Enhanced Modeling of Uplink Configured Grant Transmissions for URLLC

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***Abstract*—The third generation partnership project (3GPP) has defined ultra-reliable low latency communication (URLLC) as one of the key 5G competencies to provide high network reliability and low latency. The configured grant (CG) transmission, defined in 3GPP Release-15, is a technique that automatically transmits a URLLC packet with a configured number of repetitions to increase reliability. In this paper, we propose enhanced modeling for CG transmission considering the communication channel state to evaluate the network performance. The results show how to configure CG transmission to reach a defined probability of success for decoding the packets.**

***Index Terms*—5G, URLLC, Resource allocation, Reliability**

I. Introduction

The 5G communication system is expected to support a wide range of emerging applications. To support diverse application requirements, the international telecommunication union (ITU) categorized 5G services into enhanced mobile broadband (eMBB), massive machine type communication (mMTC), and URLLC [1]. The URLLC service supports scenarios that require high service availability with low-latency bounds. URLLC targets applications such as remote health care, vehicle-to-vehicle (V2V) communication, and industrial automation. However, depending on the application communication constraints, reliability, and latency requirements may vary [2], [3]. In general, URLLC is a challenging service that entails employing advanced techniques to support highly demanding applications. Some related challenges in achieving URLLC include quality of service (QoS) support, error handling, fast handover procedures, and scheduling [4].

A URLLC transmission can be either periodic or sporadic. In any case, multiplexing of URLLC and eMBB transmission is needed to efficiently utilize resources. In the downlink (DL), the URLLC transmission can either follow an instant or a reservation-based scheduling scheme [5]. The gNodeB (gNB), signals an eMBB user by sending a preemption indication (PI) for instant scheduling [6]. With reservation-based scheduling, resources are reserved in advance.

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In order to fulfill the URLLC requirements, there are several key aspects that need to be addressed. Locating the demodulation reference signals (DM-RS) early in the transmission (sometimes known as front-loaded reference signals), locating the CORESET[1] at the beginning of the slot, having a streamlined and implementation-friendly structure of the Physical Downlink Control Channel (PDCCH), quick feedback of hybrid automatic repeat request (HARQ) acknowledgment after the end of the DL slot, and the new radio (NR) architecture approach to low latency by allowing transmission over a fraction of a slot (starting at any orthogonal frequency division multiplexing (OFDM) symbol) are some of the enablers in NR to achieve URLLC.

The uplink (UL) URLLC scheduling is considerably more challenging compared to the DL since the preemption is signaled by the user equipment (UE) instead of the gNB. 5G NR allows CG (also known as grant free (GF)) transmission for URLLC users where the gNB configures resources for periodic UL transmission. The delay incurred through the regular Grant Based (GB) procedure is avoided with CG transmission. In the UL data transmission, CG scheduling reduces control signaling overhead which reduces the latency since no scheduling request is needed prior to data transmission. With CG scheduling, the gNB reserves resources for UL transmissions and informs the UEs about the reserved resources. When a UE wants to initiate a UL transmission, it directly utilizes the reserved resources, without sending random access (RA) and Scheduling Request (SR) and waiting for the GB procedure as shown in Figure 1. In this respect, two schemes for transmission without a dynamic grant are supported, but they differ in the ways of activation. In the first type, UL grant is provided by the Radio Resource Control (RRC) layer of the protocol stack, including activation of the grant, while in the second type, transmission periodicity is provided by RRC and L1/L2 control signaling to activate/deactivate the transmission. In both schemes it is possible to configure multiple devices with overlapping time-frequency resources in

[1] CORESET is a region (combination of Time & Frequency resources) on the resource grid where Physical Downlink Control Channel (PDCCH) resources are located.

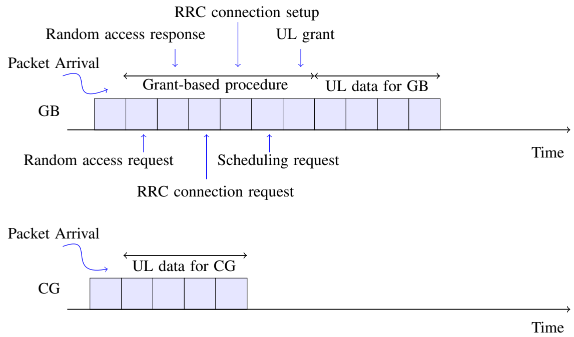

Fig. 1. UL transmissions for GB and CG.

the UL [7]. Therefore, in both cases of CG, multiple UEs can be configured by the gNB such that they can transmit their packets without a SR. Thus, collisions might occur in the UL CG transmission. On the other hand, to increase reliability, the UE is permitted to automatically repeat the transmission once or even several times without waiting for a HARQ feedback. HARQ combines error detection and correction mechanisms with re-transmission strategies to ensure that data is successfully received by the intended recipient. It operates by dividing the transmitted data into smaller units called transport blocks and encoding them with redundancy bits for error detection and correction. One of the specific schemes of HARQ to transmit replicas of the URLLC packet [8] is k-repetitions. In this scheme, the UE transmits the same packet k times with $k = 1, 2, 4, 8$ as configured by the *repK* parameter.

The proactive scheme (k_{pro}) is an extension of the k-repetition scheme with the difference that the UE can receive feedback after every repetition. The maximum number of repetitions will remain as set via the *repK* parameter. However, upon receipt of positive feedback, the UE can terminate further repetitions. Therefore, it is a dynamic scenario that is unknown when a UE will get positive feedback. In any case, knowledge of the required number of repetitions to achieve the desired reliability would be helpful in scheduling and allocation of available resources to the UE requesting for transmission. In this paper, we evaluate the network performance of UL CG transmission considering the communication channel state. The paper is organized as follows. Section II covers the related existing works. Section III presents the system model and formulates the probability of failure for URLLC transmission in 5G networks. Section IV presents evaluations of the network performance with the proposed system model and Section V concludes the paper.

II. Related Work

In recent years, extensive research has been conducted on resource allocation and reliability enhancement techniques in the context of 5G networks. This section discusses the key findings and contributions of several relevant works. In [9] resource allocation and scheduling challenges in 5G networks are explored. Their study emphasized the importance of efficient resource allocation strategies, including CG. By pre-allocating resources, such as time slots or frequency channels, to specific users or services, CG helps improve resource utilization and enhance network performance. In [10] a dynamic resource allocation scheme in 5G networks by exploiting traffic prediction and base station coordination is proposed. They leveraged the concept of CG to allocate resources effectively based on predicted traffic patterns. This approach demonstrated promising results in terms of enhancing resource utilization and improving network efficiency. In terms of reliability enhancement, in [10] a reliable data transmission scheme utilizing k-repetitions in 5G wireless networks is proposed. By transmitting the same data packet multiple times, k-repetitions mitigate the effects of channel errors or fading, thereby improving reliability. In [11] different configurations of the network to increase reliability and reduce latency are discussed. The UE can select the specific configuration to transmit the k repetitions but the channel conditions during the transmission are not modeled. In [12] the repetitions are transmitted via a shared resource without optimizing the size of the resources and the number of repetitions for the lost packets. This results in reduced communication reliability which is not suitable for URLLC use cases. In [13] the size of the shared resources is optimized but still, the gNB is unable to decode the colliding packets sent by different UEs by applying Successive Interference Cancellation (SIC). In [14] the optimized resources are analytically derived and analyzed, but the packets which are not decodable in gNB and their impact on the reliability and latency are not evaluated. In [15] a stochastic optimization for managing the possible conflicts in the resources based on the proposed queuing model is discussed. However, the proposed model does not consider the CG transmission scheme. In [16] not only an optimal solution for UL resource allocation is presented but also the optimal control convergence rate to maximize the spectral efficiency is evaluated, but the probability of resource conflicts is not addressed. In [17] it is shown how SIC can decrease the latency while the Block Error Rate (BLER) is improved. Overall, the discussed works provide valuable insights into resource allocation techniques, such as CG, and reliability enhancement methods, such as k-repetitions, in the context of 5G networks. These studies lay the foundation for the research conducted in this paper and inspire further exploration of efficient resource allocation models and reliability enhancement mechanisms in 5G systems. Different from the existing studies, we focus on combining available models for UL CG transmission and develop a model for a system that considers channel model parameters to evaluate the network performance.

III. Problem formulation and System Model

A. Problem Formulation

Reliability can decrease when the actual performed number of repetitions is smaller than the configured number. Based on 3GPP Release 15, the UE is not allowed to start the k-repetitions in an arbitrary time slot and the waiting time is sometimes increased if packets arrive after a specific CG occasion. A CG occasion refers to a specific time period within a grant period where the base station allocates resources

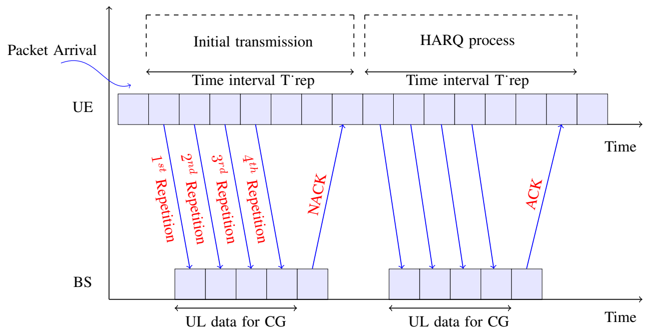

Fig. 2. k-repetition scheme with $k = 4$.

to a UE for transmitting UL data. It defines the time and frequency resources that are assigned to the UE for its UL transmission. In the CG transmission scheme, the UE can transmit k-repetitions of a data packet without waiting for feedback. However, the UEs are only allowed to transmit the repetitions in a specified periodical interval in order to avoid confusion of HARQ identities[2] in different HARQ processes [18]. Therefore, in case the UE misses the corresponding time slots, it must stop sending the repetition in that interval and resume transmission in the next possible interval. As a result, the number of repetitions might be smaller than the k configured ones. On the other hand, the reliability of the UL CG transmission will decrease when the configured number of repetitions is not met. This situation will increase the latency of the transmission because the gNB needs to reschedule the packet for the next round of re-transmission. To model the problem, we assume that in a cell N UEs are configured to transmit k-repetitions in the shared CG resources. As described already in Sect. I, there are different schemes for the repetition of packets such as blind repetitions, repetitions with a fixed number (k-repetitions), and k-repetitions with feedback (k_{pro}). In this work, we focus on k-repetitions. Figure 2 presents a simple illustration of the k-repetitions HARQ process which is done in the time interval T_{rep}. The arrival of the packets is modeled by a Poisson process with parameter λ, The probability of UE transmission in a time interval T_{CG} is

$$P_{\text{data}}(\lambda) = 1 - e^{-\lambda} \tag{1}$$

We formulate the Signal to Interference plus Noise Ratio (SINR) at the BS as

$$\text{SINR}_k = \frac{g_k h_0}{I_{\text{intra}} + \sigma^2} \tag{2}$$

where g_k denotes the power level of the k-th repetition, h_0 is the power gain of the channel, which we assume to be exponentially distributed as is the case for Rayleigh fading channels, σ^2 is the noise power, and I_{intra} is the interference caused by the UEs in the same cell [19]. We consider intra-cell interference because the UEs in the same cell associated with the same BS may cause interference with each other.

[2]HARQ ID (HARQ Identity) is a unique identifier assigned to a specific data transmission or a logical channel within a UE to differentiate and manage different ongoing HARQ processes simultaneously within the UE.

$P_{\text{tr}}(k, i)$ being the transmission probability of a UE after the i-th CG transmission [11] in a k-repetitions scheme is

$$P_{\text{tr}}(k, i, \lambda) = (1 - P_{\text{data}}(\lambda))\left(1 - (1 - P_{\text{data}}(\lambda))^{k-i}\right) \tag{3}$$

where $i \in [1, k-1]$. The probability that n UEs out of $N-1$ UEs transmit after the ith CG occasion is

$$P_{\text{N,n}}(k, i, \lambda) = \begin{pmatrix} N-1 \\ n \end{pmatrix} P_{\text{tr}}(k, i, \lambda)^n \left(1 - P_{\text{tr}}(k, i, \lambda)\right)^{N-1-n} \tag{4}$$

We derive the probability $P_{\text{st},k}(n)$ of successful transmission at the k-th repetition in the case of n interfering UEs in the same cell considering the channel conditions. We assume a successful transmission if the SINR is above a certain threshold.

$$P_{\text{st},k}(n) = \Pr[\text{SINR}_k \geq \gamma_{\text{th}} \mid n] \tag{5}$$

where γ_{th} is the SINR threshold to decode the packets of UE of interest, and SINR_k is the SINR of the k^{th} repetition. We derive the success probability under k-repetition scheme conditioning on n number of intra-cell interfering following the same steps in [14] whereas $P_{\text{st}}(n)$

$$P_{\text{st}}(n) = \sum_{j=1}^{k} (-1)^{j+1} \begin{pmatrix} k \\ j \end{pmatrix} \exp\left(-\frac{\gamma_{\text{th}} j \sigma^2}{g_k}\right) \left(\frac{1}{1+\gamma_{\text{th}}}\right)^{jn} \tag{6}$$

(See Appendix).

The aim of the CG transmission for URLLC is to make sure that a certain number of UEs can successfully deliver their payload in a certain limited time [20]. The probability of successfully decoding the packets of UE of interest is given by

$$P_{\text{s}}(k, i, \lambda, M_i) = \sum_{n=0}^{N-1} P_{\text{N,n}}(k, i, \lambda) P_{\text{st}}(n) \sum_{l=0}^{L} P_{\text{n,l}}(M_i), \tag{7}$$

where $P_{\text{n,l}}(M_i)$ is the probability that after the ith CG transmission, l out of L UEs access the same resource block as the UE of interest and gNB still can decode the packets of the UE of interest. $P_{\text{n,l}}(M_i)$ can be computed as

$$P_{\text{n,l}}(M_i) = \begin{pmatrix} n \\ l \end{pmatrix} \left(\frac{1}{M_i}\right)^l \left(1 - \frac{1}{M_i}\right)^{n-l} \tag{8}$$

where each reserved resource has M_i blocks and $\frac{1}{M_i}$ is the probability of accessing the same reserved resource block. Therefore, the probability of failure in decoding the packets of UE of interest can be computed according to

$$P_{\text{f}}(k, i, \lambda, M_i) = 1 - P_{\text{s}}(k, i, \lambda, M_i) \tag{9}$$

IV. Performance Analysis

We now evaluate the network performance of the presented model for the probability of failure in different scenarios. We evaluate our network performance based on the proposed model using the parameters described in [11]. One of the interesting scenarios to evaluate is how flexible the network is w.r.t. the number of UEs having access to the same resources. In the first scenario, these parameters include N which is simulated between $[1, 100]$ as the number of UEs, the number of repetitions is set to $k = \{2, 4, 8\}$, and $\lambda = 0.0025$. We also set $g_k = 10\,\text{dBm}$, $\sigma^2 = -90\,\text{dBm}$, and $\gamma_{th} = 3\,\text{dB}$. CG transmission number is set to $i = 1$, the number of resource blocks is set to $M_1 = 10$, and $L = 5$.

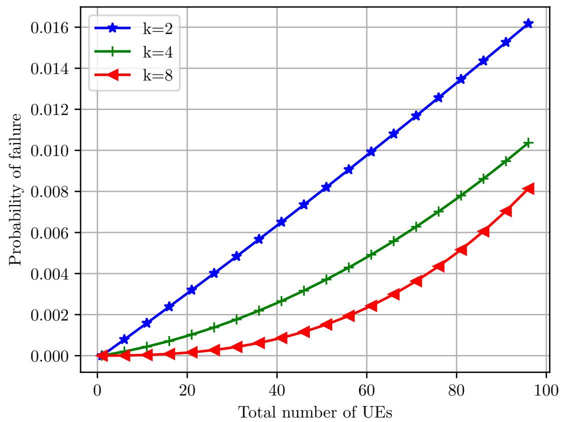

Fig. 3. P_{failure} vs. total number of UEs in different sets of repetitions

In Fig. 3, the behavior for different numbers of the repetitions for UEs that have access to the same resource blocks is evaluated. The results show that increasing the number of repetitions can support more UEs with a lower probability of failure. As shown in Fig. 3, the probability of failure for 100 UEs with $k = 2$ is twice compared to the case when $k = 8$. Thus, the scheduler of the network can select the suitable number of repetitions based on the required reliability in the network and the available time budget for re-transmissions

In the second scenario, we investigate the impact of the rate of λ on the network performance for two different values of the repetition parameter $k = 2, 4$. By stating that higher values of λ lead to a very high probability of failure and thus only a very small number of packets can be transmitted, as compared to the total throughput the network would be able to accommodate. Therefore, we simulate the Poisson process parameter over the range $\lambda = \{0.0025, 0.0065, 0.0095\}$, while keeping all other parameters the same as in the first scenario.

Our results show that as the packet arrival rate increases, the probability of failure also increases, as more packets need to be served within a fixed time period, leading to a higher chance of congestion and resource contention. However, we also observe that in some cases, the probability of failure is similar for different values of the repetition parameter, even at different packet arrival rates. For instance, for $N = [0, 60]$ UEs, the network performance with $k = 2$ and $\lambda = 0.0095$ is comparable to that of $k = 4$ and $\lambda = 0.0025$.

These findings suggest that the choice of the appropriate repetition parameter for a given network should take into account both the required reliability and the available time budget for re-transmissions, as well as the packet arrival rate. This can be achieved by an intelligent scheduler that can dynamically adjust the number of re-transmissions based on the network conditions.

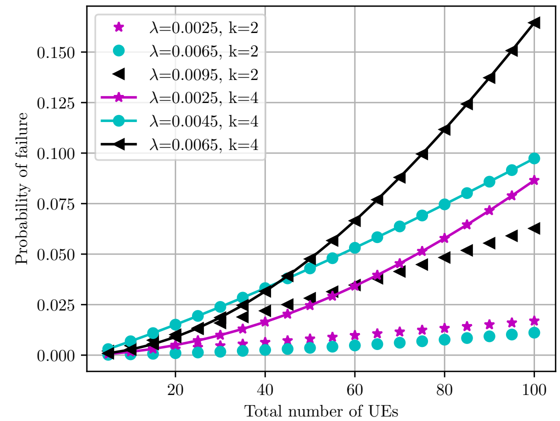

Fig. 4. P_{failure} vs. total number of UEs for different packet arrival rates

For the third scenario, we fix the number of repetitions to $k = 2$ and evaluate the network's performance with respect to γ_{th}, the SINR threshold. By setting the thresholds for higher values of thresholds, we observe that the probability of failure leads to high values. Thus, We set $\gamma_{th} = [-10, 10]\,\text{dB}$, and keep all other parameters the same as in the second scenario. The first outcome of this evaluation is that for a lower γ_{th} threshold, the failure probability is almost constant at around 2.5×10^{-3} for $\lambda = 0.0095$. Since this network requires a low γ_{th}, most of the packets are successfully transmitted, and the re-transmission parameter of $k = 2$ seems to be sufficient to control the failure probability. However, if the required γ_{th} increases to $\gamma_{th} = 10$,dB, then $k = 2$ is no longer satisfactory. Therefore, the network scheduler needs to increase the number of re-transmissions based on the available time budget. This finding suggests that a flexible scheduler could be used to allocate network resources in a reliable manner.

V. Conclusions

In this paper, we proposed a system model for CG transmission to evaluate the network performance, taking into account the state of the communication channel. By combining a channel model with the probability of accessing the same resource in a CG transmission, we established a framework to control the network's performance under varying conditions such as the number of active UEs, possible repetitions, packet arrival rate, and more.

In future work, we plan to extend our evaluation to include the impact of other system parameters such as support for SIC in the gNB. This will allow for a more accurate and comprehensive model of the network's performance. Furthermore, as a next step, we plan to implement the CG transmission technique in *nr_ulsim* [3] which is a Radio Access Network (RAN) physical layer simulator developed by the OpenAirInterface

[3]https://gitlab.eurecom.fr/oai/openairinterface5g/blob/develop/doc

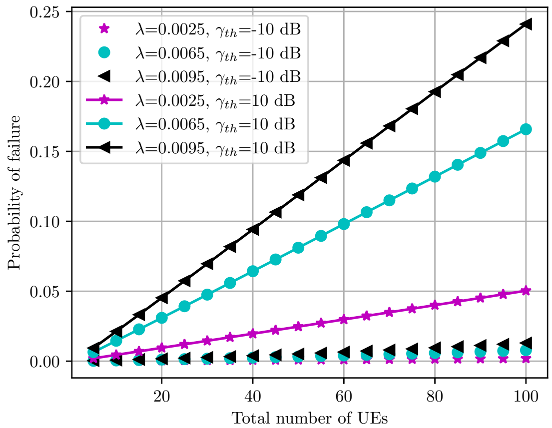

Fig. 5. P_{failure} vs. total number of UEs for different γ_{th}

Software Aliance (OSA), to evaluate the network performance of our proposed model.

Appendix

The first step to compute the probability $P_{\text{st},k}\left(n\right)$ for a successful CG transmission is to apply the SINR model from (2) in the expression for P_{st}

$$P_{\text{st},k}\left(n\right) = \Pr\left[\frac{g_k h_0}{I_{\text{intra}} + \sigma^2} \geq \gamma_{\text{th}} \mid n\right] \tag{10}$$

By considering the distribution of $h_0 \sim \text{Exp}(1)$ as it is shown in [21], in order to simplify (10), it can be rewritten as

$$\mathbb{E}_{I_{\text{intra}}}\left[\Pr[h_0 \geq \frac{\gamma_{\text{th}}}{g_k}(I_{\text{intra}} + \sigma^2) \mid n, I_{\text{intra}}]\right] \tag{11}$$

Following the approach in [21], Sect. III.B, we find

$$\Pr\left[\text{SINR}_k \geq \gamma_{\text{th}} \middle| n\right] = \exp\left(-\frac{\gamma_{\text{th}}\sigma^2}{g_k}\right)\mathcal{L}_{I_{\text{intra}}}\left(\frac{\gamma_{\text{th}}}{g_k}\middle| n\right) \tag{12}$$

where $\mathcal{L}_{I_{\text{intra}}}(x)$ is the Laplace transform of random variable I_{intra}. Based on what is presented in [14], the Laplace transform of intra-cell interference can be computed according to

$$\mathcal{L}_{I_{\text{intra}}}\left(s|n\right) = \mathbb{E}\left[\exp\left(-s\sum_{\alpha=0}^{n} g_k h_\alpha\right)\right] = \left(\frac{1}{1+sg_k}\right)^n \tag{13}$$

where $s = \frac{\gamma_{\text{th}}}{g_k}$.

For the k-repetition scheme, the CG transmission in one HARQ round trip is successful if any of the repetitions succeed. Following the same procedure for k-repetition scheme, the CG transmission success probability under k-repetitions conditioning on n number of intra-cell interfering UEs based on the SINR outage is calculated as

$$P_{\text{st}}\left(n\right) = \sum_{j=1}^{k}(-1)^{j+1}\left(\begin{array}{c} k \\ j \end{array}\right)\exp\left(-\frac{\gamma_{\text{th}} j\sigma^2}{g_k}\right)\left(\frac{1}{1+\gamma_{\text{th}}}\right)^{jn}. \tag{14}$$

References

[1] M. Series, "IMT Vision–Framework and overall objectives of the future development of IMT for 2020 and beyond," *Recommendation ITU*, vol. 2083, p. 0, 2015.

[2] A. Ghosh, A. Maeder, M. Baker, and D. Chandramouli, "5G evolution: A view on 5G cellular technology beyond 3GPP release 15," *IEEE Access*, vol. 7, pp. 127 639–127 651, 2019.

[3] 3GPP, "TS 23.501: Technical specification group services and system aspects; service requirements for the 5G system (5GS); Stage 1," Release-18, v18.2.0, 2021.

[4] M. A. Siddiqi, H. Yu, and J. Joung, "5G ultra-reliable low-latency communication implementation challenges and operational issues with IoT devices," *MDPI Electronics*, vol. 8, no. 9, p. 981, 2019.

[5] H. Ji, W. Kim, and B. Shim, "New radio technologies for ultra reliable and low latency communications," in *Proc. IEEE Region 10 Conf. (TENCON)*, 2018.

[6] T.-K. Le, U. Salim, and F. Kaltenberger, "An overview of physical layer design for ultra-reliable low-latency communications in 3GPP Releases 15, 16, and 17," *IEEE Access*, vol. 9, pp. 433–444, 2021.

[7] E. Dahlman, S. Parkvall, and J. Sköld, Eds., *5G NR: the Next Generation Wireless Access Technology*. Academic Press, 2018. [Online]. Available: https://www.sciencedirect.com/science/article/pii/B9780128143230000296

[8] 3GPP, "TS 38.214: Technical specification group radio access network; NR; Physical layer procedures for data," Release-16, v16.6.0, 2021.

[9] Y. Zhang, Y. Liu, H. Ji, and N. Zhao, "Resource allocation and scheduling in 5G networks: Challenges and research directions," *IEEE Access*, vol. 6, pp. 69 566–69 578, 2018.

[10] W. S. Yousef, M. A. El-Halaby, A. A. Abdel-Rahman, and A. A. El-Sayed, "Dynamic resource allocation in 5G networks exploiting traffic prediction and base station coordination," *Wireless Personal Communications*, vol. 105, no. 4, pp. 1051–1072, 2019.

[11] T.-K. Le, U. Salim, and F. Kaltenberger, "Enhancing URLLC Uplink Configured-grant Transmissions," in *IEEE Vehicular Technology Conference (VTC2021-Spring)*, 2021, pp. 1–5.

[12] ——, "Strategies to meet the configured repetitions in URLLC Uplink Grant-Free transmission," in *International Symposium on Wireless Communication Systems (ISWCS)*, 2019, pp. 597–601.

[13] ——, "Optimal reserved resources to ensure the repetitions in ultra-reliable low-latency communication uplink grant-free transmission," in *European Conference on Networks and Communications (EuCNC)*, 2019, pp. 554–558.

[14] Y. Liu, Y. Deng, M. Elkashlan, A. Nallanathan, and G. K. Karagiannidis, "Analyzing Grant-Free Access for URLLC Service," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 3, pp. 741–755, 2021.

[15] W. Zhang, M. Derakhshani, and S. Lambotharan, "Stochastic optimization of URLLC-eMBB Joint Scheduling With Queuing Mechanism," *IEEE Wireless Communications Letters*, vol. 10, no. 4, pp. 844–848, 2021.

[16] B. Chang, L. Zhang, L. Li, G. Zhao, and Z. Chen, "Optimizing resource allocation in URLLC for real-time wireless control Systems," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 9, pp. 8916–8927, 2019.

[17] O. Nakamura, Y. Hamaguchi, T. Takahashi, and S. Sampei, "Low latency interference cancellation for uplink URLLC repetition transmission," in *IEEE Vehicular Technology Conference (VTC2020-Spring)*, 2020, pp. 1–5.

[18] 3GPP, "TS 38.301: RRC protocol specification," Release-15, v15.3.0, 2018.

[19] N. Jiang, Y. Deng, A. Nallanathan, X. Kang, and T. Q. S. Quek, "Analyzing random access collisions in massive iot networks," *IEEE Transactions on Wireless Communications*, vol. 17, no. 10, pp. 6853–6870, 2018.

[20] G. Berardinelli, N. Huda Mahmood, R. Abreu, T. Jacobsen, K. Pedersen, I. Z. Kovács, and P. Mogensen, "Reliability analysis of uplink grant-free transmission over shared resources," *IEEE Access*, vol. 6, pp. 23 602–23 611, 2018.

[21] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on Communications*, vol. 59, no. 11, pp. 3122–3134, 2011.