# Beamforming for Reduced-Rank MIMO Interference Channels in Dynamic TDD Systems

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***Abstract*—Dynamic Time Division Duplexing (DynTDD) enables flexible time slot allocation based on changing communication needs, which distinguishes it from traditional Time Division Duplexing (TDD) systems that rely on fixed time slot allocation. However, despite the benefits offered by DynTDD, cross-link interference (CLI) can still occur when neighboring cells use different transmission directions on the same or partially-overlapping time-frequency resources. Two types of cross-link interference exist: Base Station (BS) to BS or Downlink (DL) to Uplink (UL) interference, and User Equipment (UE) to UE or UL to DL interference. To address this interference, coordinated beamforming is a crucial signal-processing technique. This study focuses on designing zero-forcing (ZF) transmit beamforming at initialization, and the iterative weighted minimum mean-square error (WMMSE) algorithm to maximize the sum rate in a Multiple Input Multiple Output (MIMO) User Equipment to User Equipment (UE-to-UE) Interference Channel (IC). The study also examines the potential advantages of non-uniform Degrees-of-Freedom (DoF) at Uplink (UL) and/or Downlink (DL) User Equipment (UEs), which can enhance the sum DoF, leading to higher sum rates at high Signal-to-Noise Ratio (SNR).**

***Index Terms*—Dynamic TDD, MIMO, rank deficient, interference alignment, Degree of Freedom, WMMSE, zero-forcing**

## I. Introduction

Multiple Input Multiple Output (MIMO) systems hold significant promise for achieving high throughput in wireless systems [1]. In the context of Downlink (DL) communications, when the transmitter possesses a certain level of Channel State Information (CSI) knowledge, it becomes possible to maximize the system throughput. Our study focuses on Dynamic Time Division Duplexing (DynTDD) systems, which offer the potential to enhance overall resource utilization [2] and significantly reduce latency [3]. However, the introduction of DynTDD introduces new challenges, primarily due to the presence of cross-link interference (CLI) encompassing Base Station to Base Station (BS-to-BS) and User Equipment to User Equipment (UE-to-UE) interference.

Previous studies have primarily focused on addressing the issue of BS-to-BS interference rather than UE-to-UE interference. This bias arises because during UL transmission, DL to UL interference can lead to significant performance degradation, unlike DL transmission where DynTDD is advantageous [4]. However, research findings in [5] indicate that the power level of UE-to-UE interference is low for UEs located in the central region of the cell but considerably high for UEs at the cell edge. Furthermore, to ensure network stability, it is crucial to handle UE-to-UE interference, especially for edge UEs. Consequently, to further enhance network capacity, concurrent transmission techniques become necessary. Multiple concurrent transmission methods, such as Zero Forcing (ZF), Interference Alignment (IA), and distributed MIMO, have been proposed, where multiple transmitters jointly encode signals for multiple receivers to align or cancel interference, enabling each receiver to decode its desired information. The feasibility conditions for IA have been extensively analyzed in [6]–[9], while [10] provides a mathematical characterization of the achievable Degrees-of-Freedom (DoF) in their proposed Distributed Interference Alignment (DIA) technique for a specific number of antennas at the BS/Mobile Station (MS).

This paper builds upon the findings of our previous studies conducted in [11] and [12], offering novel contributions to the field. Here, we leverage the non-uniformity of Degrees of Freedom (DoF) at DL UE and/or UL UE to augment the overall DoF sum, thereby increasing the rate at a high Signal-to-Noise Ratio (SNR). To validate our approach, we present numerical results and conduct sum rate simulations using a comprehensive DynTDD system. To maximize the sum rate, we have devised an algorithm that employs ZF beamformers at both DL and UL UEs during the initialization stage to eliminate UE-to-UE interference. Furthermore, we utilize ZF transmitters at the DL BS to mitigate intracell interference among DL UEs and employ Weighted Minimum Mean-Square Error (WMMSE) filters in an iterative process.

## II. System Model and Problem Formulation

Consider a MIMO system comprising two cells, each with a BS. In the DynTDD scheme, one cell operates in the DL mode, while the other operates in the UL mode. The UL and DL cells are equipped with $M_{ul}$ and $M_{dl}$ antennas, respectively. Within the UL cell, there are $K_{ul}$ interfering UEs, while the DL cell accommodates $K_{dl}$ interfered UEs. The $k^{th}$ DL UE and the $l^{th}$ UL UE are equipped with $N_{dl,k}$ and $N_{ul,l}$ antennas, respectively. In our study, we refer to our system as IBMAC (Interfering Broadcast-Multiple Access Channel), as described in [9]. It represents a two-cell system with one cell operating in DL (broadcast) mode and the other in UL mode (multiple access), where interference occurs between the two cells. However, we assume that the number of base station antennas is sufficiently large to support all UL or DL UE streams. Furthermore, we mitigate the BS-to-BS interference by leveraging a limited-rank BS-to-BS channel [10]. Consequently, the focus of the IBMAC problem is primarily on the interference originating from UL UEs to DL UEs, which we refer to as IBMAC-IC (IBMAC Interference Channel).

Let $l$ represent the index of the $l^{th}$ UL user, who sends $d_{ul,l}$ independent streams to the UL BS. The non-negative UL power allocated to user $l$ is denoted as $p_{ul,l}$. At the same time, the $k^{th}$ DL user receives $d_{dl,k}$ independent streams from the DL BS, with the non-negative DL power allocation $p_{dl,k}$. We define $\boldsymbol{V}_{dl,k} \in \mathbb{C}^{M_{dl} \times d_{dl,k}}$ as the beamformer used by the DL BS to transmit the signal $\boldsymbol{s}_{dl,k} \in \mathbb{C}^{d_{dl,k} \times 1}$ to the $k^{th}$ DL user. Similarly, $\boldsymbol{V}_{ul,l} \in \mathbb{C}^{N_{ul,l} \times d_{ul,l}}$ represents the beamformer used by the $l^{th}$ UL user to transmit the signal $\boldsymbol{s}_{ul,l} \in \mathbb{C}^{d_{ul,l} \times 1}$ to the UL BS. We assume that $E[\boldsymbol{s}_{dl,k}\boldsymbol{s}_{dl,k}^H] = \boldsymbol{I}$ and $E[\boldsymbol{s}_{ul,l}\boldsymbol{s}_{ul,l}^H] = \boldsymbol{I}$, indicating that the transmitted signals have unit power. For the reception, we consider $\boldsymbol{U}_{dl,k} \in \mathbb{C}^{N_{dl,k} \times d_{dl,k}}$ and $\boldsymbol{U}_{ul,l} \in \mathbb{C}^{M_{ul} \times d_{ul,l}}$ as the Rx beamforming (BF) matrices at the $k^{th}$ DL user and the UL BS (from the $l^{th}$ UL user), respectively. The received signal at the $k^{th}$ DL user is denoted as $\boldsymbol{y}_{dl,k}$:

$$\boldsymbol{y}_{dl,k} = \underbrace{\boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}\boldsymbol{s}_{dl,k}}_{\text{desired signal}} + \underbrace{\sum_{j=1,j\neq k}^{K_{dl}} \boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,j}\boldsymbol{s}_{dl,j}}_{\text{intracell interference}} + \underbrace{\sum_{l=1}^{K_{ul}} \boldsymbol{H}_{k,l}\boldsymbol{V}_{ul,l}s_{ul,l}}_{\text{UL To DL interference}} + \underbrace{\boldsymbol{n}_{dl,k}}_{\text{noise}} \tag{1}$$

where the matrix $\boldsymbol{H}_k^{DL} \in \mathbb{C}^{N_{dl,k} \times M_{dl}}$ represents the channel from the DL BS to the $k^{th}$ DL UE, and $\boldsymbol{H}_l^{UL} \in \mathbb{C}^{M_{ul} \times N_{ul,l}}$ is the matrix of the channel from the $l^{th}$ UL UE to the UL BS. We call $\boldsymbol{H}_k^{DL}$ and $\boldsymbol{H}_l^{UL}$ the direct channels. The interference channel between the $l^{th}$UL and DL UEs is denoted as $H_{k,l} \in \mathbb{C}^{N_{dl,k} \times N_{ul,l}}$. $n_{dl,k} \in \mathbb{C}^{N_{dl,k} \times 1}$ denotes the additive white Gaussian noise with distribution $\mathcal{CN} \in (0, \sigma_{dl,k}^2\boldsymbol{I})$ at the $k^{th}$ DL UE. ZF from UL UE $l$ to the DL UE $k$ requires:

$$\boldsymbol{U}_{dl,k}^H\boldsymbol{H}_{k,l}\boldsymbol{V}_{ul,l} = \boldsymbol{0}, \forall k \in \{1, ..., K_{dl}\}, \forall l \in \{1, ..., K_{ul}\}. \tag{2}$$

For this system the achievable rate for the UL user $l$ is given as:

$$\boldsymbol{R}_{ul,l} = \log\det\Bigg(\boldsymbol{I}_{M_{ul}} + \boldsymbol{H}_l^{UL}\boldsymbol{V}_{ul,l}\boldsymbol{V}_{ul,l}^H(\boldsymbol{H}_l^{UL})^H \Big(\sum_{i=1,i\neq l}^{K_{ul}} \boldsymbol{H}_i^{UL}\boldsymbol{V}_{ul,i}\boldsymbol{V}_{ul,i}^H(\boldsymbol{H}_i^{UL})^H + \sigma_{ul,l}^2\boldsymbol{I}_{M_{ul}}\Big)^{-1}\Bigg) \tag{3}$$

In our study we consider a ZF precoders $\boldsymbol{V}_{ul,l}$ at each UL UE given as:

$$\boldsymbol{V}_{ul,l} = \sqrt{\frac{p_{ul,l}}{\mathrm{Tr}(\boldsymbol{G}_{z,l}\boldsymbol{G}_{z,l}^H)}}\boldsymbol{G}_{z,l} \tag{4}$$

the explanation of $\boldsymbol{G}_{z,l}$ is given after equation (6).

The achievable rate for the DL user $k$ is given as:

$$\boldsymbol{R}_{dl,k} = \log\det\Bigg(\boldsymbol{I}_{N_{dl,k}} + \boldsymbol{H}_k^{DL}\boldsymbol{V}_{dl,k}\boldsymbol{V}_{dl,k}^H(\boldsymbol{H}_k^{DL})^H\Big(\sum_{j=1,j\neq k}^{K_{dl}} \boldsymbol{H}_k^{DL} \boldsymbol{V}_{dl,j}\boldsymbol{V}_{dl,j}^H(\boldsymbol{H}_k^{DL})^H + \sum_{l=1}^{K_{ul}} \boldsymbol{H}_{k,l}\boldsymbol{V}_{ul,l}\boldsymbol{V}_{ul,l}^H\boldsymbol{H}_{k,l}^H + \sigma_{dl,k}^2\boldsymbol{I}_{N_{dl,k}}\Big)^{-1}\Bigg) \tag{5}$$

In our study we choose $\boldsymbol{V}_{dl,k}$ as ZF transmit filter at the DL BS for the $k^{th}$ DL UE, which is computed as:

$$\boldsymbol{V}_{dl} = b\bar{\boldsymbol{V}} = [\boldsymbol{V}_{dl,1}, \quad \boldsymbol{V}_{dl,2}, \quad \dots, \quad \boldsymbol{V}_{dl,K_{dl}}] \tag{6a}$$

$$\bar{\boldsymbol{V}}_{dl} = \boldsymbol{H}^H\boldsymbol{F}\Big(\boldsymbol{F}^H\boldsymbol{H}\boldsymbol{H}^H\boldsymbol{F}\Big)^{-1}, b = \sqrt{\frac{\sum_{k=1}^{K_{dl}} p_{dl,k}}{\mathrm{Tr}(\bar{\boldsymbol{V}}_{dl}\bar{\boldsymbol{V}}_{dl}^H)}} \tag{6b}$$

where $\boldsymbol{H}_{[K_{dl}N_{dl,k} \times M_{dl}]} = \Big[\boldsymbol{H}_1^{DL^T}, \dots, \boldsymbol{H}_{K_{dl}}^{DL^T}\Big]^T$ contains the different DL channel matrices stacked row-wise, and $\boldsymbol{F}_{[K_{dl}N_{dl,k} \times K_{dl}d_{dl,k}]} = diag\{\boldsymbol{F}_{z,1}, \dots, \boldsymbol{F}_{z,K_{dl}}\}$ is blocked diagonal matrix.

The beamformers at the $k^{th}$ DL UE and the $l^{th}$ UL UE are denoted as $\boldsymbol{F}_{z,k}$ and $\boldsymbol{G}_{z,l}$, respectively. These beamformers are obtained through the ZF process, satisfying the condition given by equation (2). In general, the ZF process is iterative, but for certain special cases, it can be computed in closed-form. The detailed procedure to obtain $\boldsymbol{F}_{z,l}$ and $\boldsymbol{G}_{z,l}$ for a specific special case is discussed in the subsection V-A. In the context of the WMMSE study, we sometimes set $\boldsymbol{U}_{dl,k} = \boldsymbol{F}_{z,k}$ to determine the beamformers' initialization at the DL BS.

## III. IA Feasibility Conditions for DynTDD UE-to-UE generic Rank MIMO IBMAC

In our previous work [11], we established the proper conditions for the feasibility of interference alignment (IA), with the global proper conditions presented in [11, eq. (6)]. Additionally, we derived different conditions for IA feasibility from both centralized and distributed designs. Building upon this, in [12], we revisited the feasibility analysis framework proposed in [7], [8], and [6], providing a comprehensive analysis of the UE-to-UE interference. This analysis focused on the channel matrices and beamformers at the transmitter and receiver, leading to the formulation of necessary and sufficient conditions for IA feasibility in a Reduced Rank MIMO IBMAC-IC, as outlined in [12, Theorem 4].

In this section, we examine the feasibility of our combined method presented in [11, eq. (26), eq. (27)]. Specifically, we compare the Degrees of Freedom (DoF) achieved using the combined method, as expressed in [11, eq. (26), eq. (27)], with the DoF obtained from the sufficient and necessary condition for a general rank interference channel provided in [12, Theorem 4]. The latter offers a precise characterization of the achievable DoF. Based on our analysis, we present the following conjecture:

**Conjecture 1.** *For a DynTDD system, if the DoF tuple* $(d_{ul,1}, ..., d_{ul,K_{ul}}, d_{dl,1}, ..., d_{dl,K_{dl}})$ *satisfies the condition for the combined method in [11, eq.(26), eq.(27)], then this DoF is almost surely feasible.*

Subsequently, we leverage the non-uniform DoF among the DL UEs and the UL UEs, wherein the number of data streams at each DL UE, denoted as $d_{dl,k}$, or each UL UE, denoted as $d_{ul,l}$, may differ even within a uniform DynTDD system. Based on this observation, we propose the following remark:

**Remark 1.** *In DynTDD systems, if the tuple of DoF* $(d_{ul,1}, ..., d_{ul,K_{ul}}, d_{dl,1}, ..., d_{dl,K_{dl}})$ *is feasible for interference alignment (IA) according to the conditions outlined in [12, Theorem 4], and there exists a non-uniform distribution of DoF at the receivers (DL UEs) and/or transmitters (UL UEs), then the resulting sum DoF will be equal to or greater than the sum DoF achieved when assuming a feasible uniform DoF distribution.*

## IV. NUMERICAL DOF EVALUATIONS

To analyze the observations given in Conjecture 1 and Remark 1, we give Table I, in which we consider a MIMO IBMAC-IC, and we evaluate the DoF of the system $N_{ul} = 3$, $N_{dl} = 4$, $K_{ul} = 2$ and $K_{dl} = 4$. In this table, we evaluate the different conditions established in [11] and the proper and sufficient condition given by [12, Theorem 4]. While a generic tuple $(d_{dl}, d_{ul}, d_{tot})$ denotes the DoF at DL UEs, at UL UEs, and the overall UL and DL sum DoF, the details of the conditions considered in each row of Table I are mentioned in our paper [12, Section IV].

| r | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $(d_{p,dl}, d_{p,ul}, d_{p,tot})$ | (4,3,22) | (3,2,16) | (3,1,14) | ((3,2,2,2),1,11)** |
| $(d_{d,dl}, d_{d,ul}, d_{d,tot})$ | (4,3,22) | (3,1,14) | (0,3,6)* | (0,3,6)* |
| $(n_{F,d}, n_{G,d})$ | (1,2) | (1,2) | (2,0) | (1,2) |
| $(d_{c,dl}, d_{c,ul}, d_{c,tot})$ | (4,3,22) | (3,1,14) | (2,1,10) | (2,1,10) |
| $(n_{F,c}, n_{G,c})$ | (1,2) | (1,2) | (2,0) | (2,0) |
| $(d_{r,dl}, d_{r,ul}, d_{r,tot})$ | (4,3,22) | (2,3,14) | (2,1,10) | (2,1,10) |
| $(d_{t,dl}, d_{t,ul}, d_{t,tot})$ | (4,3,22) | (4,0,16)* | (4,0,16)* | (4,0,16)* |
| $(d_{T4,dl}, d_{T4,ul}, d_{T4,tot})$ | (4,3,22) | ((3,3,2,2),2,14)** | ((3,3,2,2),1,12)** | ((3,2,2,2),1,11)** |

TABLE I: DoF per user as a function of the rank of cross-link channel with $N_{ul} = 3$, $N_{dl} = 4$, $K_{ul} = 2$ and $K_{dl} = 4$.

(*): the given DoF does not satisfy the condition in [12, eq.(3)],
(**): the given DoF represents a non-uniform DoF at DL UEs, of the form $((d_{dl,1}, d_{dl,2}, d_{dl,3}, d_{dl,4}), d_{ul}, d_{tot})$

In Table I we can conclude that all the given DoF by the combined method [11, eq. (26), eq.(27)] is feasible as long as this DoF satisfies the necessary and sufficient condition in [12, Theorem 4]. For Remark 1, we can observe, in Table I for $r = 2$ and when considering the condition in [12, Theorem 4], that the non uniform tuple DoF $d_{ul,1} = d_{ul,2} = 1, d_{dl,1} = d_{dl,2} = 3, d_{dl,3} = d_{dl,4} = 2$, which gives a sum DoF equal to 12, is feasible. Otherwise, if we assume a uniform DoF (i.e. $d_{ul,1} = d_{ul,2}$ and $d_{dl,1} = d_{dl,2} = d_{dl,3} = d_{dl,4}$) we are limited to a feasible sum DoF equal to 10. So considering a different number of the data streams at Rx and Tx users could be interesting to increase the sum DoF, so the rate at high SNR.

## V. BEAMFORMER DESIGN

In this section, we begin by furnishing an example of obtaining the ZF precoders for UL UEs and the ZF decoders for DL UEs when working with closed-form case. Furthermore, we provide a demonstrative description of the methodology employed to achieve the WMSSE beamformer at the DL BS and the UL users, with the primary objective of maximizing the sum rate.

### *A. The ZF precoders at UL UEs and the ZF decoders at DL UEs*

In this subsection, we provide an explanation of how we derive the ZF precoders $\boldsymbol{G}_{z,l}$ and the ZF decoders $\boldsymbol{F}_{z,k}$ in closed-form case, which allows us to satisfy the condition of canceling all interference links from the UL UEs to the DL UEs given in equation (2).

We consider a system with $N_{ul} = 3$, $N_{dl} = 4$, $K_{ul} = 2$, and $K_{dl} = 4$, with an interference channel matrix of rank $r = 2$. We assume that the data stream is $d_{ul,1} = d_{ul,2} = 1$, $d_{dl,1} = d_{dl,2} = 3$, and $d_{dl,3} = d_{dl,4} = 2$. The following steps illustrate how we obtain $\boldsymbol{G}_{z,l}$ and $\boldsymbol{F}_{z,k}$ in closed-form cases:

**Step 0:** We generate interference channel matrices $\boldsymbol{H}_{11}, \boldsymbol{H}_{12}, \boldsymbol{H}_{21}, \boldsymbol{H}_{22}, \boldsymbol{H}_{31}, \boldsymbol{H}_{32}, \boldsymbol{H}_{41}$ and $\boldsymbol{H}_{42}$ with a rank of $r = 2$.

**Step 1:** The stream from UL UE 1 to DL UE 1 is canceled by UL UE 1. This involves performing singular value decomposition (SVD) of the interference channel matrix $\boldsymbol{H}_{11}$, resulting in:

$$[\boldsymbol{U}_{t1}\boldsymbol{S}_{t1}\boldsymbol{V}_{t1}] = SVD(\boldsymbol{H}_{11}). \tag{7}$$

$\boldsymbol{S}_{t1}$[1] is given such that:

$$\boldsymbol{S}_{t1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta_{1,1} & 0 \\ 0 & 0 & \beta_{1,2} \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

After obtaining the SVD of the interference channel matrix $\boldsymbol{H}_{11}$ and denoting the non-zero singular values by $\beta_{1,1}$ and $\beta_{1,2}$, we set $\boldsymbol{V}_{N1} = \boldsymbol{V}_{t1}$ and use it to transmit from Tx 1 (UL UE 1). This results in the following updated interference channel matrices:

$$\boldsymbol{H}_{N1,k1} = \boldsymbol{H}_{k1}\boldsymbol{V}_{N1}, \forall k \in [1, ..., K_{dl}] \tag{9}$$

The resulting $\boldsymbol{H}_{N1,11}$ has zeros at the first column, thus the interference from the UL UE 1 to the DL UE 1 is canceled by the UL UE 1.

**Step 2**: we perform interference cancellation from UL UE 2 to DL UE 2. This is achieved by performing the SVD of the interference channel matrix $\boldsymbol{H}_{22}$, which yields:

$$[\boldsymbol{U}_{t2}\boldsymbol{S}_{t2}\boldsymbol{V}_{t2}] = SVD(\boldsymbol{H}_{22}). \tag{10}$$

where the positions of the two non-zero singular values of $\boldsymbol{S}_{t2}$ are as those of $\boldsymbol{S}_{t1}$.
Then we take $\boldsymbol{V}_{N2} = \boldsymbol{V}_{t2}$ and apply it to Tx 2 (UL UE 2), so the new interference channel matrices become:

$$\boldsymbol{H}_{N2,k2} = \boldsymbol{H}_{k2}\boldsymbol{V}_{N2}, \forall k \in [1, ..., K_{dl}] \tag{11}$$

The resulting $\boldsymbol{H}_{N2,22}$ has zeros at the first column, thus the interference from the UL UE 2 to the DL UE 2 is canceled by UL UE 2.

**Step 3:** To cancel the stream from UL UE 2 to DL UE 1, we obtain the new channel matrix $\boldsymbol{H}_{N2,12}$ after completing step 2. Then, we calculate the SVD of the first column of $\boldsymbol{H}_{N2,12}$, denoted as $\boldsymbol{H}_{N2p,12}$. This step allows us to remove the interference caused by UL UE 2 on DL UE 1:

$$[\boldsymbol{U}_{1}\boldsymbol{S}_{1}\boldsymbol{V}_{1}] = SVD(\boldsymbol{H}_{N2p,12}). \tag{12}$$

[1]This distribution of singular values is used to dedicate the first effective antennas to the reception/transmission of the useful signal

$\boldsymbol{S}_1$[1] is given such that:

$$\boldsymbol{S}_1 = [0 \quad 0 \quad 0 \quad \gamma_1]^T \tag{13}$$

with $\gamma_1$ is the non-zero singular value of $\boldsymbol{H}_{N2p,12}$.
Then we take $\boldsymbol{U}_1^H$ and apply it to Rx 1 (DL UE 1), so the new interference channel matrices become:

$$\boldsymbol{H}_{n1,1l} = \boldsymbol{U}_1^H \boldsymbol{H}_{Nl,1l}, \forall l \in [1, ..., K_{ul}] \tag{14}$$

The resulting $\boldsymbol{H}_{n1,12}$ has $d_{dl,1}$ zeros at the first column, thus the interference from the UL UE 2 to the DL UE 1 is canceled at the DL UE 1.

**Step 4:** To cancel the stream from UL UE 1 to DL UE 2, we use the new channel matrix from UL UE 1 to DL UE 2 obtained after step 1, denoted by $\boldsymbol{H}_{N1,21}$. Then, we consider the first column of $\boldsymbol{H}_{N1,21}$, which corresponds to the stream from UL UE 1 to DL UE 2, denoted by $\boldsymbol{H}_{N1p,21}$. We apply the SVD to $\boldsymbol{H}_{N1p,21}$:

$$[\boldsymbol{U}_2 \boldsymbol{S}_2 \boldsymbol{V}_2] = SVD(\boldsymbol{H}_{N1p,21}). \tag{15}$$

where the positions of the non-zero singular value of $\boldsymbol{S}_2$ is as that of $\boldsymbol{S}_1$. Then we take $\boldsymbol{U}_2^H$ and apply it to Rx 2 (DL UE 2), so the new interference channel matrices become:

$$\boldsymbol{H}_{n2,2l} = \boldsymbol{U}_2^H \boldsymbol{H}_{Nl,2l}, \forall l \in [1, ..., K_{ul}] \tag{16}$$

The resulting $\boldsymbol{H}_{n2,21}$ has $d_{dl,2}$ zeros at the first column, thus the interference from the UL UE 1 to the DL UE 2 is canceled at the DL UE 2.

**Step 5**: we address the interference coming from both UL UE 1 and UL UE 2 towards DL UE 3. To cancel these two streams, we perform the singular value decomposition (SVD) of the matrix $\boldsymbol{H}_{c,3}$ which is formed by composing the interference channels from UL UE 1 and UL UE 2 to DL UE 3:

$$\boldsymbol{H}_{c,3} = \begin{bmatrix} h^{11}_{N1,31} & h^{21}_{N1,31} & h^{31}_{N1,31} & h^{41}_{N1,31} \\ h^{11}_{N2,32} & h^{21}_{N2,32} & h^{31}_{N2,32} & h^{41}_{N2,32} \end{bmatrix}^T \tag{17}$$

such that $h^{ji}_{N1,31}$ represents the element of $\boldsymbol{H}_{N1,31}$ at the $i^{th}$ column and the $j^{th}$ line:

$$[\boldsymbol{U}_3 \boldsymbol{S}_3 \boldsymbol{V}_3] = SVD(\boldsymbol{H}_{c,3}) \tag{18a}$$

$$\boldsymbol{S}_3 = \begin{bmatrix} 0 & 0 & \gamma_{3,1} & 0 \\ 0 & 0 & 0 & \gamma_{3,2} \end{bmatrix}^T \tag{18b}$$

with $\gamma_{3,1}$ and $\gamma_{3,2}$ are the non-zero singular values of $\boldsymbol{H}_{c,3}$. Then we take $\boldsymbol{U}_3^H$ and apply it to Rx 3 (DL UE 3), so the new interference channel matrices become:

$$\boldsymbol{H}_{n3,3l} = \boldsymbol{U}_3^H \boldsymbol{H}_{Nl,3l}, \forall l \in [1, ..., K_{ul}] \tag{19}$$

After applying the cancelation schemes in Steps 1-4, the resulting interference channel matrices $\boldsymbol{H}_{n3,31}$ and $\boldsymbol{H}_{n3,32}$ have a total of $d_{dl,3}$ zeros at the first column. As a result, the interference from UL UE 1 and UL UE 2 to the DL UE 3 is effectively canceled at the DL UE 3.

**Step 6**: we aim to cancel the interference from UL UE 1 and UL UE 2 at DL UE 4. To achieve this, we follow a similar approach as in Step 5 by considering the SVD of a matrix denoted as $\boldsymbol{H}_{c,4}$ which is similar to $\boldsymbol{H}_{c,3}$ with considering $\boldsymbol{H}_{N1,41}$ and $\boldsymbol{H}_{N2,42}$:

$$[\boldsymbol{U}_4 \boldsymbol{S}_4 \boldsymbol{V}_4] = SVD(\boldsymbol{H}_{c,4}). \tag{20}$$

After obtaining the SVD of the matrix $\boldsymbol{H}_{c,4}$ in the previous step, we place the two non-zero singular values of $\boldsymbol{S}_4$ in the same positions as those of $\boldsymbol{S}_3$. Then, we apply the Hermitian transpose of $\boldsymbol{U}_4$ to the received signal at DL UE 4, denoted as Rx 4. Consequently, the interference channel matrices are updated as follows:

$$\boldsymbol{H}_{n4,4l} = \boldsymbol{U}_4^H \boldsymbol{H}_{Nl,4l}, \forall l \in [1, ..., K_{ul}] \tag{21}$$

The resulting $\boldsymbol{H}_{n4,41}$ and $\boldsymbol{H}_{n4,42}$ have $d_{dl,4}$ zeros at the first column, thus the interference from the UL UE 1 and from UL UE 2 to the DL UE 4 are canceled at the DL UE 4.
Finally, $\boldsymbol{F}_{z,1} = \boldsymbol{U}_1[:, 1 : d_{dl,1}]$, $\boldsymbol{F}_{z,2} = \boldsymbol{U}_2[:, 1 : d_{dl,2}]$, $\boldsymbol{F}_{z,3} = \boldsymbol{U}_3[:, 1 : d_{dl,3}]$ and $\boldsymbol{F}_{z,4} = \boldsymbol{U}_4[:, 1 : d_{dl,4}]$; $\boldsymbol{G}_{z,1} = \boldsymbol{V}_{N1}[:, 1 : d_{ul,1}]$ and $\boldsymbol{G}_{z,2} = \boldsymbol{V}_{N2}[:, 1 : d_{ul,2}]$.

## B. WMMSE Beamformers

The derivation of the WMMSE beamformer for a MIMO Broadcast Channel system is provided previously in [13] and [14]. In our study, we have leveraged the WMMSE filter framework proposed in [13] and have extended it to account for the unique characteristics of the Dynamic TDD system. This allowed us to derive optimized beamformers at DL $\boldsymbol{V}_{dl,1}$ ... $\boldsymbol{V}_{dl,K_{dl}}$, $\boldsymbol{U}_{dl,1}$ ... $\boldsymbol{U}_{dl,K_{dl}}$ and at UL $\boldsymbol{V}_{ul,1}$ ... $\boldsymbol{V}_{ul,K_{ul}}$, $\boldsymbol{U}_{ul,1}$ ... $\boldsymbol{U}_{ul,K_{ul}}$ which maximize the weighted sum rate. The maximization problem can be written at the DL as:

$$\max_{\boldsymbol{v}} \sum_{k=1}^{K_{dl}} \alpha_k \boldsymbol{R}_{dl,k}; \quad s.t. \sum_{k=1}^{K_{dl}} \mathrm{Tr}(\boldsymbol{V}_{dl,k} \boldsymbol{V}_{dl,k}^H) \leq P_{DL-BS} \tag{22}$$

with $\alpha_k$ defines the priority for the DL user $k$ in the system, $P_{DL-BS}$ is the power budget at the DL BS, and $\boldsymbol{R}_{dl,k}$ is the rate of user $k$ which is written as shown in (5).

The MSE-matrix for user $k$ given that the MMSE-receive filter is applied can be written as:

$$\begin{aligned} \boldsymbol{E}_{dl,k} &= (\boldsymbol{I}_{d_{dl}} - \boldsymbol{U}_{dl,k}^H \boldsymbol{H}_k^{DL} \boldsymbol{V}_{dl,k})(\boldsymbol{I}_{d_{dl}} - \boldsymbol{U}_{dl,k}^H \boldsymbol{H}_k^{DL} \boldsymbol{V}_{dl,k})^H \\ &+ \sum_{j=1, j\neq k}^{K_{dl}} \boldsymbol{U}_{dl,k} \boldsymbol{H}_k^{DL} \boldsymbol{V}_{dl,j} \boldsymbol{V}_{dl,j}^H (\boldsymbol{H}_k^{DL})^H \boldsymbol{U}_{dl,k}^H \\ &+ \sum_{l=1}^{K_{ul}} \boldsymbol{U}_{dl,k} \boldsymbol{H}_{k,l} \boldsymbol{G}_l \boldsymbol{G}_l^H \boldsymbol{H}_{k,l}^H \boldsymbol{U}_{dl,k}^H + \sigma_k^2 \boldsymbol{U}_{dl,k}^H \boldsymbol{U}_{dl,k}, \end{aligned} \tag{23}$$

So the MMSE receive filter at user $k$ is given as:

$$\boldsymbol{U}_{dl,k}^{MMSE} = \boldsymbol{J}_{dl,k}^{-1} \boldsymbol{H}_k^{DL} \boldsymbol{V}_{dl,k} \tag{24a}$$

$$\begin{aligned} \boldsymbol{J}_{dl,k} &= \sum_{j=1}^{K_{dl}} \boldsymbol{H}_k^{DL} \boldsymbol{V}_{dl,j} \boldsymbol{V}_{dl,j}^H (\boldsymbol{H}_k^{DL})^H \\ &+ \sum_{l=1}^{K_{ul}} \boldsymbol{H}_{k,l} \boldsymbol{V}_{ul,l} \boldsymbol{V}_{ul,l}^H \boldsymbol{H}_{k,l}^H + \sigma_{dl,k}^2 \boldsymbol{I} \end{aligned} \tag{24b}$$

Using this MMSE receiver, the corresponding MSE matrix is given by:

$$\boldsymbol{E}_{dl,k}^{mmse} = \boldsymbol{I}_{d_{dl,k}} - \boldsymbol{V}_{dl,k}^H (\boldsymbol{H}_k^{Dl})^H \boldsymbol{J}_{dl,k}^{-1} \boldsymbol{H}_k^{DL} \boldsymbol{V}_{dl,k} \tag{25}$$

We denote $\boldsymbol{W}_{dl,k}$ as a constant weight matrix associated with user $k$, such that:

$$\boldsymbol{W}_{dl,k} = \boldsymbol{E}_{dl,k}^{mmse^{-1}} \tag{26}$$

The precoder at DL user $k$ is given such that:

$$\bar{\boldsymbol{V}}_{dl} = \left( \boldsymbol{H}^H \boldsymbol{U} \boldsymbol{W} \boldsymbol{U}^H \boldsymbol{H} + \mu_{dl} \boldsymbol{I}_{M_{dl}} \right)^{-1} \boldsymbol{H}^H \boldsymbol{U} \boldsymbol{W} \tag{27a}$$

$$b_{dl} = \sqrt{\frac{P_{DL-BS}}{\mathrm{Tr}(\bar{\boldsymbol{V}}_{dl} \bar{\boldsymbol{V}}_{dl}^H)}} \tag{27b}$$

$$\boldsymbol{V}_{dl}^{WMMSE} = b_{dl} \bar{\boldsymbol{V}}_{dl} = [\boldsymbol{V}_{dl,1}, \ \boldsymbol{V}_{dl,2}, \ \ldots, \ \boldsymbol{V}_{dl,K_{dl}}] \tag{27c}$$

with $\mu_{dl}$ a regularization parameter given by:

$$\mu_{dl} = \frac{\text{Tr}\left(\boldsymbol{W}\boldsymbol{U}^H\boldsymbol{U}\right)}{P_{DL-BS}} \tag{28}$$

The same approach used to obtain the WMMSE DL beamformers is applicable to derive the UL beamformers as well. Then at UL, the maximization of the sum rate is given by:

$$\max_{\boldsymbol{v}} \boldsymbol{R}_{ul,l}, \quad s.t. \, \text{Tr}(\boldsymbol{V}_{ul,l}\boldsymbol{V}_{ul,l}^H) \leq P_{ul,l} \tag{29}$$

$P_{ul,l}$ is the power budget at the $l^{th}$ UL UE, and $\boldsymbol{R}_{ul,l}$ is the rate of user $l$ which is written as shown in (3). The MMSE receiver at the UL BS and the weighted matrix $\boldsymbol{W}_{ul,l}$ are given such that:

$$\boldsymbol{U}_{ul,l}^{MMSE} = \boldsymbol{J}_{ul,l}^{-1}\boldsymbol{H}_l^{UL}\boldsymbol{V}_{ul,l} \tag{30a}$$

$$\boldsymbol{J}_{ul,l} = \sum_{i=1}^{K_{ul}} \boldsymbol{H}_i^{UL}\boldsymbol{V}_{ul,i}\boldsymbol{V}_{ul,i}^H(\boldsymbol{H}_i^{UL})^H + \sigma_{ul}^2\boldsymbol{I}_{M_{ul}} \tag{30b}$$

$$\boldsymbol{E}_{ul,l}^{mmse} = \boldsymbol{I}_{d_{ul,l}} - \boldsymbol{V}_{ul,l}^H(\boldsymbol{H}_l^{Ul})^H\boldsymbol{J}_{ul,l}^{-1}\boldsymbol{H}_l^{UL}\boldsymbol{V}_{ul,l} \tag{30c}$$

$$\boldsymbol{W}_{ul,l} = \boldsymbol{E}_{ul,l}^{mmse^{-1}} \tag{30d}$$

So the precoder at the $l^{th}$ UL user is:

$$\bar{\boldsymbol{V}}_{ul,l} = \Big((\boldsymbol{H}_l^{UL})^H\boldsymbol{U}_{ul,l}\boldsymbol{W}_{ul,l}\boldsymbol{U}_{ul,l}^H\boldsymbol{H}_l^{UL} + \sum_{i=1}^{K_{dl}}(\boldsymbol{H}_{i,l})^H\boldsymbol{U}_{dl,i}\boldsymbol{W}_{dl,i}\boldsymbol{U}_{dl,i}^H\boldsymbol{H}_{i,l} + \mu_{ul,l}\boldsymbol{I}_{N_{ul,l}}\Big)^{-1} (\boldsymbol{H}_l^{UL})^H\boldsymbol{U}_{ul,l}\boldsymbol{W}_{ul,l} \tag{31a}$$

$$b_{ul,l} = \sqrt{\frac{P_{ul,l}}{\text{Tr}(\bar{\boldsymbol{V}}_{ul,l}\bar{\boldsymbol{V}}_{ul,l}^H)}} \tag{31b}$$

$$\boldsymbol{V}_{ul,l}^{WMMSE} = b_{ul,l}\bar{\boldsymbol{V}}_{ul,l} \tag{31c}$$

with $\mu_{ul,l}$ a regularization parameter given by:

$$\mu_{ul,l} = \frac{\text{Tr}\left(\boldsymbol{W}_{ul,l}\boldsymbol{U}_{ul,l}^H\boldsymbol{U}_{ul,l}\right)}{P_{ul,l}} \tag{32}$$

## VI. Sum Rate Simulations

In the subsequent simulations, we assess the sum rate of the system with the following configuration: $N_{ul} = 3$, $N_{dl} = 4$, $K_{ul} = 2$, $K_{dl} = 4$, $M_{dl} = 14$, and $M_{ul} = 4$. This allows us to examine various interference channel ranks and analyze the influence of the UE-to-UE ZF method in initializing the WMMSE algorithm. We describe each notation associated with a specific simulation scenario: **1) Initialization (UE2UE ZF + BS2UE ZF)**: This simulation evaluates the following sum rate during the initialization phase using the UE-to-UE ZF method. It considers the precoders $\boldsymbol{G}_{z,l}$ at UL UEs, the decoders $\boldsymbol{F}_{z,k}$ at DL UEs, and ZF between DL UEs utilizing the ZF precoders at the DL BS (as specified in equation (6)). **2) Initialization (UE EigR + BS2UE ZF)**: In this simulation, the sum rate at the initialization phase is computed without employing the UE-to-UE ZF. The transmit and receive vectors are determined from the SVD of the direct channel matrices at the UL and DL sides. ZF is applied between DL UEs using the ZF precoders (as described in equation (6)). **3) Initialization (UE2UE ZF + BS2UE ZF) + WMMSE, iter=n**: This simulation follows the initialization process outlined in the "Initialization (UE2UE ZF + BS2UE ZF)" scenario. Subsequently, the WMMSE algorithm (described in subsection V-B) is executed. The sum rate is computed at the $n^{th}$ iteration of the WMMSE algorithm. **4) Initialization (UE EigR + BS2UE ZF) + WMMSE, iter=n**: Similar to the previous simulation, this scenario involves the initialization described in "Initialization (UE EigR + BS2UE ZF)".

We calculate the sum rate at the DL using $\boldsymbol{R}_{dl,k}$ from equation (5) and at the UL using $\boldsymbol{R}_{ul,l}$ from equation (3) through Monte Carlo averaging over 100 channel realizations. The elements of the direct channel matrices are generated as independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and unit variance, i.e., $\mathcal{CN}(0,,1)$. The receive noise covariance is normalized, such that $\boldsymbol{R}_{n_kn_k} = \boldsymbol{I}_{N_{dl,k}}$. In our simulations, we assume equal power allocation among UL UEs, denoted as $p_{ul,1} = p_{ul,2} = P$, and a total power of $K_{dl}P$ at the DL BS, where $\sum_{k=1}^{K_{dl}} p_{dl,k} = K_{dl}P = P_{DL-BS}$. We define $P = 10^{\frac{SNR}{10}}$, where $SNR$ represents the signal-to-noise ratio.

In Figure 1, we evaluate the sum rate at the DL and UL UEs for two cases based on the rank of the interference channel between the UL and DL UEs, denoted as $rank(\boldsymbol{H}_{k,l}) = r$ **a)** Reduced rank MIMO IBMAC-IC with $r = 2$ : In this case, the DoF at each UL UE is $d_{ul,1} = d_{ul,2} = 1$, and at each DL UE, we have $d_{dl,1} = d_{dl,2} = 3$ and $d_{dl,3} = d_{dl,4} = 2$, **b)** Full rank MIMO IBMAC-IC with $r = 3$ : Here, the DoF at each UL UE is $d_{ul,1} = d_{ul,2} = 1$, and for the DL UEs, we have $d_{dl,1} = 3$ and $d_{dl,2} = d_{dl,3} = d_{dl,4} = 2$. For the scenario where $r = 3$, the ZF beamformers for DL UEs $F_{z,k}$, and for UL UEs $G_{z,l}$, are determined through the utilization of the interference alignment alternating minimization algorithm presented in [15], so through an iterative process.

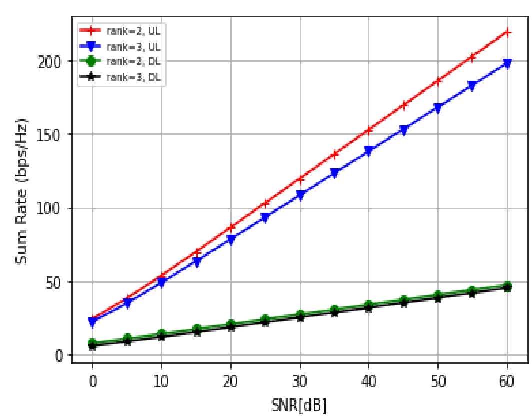

Fig. 1: sum rate performance with UE2UE ZF+ BS2UE ZF for $N_{ul} = 3$, $N_{dl} = 4$, $K_{ul} = 2$ and $K_{dl} = 4$.

In Figure 1, it can be observed that the sum rate at the UL is approximately equal in both cases. This similarity arises due to the example considered and the feasibility condition outlined in Theorem 4 of [12]. According to this condition, it is known that for the given system dimension, it is not possible to increase the DoF at UL UEs, as indicated in Table I. If the DoF were increased, IA would not be feasible.

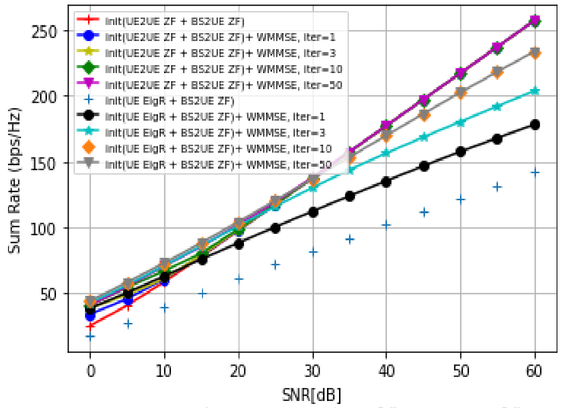


Fig. 2: sum rate performance with $N_{ul} = 3$, $N_{dl} = 4$, $K_{ul} = 2$, $K_{dl} = 4$ and $r = 2$.

Figure 2 illustrates the impact of UE-to-UE interference on the performance of the DynTDD system. The total sum rate is depicted by considering two different initialization: ”init (UE2UE ZF + BS2UE ZF)” and ”init (UE EigR + BS2UE ZF)”. Analyzing the simulation results presented in Figure 2, it becomes evident that incorporating UE-to-UE ZF interference cancellation yields a substantial enhancement in the sum rate. Up to an SNR of $15dB$, the WMMSE algorithm effectively reduces the sum rate disparity between the ”init (UE2UE ZF + BS2UE ZF)” and ”init (UE EigR + BS2UE ZF)” initialization. However, at high SNR levels, the WMMSE algorithm struggles to bridge this gap, particularly with a low number of iterations. For instance, at a sum rate of $200bps/Hz$, the WMMSE algorithm without UE-to-UE ZF in the initialization requires an additional $13dB$ and $4dB$ of SNR for iter=3 and 10, respectively, to achieve the performance of the ”init (UE2UE ZF + BS2UE ZF)” initialization. Therefore, the proposed UE-to-UE ZF decoders and precoders effectively mitigate UE-to-UE interference, leading to a remarkable overall system performance improvement.

## VII. Conclusions

This paper presents novel insights and findings building upon our previous work discussed in [11, eq. (26), eq.(27)]. We look into the feasibility of the combined method and investigate the advantages of employing a non-uniform DoF approach at DL and/or UL UEs to maximize the overall DoF and SNR rates. Specifically, the study delves into beamforming design for MIMO IBMAC-IC in DynTDD systems, aiming to enhance the sum rate for both DL and UL UEs, considering scenarios with deficient and full-rank interference channels. To address this, we propose a closed-form solution that outlines step-by-step instructions for constructing ZF beamformers at DL and UL UEs, effectively canceling all interference links. Furthermore, we employ the WMMSE iterative algorithm, which exhibits the potential for low-complexity transmit beamforming implementations, to maximize the sum rate. Simulation results indicate that the deficient rank interference channel yields a higher sum rate compared to the full rank one. From the numerical results, we observe that a non-uniform DoF distribution can improve the system DoF, regardless of whether the rank of the UE2UE IC is full or deficient. Additionally, the simulation findings demonstrate that the application of the WMMSE algorithm for sum rate maximization reveals the detrimental effects of UE-to-UE interference within the DynTDD system. However, interference alignment techniques prove to be effective in mitigating this interference and improving the system’s overall performance.

## References

[1] A. Goldsmith, *Wireless communications*. Cambridge university press, 2005.

[2] P. Jayasinghe, A. Tölli, and M. Latva-aho, “Bi-directional signaling strategies for dynamic TDD networks,” in *IEEE 16th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2015.

[3] H. H. Yang, G. Geraci, Y. Zhong, and T. Q. S. Quek, “Packet Throughput Analysis of Static and Dynamic TDD in Small Cell Networks,” *IEEE Wireless Communications Letters*, vol. 6, no. 6, 2017.

[4] R. Jalal, N. Ridha, and D. Laurent, “Interference Analysis in Dynamic TDD System Combined or Not with Cell Clustering Scheme,” in *2018 IEEE 87th Vehicular Technology Conference (VTC Spring)*, 2018.

[5] Y. Han, Y. Chang, J. Cui, and D. Yang, “A Novel Inter-Cell Interference Coordination Scheme Based on Dynamic Resource Allocation in LTE-TDD Systems,” in *IEEE 71st Vehicular Technology Conference*, 2010.

[6] T. Liu and C. Yang, “On the Feasibility of Linear Interference Alignment for MIMO Interference Broadcast Channels With Constant Coefficients,” *IEEE Trans. Signal Processing*, May 2013.

[7] M. Razaviyayn, G. Lyubeznik, and Z.-Q. Luo, “On the Degrees of Freedom Achievable Through Interference Alignment in a MIMO Interference Channel,” *IEEE Trans. Signal Processing*, Feb. 2012.

[8] Óscar González, C. Beltrán, and I. Santamaría, “A Feasibility Test for Linear Interference Alignment in MIMO Channels With Constant Coefficients,” *IEEE Trans. Information Theory*, March 2014.

[9] S.-W. Jeon, K. Kim, J. Yang, and D. K. Kim, “The Feasibility of Interference Alignment for MIMO Interfering Broadcast–Multiple-Access Channels,” *Trans. Wireless Comm’s*, July 2017.

[10] K. S. Ko, B. C. Jung, and M. Hoh, “Distributed Interference Alignment for Multi-Antenna Cellular Networks With Dynamic Time Division Duplex,” *IEEE Communications Letters*, April 2018.

[11] A. Tibhirt, D. Slock, and Y. Yuan-Wu, “Distributed Beamforming Design in Reduced-Rank MIMO Interference Channels and Application to Dynamic TDD,” *Workshop on Smart Antennas (WSA)*, Nov. 2021.

[12] ——, “Interference Alignment in Reduced-Rank MIMO Networks with Application to Dynamic TDD,” in *Resource Allocation, Cooperation and Competition in Wireless Networks (RAWNET)*, 2022.

[13] S. S. Christensen, R. Agarwal, E. De Carvalho, and J. M. Cioffi, “Weighted sum-rate maximization using weighted mmse for mimo-bc beamforming design,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 4792–4799, 2008.

[14] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, “An iteratively weighted mmse approach to distributed sum-utility maximization for a mimo interfering broadcast channel,” *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4331–4340, 2011.

[15] S. W. Peters and R. W. Heath, “Interference Alignment Via Alternating Minimization,” *Int’l Conf. Acoustics, Speech and Signal Processing (ICASSP)*, 2009.