UAV-aided Network Localization Using Radio Phase Measurements

Omid Esrafilian[1], Rajeev Gangula[2], and David Gesbert[1]
[1]Communication Systems Department, EURECOM, Sophia Antipolis, France
[2]Department of Electrical and Computer Engineering, Northeastern University, Boston, MA, USA
{esrafili, gangula, gesbert}@eurecom.fr

***Abstract*—This paper considers the problem of ground users localization aided by an unmanned aerial vehicle (UAV) flying radio capable of collecting measurements from users. To do so, we exploit the phase information of the received radio signals from users at the UAV in different locations along with time-of-arrival (ToA) measurements. The location of the UAV is also not perfectly known, hence the problem at hand turns to a simultaneous user localization and tracking of the UAV location. The phase measurements are inherently more precise than ToA measurements while bringing ambiguity since it follows a periodic behavior. To solve this issue, we unwrap the phase measurements by tracking the phase while the UAV moves. Then a time-difference-of-arrival (TDoA)-based algorithm is proposed by treating each UAV location as a virtual antenna/anchor (yet with unknown locations) and exploiting the unwrapped phase measurements to precisely localize the users and jointly track the UAV trajectory. We employed a least-squares simultaneous localization and mapping (SLAM) framework to fuse TDoA data, estimated from phase, with other measurements such as ToA, an erroneous estimate of UAV location available from GPS, and the UAV velocity measured by an inertial measurement unit (IMU) onboard. The simulations verified the performance of the developed algorithm when compared to other benchmarks including scenarios with traditional terrestrial anchor-enabled localization systems.**

***Index Terms*—UAV, localization, phase, TDoA, ToA, wireless communications**

I. Introduction

In a wireless localization system, nodes with perfectly-known positions known as anchor nodes (which can be stationary or mobile) collect various radio measurements from the emitted radio frequency (RF) signals from the users in the network and use them for localization purposes. Various measurements such as received signal strength (RSS), time-of-arrival (ToA), angle of arrival (AoA), etc., can be obtained from the RF signals by the anchor nodes [1], [2].

On the other hand, advancements in robotic technologies and the miniaturization of wireless equipment have made it possible to have flying radio networks (FRANs), where wireless connectivity to ground users can be provided by aerial base stations (BSs) or relays that are mounted on unmanned aerial vehicles (UAVs) [3], [4]. The advantage of FRANs includes fast and dynamic network deployment during an emergency or temporary crowded events, providing connectivity in areas lacking network infrastructure, etc. While in terrestrial radio access networks static BSs are used as anchor nodes, in FRANs UAV BSs can be used as mobile anchor nodes. However, when it comes to aerial anchors, the location of the UAV is of crucial importance. Unfortunately, the UAV location is not precisely known and is subject to noise. Therefore, when using aerial mobile anchors, the problem becomes not only localizing the users but also tracking the UAV location.

Localization of ground users using radio measurements collected by aerial UAV anchor nodes has recently gained interest [5]–[12]. The main advantage of using UAV anchors in localization compared to static anchors is that UAVs with their inherent 3D mobility can collect radio measurements in different geographic locations which improve the localization performance. In other words, the UAV in different locations can be considered as virtual static anchors.

UAV-aided user localization systems by exploiting RSS measurements are studied in [5]–[8]. The authors in [9] considered a multi-UAV-aided localization scenario where a combination of ToA and AoA measurements are used to localize ground users. The deployment of UAVs is also optimized for further improvement of localization performance. In [10], a hybrid ToA along with 1D AoA localization approach that merely requires elevation AoA estimations to combine with ToA measurements is proposed. The impact of the antenna radiation pattern for the channel between the UAV and the ground users in a 3D localization system using time-based (ToA and TDoA) measurements has been studied in [11], [12].

In [13], a phase-based indoor localization system is proposed. An extended Kalman filter (EKF) is employed to localize the radio devices using the phase difference measurements collected by the base stations with known locations. In [14], a time-difference-of-arrival (TDoA)-based method is used to track a mobile user with an unknown static anchor location. An EKF is used to keep track of the multi-path component (MPC) phases. However, the method proposed in [14] is suitable for indoor localization and might fail for outdoor use cases. In [15], a similar method to [14] is proposed but by exploiting the ToA measurements from a user equipped with multiple antennas. The algorithm is shown to work in indoor situations. In [16], the trajectory of a mobile user equipped with a single antenna is studied. The location of the mobile user is considered as a virtual antenna, and then a single-antenna-based AoA estimation technique is proposed to find the angle of arrival and the location of the users at each time. To make this algorithm work, a rough estimate of each virtual antenna

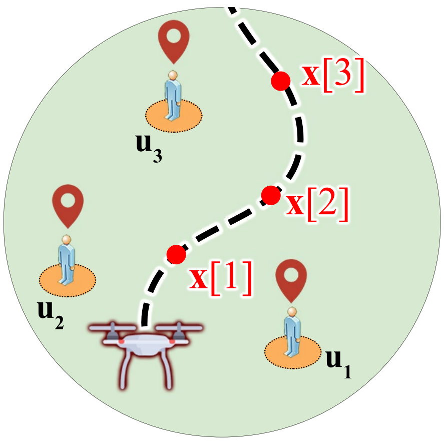

Fig. 1: UAV-aided ground user localization system.

location is required, however, if the uncertainty of the prior guess of virtual antenna location is big, the algorithm fails.

In this paper, we propose a new algorithm for UAV-aided user localization systems. The location of the UAV is not also precisely known, hence the algorithm has to track the UAV location too. To do so, we exploit the phase information of the received radio signals along with ToA measurements. The phase measurements are known to be precise but bring ambiguity since it follows a periodic behavior. To solve this issue, we unwrap the phase measurements by tracking the phase while the UAV moves. Then a TDoA-based algorithm is used by treating each UAV location as a virtual antenna/anchor and exploiting the unwrapped phase measurements to precisely localize the users and jointly track the UAV trajectory. We employed a least-squares simultaneous localization and mapping (SLAM) framework to fuse TDoA data (estimated from phase) with other measurements such as ToA, UAV location estimate available from GPS, and the UAV velocity measured by an inertial measurement unit (IMU) onboard.

To the best of our knowledge, simultaneous localization of users and tracking a mobile flying radio by exploiting phase measurements has not been studied in the literature. Specifically, our contributions are as follows:

- An anchor-free localization system by utilizing a mobile flying radio node capable of collecting radio measurements from ground nodes is proposed.
- A TDoA-based algorithm is developed capable of exploiting radio phase measurements by treating the UAV locations at different times as virtual antennas.
- A least-squares-based SLAM problem is formulated for fusing different types of measurements to jointly localize the users and track the UAV.

II. System Model and Problem Formulation

We consider a scenario similar to the one illustrated in Fig.1, where a UAV equipped with radio devices capable of measuring radio signals from K ground-level users in a given service area. The users are spread over the area and $\mathbf{u}_k = [x_k, y_k]^{\mathrm{T}} \in \mathbb{R}^2$, $k \in [1, K]$ denotes the k-th user's location. The users are considered static and their locations are unknown.

The aim of the UAV is to estimate the unknown user locations based on radio measurements taken over a mission of duration T. We discretize the UAV mission time into N equal time steps such that the UAV velocity is assumed to be constant within each time step. In the n-th time step, the UAV/drone position is denoted by $\mathbf{x}[n] = [x[n], y[n], z[n]]^{\mathrm{T}} \in \mathbb{R}^3$. We assume that the UAV is equipped with a GPS receiver, the UAV location obtained using the GPS is given by

$$\hat{\mathbf{x}}[n] = \mathbf{x}[n] + \eta, \tag{1}$$

where η is the GPS measurement noise and is modeled by a Gaussian random variable with $\mathcal{N}(0, \sigma_{gps}^2 * \mathbf{I}_2)$ [17], where $\mathbf{I}_2$ is the identity matrix of size 2×2. The UAV velocity is also available from a IMU onboard the UAV and is denoted by

$$\hat{\mathbf{v}}[n] = \mathbf{v}[n] + \nu, \tag{2}$$

where $\mathbf{v}[n]$ is the true UAV velocity at time step n, and ν is the velocity measurement noise which is modeled by a Gaussian random variable as $\mathcal{N}(0, \sigma_{vel}^2 * \mathbf{I}_2)$ [17]. Assuming that the UAV moves with constant speed within each time step, the UAV velocity can be modeled as follows

$$\mathbf{v}[n] = \frac{\mathbf{x}[n] - \mathbf{x}[n-1]}{\Delta t}, \tag{3}$$

where $\Delta t = \frac{T}{N}$. Therefore, (2) can be reformulated as

$$\hat{\mathbf{v}}[n] = \frac{\mathbf{x}[n] - \mathbf{x}[n-1]}{\Delta t} + \nu. \tag{4}$$

A. Channel Model

We now describe the radio channel model between the UAV and ground users. We assume that the UAV and the users are synchronized. Since air-to-ground channels known to exhibit few known dominant paths [18], channel between UAV position $\mathbf{x}[n]$ and user location $\mathbf{u}_k$ can be modeled as

$$g_k[n] = \sum_l a_{l,k}[n] \exp(-j\phi_{l,k}[n]) s_k \left[n - \tau_{l,k}[n]\right], \tag{5}$$

where $g_k[n], s_k[n]$ are , respectively, the received signal from user k and the transmitted signal by the k-th user, $a_{l,k}[n]$ is the overall attenuation, and $\tau_{l,k}[n]$ is the propagation delay of the l-th path between user k and the UAV at time n. The phase of the received signal from user k at time n for the l-th path is denoted by $\phi_{l,k}[n]$ which is given by

$$\phi_{l,k}[n] = 2\pi f_c \tau_{l,k}[n], \tag{6}$$

where f_c is the carrier frequency.

Assuming that the transmitter and the receiver are stationary during a short period of time, then by taking several samples form $g_k[n]$, the parameters $a_{l,k}[n], \phi_{l,k}[n], \tau_{l,k}[n]$ can be estimated using classical channel estimation methods. The estimated phase and the propagation delay can be modeled by Gaussian variables, $\hat{\phi}_{l,k}[n] \sim \mathcal{N}(\phi_{l,k}[n], \sigma_\phi^2), \hat{\tau}_{l,k}[n] \sim \mathcal{N}(\tau_{l,k}[n], \sigma_\tau^2)$, where σ_τ^2 depends on the Bandwidth of the channel. Generally speaking, $\sigma_\tau^2 \propto \frac{1}{\text{Bandwidth}} \frac{1}{\text{SNR}_k[n]}$ [19], [20]. Where $\text{SNR}_k[n]$ is the signal-to-noise-ratio (SNR) of the signal between the UAV at time step n and user k. For simplicity, in this paper we assume an average SNR over the channel to model the ToA as follows

$$\sigma_\tau^2 \propto \frac{1}{\text{Bandwidth}} \frac{1}{\overline{\text{SNR}}}, \tag{7}$$

where $\overline{\text{SNR}}$ is the average SNR.

Note that, in this paper we assume the UAV can always maintain a line-of-sight (LoS) connection to users. We denote the LoS path with $l = 0$. The true propagation delay for LoS path as a function of user and UAV locations is given by

$$\tau_{0,k}[n] = \frac{\|\mathbf{x}[n] - \mathbf{u}_k\|}{C}, \tag{8}$$

where C is the speed of light. Consequently, the phase for LoS path can be computed by substituting (8) into (6).

III. User Localization and UAV Tracking

In this section, we propose an algorithm to estimate the user locations from the radio measurements collected by the UAV. Let us denote an arbitrary set of measurements taken by the UAV during the mission as $\mathcal{G} = \{\gamma_k[n], n \in [1, N], k \in [1, K]\}$, where $\gamma_k[n]$ is a tuple of measurements collected from user k at time step n defined as follows

$$\gamma_k[n] = \left(\hat{\mathbf{x}}_n, \hat{\mathbf{v}}_n, \hat{\phi}_{0,k}[n], \hat{\tau}_{0,k}[n]\right), \tag{9}$$

where $\hat{\mathbf{x}}_n$ is the UAV location measured by the GPS, $\hat{\mathbf{v}}_n$ is the measured UAV velocity given by the IMU, and $\hat{\phi}_{0,k}[n], \hat{\tau}_{0,k}[n]$ are the estimated phase and propagation delay for LoS path between the UAV at time step n and user k, respectively. It is worth mentioning that, the true location and the velocity of the UAV are not available (i.e. the GPS and the IMU measurements are subject to the noise), therefore we not only have to localize the users but also need to track the UAV location.

To solve this problem, we employ the TDoA along with ToA measurements to localize the users. In other words, we assume that the UAV location at each time step as a virtual static anchor. However, the complexity of TDoA-based localization methods rapidly increases as the number of anchors grows (in our case, as more measurements are collected). Moreover, the TDoA measurements, computed from the unwrapped phase (explained in the following section), are potentially subject to cumulative error over time. To deal with these issues, we first divide our measurements into M partitions, then the phase is only unwrapped within each partition and consequently, the TDoA is computed for each partition individually. Each partition consists of $\delta = \frac{N}{M}, \delta > 3$ consecutive measurements. Equivalently, the measurement set $\mathcal{G}$ can be rewritten as

$$\mathcal{G} = \bigcup_{k=1}^{K} \bigcup_{m=1}^{M} \gamma_k \left[(m-1)\delta + 1 : m\delta\right], \tag{10}$$

where

$$\gamma_k \left[(m-1)\delta + 1 : m\delta\right] \triangleq \left\{\gamma_k[i], i \in \left[(m-1)\delta + 1, m\delta\right]\right\}, \tag{11}$$

is the m-th partition of data containing the collected measurements between time step $(m-1)\delta + 1$ to time step $m\delta$ from user k. Now we instead apply the TDoA-based method to each partition.

Having partitioned the measurements and assuming that collected measurements conditioned on the channel and user positions are independent and identically distributed (i.i.d), the negative log-likelihood of measurements leads to

$$\begin{aligned}\mathcal{L} = &\sum_{k=1}^{K}\sum_{n=1}^{N} \frac{1}{\sigma_{gps}^2} \|\hat{\mathbf{x}}[n] - \mathbf{x}[n]\|^2 + \\ &\sum_{k=1}^{K}\sum_{n=2}^{N} \frac{1}{\sigma_{vel}^2} \left\|\hat{\mathbf{v}}[n] - \frac{\mathbf{x}[n] - \mathbf{x}[n-1]}{\Delta t}\right\|^2 + \\ &\sum_{k=1}^{K}\sum_{n=1}^{N} \frac{1}{\sigma_\tau^2} \left|\hat{\tau}_{0,k}[n] - \frac{\|\mathbf{x}[n] - \mathbf{u}_k\|}{C}\right|^2 + \\ &\sum_{k=1}^{K}\sum_{m=1}^{M} \sum_{i=(m-1)\delta+1}^{\delta m-1} \sum_{j=i+1}^{\delta m} \frac{1}{2\sigma_\phi^2} \left|\Delta\hat{\phi}_{i,j,k} - \Delta\phi_{i,j,k}\right|^2.\end{aligned} \tag{12}$$

where $\Delta\hat{\phi}_{i,j,k} = \hat{\phi}_{0,k}[i] - \hat{\phi}_{0,k}[j]$ is the difference between the measured phase of LoS path between user k at two different UAV locations i and j. In other words, $\Delta\hat{\phi}_{i,j,k}$ can be seen as the measured time difference between the received signal from user k at two different UAV locations at time steps i and j. $\Delta\phi_{i,j,k}$ is defined similarly and can be modeled using (8) as

$$\Delta\phi_{i,j,k} = \frac{2\pi f_c}{C} \left(\|\mathbf{x}[i] - \mathbf{u}_k\| - \|\mathbf{x}[j] - \mathbf{u}_k\|\right). \tag{13}$$

The estimate of the unknown user and the UAV locations can then be obtained by solving

$$\min_{\substack{\mathbf{x}[n], \mathbf{u}_k \\ \forall n,k}} \quad \mathcal{L}. \tag{14}$$

Solving problem (14) is challenging, since it is a simultaneous user localization and UAV tracking, and the objective function is highly non-linear and non-convex. Moreover, the absolute value of phase measurements is not available due to the periodic nature of the phase measurements which bring an ambiguity into the optimization problem. In the following sections, we first try to unwrap the phase measurements to deal with the ambiguity, and then we iteratively solve the optimization problem (14).

A. Phase Unwrapping

As mentioned earlier phase measurements are inherently ambiguous and they rotate between the range of $[0, 2\pi]$, which makes the phase measurements unsuitable for our application. To deal with this problem, in this section, we propose a simple (yet efficient) algorithm to unwrap the phase measurement to a continuous form by exploiting the fact that the drone's maximum speed is limited to $v_{\max}$. To do so, for each user k, we denote $\Delta\hat{\phi}_{n,n-1,k} = \hat{\phi}_{0,k}[n] - \hat{\phi}_{0,k}[n-1]$ as the change in the LoS phase measurements within time step n. Given the drone maximum speed, we expect that the maximum change in the phase is limited by

$$|\Delta\hat{\phi}_{n,n-1,k}| \leq \Delta\phi_{\max} \triangleq \frac{2\pi f_c}{C}\frac{T}{N}v_{\max}. \tag{15}$$

Above expression implies that the phase change should be always less than $\Delta\phi_{\max}$ unless the phase overflows because of the transition from 2π to 0 (or vice versa). Therefore, in this situation the phase change is recalculated as follows:

$$\Delta\hat{\phi}_{n,n-1,k} := \begin{cases} \hat{\phi}_{0,k}[n] - \hat{\phi}_{0,k}[n-1] + 2\pi, & \text{if}\, \Delta\hat{\phi}_{n,n-1,k} < 0 \\ \hat{\phi}_{0,k}[n-1] - \hat{\phi}_{0,k}[n] + 2\pi, & \text{else.} \end{cases} \tag{16}$$

Moreover, the phase measurements are subject to noise which can cause a false phase overflow detection. To avoid this to happen, we define a threshold ϵ to account for the noise of phase measurements. Hence, before compensating for the overflow in (16), the phase is corrected by taking into account the threshold as follows:

$$\Delta\hat{\phi}_{n,n-1,k} := \begin{cases} 0, & \text{if} \quad \Delta\hat{\phi}_{n,n-1,k} < \epsilon \\ \Delta\hat{\phi}_{n,n-1,k}, & \text{else.} \end{cases} \tag{17}$$

Finally, the unwrapped phase for user k and time step n is given by

$$\tilde{\phi}_{0,k}[n] = \tilde{\phi}_{0,k}[n-1] + \Delta\hat{\phi}_{n,n-1,k}. \tag{18}$$

The different steps of phase unwrapping are explained in Algorithm 1. Note that, Algorithm 1 can be similarly used to unwrap the phase measurements within each partition of data $\gamma_k\left[(m-1)\delta + 1 : m\delta\right], m \in [1, M]$.

Algorithm 1 Phase unwrapping

$\tilde{\phi}_{0,k}[1] = 0, \forall k$
for $n = 2$ to N **do**
  compute $\Delta\hat{\phi}_{n,n-1,k} = \hat{\phi}_{0,k}[n] - \hat{\phi}_{0,k}[n-1]$.
  Remove the effect of the phase noise as (17).
  Detect and compensate for the phase overflow as (16).
  Compute $\tilde{\phi}_{0,k}[n] = \tilde{\phi}_{0,k}[n-1] + \Delta\hat{\phi}_{n,n-1,k}$
end for

B. User Localization and UAV Tracking

Having unwrapped the phase measurements we continue to localize the users and track the UAV by solving problem (14). As mentioned earlier, this problem is challenging since the objective function is non-linear and non-convex. To deal with this problem, we employ an iterative approach similar to the one presented in [21], where at each iteration the problem first is locally linearized and then is solved. The algorithm then iterates until the convergence. In the following, we first introduce a general framework for solving optimization problems similar to (14), and then we will elaborate on how our problem can be solved with this framework.

Let's assume that we want to optimize the following problem

$$\min_{\vartheta} \quad \sum_i \mathbf{e}_i^T(\vartheta_i)\mathbf{Q}_i^{-1}\mathbf{e}_i(\vartheta_i). \tag{19}$$

where $\vartheta = [\vartheta_0^T, \vartheta_1^T, \cdots]^T$ is a vector of all the unknown variables, $\mathbf{e}_i(\vartheta_i)$ is a vector function of the unknown variables ϑ_i, and $\mathbf{Q}_i$ is a known diagonal matrix. By using the first-order Taylor approximation around an initial guess $\breve{\vartheta}_i$, we can write

$$\mathbf{e}(\breve{\vartheta}_i + \Delta\vartheta_i) \approx \breve{\mathbf{e}}_i + \mathbf{J}_i\Delta\vartheta_i \tag{20}$$

where $\breve{\mathbf{e}}_i \triangleq \mathbf{e}(\breve{\vartheta}_i)$, and $\mathbf{J}_i$ is the Jacobian of $\mathbf{e}_i(\vartheta_i)$ computed in $\breve{\vartheta}_i$. By substituting (20) in (19), we have

$$\min_{\vartheta} \quad \sum_i \breve{\mathbf{e}}_i^T\breve{\mathbf{e}}_i + 2\breve{\mathbf{e}}_i^T\mathbf{Q}_i^{-1}\mathbf{J}_i\Delta\vartheta_i + \Delta\vartheta_i^T\mathbf{J}_i^T\mathbf{Q}_i^{-1}\mathbf{J}_i\Delta\vartheta_i. \tag{21}$$

We can reformulate (21) in a matrix form as follows

$$\min_{\vartheta} \quad \breve{\mathbf{e}} + 2\mathbf{b}^T\Delta\vartheta + \Delta\vartheta^T\,\mathbf{H}\,\Delta\vartheta, \tag{22}$$

where $\breve{\mathbf{e}} \triangleq [\breve{\mathbf{e}}_0^T, \breve{\mathbf{e}}_1^T, \cdots]^T$, $\mathbf{b} = [\breve{\mathbf{e}}_0^T\mathbf{Q}_0^{-1}\mathbf{J}_0, \breve{\mathbf{e}}_1^T\mathbf{Q}_1^{-1}\mathbf{J}_1, \cdots]^T$, and $\mathbf{H}$ is a block diagonal matrix defined as

$$\mathbf{H} \triangleq \text{diag}\left(\mathbf{J}_0^T\mathbf{Q}_0^{-1}\mathbf{J}_0, \mathbf{J}_1^T\mathbf{Q}_1^{-1}\mathbf{J}_1, \cdots\right). \tag{23}$$

The linear problem (21) can now be solved and the solution is given by

$$\vartheta^* = \breve{\vartheta} + \Delta\vartheta^* = \breve{\vartheta} - \mathbf{H}^{-1}\,\mathbf{b}, \tag{24}$$

where $\breve{\vartheta} = [\breve{\vartheta}_1^T, \breve{\vartheta}_2^T, \cdots]^T$ is a vector of initial guesses. This procedure then repeats until ϑ^* converges to a local minima.

We now convert problem (14) into a proper form for being solved with the above framework. To do so, we first define ϑ as follows

$$\vartheta = \left[\mathbf{x}[1]^T, \cdots, \mathbf{x}[N]^T, \mathbf{u}_1^T, \cdot\cdot^T\cdot, \mathbf{u}_K^T\right]^T. \tag{25}$$

We now reformulate problem (12) as follows

$$\begin{aligned} \mathcal{L} = &\mathbf{e}_{gps}^T\mathbf{Q}_{gps}^{-1}\mathbf{e}_{gps} + \mathbf{e}_{vel}^T\mathbf{Q}_{vel}^{-1}\mathbf{e}_{vel} + \\ &\mathbf{e}_{\tau}^T\mathbf{Q}_{\tau}^{-1}\mathbf{e}_{\tau} + \mathbf{e}_{\phi}^T\mathbf{Q}_{\phi}^{-1}\mathbf{e}_{\phi}, \end{aligned} \tag{26}$$

where

$$\begin{aligned} \mathbf{e}_{gps} &\triangleq \left[\hat{\mathbf{x}}^T[1] - \mathbf{x}^T[1], \cdots, \hat{\mathbf{x}}^T[N] - \mathbf{x}^T[N]\right]^T, \\ \mathbf{e}_{vel} &\triangleq \left[\hat{\mathbf{v}}^T[2] - \mathbf{v}^T[2], \cdots, \hat{\mathbf{v}}^T[N] - \mathbf{v}^T[N]\right]^T, \\ \mathbf{e}_{\tau} &\triangleq \left[\hat{\tau_{0,1}}[1] - \tau_{0,1}[1], \cdots, \hat{\tau_{0,K}}[N] - \tau_{0,K}[N]\right]^T, \\ \mathbf{e}_{\phi} &\triangleq \left[\mathbf{\Phi}_{1,1}^T, \cdots, \mathbf{\Phi}_{M,K}^T\right]^T, \end{aligned} \tag{27}$$

where

$$\mathbf{\Phi}_{m,k} \triangleq \begin{bmatrix} \Delta\tilde{\phi}_{1,2,k} - \Delta\phi_{1,2,k} \\ \cdots \\ \Delta\tilde{\phi}_{\delta\, m-1,\delta\, m,k} - \Delta\phi_{\delta\, m-1,\delta\, m,k} \end{bmatrix}, \quad (28)$$

where $\Delta\tilde{\phi}_{i,j,k} = \tilde{\phi}_{0,k}[i] - \tilde{\phi}_{0,k}[j]$, and

$$\mathbf{Q}_{gps} \triangleq \sigma^2_{gps} * \mathbf{I}_N, \mathbf{Q}_{vel} \triangleq \sigma^2_{vel} * \mathbf{I}_{N-1}, \\ \mathbf{Q}_{\tau} \triangleq \sigma^2_{\tau} * \mathbf{I}_N, \mathbf{Q}_{\phi} \triangleq 2\sigma^2_{\phi} * \mathbf{I}_{\overline{N}}, \quad (29)$$

where $\mathbf{I}_n$ is the identity matrix of size $n \times n$, and $\overline{N} = \frac{K\, M\, \delta\,(\delta+1)}{2}$. To solve (26) using the above framework, the GPS measurements are used for initializing the UAV location, and the users' locations are randomly initialized.

IV. Numerical Results

A service area as shown in Fig. 2 is considered where the users are randomly scattered. The standard deviation of ToA measurements is chosen as $\sigma_\tau = 8$ m, which is roughly equivalent to 40 MHz bandwidth. The standard deviation of phase measurements is selected as $\sigma_\phi = 25$ degrees. The altitude of the UAV is assumed to be fixed and set to 80 m. The GPS and the IMU have a standard deviation of $\sigma_{gps} = 2$ m, $\sigma_{vel} = 0.5$ m/s, respectively. To collect measurements from users, the UAV flies along a rectangular trajectory [1] with a fixed length of L meters, as shown in Fig. 2. We have compared the results of the proposed algorithm with the following benchmarks:

- **Benchmark 1**: The same algorithm as proposed in this paper is used without exploiting the phase measurements. Only ToA measurements are used along with IMU and GPS data.
- **Benchmark 2**: A conventional localization setting is considered where the UAV is replaced with four terrestrial BSs. The BSs are randomly scattered over the service area with perfectly known locations. To localize the users, the same framework is used by only utilizing ToA measurements collected by BSs.

In Fig. 2, the result of the proposed algorithm for the multi-user case ($K = 5$) is shown. To localize the users, the UAV trajectory has a length of $L = 160$ m. The estimation of UAV trajectory is also shown with the dashed line. It can be seen that the algorithm localizes the users and tracks the UAV accurately. The average localization accuracy over all users is 1.1 m, and the average UAV localization accuracy over the whole trajectory is 0.5 m.

In Fig. 3, the cumulative distribution function (CDF) of user localization error for our proposed algorithm over Monte-Carlo simulations is shown. The length of the UAV trajectory is set to $L = 160$ m, while in each iteration of the Monte-Carlo simulation, the location of users changes randomly. The results are also compared with different benchmarks [2]. As we can see, the exploitation of phase measurements can bring a substantial gain to localization accuracy. The result of UAV tracking is shown with the green dashed line. It is worth mentioning that with the proposed algorithm, all the measurements taken from users (TDoA and ToA measurements) can be also exploited to improve the tracking of the UAV trajectory. Therefore, we can verify by utilizing a mobile flying radio node, the user localization accuracy can be dramatically improved when compared to the traditional terrestrial anchor-enabled localization systems considered in Benchmark 2.

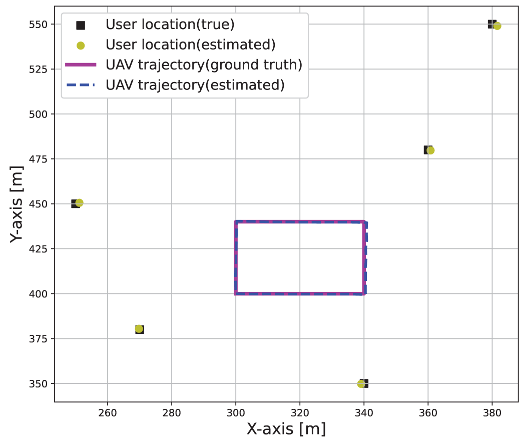

Fig. 2: The top view of the UAV trajectory and the performance of proposed localization algorithm for multi-user case.

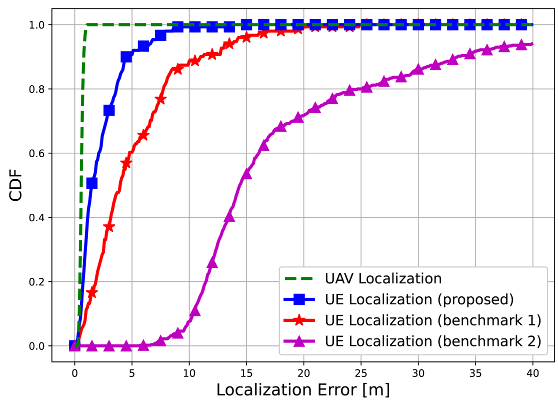

Fig. 3: The CDF of user localization error for different algorithms over Monte-Carlo simulations.

In Fig. 4, the performance of the proposed algorithm, in terms of root-mean-square error (RMSE), over several Monte-Carlo simulations is shown when the length of the UAV trajectory increases. The accuracy of user localization increases when the UAV trajectory length increases. This stems from the fact that by increasing the UAV trajectory length, the UAV can collect more measurements. We can see the out-performance of our algorithm when compared to Benchmark 1.

The result of phase unwrapping is shown in Fig. 5. As we can see the collected phase measurements are periodic and subject to noise, nevertheless, the proposed algorithm can

[1]By no means the rectangular trajectory is restrictive and any type of trajectory can be chosen.

[2]The same set of users used for all benchmarks.

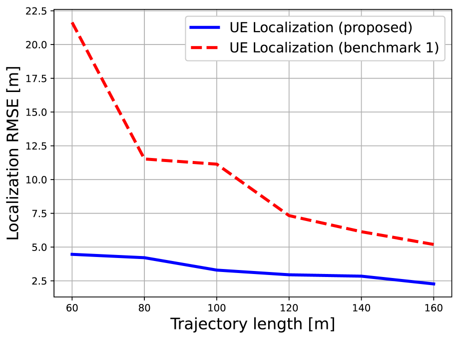

Fig. 4: The comparison of proposed localization accuracy in terms of RMSE versus increasing the UAV trajectory length.

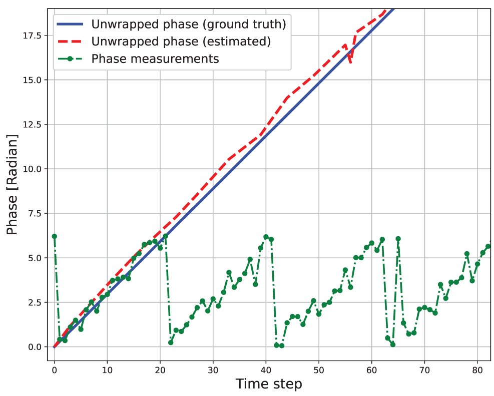

Fig. 5: The result of phase unwrapping using Algorithm 1.

successfully unwrap the phase even with large measurement noise.

V. Conclusions

This paper considered the problem of simultaneous UAV tracking as a mobile anchor (yet with an imperfect position) and localization of ground users. To do so, we exploit the phase information of the received radio signals from users at the UAV along with ToA measurements. The phase measurements are inherently more precise than ToA measurements while bringing ambiguity since it follows a periodic behavior. To solve this issue, we unwrap the phase measurements by tracking the phase while the UAV moves. Then a TDoA-based algorithm is used by treating each UAV location as a virtual anchor/antenna and exploiting the unwrapped phase measurements to precisely localize the users and jointly track the UAV trajectory. We employed a least-squares SLAM framework to fuse TDoA data, which is estimated from phase, with other measurements such as ToA, UAV location estimate available from GPS, and the UAV velocity measured by an IMU onboard. The performance of the developed algorithm is verified over various simulations and in comparison with other benchmarks.

VI. Acknowledgment

This work was funded via the HUAWEI France supported Chair on Future Wireless Networks at EURECOM.

References

[1] R. Zekavat and R. M. Buehrer, *Handbook of position location: Theory, practice and advances*. John Wiley & Sons, 2011, vol. 27.

[2] F. Mogyorósi, P. Revisnyei, A. Pašić, Z. Papp, I. Törös, P. Varga, and A. Pašić, “Positioning in 5G and 6G networks—A Survey,” *Sensors*, vol. 22, no. 13, p. 4757, 2022.

[3] D. Gesbert, O. Esrafilian, J. Chen, R. Gangula, and U. Mitra, “UAV-aided RF Mapping for Sensing and Connectivity in Wireless Networks,” *IEEE Wireless Communications*, 2022.

[4] M. Mozaffari, W. Saad, M. Bennis, Y. Nam, and M. Debbah, “A tutorial on UAVs for wireless networks: Applications, challenges, and open problems,” *IEEE Comm Surveys Tutorials*, vol. 21, no. 3, 2019.

[5] D. Ebrahimi, S. Sharafeddine, P.-H. Ho, and C. Assi, “Autonomous UAV trajectory for localizing ground objects: A reinforcement learning approach,” *IEEE Transactions on Mobile Computing*, vol. 20, no. 4, pp. 1312–1324, 2020.

[6] Y.-J. Chen, D.-K. Chang, and C. Zhang, “Autonomous tracking using a swarm of UAVs: A constrained multi-agent reinforcement learning approach,” *IEEE Transactions on Vehicular Technology*, vol. 69, no. 11, pp. 13 702–13 717, 2020.

[7] X. Jiang, N. Li, Y. Guo, D. Yu, and S. Yang, “Localization of multiple RF sources based on Bayesian compressive sensing using a limited number of UAVs with airborne RSS sensor,” *IEEE Sensors Journal*, vol. 21, no. 5, pp. 7067–7079, 2020.

[8] O. Esrafilian, R. Gangula, and D. Gesbert, “3D Map-based Trajectory Design in UAV-aided Wireless Localization Systems,” *IEEE Internet of Things Journal*, 2020.

[9] T. Liang, T. Zhang, J. Yang, D. Feng, and Q. Zhang, “UAV Aided Positioning Systems for Ground Devices: Fundamental Limits and Algorithms,” *IEEE Internet of Things Journal*, 2022.

[10] A. T. Le, X. Huang, C. Ritz, E. Dutkiewicz, A. Bouzerdoum, D. Franklin *et al.*, “Hybrid TOA/AOA localization with 1d angle estimation in UAV-assisted WSN,” in *14th International Conference on Signal Processing and Communication Systems*, 2020.

[11] P. Sinha, Y. Yapici, and I. Guvenc, “Impact of 3D Antenna Radiation Patterns on TDOA-Based Wireless Localization of UAVs,” in *IEEE INFOCOM 2019 WKSHPS*, 2019.

[12] P. Sinha and I. Guvenc, “Impact of Antenna Pattern on TOA Based 3D UAV Localization Using a Terrestrial Sensor Network,” *IEEE Transactions on Vehicular Technology*, 2022.

[13] M. Lipka, E. Sippel, and M. Vossiek, “An extended Kalman filter for direct, real-time, phase-based high precision indoor localization,” *IEEE Access*, vol. 7, pp. 25 288–25 297, 2019.

[14] M. Zhu, J. Vieira, Y. Kuang, K. Åström, A. F. Molisch, and F. Tufvesson, “Tracking and positioning using phase information from estimated multi-path components,” in *2015 IEEE International Conference on Communication Workshop (ICCW)*. IEEE, 2015, pp. 712–717.

[15] X. Li, K. Batstone, K. Åstrom, M. Oskarsson, C. Gustafson, and F. Tufvesson, “Robust phase-based positioning using massive mimo with limited bandwidth,” in *2017 IEEE 28th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*. IEEE, 2017, pp. 1–7.

[16] J. Cheng, K. Guan, and F. Quitin, “Ground User Localization in UAV-based Wireless Networks: From Channel Characterization to System Design,” 2022.

[17] Y. Lee, J. Yoon, H. Yang, C. Kim, and D. Lee, “Camera-GPS-IMU sensor fusion for autonomous flying,” in *2016 Eighth International Conference on Ubiquitous and Future Networks (ICUFN)*. IEEE, 2016, pp. 85–88.

[18] W. Khawaja, I. Guvenc, D. W. Matolak, U.-C. Fiebig, and N. Schneckenburger, “A survey of air-to-ground propagation channel modeling for unmanned aerial vehicles,” *IEEE Communications Surveys Tutorials*, vol. 21, no. 3, pp. 2361–2391, 2019.

[19] F. Liu, W. Yuan, C. Masouros, and J. Yuan, “Radar-assisted predictive beamforming for vehicular links: Communication served by sensing,” *IEEE Transactions on Wireless Communications*, vol. 19, no. 11, pp. 7704–7719, 2020.

[20] S.-H. Kong and B. Kim, “Error analysis of the OTDOA from the resolved first arrival path in LTE,” *IEEE Transactions on Wireless Communications*, vol. 15, no. 10, pp. 6598–6610, 2016.

[21] G. Grisetti, R. Kümmerle, C. Stachniss, and W. Burgard, “A tutorial on graph-based slam,” *IEEE Intelligent Transportation Systems Magazine*, vol. 2, no. 4, pp. 31–43, 2010.