Structured Multiuser Channel Estimation for Block-Synchronous DS/CDMA

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Abstract

Uplink channel estimation for a block-synchronous chip-asynchronous DS/CDMA system as proposed for the time-division duplex option of 3rd generation cellular systems is considered. Training midambles are employed for joint channel estimation of all users. The standard *unstructured* approach based on modeling the effective user channels as unknown FIR filters is compared with two *structured* methods that exploit *a priori* knowledge about the user channels such as the maximum delay-spread, the transmit chip-shaping pulse and the path delays. Since these are usually unknown, a low-complexity estimator for the path delays of all users is derived from a maximum-likelihood approach. For all channel estimators, optimal training sequences sets based on perfect root-of-unity sequences are found. For these optimal sets, it is shown that the reduction in channel estimation mean-squared error of the structured estimator versus the unstructured estimator is *exactly* the ratio of the number of structured parameters to unstructured parameters. Simulation results show that structured channel estimation provide advantages up to 4 dB in terms of output signal-to-interference plus noise ratio with respect to unstructured estimation, for linear MMSE detection. In contrast, for conventional single-user matched filtering, unstructured estimation proves to be sufficiently good.

Keywords: DS/CDMA, channel estimation, multiuser detection.

1 Introduction

One of the proposals for UMTS, the European 3rd generation mobile communication system, is based on hybrid TDMA/CDMA (briefly denoted as T-CDMA) [1, 2, 3]. In this scheme, multiple access is regulated by a TDMA slot structure similar to GSM [4], and time-division duplexing (TDD) is adopted. User signals are direct-sequence spread-spectrum, with a limited processing gain L (typically $L = 16$). In contrast to standard TDMA, in T-CDMA multiple users per cell are allowed to transmit over the same time slot.

In the uplink (mobile-to-base), users have a coarse common timing reference and are able to align their signal "blocks" (or "bursts") with the slot reference of the base station. Due to imperfect synchronization and to the different multipath channels for each user, the signal blocks experience some misalignment. This misalignment is compensated for by inserting guard intervals of appropriate duration. Therefore, the system is block-synchronous, but cannot be considered chip-synchronous.

Provided that the blocks are sufficiently short with respect to the channel coherence time [5], the user channels can be considered time-invariant over the block duration. On the other hand, due to the TDMA dynamic user allocation over the slots and to the bursty nature of transmission, tracking the channels from block to block might be infeasible. Hence, we shall consider blockwise channel estimation, where the receiver estimates the user channels block-by-block without tracking across different blocks.

We consider training-based joint channel estimation of all uplink users as in [3]. Each signal block contains a training sequence of known chips, in a fixed nominal position (typically, in the middle of the block [1, 4]). The base-station receiver makes use of these training sequences in order to estimate the channel impulse responses of all users. Least-Squares (LS) training-based channel estimation is well-known in the single-user case (see [6, 7] and references therein) and has been extended to the block-synchronous multiuser case in [3]. Most literature considers a chip-matched filter front-end, and chip-rate sampling without an explicit timing reference. Under the assumption that the chip-shaping transmit pulse is bandlimited with non-zero excess bandwidth [5], chip-rate sampling does not provide sufficient statistics for data detection unless the correct timing epoch is chosen. Notice that, due to the frequency-selective channels and chip-asynchronous transmission, chip-rate sampling is always suboptimal. Sufficient statistics for detection can be obtained by low-pass filtering and sampling at a frequency larger than the chip-rate. In this case, the chip-matched filter front-end is to be avoided, since in general it produces a colored discrete-time noise sequence. We shall consider an ideal low-pass filter front-end with bandwidth equal to half of the receiver sampling rate. In this way, the noise after sampling is white, and LS is equivalent to maximum-likelihood (ML) estimation (for Gaussian noise) without requiring any noise whitening. Moreover, the discrete-time channel responses are naturally represented as polyphase filters and each "sampling phase" can be estimated independently of the others without loss of optimality.

In standard LS (or ML) training-based channel estimation, the user channels are modeled as FIR filters with unknown coefficients [3], and no *a priori* information about the channels is exploited. We refer to this approach as "unstructured" channel estimation. More recently, *a priori* knowledge of the structure of the users channel impulse responses has been exploited in order to improve estimation. In some works (see for example [8, 9, 10, 11]), user channels are modeled as discrete multipath characterized by a set of path delays and gains and rectangular chip-shaping pulses are assumed. After chip-matched filtering, the overall impulse responses are linear combinations of delayed triangular pulses. Since triangular pulses are piecewise linear, the discrete-time overall channels are linear functions both of the multipath gains and of the delays, and this is exploited for estimation. With this approach the channels are represented directly in terms of their "physical" parameters (multipath gains and delays). Then, a minimal number of unknowns needs to be estimated. On the other hand, these methods do not generalize easily to arbitrary chip-shaping pulses (typically, root-raised-cosine (RRC) pulses with a given roll-off factor [5]). Another method (see [12, 13] and references therein) exploits the fact that the discrete-time overall user channels are found to lie in the column-space of an *a priori* known matrix determined by the chip-shaping pulse and by the maximum delay-spread. This has the advantage of being applicable to general bandlimited chip-shaping pulses but yields generally more unknowns than the methods based on physical channel parameterization.

In this paper, we propose two types of "structured" channel estimators which can be applied to any arbitrary (approximately bandlimited) chip-shaping pulse. Our first method is essentially the multiuser version of [12], and exploits only the coarse information represented by the maximum delay-spread and by the chip-shaping pulse. Our second method is based on the physical channel parameterization and exploits the knowledge of the path delays of all users. In practice, this information is not usually available. Thus, we propose a two-step approach where first the path delays are explicitly estimated, and then used in the structured channel estimator. Starting from a ML approach, we derive a low-complexity path delay estimator suited for the first step of structured channel estimation.

For both structured and unstructured channel estimation, we discuss the problem of training sequence optimization and we show that the rich family of perfect root-of-unity sequences (PRUS) [14] provides optimal training sequence sets for all desired lengths. Remarkably, the same set of sequences is optimal for both structured and unstructured estimation. If optimal sequences are used, we are able to show that the estimation mean-squared error (MSE) reduction provided by structured over unstructured estimation is given by the ratio between the number of structured and unstructured parameters. This result is not generally true for an arbitrary choice of the training sequences.

At the base-station, the channel estimates are used to compute the coefficients of a bank of receiving filters, whose output is sampled at the symbol rate and sent to a bank of single-user decoders [15, 16]. In this case, as discussed extensively in [15, 16, 17, 18], assuming single-user capacity-achieving Gaussian codes[1] with interleaving over a large number of slots, the system spectral efficiency is given by $\eta = \beta E[\log_2(1+\text{SINR})]$ bit/s/Hz, where β is a proportionality constant that depends on the chip-shaping pulse excess bandwidth [5] and on the "channel load" (i.e., the number of users divided by the spreading gain). On the other hand, if each slot is independently encoded and decoded, the codeword error rate is closely related to the information outage probability [19] and, in the limit for a large number of symbols per slot, the system spectral efficiency subject to a required outage probability ϵ is given by $\eta = \beta \log_2(1 + \gamma)$ where γ is the largest value for which $\Pr(\text{SINR} \leq \gamma) \leq \epsilon$. In all cases, the system spectral efficiency is determined by the cumulative distribution function (cdf) of the SINR at the output of the linear receivers. Classical choices for the linear filters are either the single-user matched filter (SUMF) and the linear minimum MSE (LMMSE) filter [22]. The SUMF is normally implemented in current DS/CDMA systems as a rake receiver [5], and the LMMSE one of the solutions proposed for T-CDMA [2].

Motivated by the above arguments, we compare our estimators by evaluating the SINR cdf at the output of the SUMF and LMMSE receivers when the filter coefficients are cal-

[1]With modern concatenated coding and iterative decoding techniques [20, 21] the Gaussian capacity can be closely approached by practical finite-complexity codes.

culated from the channel estimates. Simulations show that structured channel estimation provide advantages up to 4 dB in terms of output SINR with respect to unstructured estimation, for LMMSE detection. In contrast, for conventional SUMF detection, unstructured estimation proves to be sufficiently good.

This paper is organized as follows. In Section 2, the system model is defined. Section 3 presents the maximum-likelihood unstructured channel estimator and the corresponding optimal training sequences. In Section 4, we derive the structured channel estimators. In addition, it is shown that the training sequences previously found are optimal also in this case. Furthermore, a multipath delay estimator is also derived. Section 5 presents simulation results and in Section 6 we provide some concluding remarks.

Notation. i) $\mathbf{v} \sim \mathcal{N}_{\mathbb{C}}(\overline{\mathbf{v}}, \mathbf{R})$ means that the random vector $\mathbf{v}$ is complex Gaussian with circular symmetry, with mean $E[\mathbf{v}] = \overline{\mathbf{v}}$ and covariance $E[\mathbf{v}\mathbf{v}^H] - \overline{\mathbf{v}}\overline{\mathbf{v}}^H = \mathbf{R}$. ii) $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. When the dimension is clear from the context, the subscript n may be omitted. iii) $\delta_{i,j}$ denotes the Kronecker delta symbol, equal to 1 if $i = j$ and 0 otherwise. iv) $\propto$ means "proportional to". v) Superscripts ^{T} and ^{H} indicate transpose and Hermitian transpose, respectively. vi) $\mathrm{sinc}(x) \stackrel{\Delta}{=} \sin(\pi x)/(\pi x)$. vii) $\otimes$ and $\star$ denote Kronecker product [23] and convolution, respectively.

2 Signal model

Fig. 1 shows the block diagram of the complex baseband equivalent system at hand, with K users. Since blockwise channel estimation is considered, we focus on a particular "reference" time slot. The k-th user multipath channel impulse response is assumed to be time-invariant on each slot and is expressed by

$$c_k(t) = \sum_{p=0}^{P-1} c_{k,p}\delta(t - \tau_{k,p}) \tag{1}$$

We do not make any explicit distinction between the delay introduced by non-ideal user synchronization and the delay introduced by multipath propagation. These effects are incorporated in the channel impulse response so that the delays $\tau_{k,p}$ account also for synchronization errors. Without loss of generality, we assume that all users have the same number P of paths: if $c_k(t)$ has less than P paths, the gains corresponding to the missing paths are zero. Also, we assume that all users have the same *transmit* power:

different *received* signal-to-noise ratios (SNR) are taken into account by multiplying each user channel coefficients by the corresponding amplitude factor.

We assume that the system is designed to cope with a given maximum overall delay-spread Δ (a design parameter known a priori [1]). An initial synchronization procedure (not taken into account here) allows user k to transmit only if $\max_p\{\tau_{k,p}\} \leq \Delta$, i.e., if it is synchronized to the base-station slot time reference with overall delay-spread not larger than Δ [24]. In general, Δ is much shorter than the slot duration but longer than the chip interval. Then, the system is block-synchronous but not chip-synchronous.

The DS/CDMA signal transmitted by user k is given by

$$u_k(t) = \sum_m b_k[m] \left[\sum_{\ell=0}^{L-1} a_k[mL+\ell]\psi(t-(mL+\ell)T_c) \right] \tag{2}$$

where $b_k[m]$ is the m-th data symbol, belonging to some unit-energy complex signal alphabet (e.g., BPSK, QPSK, 16QAM, etc ...), T_c is the chip interval, $a_k[n]$ is the n-th (complex) chip, L is the spreading gain and $\psi(t)$ is the chip-shaping pulse, common to all users. The pulse $\psi(t)$ is obtained by truncating a bandlimited waveform over an interval of duration κT_c, where the integer κ is sufficiently large in order to make the bandwidth of $\psi(t)$ *approximately* equal to $(1+\alpha)/(2T_c)$ (with $0 < \alpha \leq 1$) and such that $\int \psi(\tau)\psi(\tau+t)^* d\tau$ satisfies *approximately* the Nyquist criterion [5]. For example, in the UMTS T-CDMA system $\psi(t)$ is obtained by truncating a RRC pulse [5] with roll-off factor $\alpha = 0.22$ [1].

Each user transmits a training sequence of known chips in a fixed position in the slot. During training no data is transmitted (i.e., the data symbols are all equal to 1) and the number of training chips is not necessarily equal to a multiple of L. Without loss of generality, we fix the time axis origin so that the training sequence is transmitted on the interval $[-(Q-1)T_c, (M-1)T_c]$, for integers Q and M, where

$$Q = \lceil \Delta/T_c \rceil + \kappa \tag{3}$$

is chosen to be not smaller than the duration, expressed in chip intervals, of the overall impulse responses

$$g_k(t) = \psi(t) \star c_k(t) \tag{4}$$

for all users.

The received signal, given by the superposition of all users plus Gaussian background noise (accounting also for outer-cell interference), can be written as

$$r(t) = \sum_{k=1}^{K} u_k(t) \star c_k(t) + \nu(t) \tag{5}$$

where $\nu(t)$ is a complex circularly-symmetric Gaussian process with autocorrelation function $E[\nu(t)\nu(t-\tau)^*] = N_0\delta(\tau)$.

The baseband receiver front-end is an ideal low-pass filter with bandwidth $W/2$ and amplitude $1/\sqrt{W}$, whose output is sampled at rate $W = N_c/T_c$ ($N_c \geq 2$ is an integer), without any explicit timing reference. The channel estimator collects $N_c M$ samples of the received signal over the interval $[0, MT_c]$. This are given by

$$r[nN_c + \ell] = \sum_{k=1}^{K} \sum_{m=0}^{Q-1} g_{k,\ell}[m] a_k[n-m] + \nu[nN_c + \ell] \tag{6}$$

for $n = 0, \ldots, M-1$ and $\ell = 0, \ldots, N_c - 1$, where $\nu[j]$ is the discrete-time low-pass filtered noise sequence with i.i.d. samples $\sim \mathcal{N}_{\mathbb{C}}(0, N_0)$, and where we define the *polyphase representation* of the discrete-time low-pass filtered overall channel impulse response

$$g_{k,\ell}[m] \stackrel{\Delta}{=} \frac{1}{\sqrt{W}} g_k((mN_c + \ell)/W) \qquad m = 0, \ldots, Q-1 \tag{7}$$

for $\ell = 0, \ldots, N_c - 1$. For later use, we collect the samples of the ℓ-th sampling phase of the k-th user channel in the vector $\mathbf{g}_{k,\ell} = (g_{k,\ell}[0], \ldots, g_{k,\ell}[Q-1])^T$. When Q satisfies (3), for all realizations of the users synchronization errors and channel multipath delays the samples (6) contains only the contribution of known training chips (see Fig. 2).

3 Unstructured ML channel estimation

In this section we review standard ML channel estimation as proposed in [3] and we derive optimal sets of training sequences.

The channel estimator forms the vectors $\mathbf{r}_\ell = (r[\ell], r[N_c + \ell], \ldots, r[(M-1)N_c + \ell])^T$, for $\ell = 0, \ldots, N_c - 1$. After some standard algebra [3, 6, 7], these can be written in the compact matrix form

$$\mathbf{r}_\ell = \mathbf{A}\mathbf{g}_\ell + \boldsymbol{\nu}_\ell \tag{8}$$

where $\mathbf{g}_\ell = (\mathbf{g}_{1,\ell}^T, \ldots, \mathbf{g}_{K,\ell}^T)^T$ is the total user channel vector corresponding to the ℓ-th sampling phase, $\boldsymbol{\nu}_\ell = (\nu[\ell], \nu[N_c + \ell], \ldots, \nu[(M-1)N_c + \ell])^T$ is a vector of i.i.d. noise samples, and $\mathbf{A} = [\mathbf{A}_1, \ldots, \mathbf{A}_K]$ is a $M \times KQ$ block matrix containing only training chips, whose k-th $M \times Q$ block is given by the convolution (Toeplitz) matrix

$$\mathbf{A}_k = \begin{bmatrix} a_k[0] & a_k[-1] & \cdots & a_k[-Q+1] \\ a_k[1] & a_k[0] & \cdots & a_k[-Q+2] \\ \vdots & & \ddots & \vdots \\ a_k[M-1] & a_k[M-2] & \cdots & a_k[M-Q] \end{bmatrix} \tag{9}$$

Since the noise in (6) is Gaussian and white, the ML estimation of $\{\mathbf{g}_0, \ldots, \mathbf{g}_{N_c-1}\}$ from the observation $\{\mathbf{r}_0, \ldots, \mathbf{r}_{N_c-1}\}$ splits into the individual ML estimations of $\mathbf{g}_\ell$ with observation $\mathbf{r}_\ell$, for $\ell = 0, \ldots, N_c - 1$. Moreover, since $\boldsymbol{\nu}_\ell$ has i.i.d. components, ML estimation is equivalent to the simple LS estimation [25]:

$$\widehat{\mathbf{g}}_\ell = \arg \min_{\mathbf{g}} |\mathbf{r}_\ell - \mathbf{A}\mathbf{g}|^2 \tag{10}$$

whose solution is readily given by

$$\widehat{\mathbf{g}}_\ell = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{r}_\ell \tag{11}$$

where we assume that $\mathbf{A}$ has full column-rank, i.e., $\text{rank}(\mathbf{A}) = KQ$.

The above method is referred to as "unstructured" since it treats $\mathbf{g}_\ell$ as a vector of KQ unknowns, without taking into account that the elements of $\mathbf{g}_\ell$ depend on the same set of "physical" channel parameters (the delays and channel gains of the channel model (1)).

3.1 Optimal training sequences

The estimation error vector $\mathbf{e}_\ell = \widehat{\mathbf{g}}_\ell - \mathbf{g}_\ell = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \boldsymbol{\nu}_\ell$ is $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma} \stackrel{\Delta}{=} E[\mathbf{e}_\ell \mathbf{e}_\ell^H] = N_0 (\mathbf{A}^H \mathbf{A})^{-1}$. The resulting normalized estimation MSE of the unstructured estimator is given by

$$\epsilon^2_{\text{unstr}} \stackrel{\Delta}{=} \frac{1}{KQN_c} \sum_{\ell=0}^{N_c-1} E[|\mathbf{e}_\ell|^2] = \frac{N_0}{KQ} \text{Tr}\left((\mathbf{A}^H \mathbf{A})^{-1}\right) \tag{12}$$

It is well-known [3, 6, 7], that optimal training sequences minimizing $\epsilon^2_{\text{unstr}}$ must satisfy $\mathbf{A}^H \mathbf{A} \propto \mathbf{I}$. This fact has been shown either by using matrix inequalities [6] or by using a matched filter argument [7]. Here, we provide an alternative simple proof by solving a constrained minimization problem.

The estimation MSE is inversely proportional to the chip-energy. Therefore, we impose the trace constraint $\text{Tr}(\mathbf{A}^H\mathbf{A}) = \mathcal{E}$, where $\mathcal{E}$ can be interpreted as the total energy devoted to training. Let $\{\lambda_i^2 : i = 1, \ldots, KQ\}$ denote the eigenvalues of $\mathbf{A}^H\mathbf{A}$ (all positive, by construction). Optimal sequence sets are solutions of

$$\begin{cases} \text{minimize} & \text{Tr}\left((\mathbf{A}^H\mathbf{A})^{-1}\right) = \sum_i 1/\lambda_i^2 \\ \text{subject to} & \text{Tr}(\mathbf{A}^H\mathbf{A}) = \sum_i \lambda_i^2 = \mathcal{E} \end{cases} \tag{13}$$

By applying Lagrange multipliers we obtain that the minimum is achieved by the constant eigenvalues $\lambda_i^2 = \mathcal{E}/(KQ)$. Since $\mathbf{A}^H\mathbf{A}$ is positive definite, there exists a unitary matrix $\mathbf{Q}$ such that $\mathbf{A}^H\mathbf{A} = \mathcal{E}/(KQ)\mathbf{Q}\mathbf{I}\mathbf{Q}^H$, which yields $\mathbf{A}^H\mathbf{A} = \mathcal{E}/(KQ)\mathbf{I}$. Hence, in order to minimize the MSE, $\mathbf{A}^H\mathbf{A}$ must be proportional to the identity matrix.

The construction of optimal training sequences satisfying $\mathbf{A}^H\mathbf{A} \propto \mathbf{I}$ has been investigated in several papers (see [3, 6, 7, 26, 14, 27] and references therein). Most related research considers sequences constructed from simple symbol alphabets, like BPSK and QPSK. Unfortunately, this requirement is too restrictive and optimal sequences cannot be found for most training lengths M. In some works (see [3, 26]), exhaustive search for BPSK sequences minimizing the off-diagonal elements of $\mathbf{A}^H\mathbf{A}$ is made for small M, and heuristic construction of good sequences with small off-diagonal elements of $\mathbf{A}^H\mathbf{A}$ is carried out for large M. Other works propose the use of non-constant envelope sequences [7].

We observe that requiring simple symbol alphabets like BPSK or QPSK does not bring a real complexity advantage, especially if modern DSP architectures are used for implementation. Rather, a constant envelope allows more efficient utilization of the transmitter power amplifier [4], and has a major impact on the battery life of the mobile terminals. In this paper, we require the chips to belong to a N-th root-of-unity alphabet $\mathcal{A}_N = \{e^{j2\pi i/N} : i = 0, \ldots, N-1\}$, for some integer N. This choice is enough to obtain optimal training sequences for any desired training length M, while preserving the constant envelope.[2] In particular, an optimal set of training sequences can be derived from a single PRUS [14]:

Definition: Perfect Root-of-Unity Sequences. *The sequence* $\mathbf{x} = (x_0, \ldots, x_{M-1}) \in$

[2]For the sake of precision, we have to say that even if the chips have constant magnitude, the transmit signal after the chip-shaping filter is not constant-envelope any longer. Then, for a given non-linear amplifier, there might be sequences that perform better than others (e.g., avoiding phase transitions of π between consecutive symbols). This type of “fine-tuning” optimization is beyond the scope of this paper, and is left for future work.

$\mathbb{C}^M$ *is a PRUS if* $x_m \in \mathcal{A}_N$*, for all* $m = 0, \ldots, M-1$ *and some integer* N*, and if its periodic autocorrelation satisfies*

$$\phi_x(n) = \sum_{m=0}^{M-1} x_m x^*_{[m-n \mod M]} = M\delta_{n,0}$$

□

Explicit and simple constructions of PRUSs of any length are provided in [14, 27]. Let $\mathbf{x}$ be a PRUS of length $M \geq KQ$. Then, the k-th user training sequence $(a_k[-Q+1], \ldots, a_k[M-1])$ is obtained from $\mathbf{x}$ as

$$a_k[m] = \sqrt{\mathcal{E}_c} x_{[m-(k-1)Q \mod M]} \tag{14}$$

for all $k = 1, \ldots, K$ and $m = -Q+1, \ldots, M-1$, where $\mathcal{E}_c$ is the transmit chip-energy. By using (14) into (9), it is easy to check that the columns of $\mathbf{A}$ are distinct cyclic shifts of the same PRUS $\mathbf{x}$. By construction, we obtain $\mathbf{A}^H\mathbf{A} = M\mathcal{E}_c\mathbf{I}$, as desired. The resulting minimum estimation error is given by

$$\epsilon^2_{\text{unstr}} = \frac{L}{M}\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1} \tag{15}$$

where $\mathcal{E}_s = L\mathcal{E}_c$ is the nominal average transmit energy per symbol.

From the implementation point of view, the proposed channel estimation scheme is extremely simple. Since the columns of $\mathbf{A}$ are cyclic shifts of $\mathbf{x}$, the vector $\mathbf{A}^H\mathbf{r}_\ell$ can be seen as the result of the cyclic convolution of $\mathbf{x}$ with $\mathbf{r}_\ell$. Then, it can be computed efficiently in the frequency domain, by using FFT. In particular, the discrete Fourier transform of $\mathbf{x}$ can be precomputed and stored in the receiver. Moreover, no matrix storage or matrix-vector product are needed since $(\mathbf{A}^H\mathbf{A})^{-1} \propto \mathbf{I}$. Finally, the $\widehat{\mathbf{g}}_\ell$'s for $\ell = 0, \ldots, N_c - 1$ can be computed in parallel, by N_c identical processors.

4 Structured channel estimation

In this section we propose estimation methods that exploit some *a priori* information of the "structure" of the channel. Our first method (referred to as *Type I structured estimation*) exploits coarse *a priori* knowledge about the channel responses represented by the maximum delay-spread Δ and by the chip-shaping pulse $\psi(t)$. Our second method (referred to as *Type II structured estimation*) exploits finer *a priori* knowledge about the channel responses, represented by the sets of delays $\{\tau_{k,p} : p = 0, \ldots, P-1\}$ for all $k = 1, \ldots, K$, and by the number of paths P.

4.1 Channel structure

By using the fact that $\psi(t)$ is bandlimited[3] and by using (1) and (4) in (7) we can write explicitly

$$g_{k,\ell}[m] = \sum_{i=0}^{\kappa N_c - 1} \psi(i/W)\widetilde{c}_k[mN_c + \ell - i] \tag{16}$$

where we define the low-pass filtered and sampled channel response

$$\widetilde{c}_k[j] \stackrel{\Delta}{=} \sum_{p=0}^{P-1} \frac{c_{k,p}}{\sqrt{W}} \mathrm{sinc}(j - \tau_{k,p}W) \tag{17}$$

Let $\widetilde{\mathbf{c}}_k \stackrel{\Delta}{=} (\widetilde{c}_k[0], \ldots, \widetilde{c}_k[D-1])^T$ be the vector of significant channel response samples, where D is a suitable integer known *a priori* (it depends on Δ), let $\mathbf{c}_k \stackrel{\Delta}{=} (c_{k,0}, \ldots, c_{k,P-1})^T$ and define the $Q \times D$ convolution matrix $\boldsymbol{\Psi}_\ell$ with (i,j)-th element

$$[\boldsymbol{\Psi}_\ell]_{i,j} = \psi\left(\frac{iN_c + \ell - j}{W}\right) \tag{18}$$

for $i = 0, \ldots, Q-1$ and $j = 0, \ldots, D-1$, and the $D \times P$ interpolation matrix $\boldsymbol{\Phi}_k$ with (j,p)-th element

$$[\boldsymbol{\Phi}_k]_{j,p} = \mathrm{sinc}\left(j - \tau_{k,p}W\right)/\sqrt{W} \tag{19}$$

Then, (16) and (17) can be written in matrix form as

$$\begin{aligned} \mathbf{g}_{k,\ell} &= \boldsymbol{\Psi}_\ell \widetilde{\mathbf{c}}_k \\ \widetilde{\mathbf{c}}_k &= \boldsymbol{\Phi}_k \mathbf{c}_k \end{aligned} \tag{20}$$

respectively. We define $\widetilde{\mathbf{c}} \stackrel{\Delta}{=} (\widetilde{\mathbf{c}}_1^T, \ldots, \widetilde{\mathbf{c}}_K^T)^T$, $\mathbf{c} \stackrel{\Delta}{=} (\mathbf{c}_1^T, \ldots, \mathbf{c}_K^T)^T$, the block-diagonal matrix $\boldsymbol{\Phi} \stackrel{\Delta}{=} \mathrm{diag}(\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_K)$ of dimension $KD \times KP$ and the Kronecker product matrix $\underline{\boldsymbol{\Psi}}_\ell \stackrel{\Delta}{=} \mathbf{I}_K \otimes \boldsymbol{\Psi}_\ell$ of dimension $KQ \times KD$. We can write $\widetilde{\mathbf{c}} = \boldsymbol{\Phi}\mathbf{c}$ and, by recalling the definition of $\mathbf{g}_\ell$, we get $\mathbf{g}_\ell = \underline{\boldsymbol{\Psi}}_\ell \widetilde{\mathbf{c}}$. Finally, we stack the different sampling phases $\mathbf{g}_\ell$ into a single vector $\mathbf{g} \stackrel{\Delta}{=} (\mathbf{g}_0^T, \ldots, \mathbf{g}_{N_c-1}^T)^T$ to obtain

$$\mathbf{g} = \underline{\boldsymbol{\Psi}}\widetilde{\mathbf{c}} = \underline{\boldsymbol{\Theta}}\mathbf{c} \tag{21}$$

where we define the $KQN_c \times KD$ block matrix

$$\underline{\boldsymbol{\Psi}} \stackrel{\Delta}{=} \left[\begin{array}{c} \underline{\boldsymbol{\Psi}}_0 \\ \vdots \\ \underline{\boldsymbol{\Psi}}_{N_c-1} \end{array}\right]$$

[3] We neglect the distortion due to truncation, since practical systems are designed to keep this distortion very small.

and where we define the $KQN_c \times KP$ matrix $\underline{\boldsymbol{\Theta}} \stackrel{\Delta}{=} \underline{\boldsymbol{\Psi}}\boldsymbol{\Phi}$. Equations (21) define the *a priori* structure of the channel impulse responses. The matrix $\underline{\boldsymbol{\Psi}}$ is determined by the chip-shaping pulse $\psi(t)$ and by the maximum delay-spread Δ, therefore it is known by the receiver. The matrix $\underline{\boldsymbol{\Theta}}$ is determined also by the path delays $\{\tau_{k,p} : p = 0, \ldots, P-1\}$ for all $k = 1, \ldots, K$, and by the number of paths P, which are generally not known a priori.

4.2 Type I structured estimation

Let $\mathbf{r} \stackrel{\Delta}{=} (\mathbf{r}_0^T, \ldots, \mathbf{r}_{N_c-1}^T)^T$ and $\boldsymbol{\nu} \stackrel{\Delta}{=} (\boldsymbol{\nu}_0^T, \ldots, \boldsymbol{\nu}_{N_c-1}^T)^T$. Then,

$$\mathbf{r} = [\mathbf{I}_{N_c} \otimes \mathbf{A}]\,\mathbf{g} + \boldsymbol{\nu} \tag{22}$$

where $\mathbf{A}$ is defined in (9). From the first equality of (21) we have that the desired channel vector $\mathbf{g}$ lies in the column-space of the *a priori* known matrix $\underline{\boldsymbol{\Psi}}$. If the null-space of $\underline{\boldsymbol{\Psi}}$ is non-trivial, this information can be exploited to improve channel estimation.

We observe that $\underline{\boldsymbol{\Psi}}$ might be ill-conditioned for large D and/or large N_c, as it is obtained by oversampling the bandlimited waveform $\psi(t)$. In order to cope with this possibility, we use singular-value decomposition (SVD) [28] and re-parameterize our estimation problem. We can write

$$\underline{\boldsymbol{\Psi}} = \mathbf{U}'\mathbf{S}'(\mathbf{V}')^H \tag{23}$$

where $\mathbf{S}'$ is $\rho' \times \rho'$ diagonal with positive decreasing diagonal elements (singular values), ρ' is the rank of $\underline{\boldsymbol{\Psi}}$, and $\mathbf{U}'$, $\mathbf{V}'$ are rectangular matrices with orthonormal columns and dimension $KQN_c \times \rho'$ and $KD \times \rho'$, respectively. Then, $\mathbf{g} = \underline{\boldsymbol{\Psi}}\widetilde{\mathbf{c}} = \mathbf{U}'\mathbf{d}'$, where $\mathbf{d}' \stackrel{\Delta}{=} \mathbf{S}'(\mathbf{V}')^H\widetilde{\mathbf{c}}$. Notice that any component of $\widetilde{\mathbf{c}}$ in the null-space of $\underline{\boldsymbol{\Phi}}$ has no impact on the observed channel response $\mathbf{g}$. Thus, we can first project $\widetilde{\mathbf{c}}$ onto the orthogonal complement of the null-space of $\underline{\boldsymbol{\Psi}}$ and then estimate its projection $\mathbf{d}'$. The proposed channel estimation algorithm is as follows:

1. Obtain the ML estimate of $\mathbf{d}'$ from the observation $\mathbf{r}$ as

$$\begin{aligned}\widehat{\mathbf{d}} &= \arg\min_{\mathbf{d}} |\mathbf{r} - [\mathbf{I}_{N_c} \otimes \mathbf{A}]\,\mathbf{U}'\mathbf{d}|^2 \\ &= \left((\mathbf{U}')^H[\mathbf{I}_{N_c} \otimes (\mathbf{A}^H\mathbf{A})]\mathbf{U}'\right)^{-1}(\mathbf{U}')^H[\mathbf{I}_{N_c} \otimes \mathbf{A}^H]\mathbf{r}\end{aligned} \tag{24}$$

2. Obtain the Type I structured estimate of $\mathbf{g}$ as $\widehat{\mathbf{g}} = \mathbf{U}'\widehat{\mathbf{d}}$.

Since $\underline{\boldsymbol{\Psi}}$ is known *a priori*, no real-time SVD computation is required.

4.3 Type II structured estimation

If the delays $\tau_{k,p}$ and the number of paths P are known, the receiver can compute $\underline{\boldsymbol{\Theta}}$ and impose that the channel vector $\mathbf{g}$ must lie in its column space (see the second equality of (21)). By substituting $\mathbf{g} = \underline{\boldsymbol{\Theta}}\mathbf{c}$ into (22) we get the linear model

$$\mathbf{r} = [\mathbf{I}_{N_c} \otimes \mathbf{A}]\,\underline{\boldsymbol{\Theta}}\mathbf{c} + \boldsymbol{\nu} \tag{25}$$

Also in this case it is convenient to re-parameterize the problem by using the SVD $\underline{\boldsymbol{\Theta}} = \mathbf{U}''\mathbf{S}''(\mathbf{V}'')^H$, where $\mathbf{S}''$ is $\rho'' \times \rho''$ diagonal with positive decreasing diagonal elements (singular values), ρ'' is the rank of $\underline{\boldsymbol{\Theta}}$, and $\mathbf{U}''$, $\mathbf{V}''$ are rectangular matrices with orthonormal columns and dimension $KQN_c \times \rho''$ and $KP \times \rho''$, respectively. Then, $\mathbf{g} = \underline{\boldsymbol{\Theta}}\mathbf{c} = \mathbf{U}''\mathbf{d}''$, where $\mathbf{d}'' \triangleq \mathbf{S}''\mathbf{V}''^H\mathbf{c}$. Since in general neither the $\tau_{k,p}$'s nor the number of paths P are *a priori* known, these parameters must also be estimated from the received signal. We propose a two-step approach where first an estimate $\widehat{\tau}_{k,p}$ of delays $\tau_{k,p}$ is obtained and then the above Type II structured estimator is computed assuming $\tau_{k,p} = \widehat{\tau}_{k,p}$. More precisely, we have:

1. Obtain an estimate the maximum number of paths per user $\widehat{P}$ and of the delays $\{\widehat{\tau}_{k,p} : p = 0, \ldots, \widehat{P} - 1\}$, for all $k = 1, \ldots, K$.

2. Based on the estimated delays, compute an estimate $\widehat{\underline{\boldsymbol{\Theta}}}$ of $\underline{\boldsymbol{\Theta}}$ and the $KQN_c \times \widehat{\rho}''$ factor $\widehat{\mathbf{U}}''$ in its SVD.

3. Under the assumption $\mathbf{U}'' = \widehat{\mathbf{U}}''$, obtain the ML estimate of $\mathbf{d}''$ from the observation $\mathbf{r}$ as

$$\begin{aligned}\widehat{\mathbf{d}} &= \arg\min_{\mathbf{d}} \left| \mathbf{r} - [\mathbf{I}_{N_c} \otimes \mathbf{A}]\,\widehat{\mathbf{U}}''\mathbf{d} \right|^2 \\ &= \left((\widehat{\mathbf{U}}'')^H [\mathbf{I}_{N_c} \otimes (\mathbf{A}^H\mathbf{A})] \widehat{\mathbf{U}}'' \right)^{-1} (\widehat{\mathbf{U}}'')^H [\mathbf{I}_{N_c} \otimes \mathbf{A}^H]\mathbf{r}\end{aligned} \tag{26}$$

4. Obtain the Type II structured estimate of $\mathbf{g}$ as $\widehat{\mathbf{g}} = \widehat{\mathbf{U}}''\widehat{\mathbf{d}}$.

Since $\widehat{\underline{\boldsymbol{\Theta}}}$ is not known *a priori*, a real-time SVD is needed every time a new estimate of the users path delays is available.

In general, since $\mathbf{U}'' \neq \widehat{\mathbf{U}}''$ the Type II estimator is biased. We expect that the performance of the proposed estimator depends critically on the quality of the preliminary delay estimation. On the other hand, for perfect knowledge of the $\tau_{k,p}$'s, Type II and Type

I estimators have the same form. Therefore, for perfect delay information an optimal training sequence set for the Type I is also optimal for the Type II. In the next section, we find optimal training sequence sets for Type I and Type II structured estimators (assuming perfect knowledge of the delays for the latter) and we postpone the delay estimation problem to Section 4.5.

Remark: near-far resistance of channel estimators. In multiuser detection, a receiver is said to be near-far resistant for user k if its error probability goes to zero as $N_0 \to 0$ for any choice of the other users received powers [22]. Analogously, we say that a channel estimator for user k is near-far resistant if its estimation MSE goes to zero as $N_0 \to 0$ for any choice of the other users received powers. In particular, if a multiuser channel estimator can be put in the form $\widehat{\mathbf{g}} = \mathbf{g}+\mathbf{e}$, where the error vector $\mathbf{e}$ has mean zero and covariance matrix $\boldsymbol{\Sigma}$ such that $\mathrm{Tr}(\boldsymbol{\Sigma}) \to 0$ as $N_0 \to 0$, then it is near-far resistant. According to the above definition, the unstructured and structured Type I estimators are near-far resistant while the structured Type II estimator is near-far resistant if it is unbiased, i.e., if the delays are perfectly known. $\diamondsuit$

4.4 Optimal training sequences.

We let $\mathbf{U} = \mathbf{U}'$ and $\mathbf{d} = \mathbf{d}'$ (resp., $\mathbf{U} = \widehat{\mathbf{U}}''$ and $\mathbf{d} = \mathbf{d}''$) for Type I (resp., Type II) estimation, and for Type II we assume perfect knowledge of the delays, i.e., $\widehat{\mathbf{U}}'' = \mathbf{U}''$. The error vector in the estimation of $\mathbf{d}$ is given by $\mathbf{e} = \widehat{\mathbf{d}} - \mathbf{d}$, $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{\Sigma})$, where

$$\boldsymbol{\Sigma} \stackrel{\Delta}{=} E[\mathbf{e}\mathbf{e}^H] = N_0 \left(\mathbf{U}^H \left[\mathbf{I}_{N_c} \otimes (\mathbf{A}^H\mathbf{A})\right] \mathbf{U}\right)^{-1}$$

The error vector in the estimation of $\mathbf{g}$ is given by $\mathbf{U}\mathbf{e}$, $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H)$. The resulting normalized estimation MSE is given by

$$\epsilon^2_{\text{struc}} \stackrel{\Delta}{=} \frac{1}{KQN_c}\mathrm{Tr}\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H\right) = \frac{1}{KQN_c}\mathrm{Tr}(\boldsymbol{\Sigma}) \tag{27}$$

where we used the fact that $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. Then, the optimal choice of training sequences should minimize the trace of $\boldsymbol{\Sigma}$ subject to a constraint on the total training energy. We already know that this is obtained by $\boldsymbol{\Sigma} \propto \mathbf{I}$. Fortunately, since $\mathbf{U}$ has orthonormal columns, we have immediately that if $\mathbf{A}^H\mathbf{A} \propto \mathbf{I}$ then also $\boldsymbol{\Sigma} \propto \mathbf{I}$. We conclude that optimal sequences for unstructured estimation are also optimal for Type I and Type II structured estimation (assuming ideal delay estimation for the latter). In particular, the construction of training sequences from a single PRUS of length $M \geq KQ$, as indicated in (14), can be successfully applied here.

The resulting minimum estimation error is given by

$$\epsilon^2_{\text{struc}} = \frac{L}{M} \left(\frac{\mathcal{E}_s}{N_0} \right)^{-1} \frac{\rho}{KQN_c} \tag{28}$$

where $\rho = \rho'$ (resp., $\rho = \rho''$) is the rank of $\underline{\mathbf{\Psi}}$ (resp., $\underline{\mathbf{\Theta}}$) for Type I (resp., Type II) estimation.

From an implementation point of view, with optimal sequences, the structured channel estimator is obtained by $\widehat{\mathbf{g}} \propto \mathbf{U}\mathbf{U}^H \left[\mathbf{I}_{N_c} \otimes \mathbf{A}^H\right] \mathbf{r}$, where the proportionality constant is known. This can be interpreted as the linear post-processing defined by the squared matrix $\mathbf{U}\mathbf{U}^H$ of the unstructured channel estimation $\left[\mathbf{I}_{N_c} \otimes \mathbf{A}^H\right] \mathbf{r}$ of Section 3. For Type I estimation, $\mathbf{U}\mathbf{U}^H$ can be precomputed and stored, and the extra complexity is only given by the matrix transformation. For Type II estimation, extra complexity is also incurred by explicit delay estimation and by the SVD required to calculate $\mathbf{U}$.

Remark: reduction of the estimation MSE. By comparing (15) with (28), we see that for optimal sequences, $\epsilon^2_{\text{struc}}/\epsilon^2_{\text{unstr}} = \rho/(KQN_c)$, i.e., that the estimation MSE reduction provided by the knowledge of the *a priori* structure of the channel is given by the ratio between the number of structured and unstructured unknown parameters to be estimated.

The MSE reduction provided by Type I structured estimation can be coarsely evaluated by the following intuitive argument. If the channel delay-spread is considerably larger than the chip-shaping pulse duration, we get that $Q < D \approx N_c Q$. The $Q \times D$ matrices $\mathbf{\Psi}_{k,\ell}$ defined in (18) for $\ell = 0, \ldots, N_c - 1$ are all obtained by sampling the bandlimited pulse $\psi(t)$. Each of these matrices has full row-rank Q. However, since the rows of $\mathbf{\Psi}_{k,\ell}$ for $\ell > 0$ are obtained by sampling $\psi(t - \ell/W)$at rate W (larger than the Nyquist rate), for the Nyquist sampling theorem [5] they all lie in the linear space generated by the rows of $\mathbf{\Psi}_{k,0}$. This is not *exactly* true because of truncation, however we expect that the number of dominant singular values of the $N_c Q \times D$ combined matrix $[\mathbf{\Psi}^T_{k,0}, \ldots, \mathbf{\Psi}^T_{k,N_c-1}]^T$ is $\approx Q$, so that the rank ρ' of $\underline{\mathbf{\Psi}}$ is $\approx KQ$. This yields a MSE reduction factor of about $1/N_c$. Since $N_c \geq 2$, the improvement in the estimation MSE is about 3 dB or larger.

Fig. 3 shows the squared singular values of $[\mathbf{\Psi}^T_{k,0}, \ldots, \mathbf{\Psi}^T_{k,N_c-1}]^T$ for $N_c = 2$ and 4, $Q = 32$ and RRC chip-shaping pulse with roll-off $\alpha = 0.22$ truncated over a support of $\kappa = 12$ chips. We observe that in both cases a sharp transition occurs after the first Q singular values. In the results of Section 5 we use a threshold criterion for rank determination. Namely, the rank is estimated as the largest index for which the corresponding squared singular value is not smaller than 10^{-3} times the sum of all squared singular values. In the example of Fig. 3 this criterion yields an estimated rank equal to 33, in agreement to what anticipated by the above intuitive argument.

The MSE reduction provided by Type II structured estimation (with perfect delay knowledge) is easily evaluated by noticing that the rank of $\underline{\mathbf{\Theta}}$ is not larger than the sum of all user paths, i.e., $\rho'' \leq KP$. Then, the reduction factor provided by Type II estimation is at least $P/(N_c Q)$. For channels with a small number of dominant paths much smaller than the delay-spread (expressed in chips), such as in most ETSI models [29], the potential gain of Type II estimation is large. However, this depends critically on the channel model considered and on the ability of estimating the delays of the dominant paths of all users. $\Diamond$

4.5 Delay estimation

In this section we propose a path delay estimator derived from the ML estimation of $\boldsymbol{\tau} \stackrel{\Delta}{=} \{\tau_{k,p} : k = 1, \ldots, K, \quad p = 0, \ldots, P-1\}$ from the observation $\widehat{\mathbf{g}}$, given by the estimated discrete-time low-pass filtered channel impulse responses provided either by unstructured or by Type I structured estimation. This method can be used in the preliminary delay estimation of the Type II structured channel estimator described in Section 4.3.

Consider the preliminary channel estimate $\widehat{\mathbf{g}}$ obtained either by the unstructured estimator of Section 3 or by the Type I structured estimator. For both estimators we can write

$$\widehat{\mathbf{g}} = \mathbf{g} + \mathbf{e} = \underline{\mathbf{\Theta}}\mathbf{c} + \mathbf{e} \tag{29}$$

where $\mathbf{e} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{\Sigma})$, independent of the vector of channel gains $\mathbf{c}$. We assume Rayleigh fading, so that $\mathbf{c}$ is also $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{\Lambda})$, where the covariance matrix $\mathbf{\Lambda}$ defines the average energy for each path (delay-intensity profile [5])) and the correlation between the path gains. For given delay vector $\boldsymbol{\tau}$, $\underline{\mathbf{\Theta}}$ is fixed and $\widehat{\mathbf{g}}$ is conditionally $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{R}_g(\boldsymbol{\tau}))$, where the conditional covariance matrix is given by

$$\mathbf{R}_g(\boldsymbol{\tau}) = \underline{\mathbf{\Theta}}(\boldsymbol{\tau})\mathbf{\Lambda}\underline{\mathbf{\Theta}}(\boldsymbol{\tau})^H + \mathbf{\Sigma}$$

where we indicate explicitly the dependence on the delay vector $\boldsymbol{\tau}$.

With the above statistical model, the ML estimate of $\boldsymbol{\tau}$ based on the observation $\widehat{\mathbf{g}}$ is given by

$$\widehat{\boldsymbol{\tau}} = \arg \max_{\boldsymbol{\tau}} \left\{ -\widehat{\mathbf{g}}^H \mathbf{R}_g^{-1}(\boldsymbol{\tau})\widehat{\mathbf{g}} - \log \mathrm{Det}(\mathbf{R}_g(\boldsymbol{\tau})) \right\} \tag{30}$$

This requires the maximization of the log-likelihood function over a KP-dimensional real space. Moreover, the log-likelihood function depends on $\boldsymbol{\tau}$ via a matrix inversion.

In order to decrease complexity, we make some simplifications and assumptions. First, we assume a white error vector (i.e., $\boldsymbol{\Sigma} = \sigma_e^2 \mathbf{I}$). This assumption holds exactly for the unstructured estimator of Section 3 and approximately for the structured Type I estimator if an optimal training sequence set is used. Next, we assume that the channel gains for different users are mutually independent and that each user channel obeys the uncorrelated-scattering model [5], so that $E[c_{k,p} c^*_{k',p'}] = \sigma^2_{k,p} \delta_{k,k'} \delta_{p,p'}$. Then, after suitable reordering of the components of $\widehat{\mathbf{g}}$ and by exploiting the block-matrix structure of $\underline{\boldsymbol{\Theta}}$, the ML joint delay estimator (30) reduces to K independent ML estimators for the $\boldsymbol{\tau}_k$'s (the delays of user k), given by

$$\widehat{\boldsymbol{\tau}}_k = \arg \max_{\boldsymbol{\tau}_k} \left\{ -\left(\widehat{\mathbf{g}}^{(k)}\right)^H \mathbf{R}_k^{-1}(\boldsymbol{\tau}_k)\widehat{\mathbf{g}}^{(k)} - \log \mathrm{Det}(\mathbf{R}_k(\boldsymbol{\tau}_k)) \right\} \tag{31}$$

where

$$\widehat{\mathbf{g}}^{(k)} \triangleq (\widehat{g}_{k,0}[0], \ldots, \widehat{g}_{k,N_c-1}[0], \widehat{g}_{k,0}[1], \ldots, \widehat{g}_{k,N_c-1}[1], \ldots, \widehat{g}_{k,0}[Q-1], \ldots, \widehat{g}_{k,N_c-1}[Q-1])^T$$

where

$$\mathbf{R}_k(\boldsymbol{\tau}_k) = \sum_{p=0}^{P-1} \sigma^2_{k,p} \boldsymbol{\psi}_{k,p} \boldsymbol{\psi}^H_{k,p} + \sigma_e^2 \mathbf{I} \tag{32}$$

and where the vector $\boldsymbol{\psi}_{k,p}$ contains the samples of the chip-shaping pulse delayed by $\tau_{k,p}$. Namely, the i-th element of $\boldsymbol{\psi}_{k,p}$ is given by

$$[\boldsymbol{\psi}_{k,p}]_i = \psi(i/W - \tau_{k,p})/\sqrt{W} \tag{33}$$

for $i = 0, \ldots, QN_c - 1$.

The solution of (31) still requires maximization over a P-dimensional real space. In order to obtain an additional simplification, we assume that the delays $\boldsymbol{\tau}_k$ are sufficiently separated, i.e., that $\min_{p \neq p'} |\tau_{k,p} - \tau_{k,p'}| > \kappa T_c$. Since the support of $\psi(t)$ has limited size κT_c, the vectors $\boldsymbol{\psi}_{k,p}$ and $\boldsymbol{\psi}_{k,p'}$ have disjoint support, i.e., their non-zero components are in different positions. Therefore, $\boldsymbol{\psi}^H_{k,p} \boldsymbol{\psi}_{k,p'} = \delta_{p,p'}$ (recall that, because of the bandlimited assumption, $|\boldsymbol{\psi}_{k,p}|^2 = \frac{1}{W}\sum_{i=0}^{QN_c-1}|\psi(i/W - \tau_{k,p})|^2 \simeq \int |\psi(t)|^2 dt = 1$). Under this "separated delay" assumption, by applying the matrix inversion lemma [25] to (32) we can write

$$\mathbf{R}_k^{-1}(\boldsymbol{\tau}_k) = \frac{1}{\sigma_e^2}\left[\mathbf{I} - \sum_{p=0}^{P-1}\frac{\sigma_{k,p}^2}{\sigma_e^2+\sigma_{k,p}^2}\boldsymbol{\psi}_{k,p}\boldsymbol{\psi}_{k,p}^H\right] \tag{34}$$

Also, because of the disjoint support, $\mathbf{R}_k(\boldsymbol{\tau}_k)$ is block diagonal with P blocks of size $\kappa N_c \times \kappa N_c$ with the form

$$\sigma_{k,p}^2 \mathbf{v}\mathbf{v}^H + \sigma_e^2\mathbf{I} \tag{35}$$

where $\mathbf{v}$ contains the κN_c non-zero samples of $\psi(t)$. Since in general $P\kappa N_c \leq QN_c$, $\mathbf{R}_k(\boldsymbol{\tau}_k)$ contains also some other diagonal blocks with diagonal elements all equal to σ_e^2. Hence, $\mathrm{Det}(\mathbf{R}_k(\boldsymbol{\tau}_k))$ is given by

$$\begin{aligned}\mathrm{Det}(\mathbf{R}_k(\boldsymbol{\tau}_k)) &= \sigma_e^{2QN_c}\prod_{p=0}^{P-1}\mathrm{Det}\left(\frac{\sigma_{k,p}^2}{\sigma_e^2}\mathbf{v}\mathbf{v}^H + \mathbf{I}\right)\\ &= \sigma_e^{2QN_c}\prod_{p=0}^{P-1}\left(\frac{\sigma_{k,p}^2}{\sigma_e^2}+1\right) \end{aligned}\tag{36}$$

and it is independent of $\boldsymbol{\tau}_k$. By neglecting the terms independent of $\boldsymbol{\tau}_k$, the following approximated ML estimator is obtained:

$$\widehat{\boldsymbol{\tau}}_k = \arg\max_{\boldsymbol{\tau}_k}\sum_{p=0}^{P-1}\frac{\sigma_{k,p}^2}{\sigma_e^2+\sigma_{k,p}^2}\left|\boldsymbol{\psi}_{k,p}^H\widehat{\mathbf{g}}^{(k)}\right|^2 \tag{37}$$

We have not escaped a P dimensional maximization and typically independent maximization of each term in (37) yields $\widehat{\tau}_{k,p} = \widehat{\tau}$, for all $p = 0, \ldots, P-1$, where $\widehat{\tau}$ is located with high probability in the vicinity of the maximum peak of the sampled observed channel response $\widehat{\mathbf{g}}^{(k)}$. Also, we observe that in practice, both the delay-intensity profile and P are unknown. However, we propose an approximated algorithm which requires only a sequence of P one-dimensional maximizations.

Assume P known. First, define a delay discretization step $\Delta\tau$, such that $T_c/\Delta\tau = N_\tau$ is an integer multiple of N_c. Then, for all $j = 0, \ldots, QN_\tau - 1$, define the vectors $\mathbf{v}_j$ of length QN_c with i-th component

$$[\mathbf{v}_j]_i = \psi(i/W - j\Delta\tau)/\sqrt{W}$$

for $i = 0, \ldots, QN_c - 1$. Clearly, $\mathbf{v}_j = \boldsymbol{\psi}_{k,p}$ if $j\Delta\tau = \tau_{k,p}$, for some j, while if $\tau_{k,p}$ is not an integer multiple of $\Delta\tau$, the maximum delay discretization error is $\Delta\tau/2$. Initialize the vector $\mathbf{w}_0 = \widehat{\mathbf{g}}^{(k)}$ and the set of delay indexes $\mathcal{S}_0 = \{0, \ldots, QN_\tau - 1\}$. For $p = 0, \ldots, P-1$, repeat the following steps:

1. Estimate the p-th delay as $\widehat{\tau}_{k,p} = \widehat{j}_p \Delta\tau$, where

$$\widehat{j}_p = \arg \max_{j \in \mathcal{S}_p} \left| \mathbf{v}_j^H \mathbf{w}_p \right|^2 \tag{38}$$

2. Eliminate the effect of the p-th delay from the observed channel impulse response as

$$\mathbf{w}_{p+1} = \mathbf{w}_p - \left(\frac{\mathbf{v}_{\widehat{j}_p}^H \mathbf{w}_p}{|\mathbf{v}_{\widehat{j}_p}|^2} \right) \mathbf{v}_{\widehat{j}_p} \tag{39}$$

3. Update the delay index set as

$$\mathcal{S}_{p+1} = \mathcal{S}_p - \{\widehat{j}_p - N_\tau + 1, \ldots, \widehat{j}_p + N_\tau - 1\} \tag{40}$$

Remark: algorithm interpretation. If the "separated delay" assumption holds, we could maximize (37) by searching over all possible combinations of supports $\{S_0, \ldots, S_{P-1}\}$ of size kT_c, such that $S_p \subseteq [0, QT_c]$ and $S_p \cap S_{p'} = \emptyset$, and by maximizing each term $|\boldsymbol{\psi}_{k,p}^H \widehat{\mathbf{g}}^{(k)}|^2$ with respect to $\tau_{k,p} \in S_p$. While the "separated delay" assumption is instrumental for reducing the likelihood function to a manageable form, it is not generally satisfied. We argue that as the delays are more and more separated, the true likelihood function is closer and closer to the approximation (37). Obviously, if some paths are removed from the channel response, the separation between the delays of the remaining paths is increased and the likelihood function for these delays is better approximated by (37). Then, it is meaningful to look for the delays in order, starting from the path with the largest gain, and remove it at each p-th iteration the path corresponding to the p-th estimated delay by projecting the observation vector onto the orthogonal complement of $\mathbf{v}_{\widehat{j}_p}$. All inner products $\mathbf{v}_j^H \mathbf{w}_p$ in (38) can be computed by convolving $\mathbf{w}_p$ with a polyphase version of the chip-matched filter $\psi(t)$, sampled at frequency N_τ / T_c. Notice that N_τ can be much higher than N_c, so that the timing resolution of the delay estimator is not limited by the receiver sampling frequency. For each p, we estimate $\tau_{k,p}$ as the multiple of $\Delta\tau$ for which the output of the polyphase chip-matched filter has maximum squared magnitude. Because of noise and finite timing resolution, the cancellation of the p-th path effect is not perfect. If the p-th path gain is much larger than the remaining path gains, the energy of the p-th path after imperfect cancellation might be still larger than the energy of remaining paths. This prevents the detection of other paths. We observe that, for incorrect path cancellation, the delay estimator for $\tau_{k,p+1}$ typically finds a non-existent delay (outlier) very close to one of the previously estimated delays $\widehat{\tau}_{k,0}, \ldots, \widehat{\tau}_{k,p}$. In order to prevent this effect, in (40) we eliminate from the set of allowed delay indices the indices differing from $\widehat{j}_p$ by less than N_τ. In this way, we force the delay estimator to find delays separated by at least one chip interval. Notice that also a conventional rake receiver cannot individually track delays separated by less than one chip. In fact, since the channel is convolved with $\psi(t)$, which has bandwidth only slightly larger than $1/(2T_c)$, individual estimation of delays separated by less than one chip in essence violates the Nyquist sampling theorem. $\Diamond$

In practice, P is not known *a priori*. We can either fix an arbitrary value for the maximum number $\widehat{P}$ of paths per user (in this case, we may miss significant components for some users while we waste complexity for other users) or we can introduce a stopping rule in the delay search. Namely, after removing the p-th estimated path, if $|\mathbf{w}_{p+1}|^2/|\mathbf{w}_0|^2 \geq \gamma$, delay $(p+1)$ is searched, otherwise the algorithm is terminated and the number of paths for user k is set to p. The threshold γ should be designed according to the estimation MSE of the preliminary estimate $\widehat{\mathbf{g}}$, which is known *a priori*. Basically, we should stop the delay search when $\mathbf{w}_{p+1}$ contains only the estimation noise, plus some extra noise due to imperfect cancellation of the already estimated paths.

5 Performance with linear detectors

In order to investigate the impact of channel estimation on the receiver performance, we consider simple FIR (or "one-shot") linear receivers [2, 22]. After low-pass filtering and sampling at rate W, the samples of the received signal are input to a shift-register of length $M_1 + M_2 + 1$, for suitable integers M_1 and M_2. For each m-th symbol the register contains the window of samples $[mLN_c - M_1, mLN_c + M_2]$ (referred to as the receiver "processing window") centered around the m-th symbol interval. The content of the receiver processing window is used to produce soft estimates of the users symbols, which are sent to the bank of single-user decoders. Since the channel delay-spread can be larger than the symbol interval, the processing window should span more than one symbol interval, i.e., $M_1 + M_2 + 1 \geq LN_c$ [30, 31]. After some straightforward but tedious algebra (see [32, 30] for the details), we can write the content of the receiver processing window corresponding to the m-th symbol as the vector

$$\mathbf{r}[m] = \sum_{k=1}^{K} \sum_{n=-B_1}^{B_2} \mathbf{s}_k[m,n] b_k[m-n] + \boldsymbol{\nu}[m] \tag{41}$$

where $\boldsymbol{\nu}[m] \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{I})$ and where the signal vectors $\mathbf{s}_k[m,n]$ are given by the convolution of the segment of the k-th chip sequence corresponding to the $(m-n)$-th symbol with the k-th discrete-time overall channel response. For periodic spreading, $\mathbf{s}_k[m,n]$ does not depend on m [33]. The vector channel model (41) takes into account both multiple-access and inter-symbol interference (MAI and ISI) [32]. In fact, each user contributes to $\mathbf{r}[m]$ with $B_1 + B_2 + 1$ symbols. The integers B_1 and B_2 are related to the processing windows boundaries M_1 and M_2 by [30]

$$\begin{aligned} B_1 &= \left\lfloor \frac{M_1 + N_c(L+Q-1) - 1}{N_c L} \right\rfloor \\ B_2 &= \left\lfloor \frac{M_2}{N_c L} \right\rfloor \end{aligned} \tag{42}$$

The receiver makes use of the channel estimates obtained by one of the proposed estimators and computes the signal vectors $\widehat{\mathbf{s}}_k[m,n]$ by convolving the estimated k-th channel response with the appropriate segment of the k-th chip sequence. In the case of periodic spreading, $\widehat{\mathbf{s}}_k[m,n]$ can be computed once and used for the whole block. For aperiodic spreading $\mathbf{s}_k[m,n]$ must be computed for every symbol interval in the block.

Without loss of generality, we focus on the linear detection of symbol $b_1[m]$. A soft estimate of $b_1[m]$ is obtained by the linear filtering operation $z_1[m] = \mathbf{h}_1[m]^H \mathbf{r}[m]$, where $\mathbf{h}_1[m]$ is the vector of filter coefficients for the m-th symbol of user 1. The SINR at the filter output is given by [15]

$$\mathrm{SINR}_1[m] = \frac{|\mathbf{h}_1[m]^H \mathbf{s}_1[m,0]|^2}{N_0 |\mathbf{h}_1[m]|^2 + \sum_{k=1}^{K} \sum_{n=-B_1}^{B_2} |\mathbf{h}_1[m]^H \mathbf{s}_k[m,n]|^2 - |\mathbf{h}_1[m]^H \mathbf{s}_1[m,0]|^2} \tag{43}$$

Here, we consider SUMF and LMMSE filtering [22]. Approximations of the SUMF and of the LMMSE filters are obtained from the vectors $\widehat{\mathbf{s}}_k[m,n]$ as

$$\begin{aligned} \widehat{\mathbf{h}}_{1,\mathrm{sumf}}[m] &= \widehat{\mathbf{s}}_1[m,0] \\ \widehat{\mathbf{h}}_{1,\mathrm{mmse}}[m] &= \widehat{\mathbf{R}}_r^{-1}[m] \widehat{\mathbf{s}}_1[m,0] \end{aligned} \tag{44}$$

where

$$\widehat{\mathbf{R}}_r[m] = \sum_{k=1}^{K} \sum_{n=-B_1}^{B_2} \widehat{\mathbf{s}}_k[m,n]\widehat{\mathbf{s}}_k[m,n]^H + N_0\mathbf{I}$$

is the covariance matrix of $\mathbf{r}[m]$ assuming $\widehat{\mathbf{s}}_k[m,n] = \mathbf{s}_k[m,n]$.

We consider a system with $K = 8$ users, spreading gain $L = 16$, RRC chip-shaping pulse with roll-off $\alpha = 0.22$, truncated over $\kappa = 12$ chips, and maximum channel delay-spread of $\Delta = 20T_c$. With this delay-spread, user blocks may be misaligned by more than one symbol and ISI is not at all negligible, in contrast to what is normally assumed in most DS/CDMA literature. The maximum channel length is $Q = 20+12 = 32$ chips. We choose training length $M = 256$, i.e., the minimum length necessary for estimating 8 channels of length 32. For each block and for each user k, a random channel impulse response $c_k(t)$ is generated according to the model (1), where the number of paths P is random and uniformly distributed over the integers $\{1, \ldots, 6\}$, the delays $\tau_{k,p}$ are independently and uniformly distributed on $[0, \Delta]$ and the gains $c_{k,p}$ are independent $\sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2_{k,p})$, where

$$\sigma^2_{k,p} = A_k \exp\left(-\frac{\tau_{k,p} - \tau_{\min,k}}{\tau_{\max,k} - \tau_{\min,k}}\right)$$

($\tau_{\min,k}$ and $\tau_{\max,k}$ denote the minimum and the maximum delays of user k). The amplitude factor A_k is chosen such that $\sum_{p=0}^{P-1} \sigma^2_{k,p} = E_k$, where E_k/N_0 is the received average SNR of user k.

As illustrated in the Introduction, the system spectral efficiency (assuming single-user decoding) is determined by the SINR cdf. We use Monte Carlo simulation to evaluate the cdf $F_{\mathrm{sinr}}(\gamma) = \Pr(\mathrm{SINR}_1[m] \leq \gamma)$ over 5000 independent slots. Figs. 4 and 5 show the SINR cdf for the SUMF receiver in the case of slow and fast power control, respectively. In the case of slow power control, E_k/N_0 is set to 10 dB for all users without any further normalization of the path gains. Then, the instantaneous user received power fluctuates considerably from block to block, because of the uncompensated Rayleigh fading. In the case of fast power control, E_k/N_0 is set as before but the channel gains are normalized such that $\sum_{p=0}^{P-1} |c_{k,p}|^2 = E_k$. This could be obtained in practice by a TDD system where each user measures the signal power received on the downlink and exploits reciprocity in order to compensate instantaneously for the Rayleigh fading block by block. The SINR cdf yields the block outage probability, defined as the probability that the SINR is below a fixed threshold γ. The horizontal dashed line corresponds to $F_{\mathrm{sinr}}(\gamma) = 10^{-1}$. For a 10^{-1} outage probability, non-ideal channel knowledge incurs about 1 dB loss for the unstructured estimator, about 0.6 dB for the Type I structured estimator and 0.2 dB for the Type II structured estimator (the latter employs the delay estimator described in Section 4.5, with a fixed number of paths per user $\widehat{P} = 4$). For comparison, the SINR cdf for Type II estimation with perfect knowledge of the delays is shown. The loss between perfect and imperfect delay knowledge is about 0.1 dB.

Figs. 6 and 7 show analogous results for the LMMSE receiver. In this case, for a 10^{-1} outage probability, non-ideal channel knowledge incurs about a 4.5 dB loss for the unstructured estimator, about 2.6 dB for the Type I structured estimator and 1.0 dB for the Type II structured estimator. The loss caused by imperfect delay knowledge for the Type II estimator is about 0.4 dB.

Finally, Figs. 8 and 9 show results for the LMMSE receiver assuming imbalanced user powers. Namely, we assume that $E_k/N_0 = 10$ dB for users $k = 1, \ldots, 4$ and $E_k/N_0 = 20$ dB for users $k = 5, \ldots, 8$. Slow and fast power control are considered, respectively. The imbalanced-power case is motivated here by the possibility of accommodating users with different service requirements. In this case, the performance of all estimators is almost unchanged with respect to the case of imbalanced powers, with the difference that the degradation suffered from imperfect delay knowledge in Type II estimation is slightly increased (about 0.6 dB). Performance for the SUMF in the imbalanced-power scenario is not shown since the resulting SINR is very low, and there is limited utility in using the SUMF for low-power users in the presence of high-power users [22].

Some comments are in order: 1) the estimators presented in this paper are almost insensitive to user power imbalance (although strictly-speaking Type II estimation is not near-far resistant, because of the bias introduced by non-ideal delay estimation). This is due to the fact that, since the training matrix $\mathbf{A}$ has full column-rank, the channels are estimated in mutually orthogonal subspaces, so that users with higher received power do not create larger interference for the estimation of lower power users. In comparison, conventional rake-based channel estimation is not robust to large user power differences.

2) The slight degradation of Type II estimation with respect to the case of balanced powers can be explained by the fact that only a fixed number of paths $\widehat{P}$ per user are taken into account. If a user has more than $\widehat{P}$ paths, the effect of these paths is not taken into account in the approximated LMMSE filter (44). For high-power interfering users, the effect of neglected paths is more evident. However, the degradation shown in our simulations is very small, and it could be reduced further by allowing a variable number

of estimated paths per user as explained at the end of Section 4.5.

3) Conventional unstructured estimation proves to be sufficiently good for SUMF detection. On the contrary, for LMMSE detection is makes sense to consider improved channel estimation such as the structured Type I and Type II schemes proposed here. In fact, it is well-known that in a multipath environment channel estimation errors have a much larger impact on LMMSE than on SUMF, especially for large SNR (see [34]). Intuitively, we can explain this fact by considering that the LMMSE for large SNR is similar to the decorrelator [22], which projects the received signal onto the orthogonal complement of the subspace spanned by interference. Non-perfect knowledge of the interference subspace caused by estimation errors yields a large SINR degradation. On the other hand, the SUMF is insensitive to the knowledge of the interference subspace, since it treats interference as white noise, and SINR degradation is caused only by the non-perfect knowledge of the useful signal. Notice that this effect cannot be observed just by considering the estimation MSE as performance measure.

6 Conclusions

In this paper we have considered training sequence-based joint channel estimation for a block-synchronous chip-asynchronous system motivated by the T-CDMA/TDD proposal of 3rd generation systems. We reviewed the unstructured channel estimator currently proposed for T-CDMA/TDD and we derived new structured channel estimators, exploiting various levels of *a priori* information. The proposed structured channel estimators can be applied to any arbitrary (approximately bandlimited) chip-shaping pulse, as opposed to most methods that assume rectangular pulses.

The structured channel estimator that exploits the fine structure of the multipath channels, i.e., the number of paths and their delays, requires accurate delay estimation. Then, starting from the ML criterion, we derived a low-complexity delay estimator that proves to be suited for structured channel estimation.

For all methods, we found optimal sets of training sequences, existing for any desired training length. The same sequence set is optimal for both unstructured and structured estimation. If these sequences are adopted in standardization, it will be up to the designer of the base-station equipment to implement the lower complexity unstructured estimation or the higher complexity structured estimation, according to the desired performance/complexity trade-off.

We compared the proposed channel estimators by looking at the output SINR of simple linear detectors like the SUMF and the LMMSE receivers. Our simulations show that while unstructured estimation might be sufficiently good for the SUMF, its loss with respect to ideal channel knowledge might be too large in the case of LMMSE. On the contrary, the performance loss of structured estimation is small also with LMMSE detection. Even though not considered here, we expect that also parallel or serial interference cancellation schemes [22] are sensitive to channel estimation errors, since imperfect channel knowledge prevents complete interference cancellation. Then, the higher complexity of structured channel estimators proposed in this paper might be motivated and justified for receivers employing multiuser detection.

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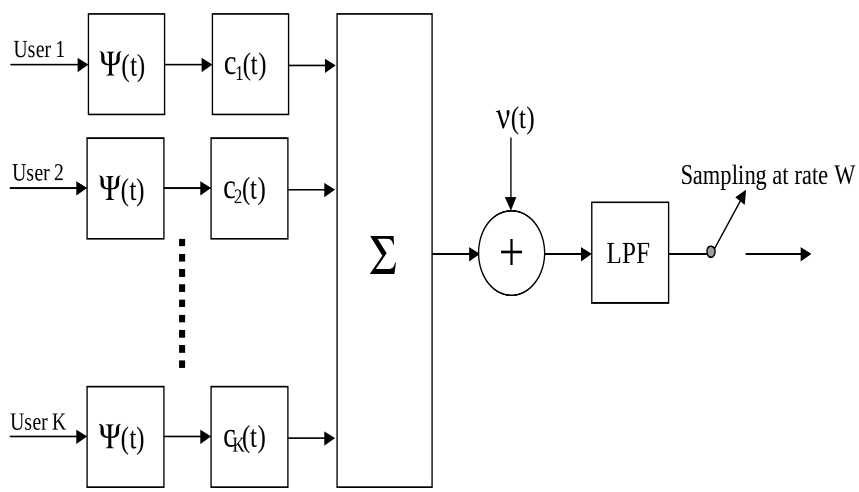

Figure 1: Baseband equivalent representation of a DS/CDMA system.

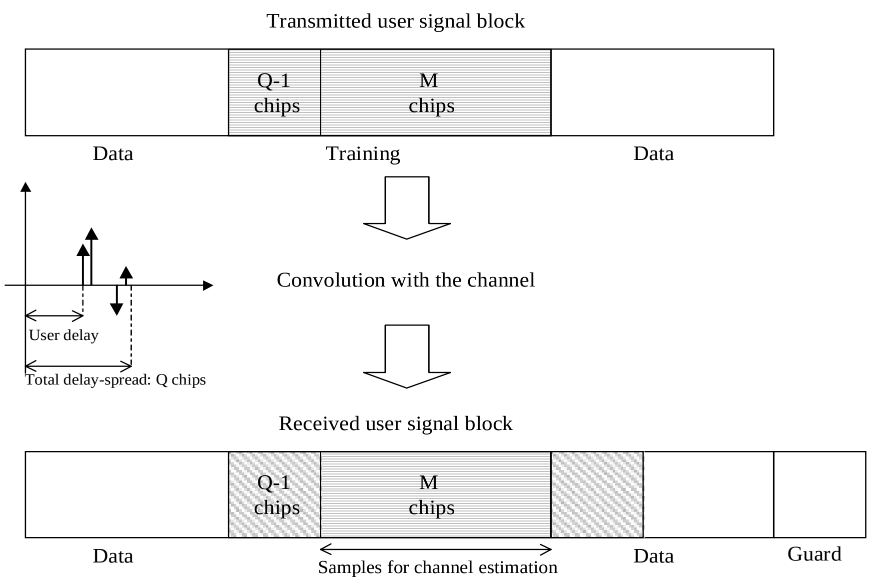

Figure 2: Signal block structure before and after convolution with the channel of length at most Q chips.

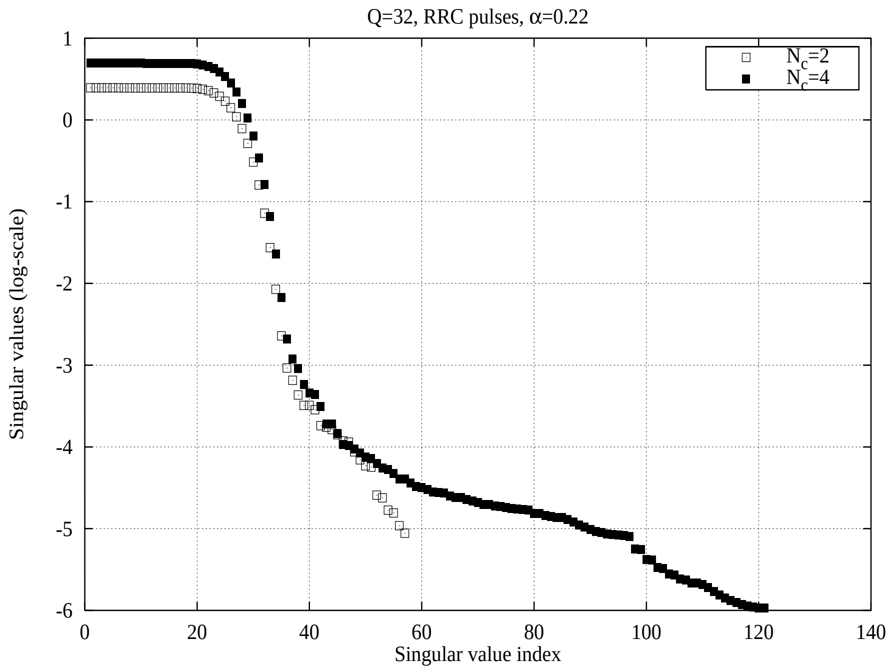

Figure 3: Normalized squared singular values for the chip-shaping pulse convolution matrix, for $Q = 32$, $N_c = 2, 4$, and RRC pulse with roll-off $\alpha = 0.22$.

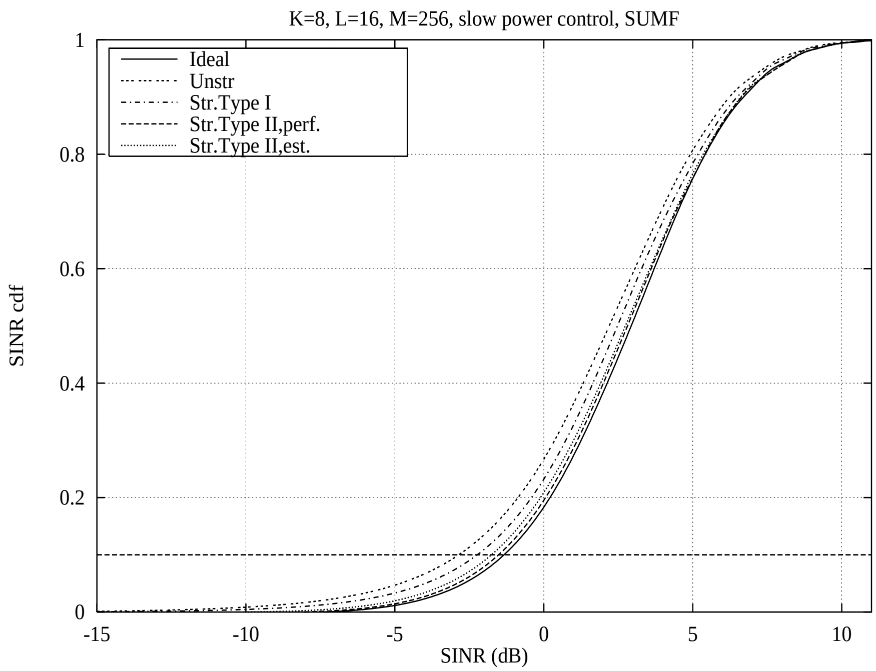

Figure 4: SINR cdf with slow power control and SUMF receiver, with different channel information: Ideal knowledge (Ideal), Unstructured estimation (Unstr.), Type I structured estimation (Str.Type I), Type II structured estimation with perfect delays (Str.Type II, perf.) and with estimated delays ,(Str. Type II, est.).

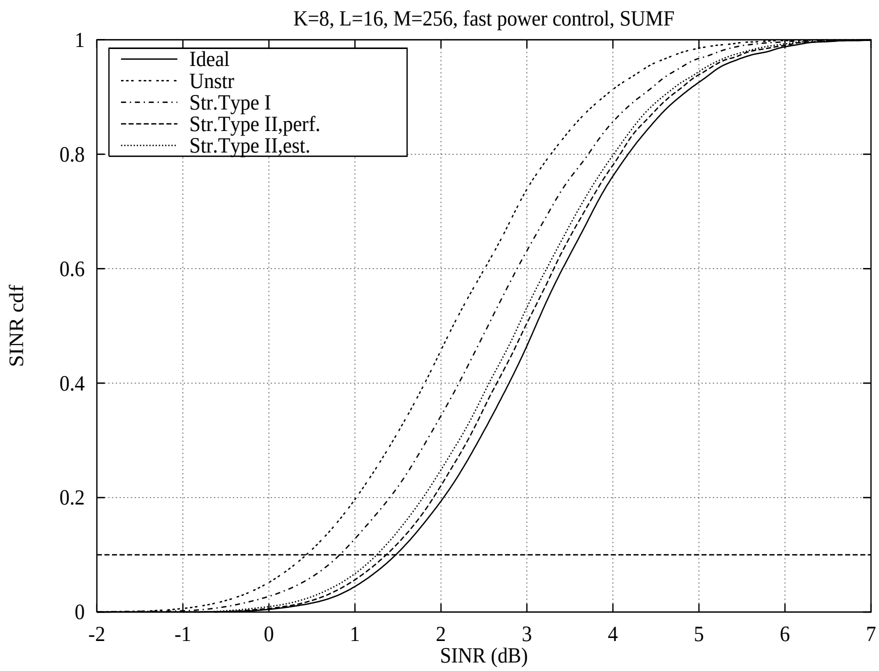

Figure 5: SINR cdf with fast power control and SUMF receiver, with different channel information: Ideal knowledge (Ideal), Unstructured estimation (Unstr.), Type I structured estimation (Str.Type I), Type II structured estimation with perfect delays (Str.Type II, perf.) and with estimated delays ,(Str. Type II, est.).

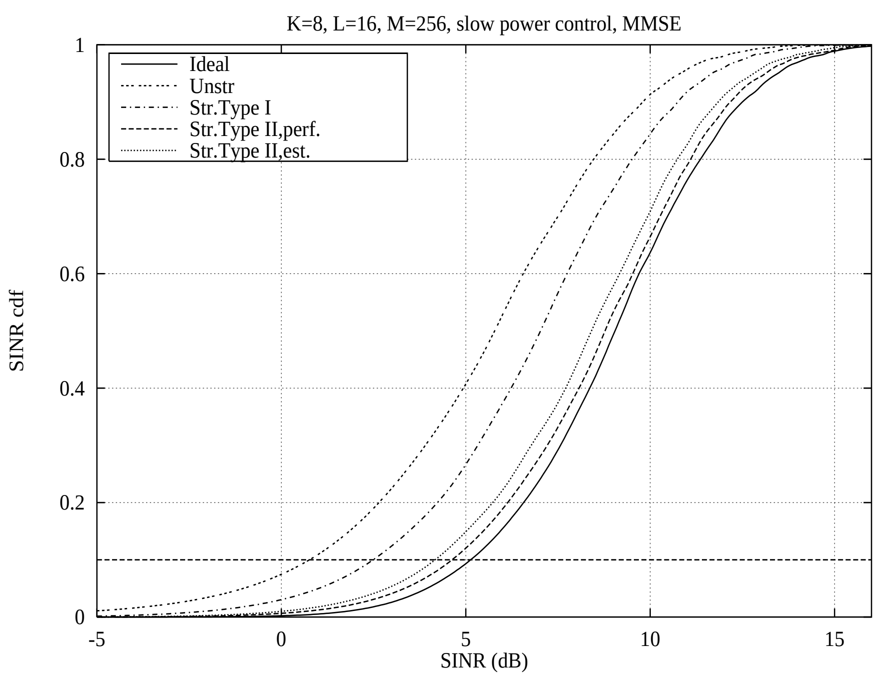

Figure 6: SINR cdf with slow power control and LMMSE receiver, with different channel information: Ideal knowledge (Ideal), Unstructured estimation (Unstr.), Type I structured estimation (Str.Type I), Type II structured estimation with perfect delays (Str.Type II, perf.) and with estimated delays ,(Str. Type II, est.).

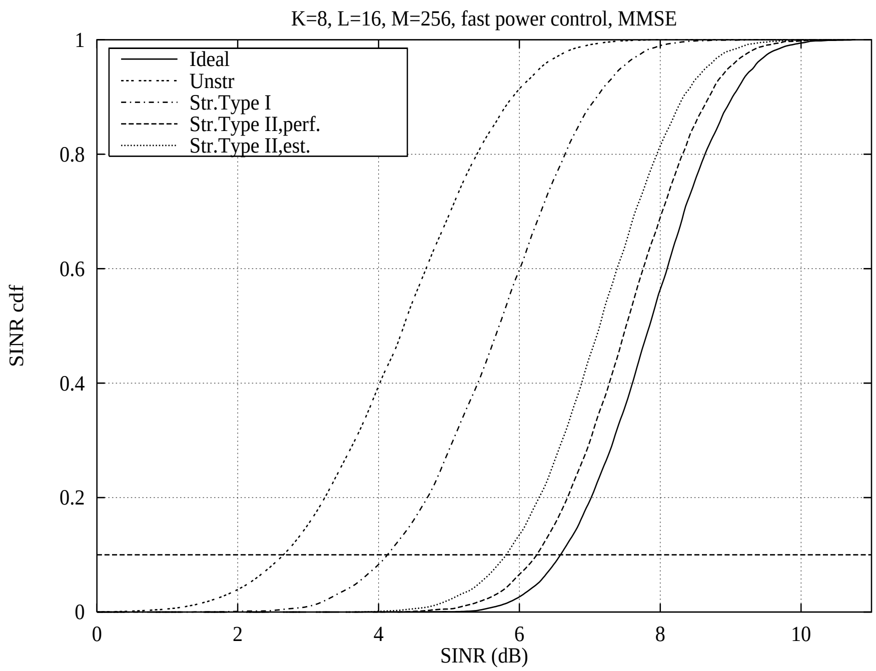

Figure 7: SINR cdf with fast power control and LMMSE receiver, with different channel information: Ideal knowledge (Ideal), Unstructured estimation (Unstr.), Type I structured estimation (Str.Type I), Type II structured estimation with perfect delays (Str.Type II, perf.) and with estimated delays ,(Str. Type II, est.).

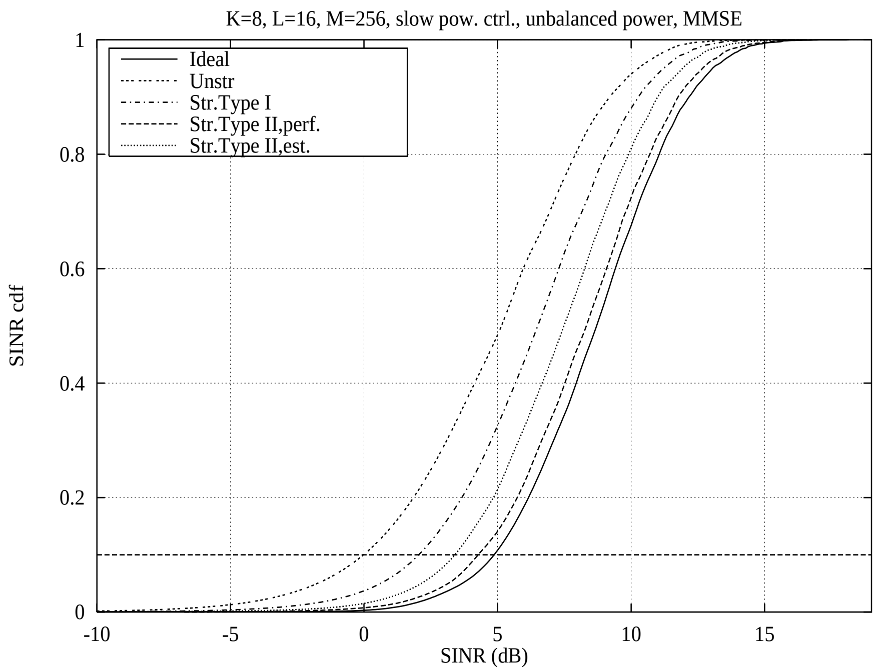

Figure 8: SINR cdf with slow power control, imbalanced powers, and LMMSE receiver, with different channel information: Ideal knowledge (Ideal), Unstructured estimation (Unstr.), Type I structured estimation (Str.Type I), Type II structured estimation with perfect delays (Str.Type II, perf.) and with estimated delays ,(Str. Type II, est.).

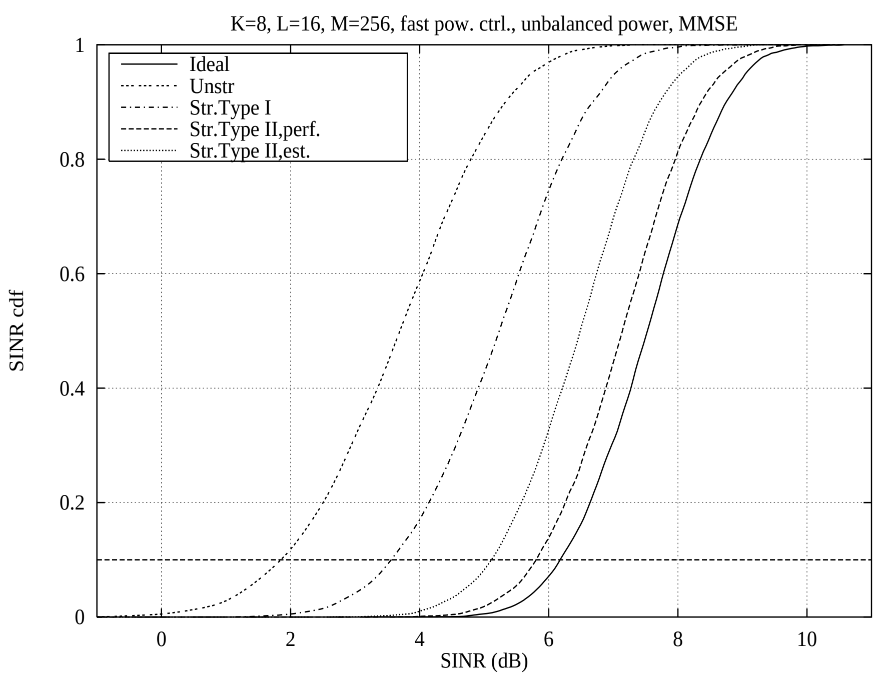

Figure 9: SINR cdf with fast power control, imbalanced powers, and LMMSE receiver, with different channel information: Ideal knowledge (Ideal), Unstructured estimation (Unstr.), Type I structured estimation (Str.Type I), Type II structured estimation with perfect delays (Str.Type II, perf.) and with estimated delays ,(Str. Type II, est.).