigning distinct sampling probabilities to different states. This allows for adjusting the sampling frequency based on the significance of each state. For instance, in a scenario where each state represents a command for a remote agent, different commands may have varying levels of importance or criticality. We assume that $p_{\alpha_i^{\mathrm{s}}}$ is the probability of joint sampling and transmission actions when the source is at the state i. Therefore, we define $p_{\alpha_i^{\mathrm{s}}}$ as follows

$$\Pr\Big[\alpha_i^{\mathrm{s}}(t+1)=1, \alpha_i^{\mathrm{tx}}(t+1)=1\Big] = p_{\alpha_i^{\mathrm{s}}}. \tag{1}$$

The probability that the source at the state i is not sampled at time slot $t+1$ is $\Pr\Big[\alpha_i^{\mathrm{s}}(t+1)=0\Big]=1-p_{\alpha_i^{\mathrm{s}}}$. For comparison purposes, we consider three relevant policies, namely *uniform, change-aware*, and *semantics-aware* proposed in [10] and [14].

[1]In practice, only the semantics-aware policy actually requires an ACK/NACK feedback channel.

IV. Performance Analysis

In this section, we analyze the performance of state-aware randomized stationary policy in terms of three metrics, namely time-averaged reconstruction error, average cost of actuation error, and average consecutive error.

A. Real-time Reconstruction Error

The real-time reconstruction error captures the discrepancy between the original source $X(t)$ and the reconstructed source $\hat{X}(t)$ at time slot t, i.e.,

$$E(t) = \Big|X(t)-\hat{X}(t)\Big|, \tag{2}$$

where at time slot t, $E(t)=0$ denotes the system is in the sync state, while the erroneous state of the system is denoted by $E(t)\neq 0$. The time-averaged reconstruction error or the probability that the system is in an erroneous state, P_E, for an observation interval $[1,T]$ with T being a large positive number, is defined as [10] and [14]

$$P_E = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{1}\left(E(t)\neq 0\right) = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{1}\Big(X(t)\neq\hat{X}(t)\Big), \tag{3}$$

where $\mathbb{1}(\cdot)$ is the indicator function. For a two-state DTMC information source, $P_E = \Pr[X(t)=0,\hat{X}(t)=1] + \Pr[X(t)=1,\hat{X}(t)=0] = \pi_{0,1}+\pi_{1,0}$. Note that $\pi_{0,1}$ and $\pi_{1,0}$ are the probabilities obtained from the stationary distribution of the two-dimensional DTMC describing the joint status of the system regarding the current state at the original source, i.e., $\big(X(t),\hat{X}(t)\big)$. To derive $\pi_{i,j}, (i,j)\in\{0,1\}$, we assume that when the sampler performs sampling, the transmitter sends the sample in the form of packets during the same time slot. For a two-state DTMC information source, the stationary distribution $\pi_{i,j}$ for the state-aware randomized stationary policy is given by

$$\pi_{0,0} = \frac{q p_{\alpha_0^{\mathrm{s}}} p_{\mathrm{s}_0}\big[q+(1-q)p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big]}{(p+q)\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)} \tag{4a}$$

$$\pi_{0,1} = \frac{pq p_{\alpha_1^{\mathrm{s}}} p_{\mathrm{s}_1}\big(1-p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big)}{(p+q)\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)} \tag{4b}$$

$$\pi_{1,0} = \frac{pq p_{\alpha_0^{\mathrm{s}}} p_{\mathrm{s}_0}\big(1-p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)}{(p+q)\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)} \tag{4c}$$

$$\pi_{1,1} = \frac{p p_{\alpha_1^{\mathrm{s}}} p_{\mathrm{s}_1}\big[p+(1-p)p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big]}{(p+q)\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)}, \tag{4d}$$

where

$$\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big) = p p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big(1-p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big) + p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big(q+(1-q)p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big). \tag{5}$$

Using (4b), (4c), and (5), the time-averaged reconstruction error in (3) can be calculated as

$$P_E = \pi_{0,1}+\pi_{1,0} = \frac{pq\Big[p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1} + p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big(1-2p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)\Big]}{(p+q)\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)}. \tag{6}$$

B. Cost of Actuation Error

This metric quantifies the importance of errors occurring during reception and takes into account various costs or penalties associated with different types of incorrect actions. To analyze the cost of actuation error, we introduce $C_{i,j}$, which represents the cost of an error when the source is in state i, and the reconstructed source is in a state j that is not equal to i. It is assumed that the value of $C_{i,j}$ remains constant over time. Now, using $C_{i,j}$, the average cost of actuation error for a two-state DTMC can be calculated as follows

$$P_E^C = C_{0,1}\pi_{0,1} + C_{1,0}\pi_{1,0}, \tag{7}$$

where using (4b) and (4c), we can write (7) as

$$P_E^C = \frac{pq\Big[C_{0,1}p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big(1-p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big)+C_{1,0}p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big(1-p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)\Big]}{(p+q)\Big[pp_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big(1-p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big)+p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big(q+(1-q)p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)\Big]}. \tag{8}$$

Remark 1. *We can analytically prove that when* $\max\{0, T_1\} \leqslant p_{\alpha_1^{\mathrm{s}}} \leqslant 1$*, the state-aware randomized stationary policy has lower average cost of actuation error as compared to the semantics-aware policy for* $\max\{0, T_2\} \leqslant p_{\alpha_0^{\mathrm{s}}} \leqslant 1$*, where* T_1 *and* T_2 *are given by*

$$\begin{aligned}
T_1 &= \frac{pC_{1,0}+C_{1,0}p_{s_0}-pC_{1,0}p_{s_0}-qC_{0,1}(1-p_{s_0})}{C_{1,0}(1-p)p_{s_0}+pC_{1,0}p_{s_1}+C_{0,1}(1-q)p_{s_1}+qC_{0,1}p_{s_0}}\\
T_2 &= p_{\alpha_1^{\mathrm{s}}}\left[C_{0,1}\big(q+(1-q)p_{s_1}\big)-pC_{1,0}(1-p_{s_1})\right]\\
&\quad\times\Big[p_{\alpha_1^{\mathrm{s}}}\big(C_{1,0}p_{s_0}(1-p)+pC_{1,0}p_{s_1}+C_{0,1}p_{s_1}(1-q)\\
&\quad+qC_{0,1}p_{s_0}\big)-pC_{1,0}-C_{1,0}p_{s_0}+pC_{1,0}p_{s_0}+qC_{0,1}(1-p_{s_0})\Big]^{-1}.
\end{aligned} \tag{9}$$

C. Consecutive Error Metric

The *consecutive error* metric, introduced in [14], measures the number of consecutive timeslots in which a system remains in an erroneous state[2]. To quantify this metric, a DTMC, depicted in Fig. 1, is employed. When the system is in a synced state, it is denoted as $C_E(t) = 0$. On the other hand, if the system is in an erroneous state for a consecutive number of time slots, it is represented by $C_E(t) = 1 \leqslant i \leqslant n-1$, where i indicates the number of consecutive erroneous time slots. Furthermore, the transition probability $P_{i,i+1}$ is given by

$$\begin{aligned}
P_{i,i+1} &= \Pr\big[C_E(t+1) = i+1 \big| C_E(t) = i\big]\\
&= \frac{\Pr\big[C_E(t) = i+1\big]}{\Pr\big[C_E(t) = i\big]}, \quad \forall i \geqslant 0,
\end{aligned} \tag{10}$$

where $\Pr[C_E(t) = i]$ for $i = 0$ is equal to $\Pr\big[C_E(t) = 0\big] = \pi_{0,0} + \pi_{1,1}$. Also, for $i \geqslant 1$ and the state-aware randomized stationary policy, $\Pr[C_E(t) = i]$ is calculated as

$$\begin{aligned}
&\Pr\big[C_E(t) = i\big]\\
&= p(1-q)^{i-1}\big(1-p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)^i\pi_{0,0} + q(1-p)^{i-1}\big(1-p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big)^i\pi_{1,1},
\end{aligned} \tag{11}$$

where $\pi_{i,j}, \forall i,j \in \{0,1\}$ was given in (4a) and (4d). Using (11), we calculate the average consecutive error $\bar{C}_E$ as

$$\begin{aligned}
\bar{C}_E &= \sum_{x=1}^{\infty} x\Pr\big[C_E(t) = x\big]\\
&= \frac{p\big(1-p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)\pi_{0,0}}{\big(q+(1-q)p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)^2} + \frac{q\big(1-p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big)\pi_{1,1}}{\big(p+(1-p)p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big)^2}.
\end{aligned} \tag{12}$$

[2]A similar metric was defined first in [16] and then in [17].

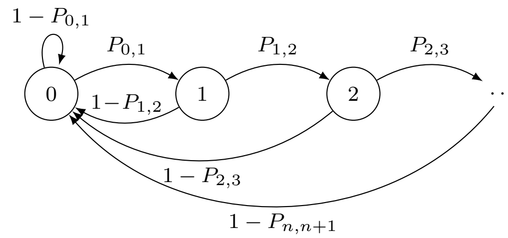


Fig. 1. DTMC describing the state of the consecutive error.

V. Optimization Problem

In this section, our objective is to find an optimal state-aware randomized stationary sampling policy, so that it minimizes the average cost of actuation error subject to a time-averaged sampling cost constraint. Here, we assume that each attempted sampling has a sampling cost δ, and that the time-averaged sampling cost cannot exceed a certain threshold $\delta_{\max}$. Therefore, the optimization problem is formulated as

$$\underset{p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}}{\text{minimize}} \quad P_E^C \tag{13a}$$

$$\text{subject to} \quad \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbb{1}\{\alpha_t^{\mathrm{s}} = 1\} \leqslant \delta_{\max}, \tag{13b}$$

where the constraint given in (13b) can be written as

$$\begin{aligned}
\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbb{1}\{\alpha_t^{\mathrm{s}}=1\} &= \delta\Pr[X(t)=0]p_{\alpha_0^{\mathrm{s}}} + \delta\Pr[X(t)=1]p_{\alpha_1^{\mathrm{s}}}\\
&= \delta\frac{qp_{\alpha_0^{\mathrm{s}}}}{p+q} + \delta\frac{pp_{\alpha_1^{\mathrm{s}}}}{p+q}.
\end{aligned} \tag{14}$$

Using (7) and (14), (13) can be simplified as

$$\underset{p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}}{\text{minimize}} \quad \frac{pq\Psi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)}{(p+q)\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)} \tag{15a}$$

$$\text{subject to} \quad qp_{\alpha_0^{\mathrm{s}}} + pp_{\alpha_1^{\mathrm{s}}} \leqslant \eta(p+q), \tag{15b}$$

where $\eta = \delta_{\max}/\delta$, $\Psi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big) = C_{0,1}p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big(1-p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big) + C_{1,0}p_{\alpha_0^{\mathrm{s}}}p_{\mathrm{s}_0}\big(1-p_{\alpha_1^{\mathrm{s}}}p_{\mathrm{s}_1}\big)$, and $\Phi\big(p_{\alpha_0^{\mathrm{s}}},p_{\alpha_1^{\mathrm{s}}}\big)$ is given by (5). To solve this optimization problem, we consider two cases: one with $pC_{1,0} \geqslant qC_{0,1}$ and one other with $pC_{1,0} < qC_{0,1}$.[3]

1) *When* $pC_{1,0} \geqslant qC_{0,1}$: in this case, the objective function in (15a) has its minimum value when $p_{\alpha_1^{\mathrm{s}}}$ is maximized. Now, using the constraint given in (15b), the maximum value of $p_{\alpha_1^{\mathrm{s}}}$ is

$$p_{\alpha_1^{\mathrm{s}}} = \frac{\eta(p+q) - qp_{\alpha_0^{\mathrm{s}}}}{p}. \tag{16}$$

We substitute (16) into (15a), resulting in an objective function that depends on $p_{\alpha_0^{\mathrm{s}}}$. To determine the value of $p_{\alpha_0^{\mathrm{s}}}$ that minimizes this objective function, we need to calculate the critical points of the objective function within the interval $\big[p_{\alpha_0^{\mathrm{s}}}^{\mathrm{LB}}, p_{\alpha_0^{\mathrm{s}}}^{\mathrm{UB}}\big]$ where $p_{\alpha_0^{\mathrm{s}}}^{\mathrm{LB}} = \max\Big\{0, \frac{\eta(p+q)-p}{q}\Big\}$, and $p_{\alpha_0^{\mathrm{s}}}^{\mathrm{UB}} = \min\Big\{1, \frac{\eta(p+q)}{q}\Big\}$. Now, we evaluate the objective function at the critical points, as well as at points $p_{\alpha_0^{\mathrm{s}}}^{\mathrm{LB}}$ and $p_{\alpha_0^{\mathrm{s}}}^{\mathrm{UB}}$. The minimum value of the objective function within the given interval corresponds to the smallest value. After determining the value of $p_{\alpha_0^{\mathrm{s}}}$ that minimizes the objective function, we can calculate $p_{\alpha_1^{\mathrm{s}}}$ by utilizing

[3]The complete steps to solve this optimization problem are detailed in [18].

the expression given in (16). We can prove that the values of $p_{\alpha_0^s}$ and $p_{\alpha_1^s}$ obtained by solving the this optimization problem, are the optimal values of the sampling probabilities when $p_{\alpha_1^s} \geqslant \frac{pC_{1,0}-qC_{0,1}}{p_{s_1}\left(pC_{1,0}+(1-q)C_{0,1}\right)}$. Otherwise, the optimal values of sampling probabilities $p_{\alpha_0^s}$ and $p_{\alpha_1^s}$ are equal to $p^*_{\alpha_0^s}=0$ and $p^*_{\alpha_1^s}=\min\left\{1, \frac{\eta(p+q)}{p}\right\}$.

2) *When* $pC_{1,0} < qC_{0,1}$: in this case, as $p_{\alpha_0^s}$ increases, the objective function in (15a) decreases. Using the constraint in (15b), the maximum value of $p_{\alpha_0^s}$ is given by

$$p_{\alpha_0^s} = \frac{\eta(p+q)-pp_{\alpha_1^s}}{q}. \quad (17)$$

Now, we substitute (17) in (15a) and obtain an objective function that depends on $p_{\alpha_1^s}$. Similar to the case when $pC_{1,0} \geqslant qC_{0,1}$, we can obtain $p_{\alpha_1^s}$ that minimizes this objective function by calculating its critical points within the interval $\left[p^{\mathrm{LB}}_{\alpha_1^s}, p^{\mathrm{UB}}_{\alpha_1^s}\right]$ where $p^{\mathrm{LB}}_{\alpha_1^s} = \max\left\{0, \frac{\eta(p+q)-q}{p}\right\}$, and $p^{\mathrm{UB}}_{\alpha_1^s} = \min\left\{1, \frac{\eta(p+q)}{p}\right\}$. Then, we obtain $p_{\alpha_0^s}$ using (17). We can similarly prove that when $p_{\alpha_0^s} \geqslant \frac{qC_{0,1}-pC_{1,0}}{p_{s_0}\left(qC_{0,1}+(1-p)C_{1,0}\right)}$, $p_{\alpha_0^s}$ and $p_{\alpha_1^s}$ derived by solving this optimization problem, are the optimal values of the sampling probabilities. Otherwise, the optimal values of the sampling probabilities are equal to $p^*_{\alpha_0^s} = \min\left\{1, \frac{\eta(p+q)}{q}\right\}$ and $p^*_{\alpha_1^s}=0$.

VI. Numerical and Simulation Results

In this section, we validate our analytical results and assess the performance of the sampling policies in terms of time-averaged reconstruction error, average cost of actuation error, and average consecutive error under various system parameters. In the uniform policy, a sample is acquired every 5 time slots. Simulation results are obtained by averaging over 10^7 time slots. In what follows, RS and RSC policies are the abbreviations for the state-aware randomized stationary policy and the state-aware randomized stationary policy in the constrained optimization problem, respectively.

In Tables I to VI, we depict the minimum average cost of actuation error under a sampling cost constraint of $\eta = 0.5$, considering various values of p, q, p_{s_0}, and p_{s_1}, while setting $C_{0,1}=1$ and $C_{1,0}=2$. As observed, when $pC_{1,0} \geqslant qC_{0,1}$, the average cost of actuation error reaches its minimum when $p_{\alpha_1^s}$ is greater than $p_{\alpha_0^s}$. Otherwise, the minimum average cost of actuation error occurs when $p_{\alpha_0^s} > p_{\alpha_1^s}$. Note also that, in the case of lower successful probabilities, the minimum average cost of actuation error is achieved at small values of $p^*_{\alpha_0^s}$ and $p^*_{\alpha_1^s}$, where $pC_{1,0} \geqslant qC_{0,1}$ and $qC_{0,1} > pC_{1,0}$, respectively. Furthermore, we observe that the optimal RSC policy demonstrates superior performance compared to the semantics-aware policy under the conditions given in Remark 1, for both slowly and rapidly changing information sources. Conversely, in cases where Remark 1 is not applicable, the semantics-aware policy outperforms the optimal RSC policy only when the source evolves slowly. Note that the optimal values highlighted in red for the semantics-aware, change-aware, and RS policies are derived using p, q, p_{s_0}, and p_{s_1} values violate the constraint requirement. This implies that in the unconstrained scenario, the optimal RS policy demonstrates either superior or equivalent performance compared to the semantics-aware policy. However, in this specific case, the optimal solution for the RS policy involves sampling and transmitting during most of the timeslots, resulting in an excessive number of generated samples.

The optimal values for sampling probabilities in the unconstrained scenario are presented in Tables III and VI. For all values of p and q, we have $pC_{1,0} \geqslant qC_{0,1}$. As a result, the optimal value of $p^*_{\alpha_1^s}$ is set to 1. Moreover, when considering values of p and q where $p_{s_1} < \frac{pC1,0-qC_{0,1}}{pC_{1,0}+(1-q)C_{0,1}}$, the optimal value of $p^*_{\alpha_0^s}$ is determined as 0; otherwise, $p^*_{\alpha_0^s}=1$. This indicates that in scenarios where the success probability of a state is low, the optimal approach is to refrain from sampling for the state that causes less significant errors in terms of actuation. Conversely, in situations with higher success probabilities, the optimal policy is to always perform sampling.

TABLE I
Minimum average cost of actuation error for RSC state-aware with $\eta=0.5$, $C_{0,1}=1$, $C_{1,0}=2$, $p_{s_0}=0.2$, $p_{s_1}=0.3$.

p	q	$p^*_{\alpha_0^s}$	$p^*_{\alpha_1^s}$	Minimum average cost of actuation error
0.1	0.01	0.083	0.542	0.091
0.3	0.1	0	0.667	0.25
0.5	0.4	0	0.9	0.444
0.7	0.8	0	1	0.533
0.9	0.95	0	1	0.513

TABLE II
Minimum average cost of actuation error for $\eta=0.5$, $C_{0,1}=1$, $C_{1,0}=2$, $p_{s_0}=0.2$, $p_{s_1}=0.3$.

p	q	Semantics-aware	Change-aware	Uniform	RSC	RS
0.1	0.01	0.055	0.628	0.131	0.091	0.055
0.3	0.1	0.267	0.613	0.417	0.25	0.25
0.5	0.4	0.489	0.596	0.638	0.444	0.444
0.7	0.8	0.571	0.588	0.683	0.533	0.533
0.9	0.95	0.587	0.589	0.677	0.513	0.513

TABLE III
Minimum average cost of actuation error for RS state-aware with $\eta=0.5$, $C_{0,1}=1$, $C_{1,0}=2$, $p_{s_0}=0.2$, $p_{s_1}=0.3$.

p	q	$p^*_{\alpha_0^s}$	$p^*_{\alpha_1^s}$	Minimum average cost of actuation error
0.1	0.01	1	1	0.055
0.3	0.1	0	1	0.25
0.5	0.4	0	1	0.444
0.7	0.8	0	1	0.533
0.9	0.95	0	1	0.513

Figs. 2, and 3 show the average consecutive error contour plots as a function of $p_{\alpha_0^s}$ and $p_{\alpha_1^s}$ for $p > q$, considering the slow and rapid changes of the source, respectively. As

TABLE IV
Minimum average cost of actuation error for RSC state-aware with $\eta=0.5$, $C_{0,1}=1$, $C_{1,0}=2$, $p_{s_0}=0.6$, $p_{s_1}=0.6$.

p	q	$p^*_{\alpha_0^s}$	$p^*_{\alpha_1^s}$	Minimum average cost of actuation error
0.1	0.01	0.730	0.477	0.049
0.3	0.1	0.155	0.615	0.241
0.5	0.4	0.171	0.763	0.422
0.7	0.8	0.200	0.842	0.501
0.9	0.95	0.127	0.893	0.503

TABLE V

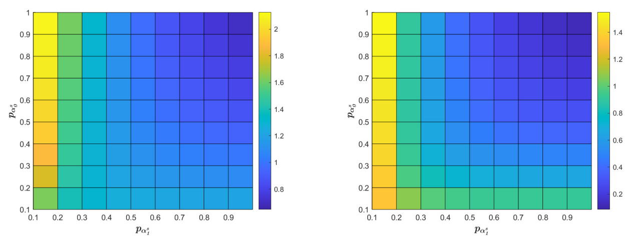
MINIMUM AVERAGE COST OF ACTUATION ERROR FOR $\eta = 0.5, C_{0,1} = 1, C_{1,0} = 2, p_{s_0} = 0.6, p_{s_1} = 0.6$.

p	q	Semantics-aware	Change-aware	Uniform	RSC	RS
0.1	0.01	0.017	0.545	0.092	0.049	0.017
0.3	0.1	0.118	0.5	0.404	0.241	0.118
0.5	0.4	0.278	0.444	0.640	0.422	0.278
0.7	0.8	0.373	0.419	0.686	0.501	0.373
0.9	0.95	0.414	0.424	0.690	0.503	0.414

TABLE VI

MINIMUM AVERAGE COST OF ACTUATION ERROR FOR RS STATE-AWARE WITH $\eta = 0.5, C_{0,1} = 1, C_{1,0} = 2, p_{s_0} = 0.6, p_{s_1} = 0.6$.

p	q	$p^*_{\alpha^s_0}$	$p^*_{\alpha^s_1}$	Minimum average cost of actuation error
0.1	0.01	1	1	0.017
0.3	0.1	1	1	0.118
0.5	0.4	1	1	0.278
0.7	0.8	1	1	0.373
0.9	0.95	1	1	0.414



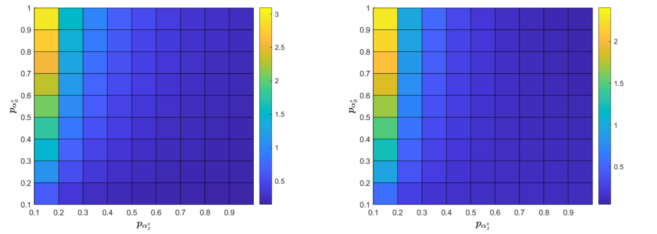
(a) $p_{s_0} = 0.2, p_{s_1} = 0.3$ (b) $p_{s_0} = 0.7, p_{s_1} = 0.8$

Fig. 2. Average consecutive error as a function of $p_{\alpha^s_0}$ and $p_{\alpha^s_1}$ for a slowly changing source with $p = 0.3$, $q = 0.2$.

illustrated in Fig. 2, when the source changes slowly, the minimum average consecutive error occurs at high values of $p_{\alpha^s_0}$ and $p_{\alpha^s_1}$. In addition, as observed in Fig. 3, when the source changes rapidly and success probabilities are low, the average consecutive error decreases with a high value of $p_{\alpha^s_1}$ and a low value of $p_{\alpha^s_0}$. Furthermore, when success probabilities are high, the average consecutive error has its minimum value as $p_{\alpha^s_0}$ and $p_{\alpha^s_1}$ increase. Also, note that these figures can be used to obtain the optimal values of sampling probabilities. Interestingly, as the success probabilities increase, we can achieve a comparable average consecutive error with smaller sampling probabilities compared to situations where the success probabilities are lower. For example, when $p = 0.3$ and $q = 0.2$, with $p_{s_0} = 0.2$ and $p_{s_1} = 0.3$, the minimum average consecutive error is approximately 0.65, which is achieved by setting $p_{\alpha^s_0} = 1$ and $p_{\alpha^s_1} = 1$. However, for $p_{s_0} = 0.7$ and $p_{s_1} = 0.8$, the similar average consecutive error value can be obtained by using $p_{\alpha^s_0} = 0.2$ and $p_{\alpha^s_1} = 1$.

VII. CONCLUSIONS

This paper considers a time-slotted communication system that involves sampling and transmission over a wireless erasure channel to track a two-state Markov process. A state-aware randomized stationary policy considering different sampling and success probabilities according to the source's states was proposed. The system performance was evaluated using the time-averaged reconstruction error, the average cost of actuation error, and the average consecutive error metrics. Furthermore, we optimized the system so as to minimize the average cost of actuation error while keeping the time-averaged sampling cost below a specified threshold. Our results demonstrated that the proposed state-aware randomized stationary policy outperforms other state-of-the-art policies, especially under cost constraints and rapid source changes, while being effective in slowly varying sources under certain conditions.



(a) $p_{s_0} = 0.2, p_{s_1} = 0.3$ (b) $p_{s_0} = 0.7, p_{s_1} = 0.8$

Fig. 3. Average consecutive error as a function of $p_{\alpha^s_0}$ and $p_{\alpha^s_1}$ for a rapidly changing source with $p = 0.8$, $q = 0.1$.

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