Channel State Information Based Ranging via EM-reVAMP Algorithm

Fangqing Xiao, Zilu Zhao, Dirk Slock
Communication Systems Department, EURECOM, France
fangqing.xiao@eurecom.fr, zilu.zhao@eurecom.fr, dirk.slock@eurecom.fr

***Abstract*—The Channel State Information (CSI) of the orthogonal frequency division multiplexing (OFDM) comprises data pertaining to the attenuation of multipath propagation. In this paper, we assume that the amplitude fading of both line-of-sight (LoS) and non-line-of-sight (NLoS) paths conforms to the Nakagami-m distribution. Via establishing a relationship between the distribution parameters and the propagation distance, we propose a CSI-based ranging method utilizing the Expectation Maximization (EM)-Revisited Approximate Message Passing (reVAMP) algorithm. This algorithm is not only applicable to the CSI-based ranging estimation but can also be extended to other parameter estimation scenarios. It effectively tackles challenges associated with generalized linear models (GLMs) that involve hidden random variables and the intractability of posterior distributions during the EM iterations.**

I. Introduction

The orthogonal frequency division multiplexing (OFDM) is employed in widely recognized standards such as IEEE 802.11, long term evolution (LTE), and 5G NR [1]. Consequently, there is a growing interest in leveraging wireless signals to extract distance estimates and provide accurate user positioning, particularly in environments where GPS signals may be unreliable, such as indoor or complex settings.

Several studies have attempted to establish the relationship between signal attenuation and propagation distance for distinguishable multipath components (MPCs)[2]. In recent years, the Nakagami-m distribution has gained significant attention due to its general flexibility and accuracy in matching various experimental data compared to Rayleigh, Rician, or log-normal distributions[3]. Due to the influence of diffuse reflections, a superposition of primary and clutter signals occurs even within a single path, making the Nakagami-m distribution a better fit than the Rayleigh distribution[4]. Additionally, the Rician and Nakagami-m distributions exhibit similar behavior at the mean attachment point.

In this paper, we propose a method for CSI-based ranging in multipath Nakagami-m fading channels. The main contributions of our work are as follows: 1) We establish the Nakagami-m fading distribution in multipath paths and propose a CSI-based ranging method for OFDM systems based on this distribution. 2) We introduce the new EM-reVAMP algorithm that directly estimates distance from the received OFDM signal. This algorithm can be extended to other scenarios. 3) We conduct simulations to validate the feasibility of the algorithm and confirm that the ranging method is indeed effective when the fading matches a certain distribution. Our research offers valuable insights as provides a foundation for accurate distance estimation and further positioning in wireless communication networks, paving the way for enhanced performance and improved user experiences.

II. System Model

A. OFDM signal model with Nakagami-m Fading

The received signal in OFDM can be represented as follows:

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{T}\boldsymbol{a} + \boldsymbol{v} = \boldsymbol{H}\boldsymbol{a} + \boldsymbol{v}; \quad \boldsymbol{v} \sim \mathcal{CN}(0, \sigma_v^2 \mathbf{I}), \tag{1}$$

where $\boldsymbol{y} \in \mathcal{C}^{N\times 1}$ is the received signal, $\boldsymbol{X} \in \mathcal{C}^{N\times N}$ is a known matrix containing the transmitted symbols on its diagonal, $\boldsymbol{T} \in \mathcal{C}^{N\times L}$ is a transformation matrix, $\boldsymbol{a} \in \mathcal{C}^{L\times 1}$ denotes the complex attenuation coefficients, $\boldsymbol{H} \in \mathcal{C}^{N\times L}$ presents the product of $\boldsymbol{X}$ and $\boldsymbol{T}$, $\boldsymbol{v} \in \mathcal{C}^{N\times 1}$ is a vector of independently and identically distributed (i.i.d.) complex zero-mean Gaussian noise samples with equal variance σ_v^2.

For each individual element a_i of $\boldsymbol{a}$, we assume its magnitude with a Nakagami-m distribution and phase with a uniform distribution from 0 to 2π. Therefore, the probability density function (PDF) of the probability density of the complex fading coefficient a_i can be expressed as as follows:

$$p(a_i|\Omega_i) = \frac{m^m |a_i|^{2m-2}}{\pi\, \Gamma(m)\, \Omega_i^m} \exp\left[-\frac{m\,|a_i|^2}{\Omega_i}\right], m \geq 0.5. \tag{2}$$

where $\Gamma(\cdot)$ denotes the gamma function, m is the shape parameter of the Nakagami-m distribution, Ω_i is the average power intensity of path i. Referring to [5], the parameter Ω_i can be defined as a function of the propagation distance d_i:

$$\Omega_i = P_t G_t G_r \left(\frac{\lambda}{4\pi d_i}\right)^n = G_0(d_0 + c\tau_i)^{-n}, \tag{3}$$

in the given equation, several variables are defined as follows: P_t represents the transmitting power, G_t denotes the transmitting antenna amplification, λ is the wavelength of the electromagnetic wave, c is the velocity of light, n represents the propagation factor influenced by the environment, d_0 indicates the Line-of-Sight (LoS) distance, and τ_i indicates the propagation delay between the i-th path and the LoS path.

B. Ranging Estimation

Our objective is to estimate d_0 directly from $\boldsymbol{y}$. To achieve this, we employed the maximum likelihood estimation (MLE) method, which transforms the problem into the following equation:

$$\hat{d}_0 = \arg\max_{d_0} \ln \mathcal{L}(d_0; \boldsymbol{y}) = \arg\max_{d_0} \ln \int p(\boldsymbol{y}|\boldsymbol{a}) p(\boldsymbol{a}|d_0) d\boldsymbol{a},$$

where $\mathcal{L}(\cdot)$ represents the likelihood function.

However, solving this optimization problem directly proves to be intractable, as both the integral and the involved MLE involving latent variables pose significant challenges.

III. EM-reVAMP Algorithm

To address the challenge at hand, we employed the expectation maximization (EM) algorithm to transform the MLE involving latent variables into an iterative problem. When considering the Nakagami-m prior distribution of $\boldsymbol{a}$ as described in equation (2), the EM algorithm can be transformed as follows:

$$\begin{aligned} d_0^{(t+1)} &= \arg\max_{d_0} \mathcal{E}_{p(\boldsymbol{a}|\boldsymbol{y},d_0^{(t)})}\left[\ln p(\boldsymbol{a},\boldsymbol{y}|d_0)\right] \\ &= \arg\min_{d_0} \sum_{i=0}^{L-1}\left[\ln \Omega_i(d_0) + \frac{\mathcal{E}_{p(\boldsymbol{a}|\boldsymbol{y},\boldsymbol{\theta}^{(t)})}[|a_i|^2]}{\Omega_i(d_0)})\right], \end{aligned} \quad (4)$$

where $\Omega_i(d_0)$ was defined in (3). However, in this scenario, the EM algorithm remains intractable because obtaining the posterior distribution $p(\boldsymbol{a}|\boldsymbol{y},\boldsymbol{\theta}^{(t)})$ is challenging due to the integration involved in equation (4). Therefore, it becomes crucial to develop an algorithm that approximates this posterior distribution with another tractable distribution. To achieve this goal, we propose the reVAMP algorithm, which can be regarded as an Expectation Propagation (EP) [6] algorithm via approximating the factorization as follows:

$$p(\boldsymbol{a},\boldsymbol{y}) \propto p(\boldsymbol{a}|\boldsymbol{y}) \simeq p(\boldsymbol{y}|\boldsymbol{a})\prod_{i=1}^{N} q(a_i)q(\boldsymbol{a}). \quad (5)$$

Combined EM and reVAMP, we propose the EM-reVAMP algorithm for estimating d_0. The algorithm, outlined in Algorithm below, utilizes reVAMP sequentially at each step of the EM algorithm to obtain approximate first-order and second-order moments.

IV. Simulation

The simulation aimed to evaluate the effect of different Signal-to-Noise Ratio (SNR) levels and the number of Non-Line-of-Sight (NLoS) paths on estimation accuracy. Specifically, we conducted the simulation with certain parameters. The LoS distance d_0 to be estimated was fixed at 10 meters. The parameter values were set as follows: the distances of NLoS paths were uniformly opted between 11 and 25 meters, N was assigned a value of 10, G_0 was set to 1, m was selected randomly from a uniform distribution ranging between 0.5 and 2, and n was chosen randomly within the range of -4 to -2. The matrix $\boldsymbol{H}$ was generated randomly with a complex normal distribution, and the fading phase of each path was uniformly distributed between 0 and 2π. The simulation results are presented in Figure 1. It demonstrates that within the range of 10-30 dB, the SNR has no significant impact on the estimation results. However, we observed that the accuracy of the estimation improves as the number of paths increases, which aligns with our initial expectations.

V. Concluding Remarks

This paper introduces a CSI-based ranging estimation method using the EM-reVAMP algorithm, which exploits the multipath effect in communication propagation and assumes the Nakagami-m distribution for path attenuation. To assess its performance, besides simulations, we are currently collecting measurement data in different environments to validate the effectiveness of our ranging technique. Furthermore, we are exploring the application of the EM-reVAMP algorithm to other scenarios involving non-Gaussian prior distributions of GLMs.

Acknowledgements EURECOM's research is partially supported by its industrial members: ORANGE, BMW, SAP, iABG, Norton LifeLock, and by the Franco-German projects 5G-OPERA and CellFree6G .

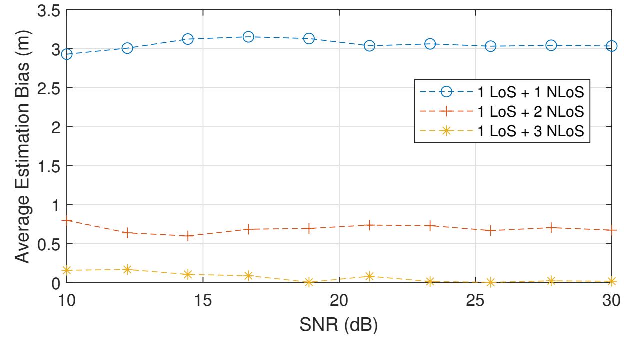

Fig. 1. The impact of SNR and the number of NLoS paths on d_0 estimation

Algorithm EM-reVAMP

Ensure: $\hat{d}_0$
Require: $\boldsymbol{y}$, $\boldsymbol{A}$, $p_{\boldsymbol{v}}(\boldsymbol{v})$, m, $\boldsymbol{\tau}_i$, G_0
Initialize: $\hat{d}_0$
repeat [For $t = 0 \ldots L-1$]
Initialize: $\boldsymbol{\tau}_p$, $\boldsymbol{p}$
  repeat
    repeat [For $i = 1 \ldots N$]
      [Update the posterior approximation]
      $\boldsymbol{C}_m = \left(\boldsymbol{A}^H\boldsymbol{C}_{vv}^{-1}\boldsymbol{A} + \boldsymbol{D}_p^{-1}\right)^{-1}$
      $\boldsymbol{m} = \boldsymbol{C}_m\left(\boldsymbol{A}^H\boldsymbol{C}_{vv}^{-1}\boldsymbol{y} + \boldsymbol{D}_p^{-1}\boldsymbol{p}\right)$
      [Update the extrinsic]
      $\boldsymbol{\tau}_m = \mathbf{diag}(\boldsymbol{C}_m)$
      $r_i = \frac{\tau_{p_i}m_i - \tau_{m_i}p_i}{\tau_{p_i} - \tau_{m_i}}$
      $\tau_{r_i} = \frac{\tau_{m_i}\tau_{p_i}}{\tau_{p_i} - \tau_{m_i}}$
      [Approximate the marginal posterior]
      $\hat{a}_i = \frac{\int a_i p(a_i|\hat{d}_0)\mathcal{CN}(a_i;r_i,\tau_{r_i})da_i}{\int p(a_i|\hat{d}_0)\mathcal{CN}(a_i;r_i,\tau_{r_i})da_i}$
      $\tau_{a_i} = \frac{\int |a_i - \hat{a}_i|^2 p(a_i|\hat{d}_0)\mathcal{CN}(a_i;r_i,\tau_{r_i})da_i}{\int p(a_i|\hat{d}_0)\mathcal{CN}(a_i;r_i,\tau_{r_i})da_i}$
      [Propagate the approximation back]
      $p_i = \frac{\tau_{r_i}\hat{a}_i - \tau_{a_i}r_i}{\tau_{r_i} - \tau_{a_i}}$
      $\tau_{p_i} = \frac{\tau_{r_i}\tau_{a_i}}{\tau_{r_i} - \tau_{a_i}}$
    until All i-s have been updated
  until Convergence
  $\hat{d}_0 = \arg\min_{d_0}\sum_{i=0}^{L-1}\left[\ln\Omega_i(d_0) + \frac{\tau_{m_i} + |m_i|^2}{\Omega_i(d_0)})\right]$
until Convergence

References

[1] A. M. Graff, W. N. Blount, P. A. Iannucci, J. G. Andrews, and T. E. Humphreys, "Analysis of ofdm signals for ranging and communications," in *Proceedings of the 34th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+)*, 2021, pp. 2910–2924.

[2] T. S. Rappaport, G. R. MacCartney, M. K. Samimi, and S. Sun, "Wideband millimeter-wave propagation measurements and channel models for future wireless communication system design," *IEEE transactions on Communications*, vol. 63, no. 9, pp. 3029–3056, 2015.

[3] S. M. Abuelenin, "On the similarity between nakagami-m fading distribution and the gaussian ensembles of random matrix theory," *arXiv preprint arXiv:1803.08688*, 2018.

[4] H. Stefanovic, A. Savic *et al.*, "Some general characteristics of nakagami-m distribution," in *Proceedings of ISCIM Conference*, 2011, pp. 721–731.

[5] V. A. Aalo, C. Mukasa, and G. P. Efthymoglou, "Effect of mobility on the outage and ber performances of digital transmissions over nakagami-m fading channels," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 4, pp. 2715–2721, 2015.

[6] T. P. Minka, "Expectation propagation for approximate bayesian inference," *arXiv preprint arXiv:1301.2294*, 2013.