Multi-Cell MIMO User Rate Balancing with Imperfect CSIT: SESIP vs. RESIP

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***Abstract*—In this work, we consider the max-min user rate balancing problem w.r.t. imperfect Channel Knowledge at the Transmitter (CSIT), namely: expected user rate balancing. This combines an operation of balancing at the user level and sum rate maximization at the level of the user streams. For the imperfect CSIT, we exploit an approximation of the expected rate as the Expected Signal and Interference Power (ESIP) rate, based on an original minorizer for every individual rate term. We study the latter with two expected rate approximations: i) Received signal level ESIP (RESIP), which may seem the most natural, and ii) Stream level ESIP (SESIP), which requires some more work for the stream level power optimization. Simulation results confirm the intuition that SESIP outperforms RESIP when the number of streams is lower than the number of receive antennas.**

***Index Terms*—Inter-cell interference coordination (ICIC), Coordinated Beamforming (CoBF), Multi-User MIMO, Rate Balancing, Imperfect CSIT**

I. Introduction

Massive Multiple-Input Multiple-Output (MIMO) has become a key solution to increase the spectral efficiency of wireless cellular systems [1]. In fact, MIMO technology for wireless communications is now incorporated into wireless broadband standards since 3G. The basic idea behind MIMO technology is that the more antennas the transmitter and the receiver are equipped with, the more the available signal paths, the better the performance in terms of data rate and energy efficiency [2]–[4].

In downlink communications, the base station (BS) with multiple transmit antennas can serve multiple users within the same time and frequency resource block. Therefore, proper resource allocation is needed to fully harvest the gain in spectral and energy efficiency; for example: user scheduling, subcarrier allocation, *power allocation and precoder (receiver) design*. The latter represents the most important aspect to enhance the performance of the system in the physical layer, and can be combined with frequency subcarrier allocation and user scheduling to further boost the performance.

The power allocation optimization can be formulated as a maximization of some utility in terms of data rate. Depending on the chosen utility function, we can achieve different points on the Pareto optimal boundary. In other words, we cannot increase the rate of any of the active users without lowering the rate of the other users [5]. The two most commonly used utility functions are $i)$ weighted sum rate [6]–[13] and $ii)$ weighted max-min fairness, also referred to as the balancing problem. The latter ensures fairness by providing the same quality-of-service for all users according to their priorities and make this value as large as possible [14]. The weighted max-min fairness problem can be expressed for different objectives such as Signal-to-Noise-plus-Interference Ratio (SINR) [15]–[18], the Mean Square Error (MSE) [19]–[21] and user rate [22]–[26]. Actually, in the single stream per usercase (e.g. in MISO systems), balancing w.r.t. SINR, MSE or user rate is equivalent (in the unweighted case).

In this work, we focus on user rate balancing with imperfect Channel State Information at the Transmitter (CSIT), which corresponds to maximizing the minimum (weighted) per user expected (ergodic) rate in the network. We consider a multi-cell multi-user MIMO system with imperfect CSIT, which combines both channel estimates and channel (error) covariance information. In [26] we have introduced an approach based on an approximation of the expected rate as the Expected Signal and Interference Power (ESIP) rate. Actually, we now call the approach of [26] Stream level ESIP (SESIP), to distinguish from a second variation on the ESIP theme to be introduced here, Received signal level ESIP (RESIP). Actually, the use of the expectation operator makes the optimization a daunting task. In the MISO case, SESIP and RESIP coincide. In the multiple receive antenna case, there is not a unique or clear way of how to approximate the expected rate. In [26] we had introduced one approach, SESIP. But more recently we realized that what we had done in earlier work, such as e.g. [27] and others, was different and was somewhat ad hoc in the MIMO (as opposed to MISO) case. E.g. in some of this previous work, the proposed beamformers actually optimize a different criterion from the one put forward. So the goal of this paper is to draw attention to this non-uniqueness of possible approaches and to clearly spell out two different approaches for the formulation of the cost function (an expected rate approximation) and for the corresponding optimizing beamformers and power distribution. The RESIP approach may seem more natural actually, but leads to complications in the power optimization. In [28], a refined analysis of the gap between expected Weighted Sum Rate (WSR) and RESIP-WSR appears, where the actual gap disappears in case of only covariance CSIT. Here we provide an initial investigation into which of the two approaches is more desirable, in terms of intuitive appeal, some heuristic considerations on approximation quality (SESIP will turn out to be better) and a few simulation results.

II. System Model

We consider a MIMO system with C cells. Each cell c has one base station (BS) of M_c transmit antennas serving K_c users, with total number of users $\sum_c K_c = K$. We refer to the BS of user $k \in \{1, \ldots, K\}$ by b_k. Each user has N_k

antennas. The channel between the kth user and the BS in cell c is denoted by $\boldsymbol{H}_{k,c} \in \mathbb{C}^{N_k \times M_c}$. We consider zero-mean white Gaussian noise $\boldsymbol{n}_k \in \mathbb{C}^{N_k \times 1}$ with distribution $\mathcal{CN}(0, \sigma_n^2 \boldsymbol{I})$ at the kth user.

We assume independent unity-power transmit symbols $\mathbf{s}_c = [\mathbf{s}_{K_{1:c-1}+1}^{\mathrm{T}} \ldots \mathbf{s}_{K_{1:c}}^{\mathrm{T}}]^{\mathrm{T}}$, i.e., $\mathbb{E}[\mathbf{s}_c \mathbf{s}_c^{\mathrm{H}}] = \boldsymbol{I}$, where $\mathbf{s}_k \in \mathbb{C}^{d_k \times 1}$ is the data vector to be transmitted to the kth user, with d_k being the number of streams allowed by user k and $K_{1:c} = \sum_{i=1}^{c} K_i$. The latter is transmitted using the transmit filtering matrix $\boldsymbol{\mathcal{G}}_c = [\boldsymbol{\mathcal{G}}_{K_{1:c-1}+1} \ldots \boldsymbol{\mathcal{G}}_{K_{1:c}}] \in \mathbb{C}^{M_c \times N_c}$, with $\boldsymbol{\mathcal{G}}_k = p_k^{1/2} \boldsymbol{G}_k$, $\boldsymbol{G}_k$ being the (unit Frobenius norm) beamforming matrix, p_k is non-negative downlink power allocation of user k and $N_c = \sum_{k:b_k=c} d_k$ is the total number of streams in cell c. Each cell is constrained with $P_{\max,c}$, i.e., the total transmit power in c is limitted such that $\sum_{k:b_k=c} p_k \le P_{\max,c}$. The received signal at user k in cell b_k is

$$\boldsymbol{y}_k = \underbrace{\boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_k \boldsymbol{s}_k}_{\text{signal}} + \underbrace{\sum_{\substack{i \neq k \\ b_i = b_k}} \boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_i \boldsymbol{s}_i}_{\text{intracel interf.}} + \underbrace{\sum_{j \neq b_k} \sum_{i:b_i=j} \boldsymbol{H}_{k,j} \boldsymbol{\mathcal{G}}_i \boldsymbol{s}_i}_{\text{intercell interf.}} + \boldsymbol{n}_k.$$

For details about the (prior) separable channel correlation model and it's impact on the posterior channel model, please see [26]. It leads to e.g.

$$\mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}_d} \boldsymbol{H}^H \boldsymbol{Q} \boldsymbol{H} = \widehat{\boldsymbol{H}}^H \boldsymbol{Q} \widehat{\boldsymbol{H}} + \mathrm{tr}\{\boldsymbol{C}_r \boldsymbol{Q}\} \boldsymbol{C}_p \tag{1}$$

$$\text{and} \quad \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}_d} \boldsymbol{H} \boldsymbol{P} \boldsymbol{H}^H = \widehat{\boldsymbol{H}} \boldsymbol{P} \widehat{\boldsymbol{H}}^H + \mathrm{tr}\{\boldsymbol{C}_p \boldsymbol{P}\} \boldsymbol{C}_r \ . \tag{2}$$

Note that $\rho_P = \frac{\mathrm{tr}\{\widehat{\boldsymbol{H}}^H \widehat{\boldsymbol{H}}\}}{\mathrm{tr}\{\boldsymbol{C}_r\} \mathrm{tr}\{\boldsymbol{C}_p\}}$ is a form of Ricean factor that represents posterior channel estimation quality. It depends on the deterministic channel estimation quality $\rho_D = 1/\sigma^2_{\widetilde{\boldsymbol{H}}}$. Below we consider $\boldsymbol{C}_r = \boldsymbol{I}$, and the only covariance $\boldsymbol{C}$ we shall need is $\boldsymbol{C}_p$, hence we drop the subscript p. Perfect CSIT algorithms can be obtained by setting $\sigma^2_{\widetilde{\boldsymbol{H}}} = 0$, leading to $\widehat{\boldsymbol{H}} = \boldsymbol{H}$ and $\boldsymbol{C}_p = 0$.

III. Expected Rate Balancing Problem

In this work, we aim to solve the weighted user-rate max-min optimization problem under per cell total transmit power constraint, i.e., the user rate balancing problem expressed as follows

$$\begin{aligned} & \max_{\boldsymbol{G},p} \min_{k} \ r_k / r_k^{\circ} \\ \text{s.t.} \ & \sum_{k:b_k=c} p_k \le P_{\max,c}, 1 \le c \le C \end{aligned} \tag{3}$$

where r_k is the kth user-rate

$$r_k = \mathrm{lndet}\Big(\boldsymbol{I} + \boldsymbol{R}_{\overline{k}}^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_k \boldsymbol{\mathcal{G}}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}}\Big) = \ln\det\Big(\boldsymbol{R}_{\overline{k}}^{-1} \boldsymbol{R}_k\Big) , \tag{4}$$

$$\boldsymbol{R}_{\overline{k}} = \sigma_n^2 \boldsymbol{I} + \sum_{l \neq k} \boldsymbol{H}_{k,b_l} \boldsymbol{\mathcal{G}}_l \boldsymbol{\mathcal{G}}_l^{\mathrm{H}} \boldsymbol{H}_{k,b_l}^{\mathrm{H}} \ , \tag{5}$$

$$\boldsymbol{R}_k = \boldsymbol{R}_{\overline{k}} + \boldsymbol{H}_{k,b_k} \boldsymbol{\mathcal{G}}_k \boldsymbol{\mathcal{G}}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}} \ , \tag{6}$$

$\boldsymbol{R}_{\overline{k}}$ and $\boldsymbol{R}_k$ are the interference plus noise and total received signal covariances, and r_k° is the rate priority (weight) for user k. Actually, in the presence of imperfect CSIT, we shall be interested in balancing the expected (or ergodic) rates

$$\begin{aligned} & \max_{\boldsymbol{G},p} \min_{k} \ \overline{r}_k / r_k^{\circ} \\ \text{s.t.} \ & \sum_{k:b_k=c} p_k \le P_{\max,c}, \ c = 1, \ldots, C \end{aligned} \tag{7}$$

where $\overline{r}_k = \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} r_k$. The final ergodic rate achieved would be $\mathrm{E}_{\widehat{\boldsymbol{H}}} \max_{\boldsymbol{G},p} \overline{r}_k$. We shall need the averaged quantities

$$\overline{\boldsymbol{S}}_{k,i} = \widehat{\boldsymbol{H}}_{k,b_i} \boldsymbol{G}_i \boldsymbol{G}_i^{\mathrm{H}} \widehat{\boldsymbol{H}}_{k,b_i}^{\mathrm{H}} + \mathrm{tr}\{\boldsymbol{G}_i^{\mathrm{H}} \boldsymbol{C}_{k,b_i} \boldsymbol{G}_i\} \boldsymbol{I}, \ \overline{\boldsymbol{S}}_k = \overline{\boldsymbol{S}}_{k,k} \tag{8}$$

$$\overline{\boldsymbol{R}}_{\overline{k}} = \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \boldsymbol{R}_{\overline{k}} = \sigma_n^2 \boldsymbol{I} + \sum_{i \neq k} p_i \overline{\boldsymbol{S}}_{k,i} \ , \ \ \overline{\boldsymbol{R}}_k = \overline{\boldsymbol{R}}_{\overline{k}} + p_k \overline{\boldsymbol{S}}_k \tag{9}$$

However, the problem presented in (7) is complex and can not be solved directly.

To solve the problem, we follow an approximation of the expected rate expression. The latter will use a rate minorizer for every r_k, similar but not identical to what is used as in the DC programming approach which for the optimization of $\boldsymbol{G}_k$ keeps r_k and linearizes the $r_{\overline{k}}$. It is worth noting that this approach does not require the introduction of Rxs. We consider again the (expected) rate balancing problem (7) where $\overline{r}_k = \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} r_k$ is now approximated by the Expected Signal and Interference Power (ESIP) rate

$$\begin{aligned} \overline{r}_k = \ & \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \mathrm{lndet}\Big(\boldsymbol{I} + p_k \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}} \boldsymbol{R}_{\overline{k}}^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{G}_k\Big) \\ \approx \ & \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \mathrm{lndet}\Big(\boldsymbol{I} + p_k \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}} (\mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \boldsymbol{R}_{\overline{k}})^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{G}_k\Big) \\ = \ & \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \mathrm{lndet}\Big(\boldsymbol{I} + p_k \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}} \overline{\boldsymbol{R}}_{\overline{k}}^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{G}_k\Big) \\ \le \ & \mathrm{lndet}\Big(\boldsymbol{I} + p_k \ \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}} \overline{\boldsymbol{R}}_{\overline{k}}^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{G}_k\Big) && (10) \\ = \ & \overline{r}_k^{s,\mathrm{S}} = f_k^{s,\mathrm{S}}(\frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}}) = \mathrm{lndet}\Big(\boldsymbol{I} + \boldsymbol{G}_k^{\mathrm{H}} \overline{\boldsymbol{B}}_k^{\mathrm{S}}(\frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}}) \ \boldsymbol{G}_k\Big) , && (11) \end{aligned}$$

$$\overline{\boldsymbol{B}}_k^{\mathrm{S}}(\overline{\boldsymbol{T}}_k) = \widehat{\boldsymbol{H}}_{k,b_k}^{\mathrm{H}} \overline{\boldsymbol{T}}_k^{-1} \widehat{\boldsymbol{H}}_{k,b_k} + \mathrm{tr}\{\overline{\boldsymbol{T}}_k^{-1}\} \boldsymbol{C}_{k,b_k} \tag{12}$$

where the $\overline{r}_k$ approximation $\overline{r}_k^{s,S}$ in (11) in general is neither an upper nor a lower bound, but in the Massive MIMO limit becomes a tight upper bound. Let us now consider an alternative development

$$\begin{aligned} \overline{r}_k \approx \ & \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \mathrm{lndet}\Big(\boldsymbol{I} + p_k \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}} \overline{\boldsymbol{R}}_{\overline{k}}^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{G}_k\Big) \\ = \ & \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \mathrm{lndet}\Big(\boldsymbol{I} + p_k \overline{\boldsymbol{R}}_{\overline{k}}^{-1} \boldsymbol{H}_{k,b_k} \boldsymbol{G}_k \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}}\Big) \\ \le \ & \mathrm{lndet}\Big(\boldsymbol{I} + p_k \overline{\boldsymbol{R}}_{\overline{k}}^{-1} \ \mathrm{E}_{\boldsymbol{H}|\widehat{\boldsymbol{H}}} \boldsymbol{H}_{k,b_k} \boldsymbol{G}_k \boldsymbol{G}_k^{\mathrm{H}} \boldsymbol{H}_{k,b_k}^{\mathrm{H}}\Big) && (13) \\ = \ & \overline{r}_k^{s,\mathrm{R}} = f_k^{s,\mathrm{R}}(\frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}}) = \mathrm{lndet}\Big(\boldsymbol{I} + p_k \overline{\boldsymbol{R}}_{\overline{k}}^{-1} \overline{\boldsymbol{S}}_k\Big) . && (14) \end{aligned}$$

This expected rate differs from the expected rate in (11), because in (13) the average is taken over the expected received signal covariance matrix of user k, $\overline{\boldsymbol{S}}_k$ of dimension $(N_k \times N_k)$, whence Received signal level ESIP or RESIP. Whereas in (11), the average is taken over the expected stream level signal covariance matrix of dimension $(d_k \times d_k)$, whence Stream level ESIP or SESIP.

Lemma 1. *The approximate $\overline{r}_k$'s, $\overline{r}_k^{s,\cdot}$, i.e. $\overline{r}_k^{s,S}$, $\overline{r}_k^{s,R}$, can be obtained as $f_k^{s,\cdot}(\frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}}) = \min_{\overline{\boldsymbol{T}}_k} \underline{f}_k^{s,\cdot}(\overline{\boldsymbol{T}}_k, \frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}})$ with*

$$\underline{f}_k^{s,\mathrm{S}} = \mathrm{lndet}\Big(\boldsymbol{I} + \boldsymbol{G}_k^{\mathrm{H}} \overline{\boldsymbol{B}}_k^{\mathrm{S}}(\overline{\boldsymbol{T}}_k) \boldsymbol{G}_k\Big) + \mathrm{tr}\{\breve{\boldsymbol{W}}_k^{\mathrm{S}}(\overline{\boldsymbol{T}}_k - \frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}})\} \tag{15}$$

$$\text{and} \ \underline{f}_k^{s,\mathrm{R}} = \mathrm{lndet}\Big(\boldsymbol{I} + \overline{\boldsymbol{T}}_k^{-1} \overline{\boldsymbol{S}}_k\Big) + \mathrm{tr}\{\breve{\boldsymbol{W}}_k^{\mathrm{R}}(\overline{\boldsymbol{T}}_k - \frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}})\} \tag{16}$$

$$\text{where} \ \breve{\boldsymbol{W}}_k^{\mathrm{S}} = \overline{\boldsymbol{T}}_k^{-1} (\widehat{\boldsymbol{H}}_{k,b_k} \boldsymbol{X}_k \widehat{\boldsymbol{H}}_{k,b_k}^{\mathrm{H}} + \mathrm{tr}\{\boldsymbol{X}_k \boldsymbol{C}_{k,b_k}\} \boldsymbol{I}) \overline{\boldsymbol{T}}_k^{-1} \tag{17}$$

$$\text{with} \ \boldsymbol{X}_k = \boldsymbol{G}_k \Big(\boldsymbol{I} + \boldsymbol{G}_k^{\mathrm{H}} \overline{\boldsymbol{B}}_k^{\mathrm{S}}(\overline{\boldsymbol{T}}_k) \boldsymbol{G}_k\Big)^{-1} \boldsymbol{G}_k^{\mathrm{H}} \tag{18}$$

$$\text{and} \ \breve{\boldsymbol{W}}_k^{\mathrm{R}} = \overline{\boldsymbol{T}}_k^{-1} - \big(\overline{\boldsymbol{T}}_k + \overline{\boldsymbol{S}}_k\big)^{-1} \tag{19}$$

The optimizer is $\overline{\boldsymbol{T}}_k = \frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}}$. Also, $\underline{f}_k^s$ is a minorizer for $f_k^s(\frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}})$ as a function of $\frac{1}{p_k} \overline{\boldsymbol{R}}_{\overline{k}}$.

Indeed, since $f^s_k(.)$ is a convex function, it gets minorized by its tangent at any point:

$$f^s_k(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}) \geq \underline{f}^s_k = f^s_k(\overline{\boldsymbol{T}}_k)+\mathrm{tr}\{\frac{\partial f^s_k(\overline{\boldsymbol{T}}_k)}{\partial \overline{\boldsymbol{T}}_k}(\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}-\overline{\boldsymbol{T}}_k)\} \quad (20)$$

and $\breve{\boldsymbol{W}}_k = -\frac{\partial f^s_k(\overline{\boldsymbol{T}}_k)}{\partial \overline{\boldsymbol{T}}_k}$. Note that for the Perron-Frobenius theory, we need a function that is linear in $\frac{\boldsymbol{p}_{\overline{k}}}{p_k}$, hence we need to work with $\frac{1}{p_k}\overline{\boldsymbol{R}}_{\overline{k}}$ instead of $\overline{\boldsymbol{R}}_{\overline{k}}$.

IV. Proposed Solution

The user expected rate balancing problem (7) can be reformulated as

$$\begin{aligned}&\min_{t,\boldsymbol{G},\boldsymbol{p}} -t\\ &\text{s.t. } t\,r^\circ_k - \underline{f}^s_k \leq 0,\ \boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c} \leq 0\,, \forall k, c. \end{aligned} \quad (21)$$

Introducing Lagrange multipliers to augment the cost function with the constraints leads to the Lagrangian

$$\begin{aligned}&\max_{\lambda',\mu}\min_{t,\boldsymbol{G},\boldsymbol{p}} \mathcal{L}\\ &\mathcal{L} = -t + \sum_k \lambda'_k(t\,r^\circ_k - \underline{f}^s_k) + \sum_c \mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \end{aligned} \quad (22)$$

where the Lagrangian for stream level ESIP is

$$\begin{aligned}\mathcal{L}^{\mathrm{S}} =& -t - \sum_k \breve{\lambda}'_k\big(\,\mathrm{lndet}\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}^{\mathrm{S}}_k\boldsymbol{G}_k\Big) - \frac{1}{p_k}\mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{S}}_k\overline{\boldsymbol{R}}_{\overline{k}}\}\\ &+\mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{S}}_k\overline{\boldsymbol{T}}_k\} - t\,r^o_k\big) + \sum_c \mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \qquad (23)\\ =& -t + \sum_k \breve{\lambda}_k(\frac{1}{p_k\,\breve{\xi}^{\mathrm{S}}_k}\mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{S}}_k\overline{\boldsymbol{R}}_{\overline{k}}\}-1) + \sum_c \mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \qquad (24)\end{aligned}$$

$$\begin{aligned}\text{with } \breve{\xi}^{\mathrm{S}}_k &= \mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{S}}_k\overline{\boldsymbol{T}}_k\} + \mathrm{lndet}\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}^{\mathrm{S}}_k\boldsymbol{G}_k\Big) - t\,r^o_k,\\ \breve{\lambda}'_k &= \breve{\lambda}_k/\breve{\xi}^{\mathrm{S}}_k, \overline{\boldsymbol{B}}^{\mathrm{S}}_k = \overline{\boldsymbol{B}}^{\mathrm{S}}_k(\overline{\boldsymbol{T}}_k),\end{aligned} \quad (25)$$

and the Lagrangian for received signal level is

$$\begin{aligned}\mathcal{L}^{\mathrm{R}} =& -t - \sum_k \breve{\lambda}'_k\big(\,\mathrm{lndet}\Big(\boldsymbol{I}+\overline{\boldsymbol{T}}_k^{-1}\,\overline{\boldsymbol{S}}_k\Big) - \frac{1}{p_k}\mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{R}}_k\overline{\boldsymbol{R}}_{\overline{k}}\}\\ &+\mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{R}}_k\overline{\boldsymbol{T}}_k\} - t\,r^o_k\big) + \sum_c \mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \qquad (26)\\ =& -t + \sum_k \breve{\lambda}_k(\frac{1}{p_k\,\breve{\xi}^{\mathrm{R}}_k}\mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{R}}_k\overline{\boldsymbol{R}}_{\overline{k}}\}-1) + \sum_c \mu_c(\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} - P_{\max,c}) \qquad (27)\end{aligned}$$

$$\text{with } \breve{\xi}^{\mathrm{R}}_k = \mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{R}}_k\overline{\boldsymbol{T}}_k\} + \mathrm{lndet}\Big(\boldsymbol{I}+\overline{\boldsymbol{T}}_k^{-1}\,\overline{\boldsymbol{S}}_k\Big) - t\,r^o_k, \quad (28)$$

The balancing of the rates in (7) is equivalent to balancing the weighted interference plus noise powers in (24) or (27),

$$\max_{\breve{\lambda}}\min_{\boldsymbol{G},\boldsymbol{p}} \sum_k \frac{\breve{\lambda}_k}{\breve{\xi}_k}\frac{\mathrm{tr}(\breve{\boldsymbol{W}}_k\overline{\boldsymbol{R}}_{\overline{k}})}{p_k}\text{s.t.} \sum_{c=1}^{C}\theta_c\boldsymbol{c}_c^{\mathrm{T}}\boldsymbol{p} \leq \sum_{c=1}^{C}\theta_c P_{\max,c} \quad (29)$$

where $\boldsymbol{c}_c$ is a column vector with $\boldsymbol{c}_c(j) = 1$ for $K_{1:c-1}+1 \leq j \leq K_{1:c}$, and 0 elsewhere. This problem formulation is a relaxation of (7), and $\boldsymbol{\theta} = [\theta_1\cdots\theta_C]^{\mathrm{T}}$can be interpreted as the weights on the individual power constraints in the relaxed problem. The power constraint in (29) can be interpreted as a single weighted power constraint

$$(\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}})\,\boldsymbol{p} \leq \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max} \quad (30)$$

with $\boldsymbol{C}_C = [\boldsymbol{c}_1\cdots\boldsymbol{c}_C] \in \mathbb{R}_+^{K_{1:C}\times C}$ and $\boldsymbol{p}_{\max} = [P_{\max,1}\cdots P_{\max,C}]^{\mathrm{T}}$, from which we get $\mu_c = \mu\theta_c$.

Now, define the following matrix (reparameterize $\boldsymbol{p} = \frac{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}}\boldsymbol{p}'}\boldsymbol{p}'$ where now $\boldsymbol{p}'$ is unconstrained, and rewriting $\boldsymbol{p}'$ as $\boldsymbol{p}$)

$$\boldsymbol{\Lambda} = \breve{\boldsymbol{\xi}}^{-1}\breve{\boldsymbol{\Psi}} + \frac{1}{\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{p}_{\max}}\breve{\boldsymbol{\xi}}^{-1}\breve{\boldsymbol{\sigma}}\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{C}_C^{\mathrm{T}} \text{ with} \quad (31)$$

$$[\breve{\boldsymbol{\Psi}}]_{ij} = \begin{cases}\mathrm{tr}\{\breve{\boldsymbol{W}}_i(\widehat{\boldsymbol{H}}_{i,b_j}\boldsymbol{G}_j\boldsymbol{G}_j^{\mathrm{H}}\widehat{\boldsymbol{H}}^{\mathrm{H}}_{i,b_j}+\mathrm{tr}\{\boldsymbol{G}_j^{\mathrm{H}}\boldsymbol{C}_{i,b_j}\boldsymbol{G}_j\}\boldsymbol{I})\}, & i \neq j\\ 0, & i = j\end{cases} \quad (32)$$

$$\boldsymbol{\sigma}_i = \sigma_n^2\,\mathrm{tr}\{\breve{\boldsymbol{W}}_i\}, \breve{\boldsymbol{\xi}} = \mathrm{diag}(\breve{\xi}_1,\ldots,\breve{\xi}_K)\,, \quad (33)$$

we can reformulate (29) as

$$\Delta = \max_{\lambda:\sum_k\lambda_k=1}\min_{\boldsymbol{p}}\sum_k \lambda_k\frac{[\boldsymbol{\Lambda}\,\boldsymbol{p}]_k}{p_k} \quad (34)$$

which is the Donsker–Varadhan–Friedland formula [29, Chapter 8] for the Perron Frobenius eigenvalue of Λ. A related formula is the Rayleigh quotient

$$\Delta = \max_{\boldsymbol{q}}\min_{\boldsymbol{p}}\frac{\boldsymbol{q}^T\boldsymbol{\Lambda}\,\boldsymbol{p}}{\boldsymbol{q}^T\boldsymbol{p}} \quad (35)$$

where $\boldsymbol{p}, \boldsymbol{q}$ are the right and left Perron Frobenius eigenvectors. Comparing (35) to (34), then apart from normalization factors, we get $\lambda_k/p_k = q_k$ or hence $\lambda_k = p_k q_k$.

The Tx BF and stream power optimization will be based on $\sum_i \frac{\breve{\lambda}_i}{\breve{\xi}_i}\underline{f}^s_i$, for both SESIP and RESIP approximates, which becomes (apart from noise terms) as described in the following.

A. SESIP Tx BF

We have from (23)

$$\sum_k \frac{\breve{\lambda}_k}{\breve{\xi}_k}\underline{f}^{s,\mathrm{S}}_k = \sum_k \frac{\breve{\lambda}_k}{\breve{\xi}^{\mathrm{S}}_k}\,\mathrm{lndet}\Big(\boldsymbol{I}+\boldsymbol{G}_k^{\mathrm{H}}\overline{\boldsymbol{B}}^{\mathrm{S}}_k\boldsymbol{G}_k\Big) - \sum_k\mathrm{tr}\{p_k\boldsymbol{G}_k^H\overline{\boldsymbol{A}}^{\mathrm{S}}_k\boldsymbol{G}_k\} \quad (36)$$

$$\text{with } \overline{\boldsymbol{A}}^{\mathrm{S}}_k = \sum_{i\neq k}\frac{\breve{\lambda}_i}{p_i\,\breve{\xi}^{\mathrm{S}}_i}\Big(\widehat{\boldsymbol{H}}^{\mathrm{H}}_{i,b_k}\breve{\boldsymbol{W}}^{\mathrm{S}}_i\widehat{\boldsymbol{H}}_{i,b_k} + \mathrm{tr}\{\breve{\boldsymbol{W}}^{\mathrm{S}}_i\}\boldsymbol{C}_{i,b_k}\Big)\,. \quad (37)$$

For the optimal Tx BF $\boldsymbol{G}_k$, the gradient of $\mathcal{L}^{\mathrm{S}}$ yields

$$\frac{\partial\mathcal{L}^{\mathrm{S}}}{\partial\boldsymbol{G}^*} = 0 = \frac{\breve{\lambda}_k}{\breve{\xi}^{\mathrm{S}}_k}\overline{\boldsymbol{B}}^{\mathrm{S}}_k\boldsymbol{G}_k\,(\boldsymbol{I}+\boldsymbol{G}_k^H\overline{\boldsymbol{B}}^{\mathrm{S}}_k\boldsymbol{G}_k)^{-1} - p_k(\overline{\boldsymbol{A}}^{\mathrm{S}}_k + \mu_{b_k}\boldsymbol{I})\,\boldsymbol{G}_k\,. \quad (38)$$

The solution is the d_k maximal generalized eigen vectors

$$\boldsymbol{G}'_k = V_{1:d_k}(\overline{\boldsymbol{B}}^{\mathrm{S}}_k, \overline{\boldsymbol{A}}^{\mathrm{S}}_k + \mu_{b_k}\boldsymbol{I}), \boldsymbol{G}_k = \boldsymbol{G}'_k\overline{\boldsymbol{P}}^{1/2}_k, \boldsymbol{\mathcal{G}}_k = \boldsymbol{G}_k\sqrt{p_k} \quad (39)$$

where the $\overline{\boldsymbol{P}}_k = \mathrm{diag}(p_{k,1},\ldots,p_{k,d_k})$, $\mathrm{tr}\{\overline{\boldsymbol{P}}_k\} = 1$, are the relative stream powers. Indeed, (38) represents the definition of generalized eigen vectors. Consider

$$\Sigma^{(1)}_k = \boldsymbol{G}'^{\mathrm{H}}_k\overline{\boldsymbol{B}}^{\mathrm{S}}_k\boldsymbol{G}'_k,\ \Sigma^{(2)}_k = \boldsymbol{G}'^{\mathrm{H}}_k\overline{\boldsymbol{A}}^{\mathrm{S}}_k\boldsymbol{G}'_k \quad (40)$$

then the generalized eigen vectors $\boldsymbol{G}'_k$ of $\overline{\boldsymbol{B}}_k, \overline{\boldsymbol{A}}^{\mathrm{S}}_k + \mu_{b_k}\boldsymbol{I}$ lead to diagonal matrices $\Sigma^{(1)}_k$, $\Sigma^{(2)}_k + \mu_{b_k}\boldsymbol{G}'^{\mathrm{H}}_k\boldsymbol{G}'_k$. Note that the normalized $\boldsymbol{G}'_k$ are not orthogonal. Then (38) represents the generalized eigen vector condition with associated generalized eigen values in the diagonal matrix $\frac{p_k\,\breve{\xi}^{\mathrm{S}}_k}{\breve{\lambda}_k}(\boldsymbol{I}+\Sigma^{(1)}_k\overline{\boldsymbol{P}}_k)$. Also, plugging in generalized eigen vectors into (36) reveals that one should choose the eigen vectors associated to d_k maximal eigen values to maximize (36). Now, premultiplying both sides of (38) by $p_k\boldsymbol{G}^{\mathrm{H}}_k$, summing over all users $k : b_k = c$, taking trace and identifying the last term with $\sum_{k:b_k=c} p_k \mathrm{tr}\{\boldsymbol{G}_k^{\mathrm{H}}\boldsymbol{G}_k\} = P_{max,c}$ allows to solve for

$$\mu_c = \frac{1}{P_{max,c}} \left\lfloor \sum_{k:b_k=c} \mathrm{tr}\{\frac{\breve{\lambda}_k}{\breve{\xi}_k^{\mathrm{S}}} \Sigma_k^{(1)} \overline{\boldsymbol{P}}_k (\boldsymbol{I} + \Sigma_k^{(1)} \overline{\boldsymbol{P}}_k)^{-1} - p_k \Sigma_k^{(2)} \overline{\boldsymbol{P}}_k\} \right\rfloor_+ . \tag{41}$$

The $\overline{\boldsymbol{P}}_k$ are themselves found from an interference leakage aware water filling (ILAWF) operation. Substituting $\boldsymbol{G}_k^{'}$ into term k of (36), dividing by p_k, and accounting for the constraint $\mathrm{tr}\{\overline{\boldsymbol{P}}_k\} = 1$ by Lagrange multiplier ν_k, we get the Lagrangian

$$\frac{\breve{\lambda}_k}{p_k\, \breve{\xi}_k^{\mathrm{S}}} \ln\det\left(\boldsymbol{I} + \Sigma_k^{(1)} \overline{\boldsymbol{P}}_k\right) - \mathrm{tr}\{(\Sigma_k^{(2)} + \nu_k \boldsymbol{I}) \overline{\boldsymbol{P}}_k\}$$

Maximizing w.r.t. $\overline{\boldsymbol{P}}_k$ leads to the ILAWF

$$\overline{\boldsymbol{P}}_k = \left\lfloor \frac{\breve{\lambda}_k}{p_k\, \breve{\xi}_k^{\mathrm{S}}} \left(\mathrm{diag}(\Sigma_k^{(2)}) + \nu_k \boldsymbol{I}\right)^{-1} - \Sigma_k^{-(1)} \right\rfloor_+ \tag{42}$$

where the Lagrange multiplier ν_k is adjusted (e.g. by bisection) to satisfy $\mathrm{tr}\{\overline{\boldsymbol{P}}_k\} = 1$. Elements in $\overline{\boldsymbol{P}}_k$ corresponding to zeros in $\Sigma_k^{(1)}$ should also be zero.

B. RESIP Tx BF

We have from (26)

$$\sum_k \frac{\breve{\lambda}_k}{\breve{\xi}_k^{\mathrm{R}}} \underline{f}_k^{s,\mathrm{R}} = \sum_k \frac{\breve{\lambda}_k}{\breve{\xi}_k^{\mathrm{R}}} \mathrm{lndet}\left(\boldsymbol{I} + \overline{\boldsymbol{T}}_k^{-1} \overline{\boldsymbol{S}}_k\right) - \sum_k \mathrm{tr}\{p_k \boldsymbol{G}_k^H \overline{\boldsymbol{A}}_k^{\mathrm{R}} \boldsymbol{G}_k\}, \tag{43}$$

$$\text{with } \overline{\boldsymbol{A}}_k^{\mathrm{R}} = \sum_{i \neq k} \frac{\breve{\lambda}_i}{p_i\, \breve{\xi}_i^{\mathrm{R}}} \left(\widehat{\boldsymbol{H}}_{i,b_k}^{\mathrm{H}} \breve{\boldsymbol{W}}_i^{\mathrm{R}} \widehat{\boldsymbol{H}}_{i,b_k} + \mathrm{tr}\{\breve{\boldsymbol{W}}_i^{\mathrm{R}}\} \boldsymbol{C}_{i,b_k}\right). \tag{44}$$

For the optimal Tx BF $\boldsymbol{G}_k$, the gradient of $\mathcal{L}^{\mathrm{R}}$ yields

$$\frac{\partial \mathcal{L}^{\mathrm{R}}}{\partial \boldsymbol{G}^*} = 0 \Leftrightarrow \frac{\breve{\lambda}_k}{\breve{\xi}_k^{\mathrm{R}}} \overline{\boldsymbol{B}}_k^{\mathrm{R}} \boldsymbol{G}_k - p_k (\overline{\boldsymbol{A}}_k^{\mathrm{R}} + \mu_{b_k} \boldsymbol{I}) \boldsymbol{G}_k = 0, \tag{45}$$

$$\begin{aligned} \text{with } \overline{\boldsymbol{B}}_k^{\mathrm{R}} &= \widehat{\boldsymbol{H}}_{k,b_k}^{\mathrm{H}} (\boldsymbol{I} + \overline{\boldsymbol{T}}_k^{-1} \overline{\boldsymbol{S}}_k)^{-1} \overline{\boldsymbol{T}}_k^{-1} \widehat{\boldsymbol{H}}_{k,b_k} \\ &+ \mathrm{tr}\{(\boldsymbol{I} + \overline{\boldsymbol{T}}_k^{-1} \overline{\boldsymbol{S}}_k)^{-1} \overline{\boldsymbol{T}}_k^{-1}\} \boldsymbol{C}_{k,b_k}. \end{aligned} \tag{46}$$

The solution is the d_k maximal generalized eigen vectors

$$\boldsymbol{G}_k^{'} = V_{1:d_k}(\overline{\boldsymbol{B}}_k^{\mathrm{R}}, \overline{\boldsymbol{A}}_k^{\mathrm{R}} + \mu_{b_k} \boldsymbol{I}), \boldsymbol{G}_k = \boldsymbol{G}_k^{'} \overline{\boldsymbol{P}}_k^{1/2}, \boldsymbol{\mathcal{G}}_k = \boldsymbol{G}_k \sqrt{p_k}. \tag{47}$$

Then, we can solve for μ_c by multiplying (45) from the left by $\boldsymbol{G}_k^{\mathrm{H}}$ and summing over the users in cell c, i.e.,

$$\mu_c = 1/P_{\max,c} \sum_{k:b_k=c} \left[\frac{\breve{\lambda}_k}{\breve{\xi}_k^{\mathrm{R}}} \boldsymbol{G}_k^H \overline{\boldsymbol{B}}_k^{\mathrm{R}} \boldsymbol{G}_k - p_k \boldsymbol{G}_k^H \overline{\boldsymbol{A}}_k^{\mathrm{R}} \boldsymbol{G}_k\right]. \tag{48}$$

Now, we have to find the corresponding $\bar{\boldsymbol{P}}_k$. Substituting $\bar{\boldsymbol{P}}_k$ in (43), we can write the following

$$\mathrm{lndet}\left(\boldsymbol{I} + \sum_{i=1}^{d_k} \overline{p}_{k,i} [\boldsymbol{J}_k]_{i,i}\right) - \sum_{i=1}^{d_k} \overline{p}_{k,i} a_{k,i} \tag{49}$$

where
$\bar{\boldsymbol{P}}_k = \mathrm{diag}(\bar{\boldsymbol{p}}_k), \bar{\boldsymbol{p}}_k = [\overline{p}_1 \ldots \overline{p}_{d_k}], a_{k,i} = \frac{p_k \breve{\xi}_k^{\mathrm{R}}}{\breve{\lambda}_k} [\boldsymbol{G}_k^{'}]_{:,i}^{\mathrm{H}} \overline{\boldsymbol{A}}_k^{\mathrm{R}} [\boldsymbol{G}_k^{'}]_{:,i}$,
and
$[\boldsymbol{J}_k]_{i,i} = \overline{\boldsymbol{T}}_k^{-1} (\widehat{\boldsymbol{H}}_{k,b_k} [\boldsymbol{G}_k^{'}]_{:,i} [\boldsymbol{G}_k^{'}]_{:,i}^{\mathrm{H}} \widehat{\boldsymbol{H}}_{k,b_k}^{\mathrm{H}} + [\boldsymbol{G}_k^{'}]_{:,i}^{\mathrm{H}} \boldsymbol{C}_{k,b_k} [\boldsymbol{G}_k^{'}]_{:,i} \boldsymbol{I})$.

Let $\boldsymbol{U}_k(\bar{\boldsymbol{p}}_k) = \sum_{i=1}^{d_k} \overline{p}_{k,i} [\boldsymbol{J}_k]_{i,i}$ and $\boldsymbol{a}_k = [a_1 \ldots a_{d_k}]$, we can rewrite (49) as

$$\mathrm{lndet}\left(\boldsymbol{I} + \boldsymbol{U}_k(\bar{\boldsymbol{p}}_k)\right) - \boldsymbol{a}_k \bar{\boldsymbol{p}}_k^T - \nu_k \boldsymbol{1}_{d_k} \bar{\boldsymbol{p}}_k^T. \tag{50}$$

with ν_k being the Lagrangian multiplier for the constraint $||\bar{\boldsymbol{p}}_k||_1 = 1$ and $\boldsymbol{1}_{d_k}$ is a line vector of ones, of length d_k.

In the following, we omit the user indices k for simplicity. Consider the Taylor series expansion for matrices $\boldsymbol{X}, \boldsymbol{Y}$ of dimension N_k,

$$\ln\det(\boldsymbol{X} + \boldsymbol{Y}) \approx \ln\det(\boldsymbol{X}) + \mathrm{tr}\{\boldsymbol{X}^{-1}\boldsymbol{Y}\} - \frac{1}{2}\mathrm{tr}\{\boldsymbol{X}^{-1}\boldsymbol{Y}\boldsymbol{X}^{-1}\boldsymbol{Y}\}.$$

Let $\bar{\boldsymbol{p}} = \hat{\bar{\boldsymbol{p}}} + \tilde{\bar{\boldsymbol{p}}}$ and choose $\boldsymbol{X} = \boldsymbol{I} + \boldsymbol{U}(\hat{\bar{\boldsymbol{p}}})$ and $\boldsymbol{Y} = \boldsymbol{U}(\tilde{\bar{\boldsymbol{p}}})$, we obtain the following Lagrangian

$$\tilde{\bar{\boldsymbol{p}}}\mathbf{v}^T - \frac{1}{2}\tilde{\bar{\boldsymbol{p}}}\boldsymbol{Z}\tilde{\bar{\boldsymbol{p}}}^T - \boldsymbol{a}\tilde{\bar{\boldsymbol{p}}}^T - \nu\boldsymbol{1}\tilde{\bar{\boldsymbol{p}}}^T \tag{51}$$

where $\mathbf{v} = [\mathrm{v}_1 \ldots \mathrm{v}_{d_k}]$ with $\mathrm{v}_i = \mathrm{tr}\{(\boldsymbol{I} + \boldsymbol{U}(\hat{\bar{\boldsymbol{p}}}))^{-1}[\boldsymbol{J}]_{i,i}\}$, and $\boldsymbol{Z}$ is a matrix with the elements $[\boldsymbol{Z}]_{i,j} = \mathrm{tr}\{(\boldsymbol{I} + \boldsymbol{U}(\hat{\bar{\boldsymbol{p}}}))^{-1}[\boldsymbol{J}]_{i,i}(\boldsymbol{I} + \boldsymbol{U}(\hat{\bar{\boldsymbol{p}}}))^{-1}[\boldsymbol{J}]_{j,j}\}$. Taking the gradient of (51) w.r.t. $\tilde{\bar{\boldsymbol{p}}}$, we get

$$\mathbf{v}^T - \boldsymbol{Z}\tilde{\bar{\boldsymbol{p}}}^T - \boldsymbol{a}^T - \nu\boldsymbol{1}^T = 0 \tag{52}$$

$$\Leftrightarrow \tilde{\bar{\boldsymbol{p}}}^T = \boldsymbol{Z}^{-1}(\mathbf{v}^T - \boldsymbol{a}^T - \nu\boldsymbol{1}^T) \tag{53}$$

thus

$$\bar{\boldsymbol{p}}^T = [\hat{\bar{\boldsymbol{p}}}^T + \boldsymbol{Z}^{-1}(\mathbf{v}^T - \boldsymbol{a}^T - \nu\boldsymbol{1}^T)]_+ \tag{54}$$

and ν gets determined by $\bar{\boldsymbol{p}}\boldsymbol{1}^T = 1$. Having $\hat{\bar{\boldsymbol{p}}}$ being $\bar{\boldsymbol{p}}$ at the current iteration i, we can write

$$\bar{\boldsymbol{p}}^{T(i+1)} = [\bar{\boldsymbol{p}}^{T(i)} + \boldsymbol{Z}^{-1}(\mathbf{v}^T - \boldsymbol{a}^T - \nu\boldsymbol{1}^T)]_+. \tag{55}$$

The SESIP approach and its corresponding algorithm have been introduced in [26]. The algorithm for RESIP approach follows the same update steps.

C. Discussion

The approaches SESIP and RESIP lead to two precoder designs resulting from taking the expectation of d_k dimensional vs. N_k dimensional matrices, respectively. Actually, the smaller the dimension, the more averaging occurs of the given amount of random entries, and so bringing in the expectation $\mathbb{E}$ inside $\log\det()$ should be a tighter upper bound for the smaller dimension case. Nevertheless, when $d_k = N_k$, this reasoning stops applying and either one can be larger or smaller. Simulation results, however, have shown that the difference is negligible when $d_k = N_k$, S/R-ESIP then becoming equivalent.

V. Results

In this section, we numerically evaluate the performance of RESIP-based vs. SESIP-based approaches. We consider for the multipath channel model the prior Tx side channel covariance matrix

$$\boldsymbol{C}_t = \sum_{n=1}^{N_p} \frac{\alpha_i}{\boldsymbol{v}_i^H \boldsymbol{v}_i} \boldsymbol{v}_i \boldsymbol{v}_i^H \tag{56}$$

with $\mathrm{tr}\{\boldsymbol{C}_t\} = \sum_{n=1}^{N_p} \alpha_i = M_c$, $\alpha_i = c^{i-1}\alpha_1$ and the $\boldsymbol{v}_i$ are i.i.d. vectors of M_c i.i.d. elements $\mathcal{CN}(0,1)$. We take $N_p = M_c/K$ and $c = 0.5$.

Figure 1 shows the difference between the approximates received signal level and stream level ESIP, by considering $N_k \neq d_k$. We can see that for $N_k = d_k$, both R- and S-ESIP are equivalent, whereas for $N_k \geq d_k$, SESIPrate outperforms RESIPrate, and the more N_k/d_k increases, the more we have gap, especially at intermediate values for SNR.

In Figure 2, we evaluate the average rate w.r.t. SNR, in a broadcast channel (only one communicating cell), for varying levels of channel estimation error $\sigma^2_{\widetilde{\boldsymbol{H}}}$. It is clear that the gap between SESIP and RESIP increases when $\rho_D = 1/\sigma^2_{\widetilde{\boldsymbol{H}}}$ decreases.

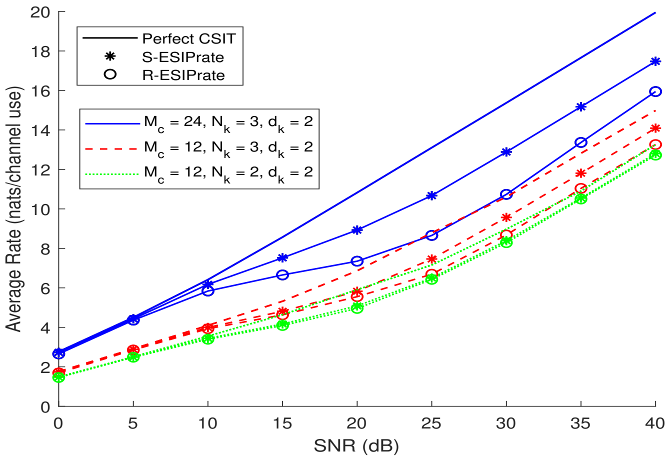

Fig. 1: Average Rate with Imperfect CSIT w.r.t. SNR: RESIP vs. SESIP, $C = 2, K_c = 3$, and $\rho_D = 10$.

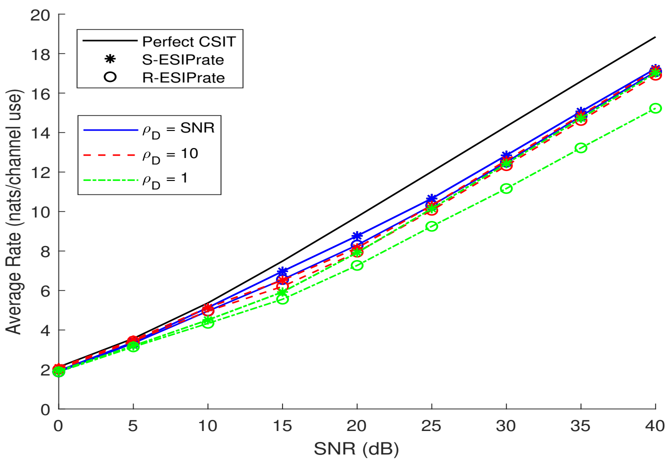

Fig. 2: Average Rate with Imperfect CSIT w.r.t. SNR: RESIP vs. SESIP, $C = 1, K = 3, N_k = 3$, and $d_k = 2$.

VI. Concluding Remarks

We have introduced two approximation approaches for the expected rate in the MIMO case with imperfect CSIT, and we have used them in a rate balancing set-up. A priori, it is not that obvious which approach is the better one. The two MIMO expected rate approximations coincide in the MISO case and in [28] an analysis was provided for the approximation quality in that case. Equations (13) introduce the RESIP approach, which might at first seem more straightforward since it corresponds to a matrix version of SINR in which then numerator and denominator are replaced by averages. However, the power optimization turns out to be less straightforward in the RESIP approach, leading to (49)-(55), unlike the SESIP approach, (42), for which it remains similar to the perfect CSIT case. Section IV.C provides the discussion on comparing approximation quality. Actually, it can also be shown that SESIP and RESIP coincide at low or (very) high SNR and only differ at intermediate SNR. Section V provides a few simulation results which confirm the discussion in section IV.C.

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