

Role of context in determining transfer of risk knowledge in roundabouts

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Abstract

The ability to predict the exit patterns of vehicles in a roundabout shows potential to improve the safety and efficiency of roundabout crossings by connected vehicles. Namely, vehicles seeking to enter a roundabout in the presence of incoming vehicles, may take educated decisions on whether to enter the roundabout based on the likelihood of the incoming vehicles not to exit and cause a merging conflict. In previous work, a machine learning model was trained to assess the probability of a vehicle to exit a roundabout based on its observed position relatively to the next exit. Yet, the transferability of the knowledge of exit probability models was not investigated, i.e., whether the knowledge of an existing exit probability model can be accurately transferred in unseen roundabouts, both for model usage and training. In this paper, we compute a metric similarity of exit probability models trained from eight real roundabouts. In turn, we identify the contextual features of two roundabouts which impact the similarity of the resulting exit probability models, and define three levels of context similarity, i.e., strict, moderate, and low. Lastly, significant accuracy improvements are obtained by constraining the knowledge transfer of exit probability models to roundabouts which feature a similar context. On the one hand, applying exit probability models on distinct roundabouts with a moderately similar context yielded an average accuracy of $80.4 \pm 4.6\%$, which is equivalent to the most accurate non-similar models. On the other hand, training a model for an unseen roundabout using exclusively training data extracted from roundabouts with a moderately similar context featured a $80 \pm 5\%$ accuracy, which represents a consistent accuracy increase of $8.5 \pm 4.8\%$ compared with knowledge transfer without context constraints.

1 Introduction

Connected and Autonomous Vehicles (CAVs) are intelligent vehicles which aim at enabling highly-automated driving, eventually eliminating the need for human supervisors. Through Artificial Intelligence (AI) techniques, CAVs analyze and extract knowledge from their environment to understand and safely interact with other road users. For example, [1] defined a deep learning model to recognize and classify objects on the roads based on camera images and LiDAR point clouds. What is more, [2] described a reinforcement learning approach for a CAV to drive autonomously and avoid obstacles. Nonetheless, CAVs can face complex situations which may compromise automated driving features. As described in [3], a key risk factor is the potentially erratic or unpredictable behavior of other road users, such as human-driven vehicles. Especially in unsignalized intersections which require negotiation with human actors, means of understanding and predicting the intentions of other vehicles are instrumental in the implementation of self-driving algorithms. For example, it has been implemented for unsignalized intersections through an AI model in [4].

Specifically, roundabouts are unsignalized yield-type intersections which involve several types of conflicts. As illustrated by Figure 1, a roundabout may involve two main types of conflicts. On the one hand, sequential conflicts may occur and potentially trigger rear-end collisions. On the other hand, merging conflicts may occur when two vehicles wish to drive in a single point at a single time. In particular, roundabout merging conflicts may involve a vehicle entering the roundabout despite an incoming vehicle. In the case of multi-lane roundabouts, lane changes in the circular part of the roundabout, or two vehicles merging into a single-lane exit are possible merging conflicts.

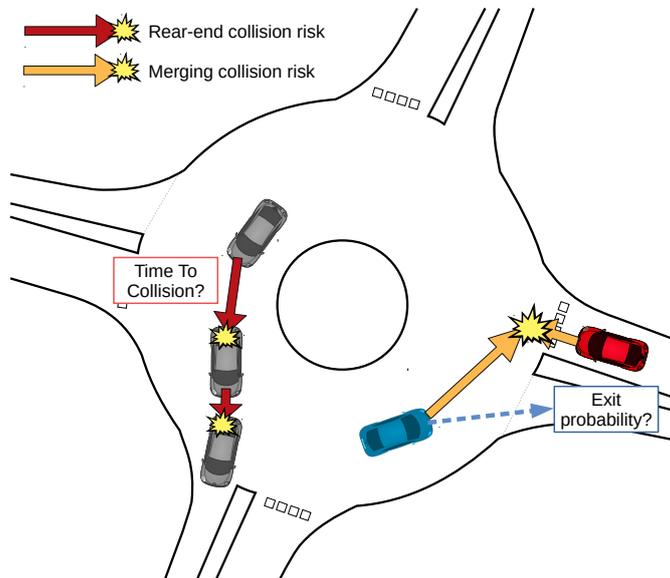


Figure 1: Types of Conflicts in a Roundabout

In particular, mitigating the risk of merging conflicts between vehicles which enter and vehicles which are already present in the circular part of the roundabout requires understanding the intentions of other vehicles. Namely, when a vehicle wishes to enter a roundabout but senses an incoming vehicle, a relevant metric to

quantify the intentions of the incoming vehicle is the probability of the incoming vehicle to exit the roundabout at the next available exit, as illustrated in Figure 1. Based on the probability of any incoming vehicle to exit at the next available exit, entering vehicles can take educated decisions on whether to engage in the roundabout. This applies both to CAVs or as an intelligent assistance system for human drivers.

AI algorithms, such as Machine Learning (ML), can be used to learn the exit patterns of vehicles in a roundabout, and in turn estimate the probability of a vehicle to exit. In [5], we defined a model to assess the probability of a vehicle to exit a roundabout, based on its position in the roundabout relatively to the next exit. The model is trained from real vehicle track data extracted from a roundabout of the RounD dataset, as introduced by Krajewski *et al.* [6]. On this roundabout, the exit behavior of vehicles can be predicted with a 91% accuracy on validation data from the considered roundabout.

Yet, training exit probability model is a costly process, which requires a set of training data extracted from vehicle tracks in a roundabout. For example, in the RounD dataset, obtained vehicle tracks and in turn training data involved complex logistics, flying a drone multiple times over each roundabout and post-processing the obtained camera images to detect vehicles. In turn, it is a desirable property to be able to (i) use an exit probability model which was already trained on a distinct roundabout to compute exit probabilities on another, or at least, (ii) complete the training data for an exit probability model from training data already sensed in another roundabout.

However, in [5], an exit probability model was trained for a single roundabout, and no model was trained nor evaluated on other roundabouts. As such, the modalities of knowledge transfer among roundabouts for exit probability models are unclear. For example, it is unknown whether exit probability models would perform accurately when applied to different roundabouts than the one they were trained from. Instead, what if classes of 'similar' roundabouts could be identified, among which a pre-existing exit probability model can be applied accurately? By first training exit probability models on multiple distinct roundabouts, and then extracting patterns which impact the similarity of two exit probability models, we identify a relevant description of their *context* of training and usage. Lastly, we evaluate the role of the *context* in which two exit probability models were trained on the accuracy of knowledge transfer, both for context-aware exit probability model (i) application and (ii) training.

Namely, in this paper, we train exit probability models as defined in [5] for eight distinct roundabouts, extracted from the RounD [6] and the INTERACTION [7] datasets of drone-captured vehicle tracks. Further, the similarity of exit probability models is computed in an information theoretic sense. Then, the obtained similarity values for each pair of roundabouts are used to extract the roundabout contextual elements which mostly impact the similarity between the resulting exit probability models, such as the geometry or traffic features of roundabouts. The obtained similarity factors are used to define a similarity condition which defines if two distinct roundabouts feature a similar context for exit probability models. Lastly, we verify that (i) exit probability models trained on a specific roundabout R can be applied accurately on distinct roundabouts featuring a similar context, and that (ii) training data for a new roundabout can be accurately completed from training data extracted

from a roundabout featuring a similar context. The contributions of this paper are the following:

- Exit probability models are trained and evaluated on eight distinct roundabouts, extracted from the Round [6] and the INTERACTION [7] datasets.
- An information theoretic approach based on the mutual information between two exit probability models is used to extract the pairwise similarity between the considered roundabouts.
- Contextual factors which could explain the similarity between exit probability models trained on two distinct roundabouts are evaluated. Especially, roundabout geometry and traffic features are considered as potential explanatory features.
- Similarity conditions between the exit probability model training contexts associated with two roundabouts is defined. The conditions are evaluated both for model application and model training purposes. Namely, we show that:
 - A model which was trained on a specific roundabout R can be used on a distinct roundabouts featuring a similar training context, with greater accuracy than models which were trained on non-similar roundabouts.
 - When training an exit probability model for an unseen roundabout R , for which a limited amount of training data is available, completing the available training data with data extracted from roundabouts featuring a similar training context improves the accuracy of the trained model compared with using data from non-similar roundabouts.

What is more, the obtained results open perspectives on the significant role of context in knowledge distribution in vehicular and transportation applications. As the context of driving, and in turn of information sensing, of CAVs is rapidly evolving due to vehicular mobility, mechanisms are needed to describe, distribute, and store AI knowledge models based on their context of training and usage. In turn, distributing knowledge to the vehicles which feature the right driving context could open new opportunities for accurate distributed knowledge networking in vehicular networks.

The rest of the article is organized as follows: Section 2 introduces the roundabouts considered for exit probability model training, and presents the resulting trained models. Section 3 investigates the information theoretic similarity of each pair of the obtained exit probability models, and identifies clusters of similar exit probability models. Based on this understanding, Section 4 identifies the roundabout contextual factors which can be associated with a greater similarity between exit probability models trained on two distinct roundabouts, roundabout geometric features as well as traffic conditions are considered as potential explanatory factors. Finally, Section 5 evaluates the accuracy improvement linked to using and training exit probability models in a relevant context, while Section 6 summarizes the article.

2 Considered Roundabouts

As a first step to investigate the contextual factors which can be related to the similarity of exit probability models trained for two distinct roundabouts, we train exit probability knowledge models on a set of real roundabouts. In turn, the obtained models are described and show the diversity of vehicle exit behavior in distinct roundabouts.

2.1 Description

To train exit probability models using realistic vehicle tracks, we used the roundabouts defined in the RounD and INTERACTION datasets. Both datasets consist of drone-captured real vehicle tracks obtained from several roundabouts. RounD [6] contains tracks extracted from three German roundabouts in the region of Aachen, while INTERACTION [7] features tracks from seven Roundabouts, based in China, Germany, and the USA.

The RounD and the INTERACTION datasets include position, heading, and velocity data at a framerate of, respectively, 25 and 10 traffic snapshots per second. Vehicle tracks are separated in multiple short recordings, typically 15 minute long for RounD and 5 minute long for INTERACTION. Each of the eight roundabouts provided in the datasets, which we consider in this study, are illustrated through aerial pictures in Figure 2. What is more, Table 1 lists a set of characteristics extracted from each roundabout, as follows:

Identifier The unique name of the roundabout.

Dataset The dataset in which the roundabout is represented.

Country The country in which the roundabout is located.

Entries The number of entry legs of the roundabout, observed from the tracks data.

Lanes The number of circulating lanes in the circular part of the roundabout.

Radius The radius of the roundabout, i.e., the distance between the center point of the roundabout and the inner extremity of the innermost lane of the circular part of the roundabout.

Width The width of the roundabout, which is the width of the driveable circular area of the roundabout, i.e., the distance between the inner extremity of the innermost lane and the outer extremity of the outermost lane.

The number of entry legs observed on the 'RounD_2' roundabout is marked with an asterisk. While the roundabout physically features four entry legs, the track data provided with the RounD dataset virtually does not include traffic from the bottom-right entry, as illustrated by Figure 3. Considering the 263 available vehicle tracks for 'RounD_2', no recorded vehicle exits the roundabout and only two vehicles enter. As the vehicular mobility effectively ignores the bottom-right exit of the roundabout, we flag the 'RounD_2' as practically a 3-entry roundabout in this study.

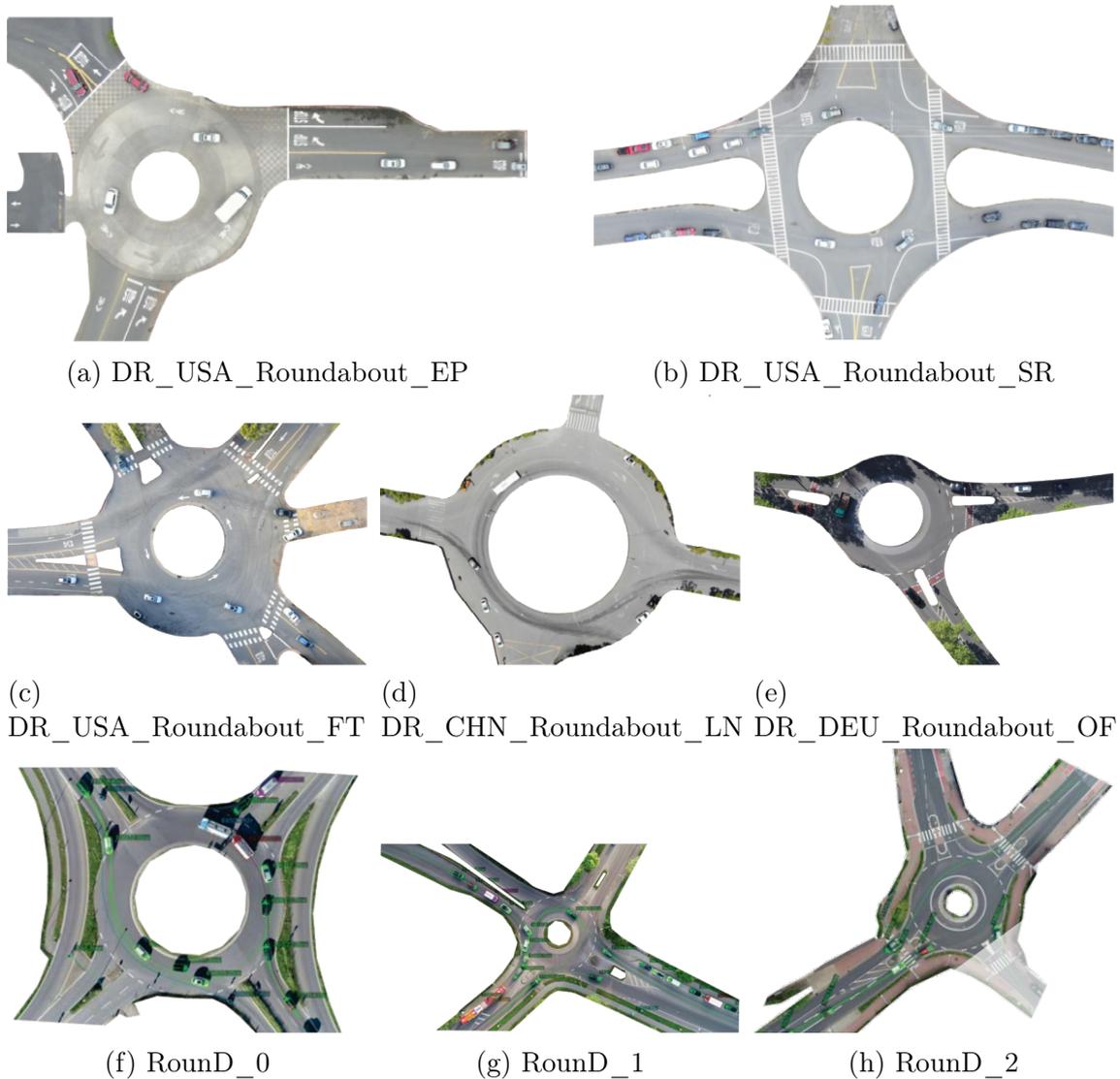


Figure 2: Roundabouts Considered for the Transfer of Exit Probability Knowledge [7, 6]

Table 1: Features of the Considered Roundabouts

Identifier	Dataset	Country	Entries	Lanes	Radius	Width
DR_USA_Roundabout_EP	Interaction	USA	4	1	6.75m	6.75m
DR_USA_Roundabout_SR	Interaction	USA	4	1	13.5m	4.5m
DR_USA_Roundabout_FT	Interaction	USA	7	1	9m	9m
DR_CHN_Roundabout_LN	Interaction	China	4	2	23m	9m
DR_DEU_Roundabout_OF	Interaction	Germany	3	1	8.75m	4.5m
RounD_0	Round	Germany	4	2	15m	9m
RounD_1	Round	Germany	4	1	8m	4.5m
RounD_2	Round	Germany	3*	1	6.75m	4.5m

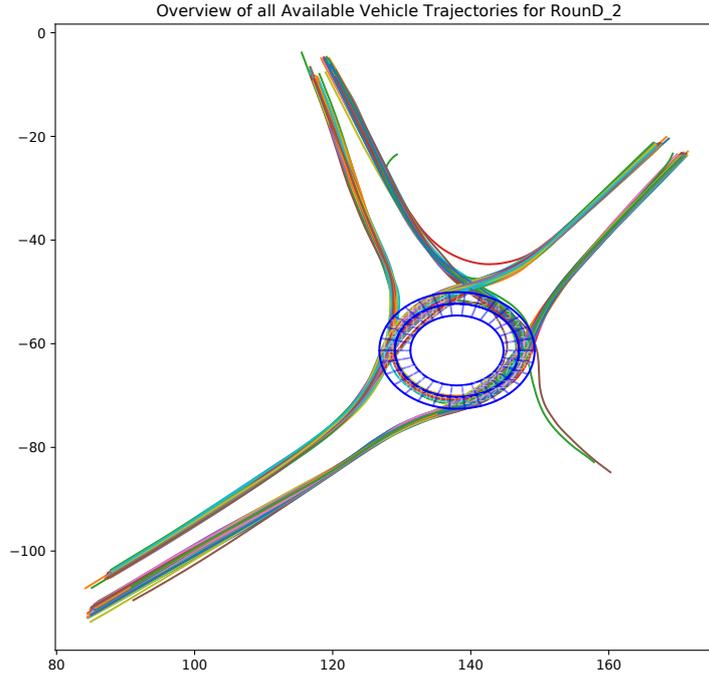


Figure 3: Vehicle Tracks of the 'RoundD_2' Roundabout

2.2 Exit Probability Model Training

To evaluate the relevant context of use of exit probability models, as well as their applicability on distinct roundabouts, a model is trained for each of the roundabouts listed in Table 1. The procedure for exit probability model training is described in [5], and summarized in this section.

2.2.1 Definition

As in [5], we define ML exit probability models which take (i) the relative heading, (ii) the distance to the next exit, and (iii) the lateral position of a vehicle in the roundabout as input, and output the probability of exit at the next exit of that vehicle. What is more, in this study, the inputs are normalized to avoid biases related to the shape of the roundabouts and increase the applicability of a model on a distinct roundabout than the roundabout it was trained from. Namely, for each frame of a recording, and for each vehicle in the circular lanes of the roundabout, the following training input is gathered, as illustrated by the matching items of Figure 4.

1. The *heading* of the vehicle, relatively to the curvature of the roundabout. It takes values in the range of $\alpha \in [-180, 180]$ degrees.
 - $\alpha = 0$ indicates that the vehicle is following the curve of the roundabout.
 - $\alpha > 0$ indicates that the vehicle is driving towards the inner part of the roundabout.

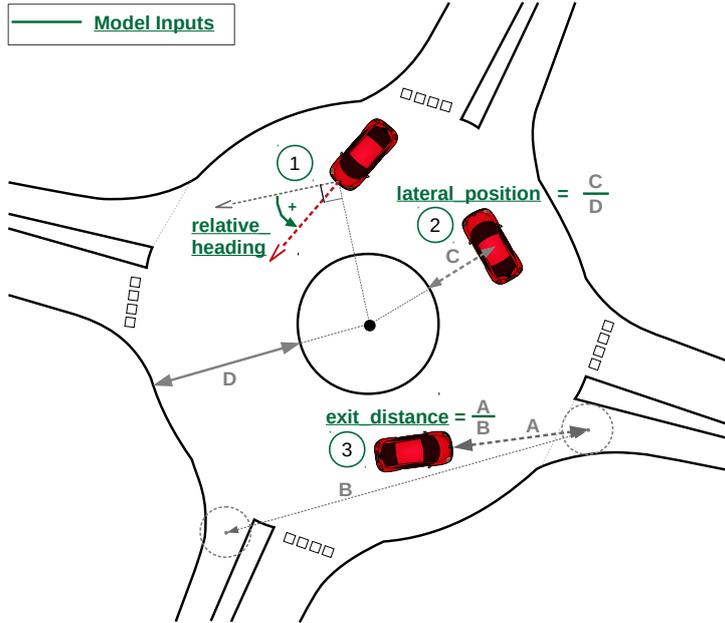


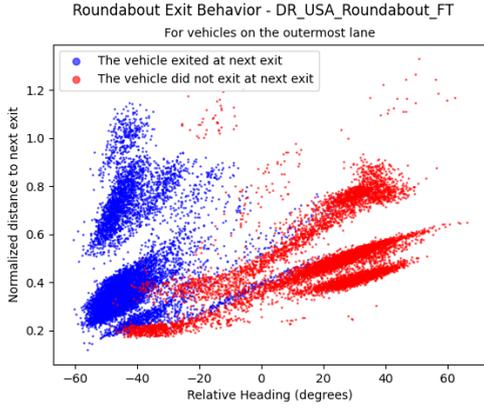
Figure 4: Input Features of the Defined Exit Probability Models

- $\alpha < 0$ indicates that the vehicles is driving towards the outer part of the roundabout.
2. The normalized straight line *distance* between the front bumper of the considered vehicle and the next exit. The distance is normalized by the distance between the next exit and the previous exit.
 3. The normalized *lateral position* of the considered vehicle in the circular part of the roundabout. The *lateral position* takes discrete values. Namely, each roundabout is divided into a finite amount of 2.25m-large virtual lanes, as described in [5]. The *lateral position* of a vehicle is the identifier of the virtual lane it is located in, normalized by the number of virtual lanes in the roundabout.

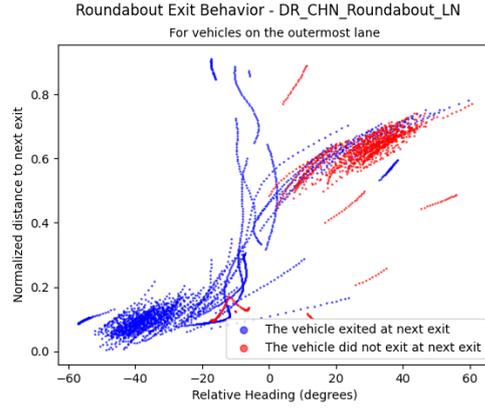
In turn, Figures 5a and 5c illustrate the training data which was obtained from the vehicle track recordings of, respectively, the DR_USA_Roundabout_FT and DR_CHN_Roundabout_LN roundabouts. Each point is associated with an observed position of a vehicle in the considered roundabout. The blue-colored points correspond to [heading, distance, lateral position] positions of a vehicle which subsequently exited at the next available exit. On the contrary, red-colored points represent positions from which a vehicle stayed in the roundabout at the next exit. Clear patterns can be observed in the training data, with lower values of relative heading typically associated with roundabout exits, and vice versa.

2.2.2 Training

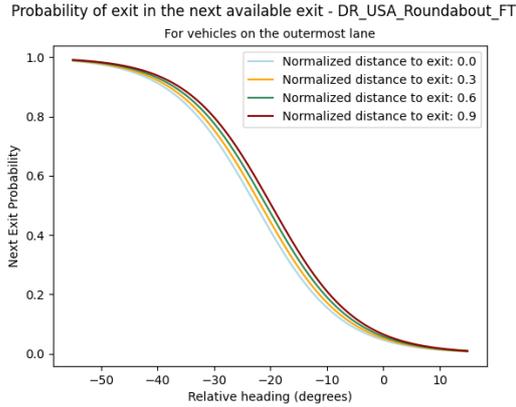
Finally, a logistic regression model is trained which returns the probability of a vehicle to exit the roundabout at the next available exit from the three aforementioned inputs. Figures 5b and 5d illustrate the obtained models. They associate the position input of a vehicle in a roundabout with its probability of exit at the next available exit for, respectively, the USA-based and the Chinese roundabout.



(a) Exit Behavior Scatter

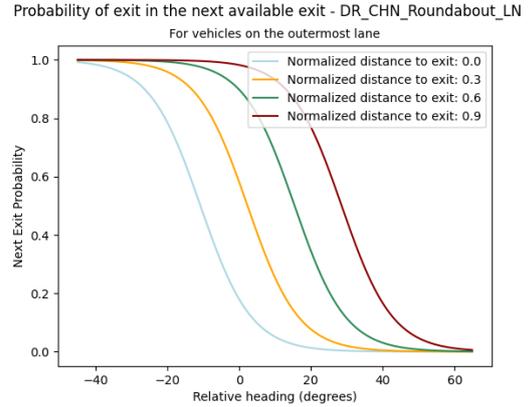


(c) Exit Behavior Scatter



(b) Exit Probability Model

DR_USA_Roundabout_FT



(d) Exit Probability Model

DR_CHN_Roundabout_LN

Figure 5: Extract of two Trained Exit Probability Models and their Matching Training Input

We observe different patterns of exit probability, as the models are adapted to the driver behaviors of each roundabout. In the USA-based roundabout, in the outermost lane, the normalized distance to the exit has a limited impact on the probability of exit, as opposed to the relative heading of the vehicle. On the contrary, in the Chinese roundabout, the relative distance to the next exit has a strong impact on the exit probability. A possible explanation for this difference in patterns is the difference in the distance between two consecutive entries in both roundabouts, which is shorter in the USA-based roundabout.

Based on the exit probability models trained for each of the roundabouts listed in Table 1, we analyze the similarity between each trained models. In turn, we analyze the contextual characteristics of a roundabout which are the most relevant to describe the *context of training* of an exit probability model.

3 A Similarity Metric for Roundabout Exit Probability Models

In Section 2, we have described the training of exit probability models on several roundabouts, to produce accurate exit probability values tailored for each roundabout. Nonetheless, training an exit probability model for a new roundabout is a costly process. It requires sensing a relatively large set of training data, which is done in the considered roundabouts by flying a drone multiple times over a roundabout, as well as calibrating and running a vehicle detection algorithm on the obtained camera images. As such, it is unpractical to train a new model for new roundabout on the map, especially in countries where roundabouts are a common type intersection.

Rather, a more efficient approach would be to reuse exit probability models which have already been trained on a different roundabout. Yet, as illustrated by Figures 5b and 5d, exit probability models learn different patterns on different roundabouts. As such, the applicability of a model on another roundabout is uncertain. In turn, in this section, we compute a pairwise similarity metric for exit probability models trained for each roundabouts listed in Table 1. Then, exit probability models are clustered based on their similarity to exhibit the roundabouts which feature the most similar and interchangeable models.

The aim of this preliminary work is to use the obtained clustering to extract generic patterns and contextual features which make two roundabout similar. Based on this, semantic rules can be defined which determine if an existing model is suitable to be used on a new roundabout. As such, the metric which defines the similarity between two exit probability models should be generic and objective. What is more, it is not satisfactory to directly compare the accuracy of two models as:

- It is biased by the selected validation set.
- Two equivalent accuracy scores may be obtained by two completely different set of predictions.

3.1 Mutual Information

Mutual Information (MI) is a similarity score which measures the mutual dependence between two random variables. It was originally introduced by C. Shannon in [8], and measures the theoretical *amount of information* which can be obtained about one random variable by observing the outcomes of the other. MI is symmetric and non-negative, the MI $I(X, Y)$ of two random variables X and Y equals zero if X and Y are independent. It is defined as:

$$I(X, Y) = H(X) - H(X|Y)$$

, with:

- $H(X)$ the entropy of X .
- $H(X|Y)$ the entropy of X conditioned on Y .

The entropy $H(X)$ of a random variable X encodes the average level of information which is inherent to the possible outcomes of X . Namely, let us consider X

to be a discrete random variable which can produce outcomes in the set $(x_i)_{i \in [1, n]}$, with respective probabilities $(P(x_i))_{i \in [1, n]}$. The entropy of X is defined as:

$$H(X) = - \sum_{i=1}^n P(x_i) \cdot \log_2(P(x_i))$$

As we expressed the entropy using base 2 logarithms, the unit of $H(X)$ is the shannon bit. Then, considering an extra random variable Y producing outcomes in $(y_i)_{i \in [1, n]}$, the entropy of X conditioned on Y is defined as:

$$H(X|Y) = - \sum_{i=1}^n \sum_{j=1}^m P(x_i, y_j) \log_2\left(\frac{P(x_i, y_j)}{P(y_j)}\right)$$

MI scores can be computed for two classifiers, by considering the predictions of classifiers as random variables. The exit probability models trained in Section 2 are classifiers which associate a class label, i.e., the exit probability, to a set of inputs, i.e., heading, distance to exit and lateral position. In turn, the MI between two exit probability models can be computed by considering the predicted exit probabilities as a random variable. MI has been computed for classifiers and used as a metric of similarity to ensure the diversity in classifier combinations applications, in [9] and [10].

Namely, the MI between two exit probability models e_1 and e_2 can be computed through the following procedure:

1. Let $K \equiv (k_i)_{i \in [1, l]}$, $l \in \mathbb{N}$ be a set of input [**heading, distance, lateral position**] observations. Each input entry k_i can be passed as input to e_1 and e_2 to produce an output probability, respectively, $e_1(k_i)$ and $e_2(k_i)$.
2. The set of probability predictions of e_1 and e_2 on K input entries, i.e., $E_1 \equiv (e_1(k_i))_{i \in [1, l]}$ and $E_2 \equiv (e_2(k_i))_{i \in [1, l]}$, are assimilated to two random variables named E_1 and E_2 .
3. The MI of the e_1 and e_2 models is expressed as $I(E_1, E_2)$.

MI measures the theoretical amount of information which can be obtained about the probability predictions of a model from the observation of the predictions of another model. As such, it is a relevant metric to estimate a generic value of similarity between two exit probability models. In turn, we implement the following procedure to compute the MI between each pair of roundabouts (R_1, R_2) , $R_1 \neq R_2$, extracted from Table 1:

1. Two exit probability models M_{R_1} and M_{R_2} are trained using 5000 training input entries extracted from the track data of, respectively, R_1 and R_2 . For each roundabout, a 1000-entry validation set is also extracted from the remaining track data, respectively, V_{R_1} for R_1 and V_{R_2} for R_2 .
2. V_{R_1} and V_{R_2} are gathered and randomly shuffled into a 2000-entry common validation set $V_{R_1, R_2} = V_{R_1} \cap V_{R_2}$.
3. The exit probability associated with the entries of V_{R_1, R_2} are computed for both the M_{R_1} and M_{R_2} models. Let $(m1_i)_{i \in [1, 2000]}$ and $(m2_i)_{i \in [1, 2000]}$ be, respectively, the set of exit probability predictions produced by M_{R_1} and M_{R_2} .

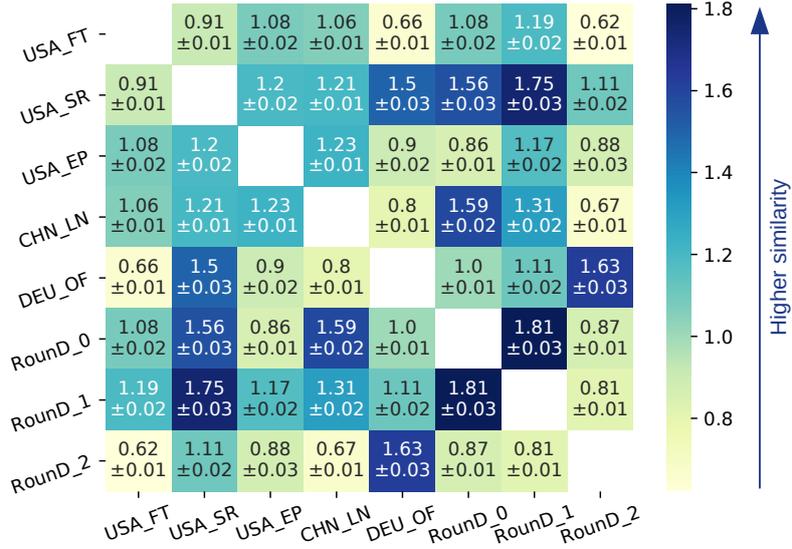


Figure 6: Computed Mutual Information for each Pair of Exit Probability Models

4. We assimilate $(m1_i)$ and $(m2_i)$ with the outcomes of two random variables $M1$ and $M2$. In turn, the MI of M_{R_1} and M_{R_2} is computed as $I(M1, M2)$.
5. The MI is $I(M1, M2)$ computed using the EDGE estimator as introduced in [11].

Figure 6 illustrates the pairwise values of MI computed for the roundabouts listed in Table 1. The identifiers for the INTERACTION roundabouts have been shortened to include only the country and roundabout codes. Finally, each value has been computed 20 times with different samplings of training and validation sets for each exit probability models, to produce 95% confidence intervals. The results give hints on the level of similarity of the considered roundabouts. Yet, it is not straightforward to interpret the strength of the similarity, and compare the MI values as-is. In turn, we proceed to normalize the MI values, to ease their interpretation.

3.2 Normalization

MI values computed in Figure 6 are not upper bounded. As such, it is challenging to interpret the strength of the mutual information between two exit probability models. In turn, we aim at normalizing the MI values in a $[0, 1]$ interval, such that a maximal value indicates a complete determination of one random variable through the knowledge of the other. The Uncertainty Coefficient (UC) is a normalization of MI which matches these requirements. The UC of a random variable X given Y is expressed as:

$$UC(X|Y) = \frac{I(X, Y)}{H(X)}$$

It comes from the definition that the UC is bounded in the $[0, 1]$ interval:

1. $UC(X|Y) \geq 0$, as $I(X, Y) \geq 0$ and $H(X) \geq 0$.
2. $UC(X|Y) \leq 1$, as $I(X, Y) = H(X) - H(X|Y) \implies I(X, Y) \leq H(X)$.

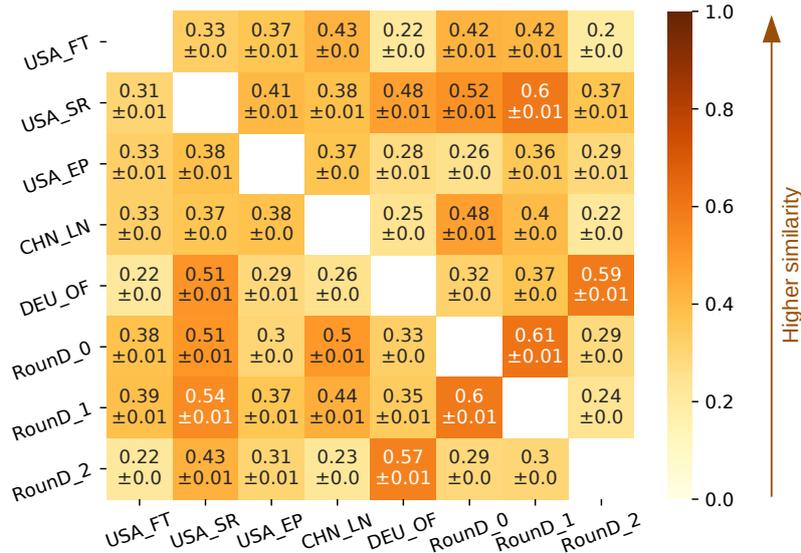


Figure 7: Computed Uncertainty Coefficient for each Pair of Exit Probability Models

$UC(X|Y)$ represents the fraction of the information which can be predicted on the X variable provided full knowledge of the outcomes of Y . As such, it provides a scale of the level of similarity between two variables. Unlike the MI, the UC is not symmetric, i.e., the proportion of information obtained on X from the observation of Y may differ from the proportion of information obtained on Y from the observation of X .

Provided with the values of pairwise MI listed in Figure 6, the UC value $UC(M1, M2)$ associated with the exit probability models trained on roundabouts $R1$ and $R2$ can be inferred from $H(M1)$. As introduced in Section 3.1, the entropy is defined for random variables taking discrete outcomes. In turn, the exit probability predictions of the considered models, i.e., the outcomes of $M1$ and $M2$, are discretized by rounding the probability value to a single digit. Namely, the predicted probabilities may take 11 possible values, in $(\frac{p}{10}, p \in \{0, 1, \dots, 10\})$. In turn, $H(M1)$ is computed using the formula $H(M1) = -\sum_{p=0}^{10} P(M1 = \frac{p}{10}) \cdot \log_2(P(M1 = \frac{p}{10}))$. The discretization is also used to compute MI values.

Figure 7 show the obtained UC values for pairs of exit probability classifiers associated with the roundabouts in Table 2. For each pair of roundabout (R_1, R_2) , with R_1 on the y-axis and R_2 on the x-axis, the associated $UC(M1|M2)$ is provided with a 95% confidence interval, similarly to Figure 6. The results show that the highest value of similarity for a pair of roundabout are obtained by the (Round_0, Round_1) and (DR_DEU_Roundabout_OF, Round_2), with about 60% of similarity, e.g., knowledge of the predictions of Round_0 theoretically allows to predict 60% of the predictions of Round_1. On the other hand, DR_USA_Roundabout_FT and Round_2 feature a lower UC of about 20%.

4 Roundabout Context Semantic Description

The pairwise MI and UC values of exit probability models listed, respectively in Figures 6 and 7 can be used to form clusters of roundabouts based on the similarity of their associated exit probability model. Clustering can be produced using

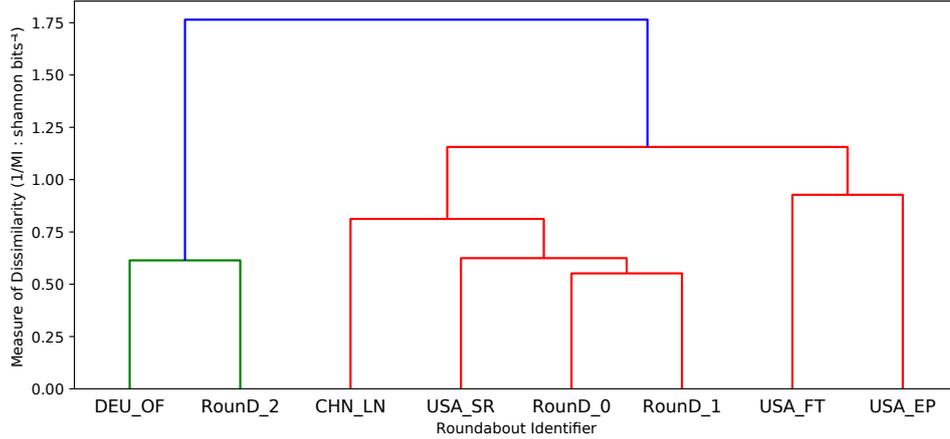


Figure 8: Hierarchical Clustering of Roundabouts based on the Similarity of their Associated Exit Probability Classifiers

distance matrix-based clustering algorithms such as hierarchical clustering [12] or DBSCAN [13].

Figure 8 shows a possible clustering based on the MI similarity, which is used because of its property of symmetry. A distance matrix representing the level of dissimilarity of exit probability classifiers is produced from the inverse of MI values $\frac{1}{MI}$. In turn, clusters are computed using the *ward* linkage method of hierarchical clustering [12]. The clusters are illustrated by a dendrogram, the y-axis represents the level of dissimilarity associated with each clusters. As such, the results show the high similarity of DR_DEU_Roundabout_OF and Round_2, with a MI value of $\frac{1}{0.55} = 1.81$. It also shows their relative dissimilarity with the other exit probability classifiers.

Clustering roundabouts by the similarity of their associated exit probability models opens perspectives of knowledge transfer, or use a model which was trained on a distinct roundabout to predict exit probabilities. Yet, the clustering is based on exit probability models which were already trained, which limits the interest of knowledge transfer, as models which were specifically trained for each roundabout are already available. In turn, in this section, we analyze the factors in terms of the geometry of the traffic in roundabouts which may explain high or low values of similarity between the resulting exit probability models. Generally, this analysis aims at identifying a generic semantic description of the context of training of exit probability models, to infer specific types of roundabouts for which exit probability models are expected to feature a high similarity.

4.1 Geometric Semantics

As a first step, we analyze to what extent the the geometric features of two roundabouts can impact and explain the similarity of their associated exit probability models. In turn, we analyze the relationship between the similarity of various geometric features of each pair of roundabouts (R_1, R_2) in Table 1 and the UC of their associated exit probability models $UC(M1|M2)$.

As such, we investigate the relationship between $UC(M1|M2)$ and the following geometric and geographical features of the R_1 and R_2 roundabouts:

Table 2: Training Set to Investigate the Relationship between Roundabout Geometric Disparities and UC

Pair of Roundabouts	Same Country	Δ Entries	Δ Lanes	Δ Radius	Δ Width	UC
USA_FT / USA_SR	True	3	0	4.50	4.50	0.31
USA_SR / USA_FT	True	3	0	4.50	4.50	0.33
USA_FT / USA_EP	True	3	0	2.25	2.25	0.33
USA_EP / USA_FT	True	3	0	2.25	2.25	0.37
USA_FT / CHN_LN	False	3	1	14.00	0.00	0.33
CHN_LN / USA_FT	False	3	1	14.00	0.00	0.43
USA_FT / DEU_OF	False	4	0	0.25	4.50	0.22
...						
RounD_0 / RounD_1	True	0	1	7.25	4.50	0.60
RounD_1 / RounD_0	True	0	1	7.25	4.50	0.61
RounD_0 / RounD_2	True	1	1	8.50	4.50	0.29
RounD_2 / RounD_0	True	1	1	8.50	4.50	0.29
RounD_1 / RounD_2	True	1	0	1.25	0.00	0.30
RounD_2 / RounD_1	True	1	0	1.25	0.00	0.24

Same country Whether both roundabouts are located in the same country.

Δ Entries The absolute difference of the number of entry legs of the two roundabouts.

Δ Lanes The absolute difference of the number of circulating lanes in the circular part of the two roundabouts

Δ Radius The absolute difference of the radius of the two roundabouts, expressed in meters.

Δ Width The absolute difference of the width of the two roundabouts, expressed in meters.

The 'radius' and 'width' features of a roundabout are described in detail in Section 2.1. Table 2 shows an extract of the 56 entries which associate the geometric and geographical differences of pairs of roundabouts with the UC of their associated exit probability models.

Based on the obtained table, we proceed to investigating any potential relationship between the considered geometric features and UC values through two complementary approaches. On the one hand, we analyze the correlation between UC and the considered features. On the other hand, we train a decision tree to estimate UC values based on an input set of geometric and geometrical differences. Because of its explainability, the obtained decision tree is used to identify further patterns of relationship between the UC and the considered features.

4.1.1 Correlation Analysis

To begin with, based on the data gathered in Table 2, we analyze the correlation between the considered geometric and geographical features, and the UC column. Figure 9 presents the obtained correlation matrix, listing the correlation coefficients for each feature pair.

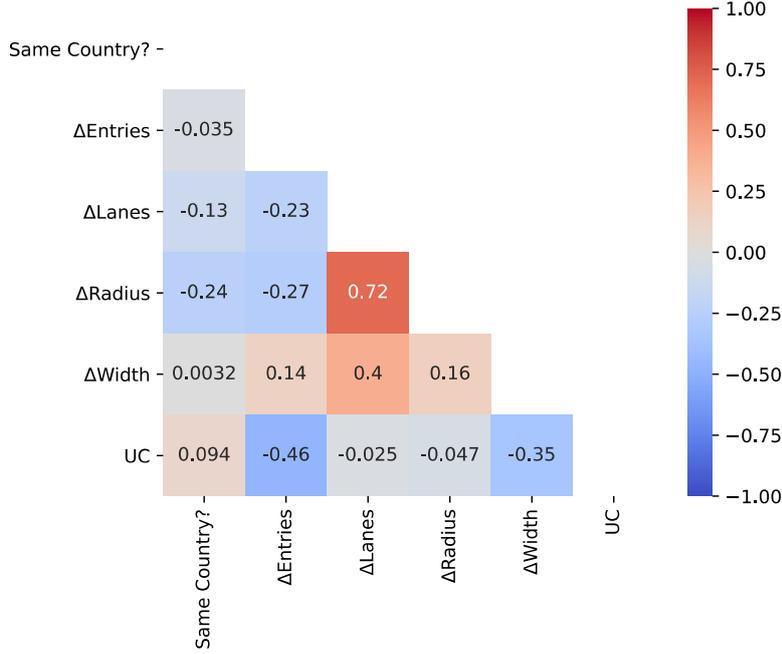


Figure 9: Correlation Coefficients for UC and Roundabout Geometric Disparities

The results indicate a moderate negative correlation between $\Delta Entries$ and UC values, i.e., $r = -0.46$. Namely, lower absolute differences in the number of entry legs of two roundabouts tend to be related with a higher UC for their associated exit probability models. The correlation between $\Delta Entries$ and UC values is further detailed in Figure 10. It shows a scatter plot of $(\Delta Entries, UC)$ pairs extracted from Table 2, as well as a visualization of their distribution. It can be observed that pairs of roundabouts which feature the largest difference in the number of entry legs feature the lowest UC values. On the contrary, the pairs of roundabouts with the same number of entry legs feature the highest values of UC in average. As such, $\Delta Entries$ can be identified as a significant factor to explain a high value of UC similarities between the exit probability models associated with two roundabouts.

Additionally, the difference of the width of two roundabouts $\Delta Width$ is negatively correlated with UC values, with a correlation coefficient $r = -0.35$. The other considered features have a low to negligible correlation coefficient with UC values. As such, the two main geometric features which can be associated with higher UC values are low absolute differences of (i) the number of entries and of (ii) the width of two roundabouts.

4.1.2 Regression Tree Analysis

In parallel to the previously discussed correlation analysis, we perform a complementary analysis based on the training of a regression tree to infer a level of UC similarity based on the geometric features of two roundabouts. Namely, we use Table 2 as a supervised learning training set for a regression tree, associating the geometric and geographical features to the UC class.

Figure 11 illustrates the obtained regression tree. It encodes a set of explainable rules and conditions to predict UC values from relevant geometric and geographical features of two roundabouts. As such, it emphasizes the role of $\Delta Entries$ in the

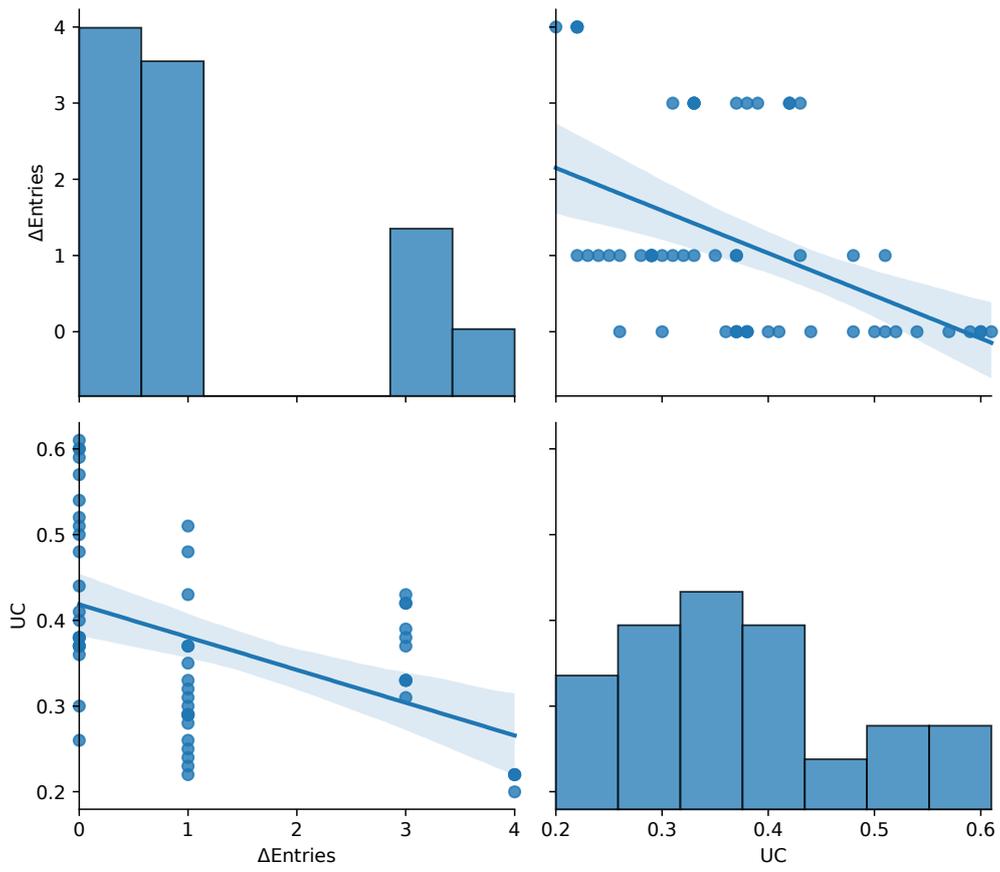


Figure 10: Relationship between UC and the Difference of the Number of Entry Legs

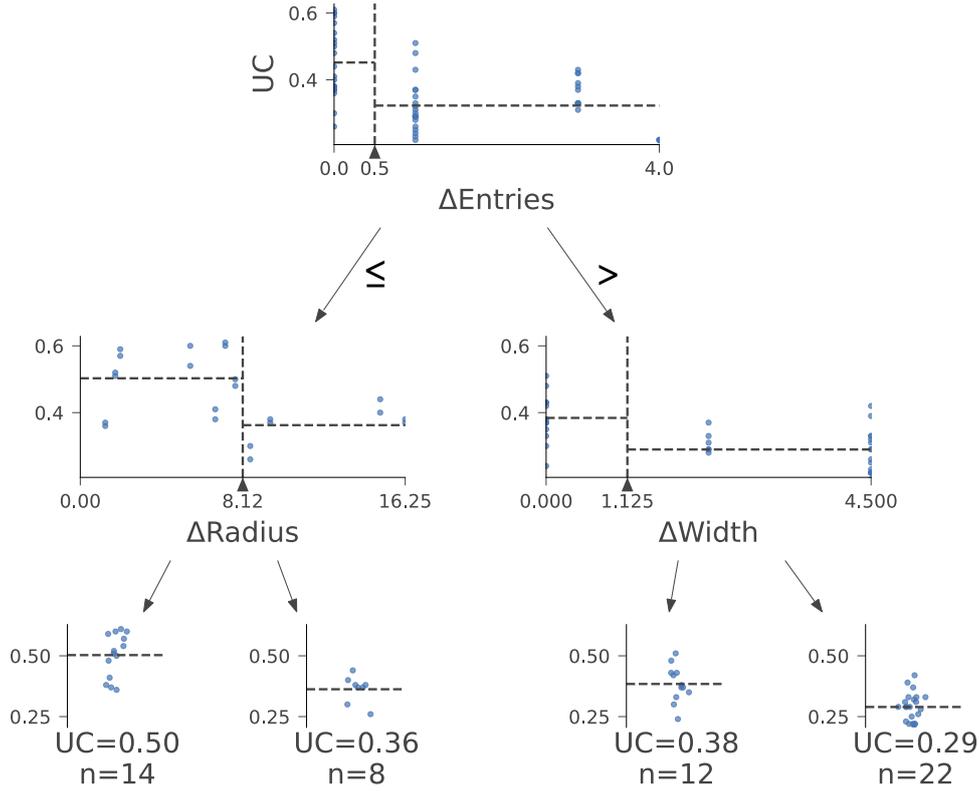


Figure 11: Regression Tree Trained to Predict UC from Roundabout Geometric Disparities

determination of similarity values, as previously identified by correlation analysis. Namely, the highest UC similarity of the exit probability models associated with two roundabouts, i.e., $UC = 0.5$ in average, are reached for $\Delta Entries = 0$, combined with $\Delta Radius \leq 8.12m$, i.e., a same number of entry legs and a radius difference below a threshold identified at $8.12m$. The role of low values of $\Delta Width$ to reach high similarity, as identified in the previous correlation analysis, is also observed: For two roundabouts with a different number of entry legs, the highest UC similarity values, i.e., $UC = 0.38$ in average, are achieved for width differences below a $1.125m$ threshold.

The regression tree analysis emphasizes the role of $\Delta Entries$ and $\Delta Width$ as high impact factors for exit probability models UC similarity. What is more, $\Delta Radius$ is identified as a key factor to reach highest similarity values, for roundabouts which feature the same number of entries. For other roundabouts, low values of $\Delta Width$ are a decisive factor.

4.2 Traffic Semantics

In Section 4.1, three geographical features of roundabouts which highly impact the similarity of exit probability models were identified. Namely, low differences of (i) the number of entry legs, (ii) the radius and, (iii) the width of two roundabouts can be linked to higher UC values of the resulting exit probability models. In turn, we investigate another key aspect of roundabout mobility, possibly influencing the exit behavior patterns of vehicles, i.e., traffic. In this section, we analyze the impact of traffic on the exit probability context of a roundabout. Namely, we consider whether

similar traffic conditions in two roundabouts can lead to higher UC similarity of the resulting exit probability models.

We follow the following process to investigate the relationship between the traffic conditions in two roundabouts and the similarity of the resulting exit probability models:

- A normalized metric is selected and computed to accurately compare the traffic conditions in two roundabouts.
- The track data available for each of the considered roundabouts listed in Table 1 is partitioned into different levels of traffic congestion.
- Exit probability models are trained for each roundabout and for each normalized value of traffic conditions.
- A correlation analysis is performed to investigate the correlation between the UC of the obtained models and the difference of the normalized traffic conditions in two roundabouts.

4.2.1 Computation of Normalized Traffic Metrics

As a first step, a metric is selected to represent the level of traffic congestion in a roundabout. A key requirement is the selection of a normalized metric which defines the level of congestion in a roundabout with reduced bias from the geometry of the roundabout. Namely, a value of traffic flow which causes a traffic congestion in a small scale roundabout may have a low impact on traffic fluidity in a larger roundabout. In turn, a normalized metric is required, which allows a relevant comparison among different roundabouts.

In turn, we select the *traffic flow* of a roundabout as a traffic level metric, normalized by the *capacity* of each of the considered roundabouts listed in Table 1. On the one hand, the *traffic flow* of a roundabout is defined as the sum of the flow of vehicles at each entry leg. In turn, the flow at a specific entry is defined as the number of vehicles which enter the circular part of the roundabout from this entry in a specific time interval. In this work we use the unit of *vehicles per hour* for flow computation. On the other hand, the capacity of a roundabout can be defined as the maximal *traffic flow* which can be handled by the roundabout before saturation.

Flow Computation To begin with, we define a procedure to measure the evolution of the flow of vehicles at each entry leg of a roundabout. As illustrated by Figure 12, showing the example of the CHN_LN Interaction roundabout, we define a set of sensing areas in each roundabout. In turn, each unique vehicle which crosses a sensing area is counted and used to compute the number of vehicle having crossed in a specific time interval. Sensing areas are defined for the entry legs, the exit legs, and the circulating lanes and illustrated by, respectively, the green, red, and blue-colored shapes in Figure 12.

In turn, the available vehicle track recordings for each of the considered roundabouts are divided into time intervals of duration t_{sub} . For each time interval, the number of unique vehicles having entered the roundabout through any of the entry sensors are counted. Then, they are divided by the t_{sub} duration, to obtain a value of the average *traffic flow* of the considered roundabout over the considered t_{sub} -long period, i.e., the sum of the flow of each entry leg over the period.

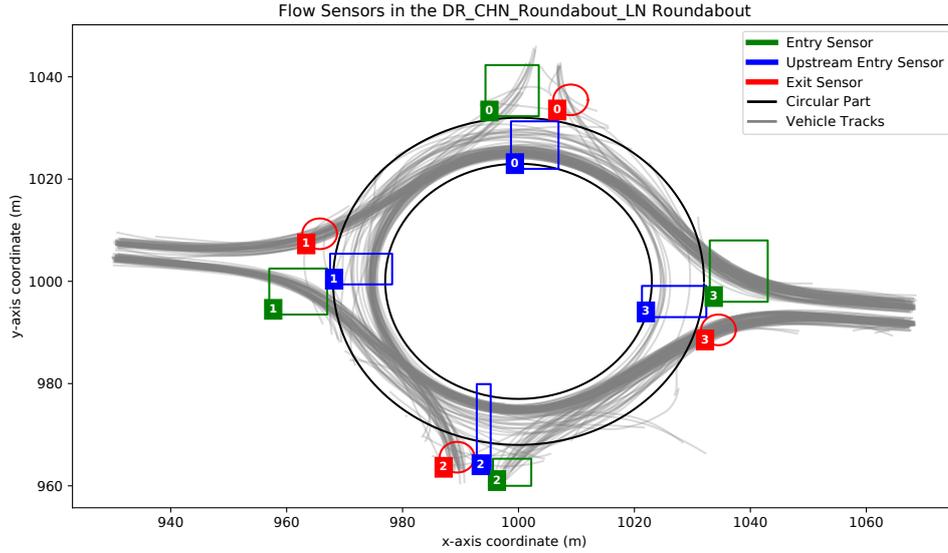


Figure 12: Roundabout Model and Sensing Areas Defined to Compute Flow Values

Capacity Computation The capacity of a roundabout is the maximal *traffic flow* value it can support before saturation. Approaches to estimate the capacity of a roundabout from its observed traffic conditions and shape have been defined, as summarized in [14, 15]. Different approaches have been defined by organizations targeting the roundabouts of different countries. In this study, we consider the roundabouts listed in Table 1 which are mostly located in Germany and the USA. In turn, we investigate the two following approaches to compute capacity values in the considered roundabouts:

- The Highway Capacity Manual (HCM) is a publication of the USA-based Transportation Research Board, which describes concepts and procedures for computing the capacity of various intersections. It is designed for USA-based intersections. The latest update of the HCM, published in 2016 as described in [16] and subsequently referred to as 'HCM2016', describes a procedure for computing the capacity of each entry leg of a roundabout. In turn, the capacity of the roundabout is the sum of the capacity of each entry leg. Specifically, the capacity of an entry leg can be computed following the HCM2016 model using Equation 1.

$$C = A \cdot \exp(-B \cdot Q_c) \quad (1)$$

With:

- C , the capacity of the entry in vehicles per hour.
- $A = \frac{3600}{t_f}$, $B = \frac{t_c - \frac{t_f}{2}}{3600}$
- t_f and t_c , respectively, the follow-up and critical headway in the roundabout.

- * HCM2016 provides a table of values for t_c and t_f , depending on the number of lanes of (i) the considered entry and (ii) the circular part of the roundabout.
- Q_c , the flow in vehicles per hour in the circular part of the roundabout, *upstream* the considered entry.
 - * This represents the flow of vehicles in the circular part of the roundabout before it merges with the considered entry, and excluding the flow of vehicles which exit the roundabout before the entry. Namely, Figure 12 illustrates the computation of the upstream flow of entries in numbered blue-colored rectangles.
- As an alternative, a model focusing on German roundabouts was described in [17]. We refer to this approach as the 'German' model. Following this model, the capacity at an entry of a roundabout can be computed using Equation 2.

$$c = \left(1 - \frac{\Delta \cdot q_c}{n_c}\right)^{n_c} \cdot \frac{n_e}{t_f} \cdot \exp(-(t_0 - \Delta) \cdot q_c) \quad (2)$$

Where:

- $c = \frac{C}{3600}$, the capacity of the entry in vehicles per second.
- n_e and n_c , the number of driving lanes in, respectively, the considered entry leg and the circular part of the roundabout.
- $t_0 = t_c - \frac{t_f}{2}$
 - * The 'German' model provides values, i.e., $t_c = 4.12s$ and $t_f = 2.88s$.
- Δ , the minimum time headway in the circular part of the roundabout.
 - * The 'German' model provides the $\Delta = 2.10s$ value.
- $q_c = \frac{Q_c}{3600}$, the flow *upstream* the entry, as captured in the blue-colored rectangles of Figure 12, in vehicles per second.

In turn, we partition the track data available for each of the roundabouts listed in Table 1 in subdivisions of a 1 minute duration. For each subdivision of track data, the capacity as well as the flow of the roundabout, i.e., summed over each entry of the roundabout, is computed. Two versions of the capacity are computed, based on the HCM2016 and 'German' approaches. Then, a normalized traffic level metric which we note γ is obtained by dividing the flow by the capacity of a roundabout for each subdivision.

The obtained values provide an indication of the level of traffic congestion γ for each 1-minute subrecording of each roundabout. Figure 13 and Figure 14 illustrate the distribution of the obtained γ values for the subrecordings of, respectively, the RoundD_0 and the USA_FT roundabouts. The orange-colored graph show the distribution of γ values computed using the 'German' model for capacity computation, and the dark blue-colored graph illustrates the γ values obtained through the HCM2016 model. We observe that the HCM2016 and the 'German' models do not produce significant differences in γ values, which in turn leads to similar distributions. As such, we select a single model, e.g., the 'German' model, to compute γ values of traffic level. In turn, the obtained values are used to analyze the relationship between the traffic conditions in two roundabouts and the *UC* similarity of the resulting exit probability models.

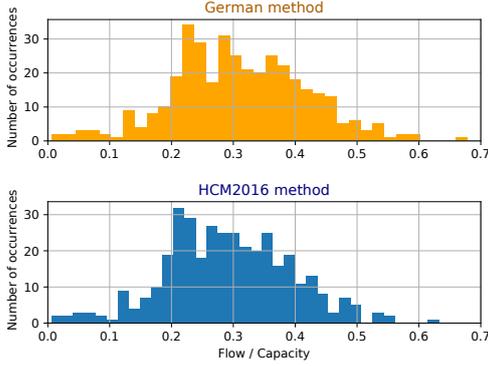


Figure 13: Distribution of the Levels of Traffic Congestion γ recorded in RoundD_0

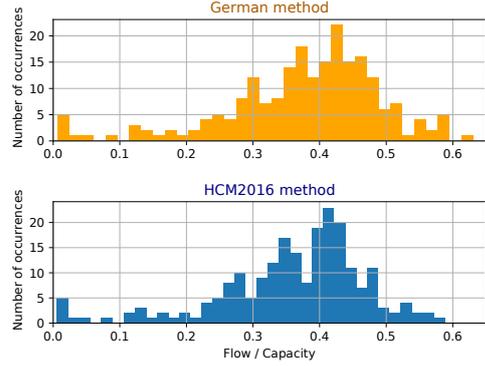


Figure 14: Distribution of the Levels of Traffic Congestion γ recorded in USA_FT

4.2.2 Impact of Traffic Conditions on Exit Probability UC

As described in Section 4.2.1, the vehicle track data available for each of the considered roundabouts were split in subrecordings of a duration of 1 minute each. In turn, the traffic level metric γ was computed for each subrecording. Practically, the tracks data of a roundabout is divided into a set R of (r_i, γ_i) pairs, with r_i a subrecording extracted from the roundabout, and γ_i the average traffic level in this subrecording.

Then, the subrecordings R of each roundabout are partitioned by traffic conditions. Namely, R is divided into several subsets of subrecordings which contain subrecording with increasingly congested traffic conditions, i.e., $R = [R_{0.0 \leq \gamma < 0.1}, R_{0.1 \leq \gamma < 0.2}, R_{0.2 \leq \gamma < 0.3}, R_{0.3 \leq \gamma < 0.4}, R_{0.4 \leq \gamma < 0.5}, R_{0.5 \leq \gamma < 0.6}]$, with $\forall x, y \in [0, 1], x < y, \forall i \in \{0, \dots, \|R\|\}, x \leq \gamma_i < y \iff (r_i, \gamma_i) \in R_{x \leq \gamma < y}$. For example, $R_{0.5 \leq \gamma < 0.6}$ contains all the subrecordings (r_i, γ_i) which feature a high level of traffic congestion $\gamma_i \in [0.5, 0.6]$. As such, a discrete partition of subrecordings is obtained, forming an incremental scale of six levels of traffic congestion from $\gamma \in [0, 0.1]$ value to $\gamma \in [0.5, 0.6]$. Higher levels of traffic congestion are not considered as only a negligible amount of subrecordings featured $\gamma > 0.6$.

Based on the obtained partition of subrecordings, distinct exit probability models are trained for each roundabout and for each value of the discrete scale of γ traffic metric. For example, given the tracks R of a roundabout, a model $M_{R_{0.5 \leq \gamma < 0.6}}$ will be trained exclusively using track data extracted in congested traffic conditions, iif $R_{0.5 \leq \gamma < 0.6}$ contains at least 5000 training entries. Generally, for each roundabout, models are trained for each partition of traffic congestion which features at least 5000 training entries.

In turn, the UC value is computed for each pair of the obtained exit probability models, following the same process as in Section 3. Table 3 illustrates an extract of the obtained 225 rows. The 'pair of exit probability models' describes the used roundabouts as well as their associated traffic level γ . Then, 'Same Roundabout' refers to whether the pair of exit probability models was trained from the same roundabout, i.e., only with a traffic condition difference. The ' $\Delta\gamma$ ' column refers to the absolute amount of difference between the traffic levels of the two considered sets of roundabout tracks. Namely, provided a pair of track data sets $[(R_1, a \leq \gamma < b), (R_2, c \leq \gamma < d)]$, it is defined as $\Delta\gamma = |a - c| = |b - d|$. Finally, the 'UC' column

Table 3: Training Set to Investigate the Relationship between Roundabout Traffic Disparities and UC

Pair of Exit Probability Models	Same Roundabout	$\Delta\gamma$	UC
(Round_2, $0.2 \leq \gamma < 0.3$) / (Round_1, $0.3 \leq \gamma < 0.4$)	False	0.1	0.28 ± 0.003
(USA_SR, $0. \leq \gamma < 0.4$) / (USA_SR, $0.2 \leq \gamma < 0.3$)	True	0.1	0.59 ± 0.006
(USA_FT, $0. \leq \gamma < 0.4$) / (USA_EP, $0.1 \leq \gamma < 0.2$)	False	0.2	0.35 ± 0.001
(Round_1, $0.2 \leq \gamma < 0.3$) / (Round_2, $0.2 \leq \gamma < 0.3$)	False	0.0	0.26 ± 0.002
(Round_0, $0.1 \leq \gamma < 0.2$) / (USA_FT, $0.2 \leq \gamma < 0.3$)	False	0.1	0.40 ± 0.001
(Round_0, $0.5 \leq \gamma < 0.6$) / (Round_1, $0.3 \leq \gamma < 0.4$)	False	0.2	0.53 ± 0.002
(USA_EP, $0.2 \leq \gamma < 0.3$) / (DEU_OF, $0.4 \leq \gamma < 0.5$)	False	0.2	0.31 ± 0.003
...			
(USA_EP, $0.2 \leq \gamma < 0.3$) / (CHN_LN, $0.1 \leq \gamma < 0.2$)	False	0.1	0.41 ± 0.003
(USA_FT, $0.3 \leq \gamma < 0.4$) / (DEU_OF, $0.3 \leq \gamma < 0.4$)	False	0.0	0.23 ± 0.001
(CHN_LN, $0.1 \leq \gamma < 0.2$) / (Round_0, $0.1 \leq \gamma < 0.2$)	False	0.0	0.48 ± 0.003
(CHN_LN, $0.0 \leq \gamma < 0.1$) / (USA_EP, $0.2 \leq \gamma < 0.3$)	False	0.2	0.41 ± 0.003
(USA_FT, $0.2 \leq \gamma < 0.3$) / (USA_EP, $0.1 \leq \gamma < 0.2$)	False	0.1	0.35 ± 0.002
(Round_0, $0.3 \leq \gamma < 0.4$) / (USA_FT, $0.4 \leq \gamma < 0.5$)	False	0.1	0.40 ± 0.002

lists the associated UC values for each pair with a 95% confidence interval, obtained by running 20 computations per pair from different random samplings of training entries.

Provided with the data of Table 3, we analyze the correlation between the difference in the traffic conditions $\Delta\gamma$ of two set of tracks data and the UC value of their resulting exit probability models. Namely, we compute the correlation coefficient between 'Same Roundabout' and 'UC', as well as ' $\Delta\gamma$ ' and 'UC'. The aim of this computation is both to:

- Investigate whether traffic plays an important role in the exit probability model UC values.
- Compare the influence of traffic to that of geometrical features. Do two distinct roundabouts with similar traffic conditions lead to higher UC values than the same roundabout under different traffic conditions?

On the one hand, the correlation coefficient of $\Delta\gamma$ with UC equals to $r = 0.068$, and $r = -0.035$ when the entries which involve the same roundabout, i.e., 'Same Roundabout' is *True*, are removed. On the other hand, 'Same Roundabout' and 'UC' are highly correlated with $r = 0.77$. As such, no link can be observed between the traffic conditions from which two exit probability models are trained and their UC similarity. On the contrary, regardless of traffic conditions, models trained on a single roundabout feature a high UC correlation, suggesting a strong impact of roundabout geometry rather than traffic conditions on the UC similarity of exit probability models.

Specifically, Figure 15 illustrates the detailed relationship between the $\Delta\gamma$ and UC . It shows both:

- The distribution of the $\Delta\gamma$ and UC values in Table 3, in the top-left and bottom-right corners.
- Scatter plots illustrating the $(\Delta\gamma, UC)$ pairs from Table 3, in the top-right and bottom-left corners.

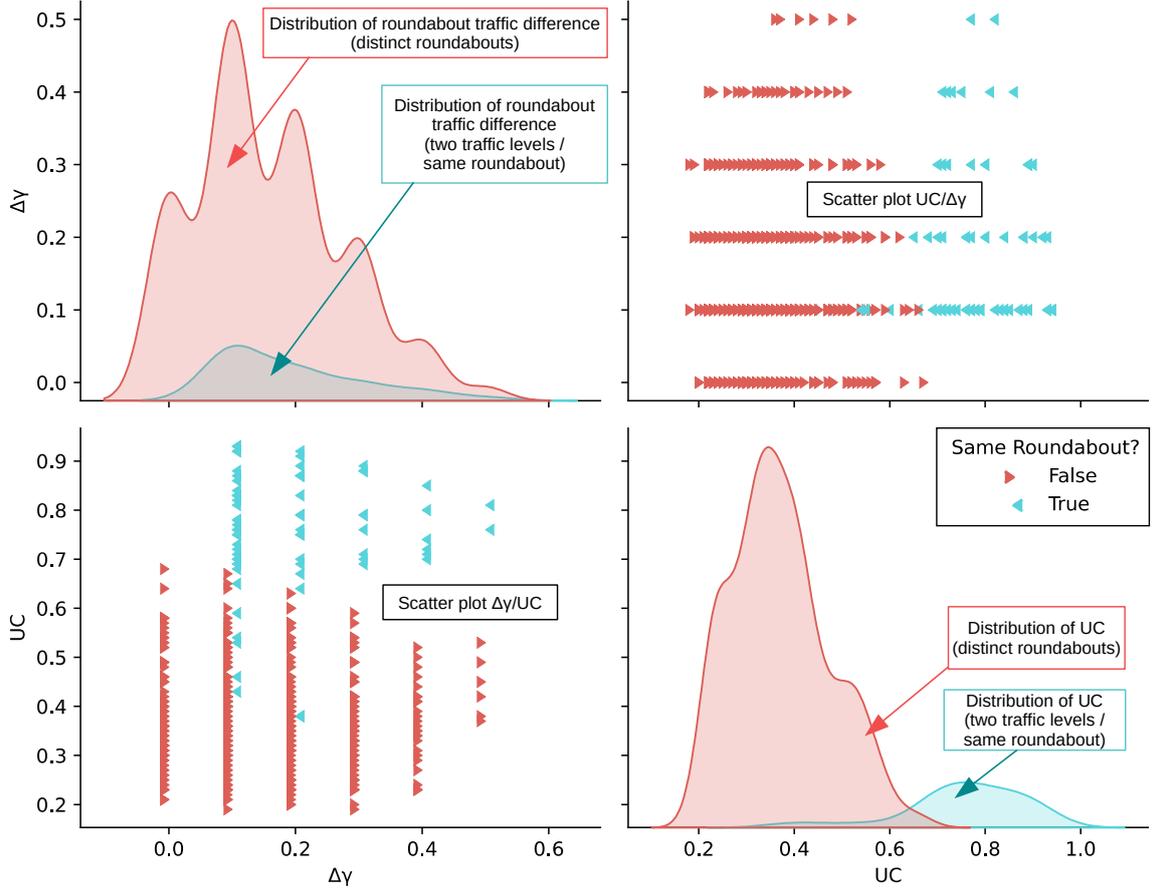


Figure 15: Relationship between UC and the Difference of Traffic Congestion Levels $\Delta\gamma$

What is more, the data illustrated by Figure 15 is partitioned in two classes, i.e., data points which were obtained from (i) the same roundabout with different traffic conditions in light-blue colors, and (ii) distinct roundabouts with potentially similar traffic conditions in red colors.

In turn, the obtained visualization illustrates the lack of correlation between $\Delta\gamma$, i.e., the difference in the traffic congestion level of input track data, and the UC similarity of the resulting exit probability models. This pattern is observed regardless of whether data points from only the same roundabouts, only distinct roundabouts, or any roundabouts are considered. On the contrary, Figure 15 illustrates the impact of the geometric characteristics of two roundabouts in the UC similarity of resulting exit probability models. Namely, models trained on the same roundabout, i.e., with the same geometric shape, as shown in light-blue points, largely lead to higher UC values than models trained on distinct roundabouts, regardless of traffic conditions.

Nonetheless, in the case of distinct roundabouts, it can be observed that the highest obtained UC values decrease as $\Delta\gamma$ increases. Namely, the distinct roundabouts which lead to the highest UC values feature low traffic differences. In turn, while they do not indicate high similarity values alone, low differences in traffic congestion levels could act as a secondary factor and a prerequisite to reach the highest UC values, i.e., in the $[0.65, 0.7]$ range in this case.

4.3 Discussion

In this section, the characteristics and features of two roundabouts which lead to an increased expected similarity of their associated resulting exit probability models were identified. Generally, we identified contextual features which are relevant to semantically describe the context of training and usage of exit probability models. The pertinent features include the following geometric features: the number of entry legs, as well as radius and width of a roundabout. Traffic conditions played a minor role in the similarity of exit probability models.

In turn, classes of exit probability models can be described and extracted to, for example, (i) apply existing models to unknown roundabouts with a similar context, or (ii) complete the training data of an exit probability model from tracks data extracted from distinct roundabouts with a similar context. As such, as a step forward and in the next section:

- We define a set of *similarity conditions*, based on the semantic description of the context of two roundabouts, to assess whether their context can be considered *similar*.
- To verify the relevance of the defined *similarity conditions*, we investigate the accuracy of knowledge transfer based on the context similarity of two distinct roundabouts.
 - The accuracy of creating exit probability knowledge on an unknown roundabout for which no exit probability model has been trained is compared considering two approaches, namely, using existing exit probability models trained in (i) *similar* contexts, and (ii) non-similar or randomly selected contexts.
 - The accuracy of model training for a new roundabout is compared considering two approaches, namely, completing the training data with vehicle tracks extracted from roundabouts with (i) a *similar* context, and (ii) a non-similar or randomly selected context.

5 Accuracy of Context-Based Roundabout Exit Probability Knowledge Transfer

In Section 4, we identified a set of factors and rules which impact the information theoretic similarity between two roundabouts. Namely, geometric factors such as the number of entry legs, the radius and width of roundabouts were found to significantly influence the similarity of the resulting exit probability models. On the contrary, traffic conditions had no clear influence. In turn, we use the obtained similarity factors to define a general semantic description of the context of training and usage of exit probability models. We aim at extracting classes of exit probability models with similar context, which support knowledge transfer with increased performance.

Namely, it is a costly procedure to train an exit probability model for a new, unknown roundabout. In the datasets used in this paper, the input vehicle track data required for training is obtained by flying a drone multiple times over each roundabout and extract the position of vehicles from camera images, which implies

complex logistics. Instead, the definition of contextual semantic rules to identify existing exit probability models which can be accurately applied to distinct unknown roundabouts would allow the exit probability estimation on a large set of roundabouts from a much smaller set of exit probability models.

In this section, we define semantic rules to assess the similarity of the context of two roundabouts. In turn, we assess whether the context similarity of two roundabouts can be related to a higher accuracy of the (i) application and (ii) training of an exit probability model from one roundabout to the other:

- To begin with, we define a set of context similarity conditions based on the context-influencing features identified in Section 4. A context similarity condition is a function which takes two exit probability model contexts as input and returns whether or not they should be considered *similar*. We define three conditions which are increasingly strict in terms of their requirements for two contexts to be considered similar.
- In turn, we show that estimating exit probabilities on an unknown roundabout, i.e., for which no model was directly trained, features a higher accuracy when using existing models trained on distinct roundabouts with a *similar* context, as opposed to randomly-selected other models, potentially with non-similar context.
- Lastly, we consider exit probability models being trained for new roundabouts for which a limited amount of training data is available. We show that the models perform more accurately when the available training data is completed from vehicle tracks extracted from roundabouts which feature a *similar* context, than from other randomly-selected roundabouts.

5.1 Context Similarity of Exit Probability Models

To begin with, based on the relevant features to describe the context and the similarity between two roundabouts, as identified in Section 4, we define a set of *similarity conditions*. They are binary, semantically described functions which compare the context of two roundabouts and assess whether the contexts can be considered similar for the use case of exit probability knowledge usage and training.

As identified in Section 4, the key contextual features which influence the information theoretic similarity of two roundabouts are geometric, specifically, the difference of (i) the number of entry legs, (ii) the width, and (iii) the radius of the two roundabouts. On the one hand, the difference of entry legs and width is negatively correlated with the *UC* similarity of the resulting exit probability models, i.e., lower differences can be related to an expected higher similarity. On the other hand, higher *UC* similarity values are obtained below a threshold of radius difference, which was identified at $8.12m$ for the roundabouts considered in this study, listed in Table 1.

In turn, we define a set of similarity conditions of gradual strictness. Namely, the 'Strict Similarity' Condition describes a strict condition of similarity between the context of two roundabouts, which sets constraints on all three of the radius, width, and number of entry legs of the roundabout. In turn, the 'Moderate Similarity' Condition is an instance of a moderate similarity condition. It follows the path

to the leftmost leaf of the regression tree of Figure 11 which associates higher UC similarity values with the $\Delta Entries$ and $\Delta Radius$ conditions, but has no constraint on $\Delta Width$. Yet, the constraint on $\Delta Radius$ is lowered from $8.12m$ in the regression tree to $6m$. Finally, the 'Weak Similarity' Condition is a weaker condition, which follows the condition identified in the regression tree of Figure 11, without lowering the $\Delta Radius$ threshold.

The specific choice of the thresholds on $\Delta Entries$, $\Delta Radius$, and $\Delta Width$ in the 'Strict Similarity', 'Moderate Similarity', and 'Weak Similarity' Conditions is given as an example of similarity conditions of gradual strictness. Values are chosen such that the number of roundabouts which match the similarity conditions grow with each stricter condition, based on the roundabout contextual data listed in Table 1.

$$\Delta Entries = 0 \wedge \Delta Radius \leq 2.0m \wedge \Delta Width \leq 2.0m \quad (\text{Strict Similarity})$$

$$\Delta Entries = 0 \wedge \Delta Radius \leq 6.0m \quad (\text{Moderate Similarity})$$

$$\Delta Entries = 0 \wedge \Delta Radius \leq 8.12m \quad (\text{Weak Similarity})$$

5.2 Performance of Context-Aware Model Application

In this section, we assess whether the context similarity of two roundabouts, assessed by the similarity conditions defined in Section 5.1, affects the accuracy of using exit probability models. Namely, we aim to compare the accuracy of the exit probability which is computed by an exit probability model which is applied on a distinct roundabout that the one it was trained from, when it features (i) a similar context, and (ii) a randomly-selected or non-similar context. Generally, this section investigates the role of context-aware exit probability model application on the accuracy of the produced knowledge.

5.2.1 Evaluation Procedure

For each roundabout R_{target} and for each similarity condition E , i.e., one of the three similarity conditions defined in Section 5.1, the accuracy of exit probability knowledge created from models trained in roundabouts with a *similar* context is compared to that of models created in *non-similar* or *random* contexts, in the following process:

1. For each roundabout $R, R \neq R_{target}$ listed in Table 1:
 - (a) A set V_{target} of 1000 validation entries is extracted from the vehicle tracks of the target roundabout R_{target} .
 - (b) An exit probability model is trained from R vehicle tracks and applied to the validation data V_{target} . Namely, the model predicts an exit iff the estimated exit probability is strictly greater than 0.5.
 - (c) The obtained exit predictions are matched to the ground truth, i.e., the observed real behavior of vehicles in the V_{target} validation data, to compute various accuracy metrics.

2. The roundabouts and results are partitioned into three distinct groups:
 - Roundabouts R which feature a similar context than R_{target} , i.e., $E(R, R_{target})$.
 - Roundabouts R which do not feature a similar context than R_{target} , i.e., $\neg E(R, R_{target})$.

In turn, the accuracy score associated with the roundabouts in each group are compared to assess whether context-aware exit probability knowledge creation improves the accuracy of the produced knowledge.

5.2.2 Considered Accuracy Metrics

In turn, we consider several accuracy metrics to assess the performance of the exit probability prediction of the models associated with each roundabout R , featuring similar or non-similar contexts, on the tracks data of the target roundabout R_{target} :

- *Accuracy*, i.e., the proportion of correct predictions among the total 1000 validation entries.
- *Precision*, i.e., $\frac{TP}{TP+FP} \in [0, 1]$, with TP and FP the number of, respectively, true positives and false positives. Namely, a true positive is a case where the model predicted an exit, which did happen. On the contrary, a false positive is a case where the model predicted an exit, but the considered vehicle did not exit. In turn, precision penalizes false positives. It is a key asset in this use case as a serious collision risk may occur if a vehicle enters a roundabout based on a false prediction of an incoming vehicle to exit the roundabout before any chance of conflict.
- *F1-score*, i.e., the harmonic mean of *precision* and *recall*. Recall is a metric which is similar to precision. It penalizes false negatives, i.e., false predictions that a vehicle will stay in the roundabout, which are less critical than false positives. While they may increase queues because of an entering vehicle repeatedly predicting incoming vehicles to stay in the roundabout, it does not induce an immediate collision risk. Nonetheless, it is relevant to consider the F1-score as a mean of both precision and risk, to penalize conservative models which reach a high precision by predicting that vehicles will stay in the roundabout in clear exit situations.

5.2.3 Results

Strict Context Similarity Condition To begin with, Figure 16 illustrates the obtained results in terms of *accuracy*, when using the 'Strict Similarity' Condition as the similarity condition to determine whether the context of two roundabouts can be considered *similar*. The results are shown for each of the eight considered target roundabouts on the x-axis. In turn, for each target roundabouts, blue and red-colored points show the accuracy obtained when predicting the exit of vehicles using, respectively, models trained in a distinct roundabout with a similar, and non-similar context. What is more, the black-colored dot represents the accuracy obtained when estimating probabilities using a model from tracks data extracted from the same target roundabout, for reference. Each value point is accompanied

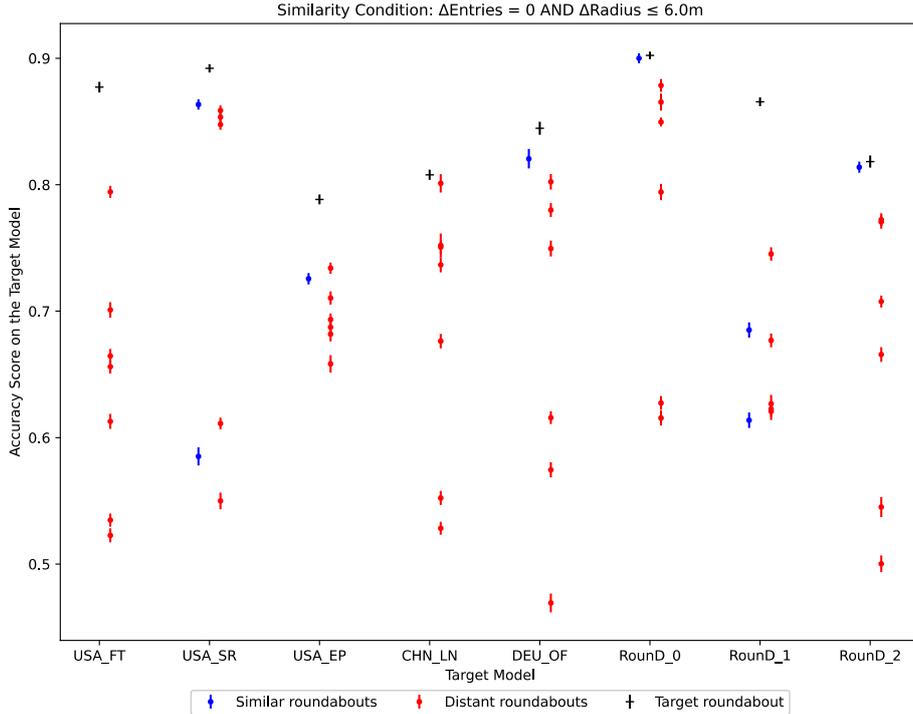


Figure 17: Accuracy of Exit Probability Knowledge Transfer for Model Application using the 'Moderate Similarity' Condition

tion, models trained in similar contexts yield higher accuracy metrics than others for DEU_OF, Round_0, Round_2, and similar values of accuracy metrics for USA_EP. Yet, noise can be observed for USA_SR and Round_1, in which similar context models perform less accurately than some non-similar models, potentially due to the weaker constraints of the considered similarity condition.

As an interpretation, the weaker Moderate Similarity Condition increases the amount of matches and similar contexts for roundabouts listed in Table 1. On the other hand, it also introduces noise, from which a few models are flagged as having a similar context to a target roundabout perform less accurately than non-similar models. For example, out of the two similar models identified for USA_SR, one model is performing more accurately than other models, while the other yields relatively low accuracy metrics. As such, selecting a single model based on a moderate condition of similarity involves a risk of relying on a 'noisy' model which yields relatively low accuracy metrics.

To alleviate the risk of selecting a single model which yields low accuracy despite being considered moderately similar, we group the exit probability predictions of several similar models in an *ensemble voting* approach, as described in [18]. The aim of this approach is to reduce the impact of potential noise in the probability produced by a single model. Namely, the following process is implemented:

- The n models M_1, \dots, M_n which have been trained in a similar context to R_{target} , according to the considered similarity condition E (here the 'Moderate Similarity' Condition), are identified and listed in a $M_{similar}$ set of n elements.
- To evaluate the set of 1000 validation entries V_{target} extracted from R_{target} tracks data, as defined in Section 5.2.1:

1. Each validation entry $v \in V_{target}$ is fed as input to each model of $M_{similar}$, to produce an exit probability value, i.e., $\forall i \in [1, n], p_i = M_i(v)$, with $M_i(v)$ the output of the application of M_i on v , i.e., a value of exit probability $p_i \in [0, 1]$.
2. The ensemble voting technique of voting regression is used for averaging the predictions of a set of classifiers with the same interface. We implement voting regression for all the predictions of exit probability produced by the models M_1, \dots, M_n . Namely, the probability of exit associated with the validation entry v is the average of the probabilities p_1, \dots, p_n : $p_{voting} = \frac{\sum_{i=1}^n p_i}{n}$.
3. p_{voting} is considered as the prediction of the ensemble of the available models which have been trained in a similar context than R_{target} . In turn, its accuracy on V_{target} is evaluated, following the process described in Section 5.2.1.

To compare the accuracy of the ensemble prediction of similar models with other models, the procedure is repeated for the set of models which have been trained in a non-similar context than R_{target} , as well as for the set of all models except the one trained on R_{target} tracks data. Figure 18 illustrates the obtained results, still using the 'Moderate Similarity' Condition. In addition to the content presented in Figure 17 and for each roundabout, Figure 18 shows the accuracy of the ensemble voting predictions of the grouped (i) similar models, (ii) non-similar models, and (iii) all models except target, respectively, in light-blue, red, and green pentagons. Each point features a 95% confidence interval, obtained by training each model 20 times with different random samplings of training data extracted from the vehicle tracks.

We observe that the ensemble voting alleviates the noise related to the 'Moderate Similarity' Condition. Namely, while models which are similar to the context of RoundD_1 yield relatively low accuracy when taken independently, their ensemble predictions reach a significant improvement of accuracy scores compared with any other model or ensemble of models. Similarly, the ensemble predictions of the models similar to the context of USA_SR reach a significantly improved accuracy, compared with the relatively low accuracy reached by a similar model taken independently.

Generally, the average accuracy of ensemble voting from similar exit probability model reaches $80.4 \pm 4.6\%$. It is equivalent to the single most accurate non-similar model of each roundabout, with a $0.8 \pm 4.3\%$ change in accuracy in average. In turn, ensemble voting is a promising approach to consistently get accurate results when using moderate to weak similarity conditions, which can be required in cases where the amount of available models is limited.

Weak Context Similarity Condition Lastly, we investigate the impact of using a weak context similarity condition as defined in the 'Weak Similarity' Condition, e.g., in cases where a strongly limited amount of trained models are available to assess exit probabilities in an unknown roundabout. Figure 19 shows the accuracy of context-based exit probability knowledge creation, as in Figure 18, but using the 'Weak Similarity' Condition as a context similarity condition. As a result, more pairs of exit probability models match the similarity condition, for the USA_SR, USA_EP, CHN_LN, Round_0 and Round_1 roundabouts.

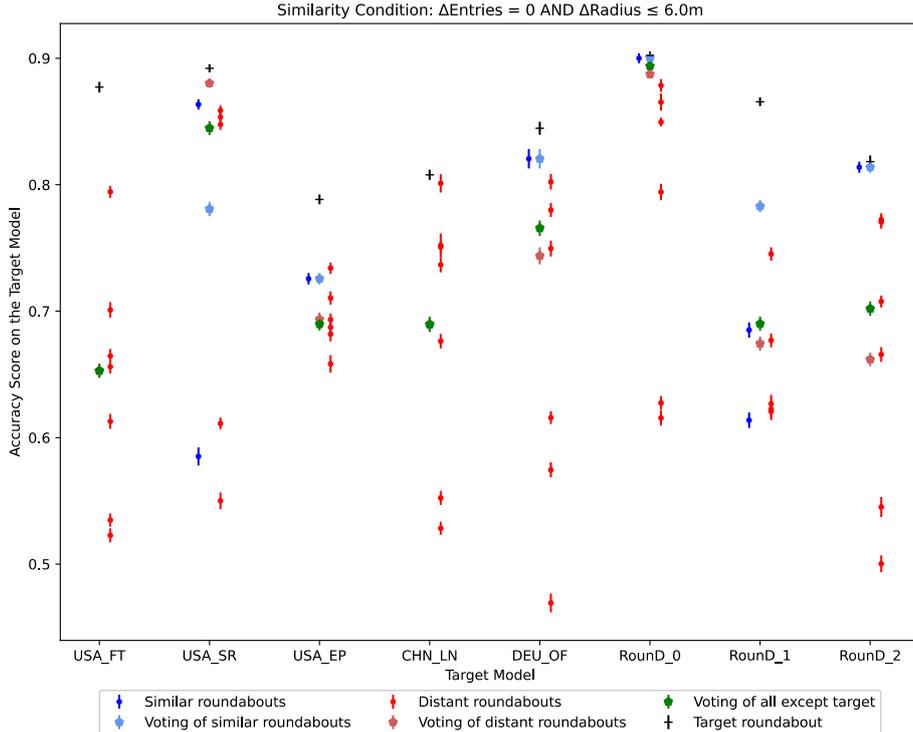


Figure 18: Accuracy of Ensemble Exit Probability Knowledge Transfer for Model Application using the 'Moderate Similarity' Condition

Nonetheless, the models which were newly flagged as 'similar' to distinct roundabout contexts perform less accurately than models flagged as 'similar' following stricter similarity conditions. This shows that a threshold exists in the level of 'strictness' of context similarities in terms of the accuracy of the produced knowledge. Namely, context-aware exit probability knowledge creation performs more accurately than context-agnostic knowledge creation, provided models with a sufficiently strong context similarity are used.

5.2.4 Impact & Discussion

In this section, we investigated context-aware knowledge transfer through the application of an existing exit probability model on a distinct roundabout for which no model was trained. We observe increase accuracy metrics of the produced exit probability knowledge when the used model has been trained in a *similar* context than the roundabout it is applied to.

To define whether the context associated with two roundabouts is *similar*, we described a set of three gradually less strict similarity conditions, in the 'Strict Similarity', 'Moderate Similarity', and 'Weak Similarity' Conditions. In turn, we observed the following results:

- Using a strict context similarity condition, i.e., the 'Strict Similarity' Condition, a low amount of the roundabouts listed in Table 1 feature a similar context. In turn, applying models on similar roundabouts significantly improved the accuracy metrics of the produced exit probability knowledge.
- Using a moderate context similarity condition, i.e., the 'Moderate Similarity'

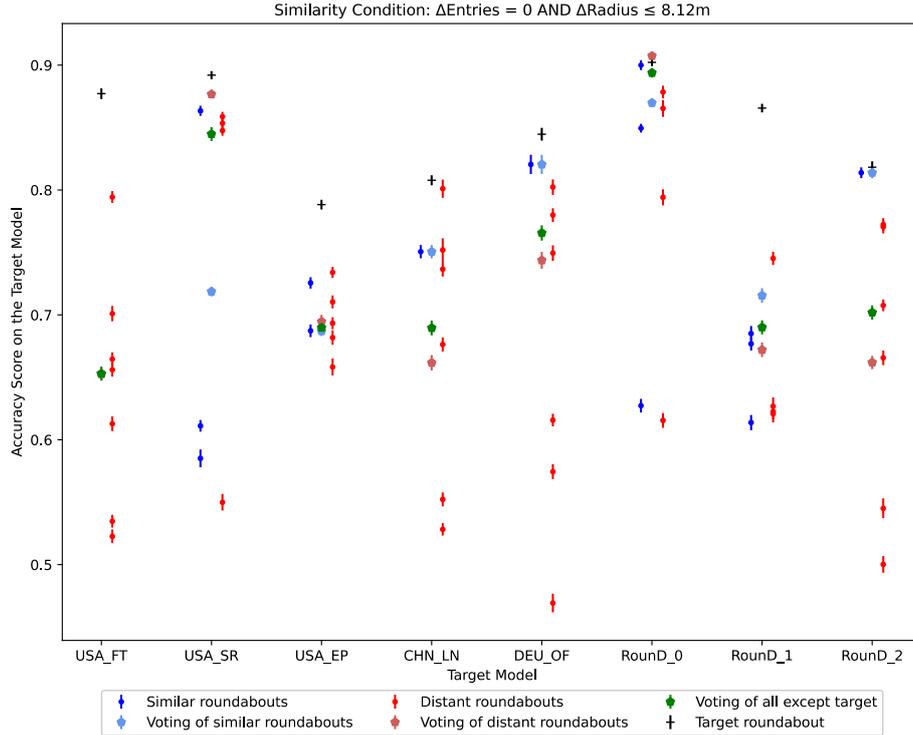


Figure 19: Accuracy of Ensemble Exit Probability Knowledge Transfer for Model Application using the 'Weak Similarity' Condition

Condition, more of the roundabouts listed in Table 1 feature a similar context. Yet, some noise is introduced in the results, i.e., some similar models perform less accurately than non-similar models, as the context similarity gets weaker. This can be mitigated by grouping the exit probability predictions of several similar models through ensemble voting.

- Using a weak context similarity condition, i.e., the 'Weak Similarity' Condition, further roundabouts listed in Table 1 feature a similar context. Yet, no pattern is identified which improves the accuracy of similar models using this overly weak similarity condition.

The obtained results show the impact and role of context in knowledge creation and knowledge transfer in the use case of roundabout exit probability models, and provide hints on its general role in other highly context-dependent models in the vehicular and transportation domains. As such, we argue that an analysis of the context of training of models, as well as condition for context similarity should be tuned for each context-dependent model.

What is more, the results illustrate the need for a balance to be found between the number of available models for knowledge transfer and the level of strictness of the context similarity condition. Namely, if the context similarity condition is too strict, few to no available models may be similar to a specific context. On the other hand, if the context similarity condition is too permissive, models judged as similar models are less likely to perform accurately, relatively to other models.

Generally, this section demonstrates the impact and potential for a semantic description of the context of training and application of context-dependent models, notably in the vehicular and transportation domains. Provided with a semantic

description of the current driving context as well as the context of application of available models, vehicles could efficiently network models to be applied in the right context, to reach a higher accuracy.

5.3 Performance of Context-Aware Model Training

In Section 5.2, the similarity of the context of training of an exit probability model was shown to positively impact context-aware knowledge application, i.e., the use of a fully trained model on a distinct roundabout with a similar context. In turn, in this section, we investigate the performance of context-aware model training.

As shown in Section 5.2, while using models trained in a similar context to target roundabouts approached the accuracy of models trained directly on tracks data of the target roundabout, it did not exceed it. Namely, training a model specifically for a target roundabout, using vehicle track data extracted from it, performed with optimal accuracy. Yet, training a model for a specific roundabout requires gathering training data, which is a complex and costly task, potentially flying drones over roundabouts.

In turn, we investigate the possibility of context-aware model training. Namely, we consider the case of training an exit probability model for an unknown roundabout for which little to no training data is available. In turn, we complete the available training data with vehicle tracks extracted from distinct, known roundabouts which feature a similar context. Finally, the accuracy of the obtained model is compared with the case in which training data is completed from randomly-selected roundabouts, i.e., not necessarily with a similar context.

5.3.1 Evaluation Procedure

We contribute an algorithm of context-aware knowledge transfer for model training in the use case of roundabout exit probability models. The algorithm is given in Figure 20. It involves the completion of the limited available training data of an exit probability model from tracks extracted from roundabouts which feature a similar context.

In Section 5.2, the 'Moderate Similarity' Condition showed a balance between the number of matching roundabout contexts and the accuracy of context-aware knowledge application. In turn, we choose it as a similarity condition E to investigate the performance of context-aware exit probability model training through the procedure described in Figure 20.

To procedure of Figure 20 is applied for each $\delta \in \{0.0, 0.1, 0.2, \dots, 1.0\}$, and for each roundabout R_{target} of Table 1 which has at least one distinct roundabout with a similar context. This excludes USA_FT and CHN_LN. This process is repeated 20 times with different random samplings of training data, to compute and compare the 95% confidence intervals of the accuracy scores of models trained from data sourced from (i) similar context, (ii) non-similar context, and (iii) all other roundabouts.

5.3.2 Results

Figure 21 illustrates the obtained results for each of the considered roundabouts in six graphs, which can be read as follows:

1. We name Rs the set of the considered roundabouts, i.e., listed in Table 1. We consider the use case in which an exit probability model is being trained for an unknown roundabout $R_{target} \in Rs$, for which a limited amount of training data is available. In turn, it must be completed with training data extracted from other roundabouts.
2. Using an input similarity condition E , we compute:
 - (a) $Rs_{similar}$, the set of roundabouts which have a context which is *similar* to R_{target} according to E : $\forall R \in Rs \setminus R_{target}, E(R, R_{target}) \iff R \in Rs_{similar}$.
 - (b) $Rs_{distant}$, the set of roundabouts which have a context which is *non-similar* to R_{target} , i.e., contextually 'distant', according to E :
 $\forall R \in Rs \setminus R_{target}, \neg E(R, R_{target}) \iff R \in Rs_{distant}$.
 - (c) $Rs_{others} \equiv Rs_{similar} \cup Rs_{distant} \equiv Rs \setminus R_{target}$.
3. As in Section 2, we train an exit probability model targeting R_{target} with a total of 5000 training entries. Yet, we consider that only a limited amount of training data has been sensed for R_{target} , i.e., only a fraction $\delta \in [0, 1[$ of the required number of training entries is available in the training data of R_{target} . In turn, we complete the $\delta \cdot 5000$ available entries with $(1 - \delta) \cdot 5000$ training entries extracted from the vehicle tracks of other known roundabouts.
4. Namely, an exit probability model is trained for R_{target} by completing the training data using $(1 - \delta) \cdot 5000$ training entries extracted from other known roundabouts with a similar context, i.e., in $Rs_{similar}$, as follows:
 - (a) Let $N = \|Rs_{similar}\|$ the number of known roundabouts which feature a similar context to R_{target} , according to the similarity condition E .
 - (b) An even number of training data is extracted from each similar roundabout $R_i \in Rs_{similar}$. Namely, $\forall i \in [1, N], \frac{(1-\delta) \cdot 5000}{N}$ entries are extracted from R_i .
 - (c) The set of training data composed of the $\delta \cdot 5000$ entries from R_{target} as well as the $(1 - \delta) \cdot 5000$ entries from the roundabouts of $Rs_{similar}$ is randomly shuffled, and used to train a model to be applied to R_{target} tracks.
5. The training process described in Item 4 is repeated to train two new models, replacing $Rs_{similar}$ by, respectively, $Rs_{distant}$ and Rs_{others} .
6. The accuracy metrics of each of the three obtained exit probability models, respectively, trained from fractions of training data from (i) similar, (ii) non-similar, and (iii) all other models are computed, using a 1000-entry evaluation set extracted from the tracks data of R_{target} . In turn, the accuracy of context-aware model trained can be compared with other approaches which do not take the context of collection of the training data into account.

Figure 20: Procedure for Exit Probability Training Knowledge Transfer and Comparison with Context-Agnostic Approaches

- The x-axis shows the evolution of the proportion δ of training data which is directly extracted from the target roundabout. In turn, the remaining $(1 - \delta)$ fraction of data is completed from distinct roundabouts. Namely, $\delta = 0.0$ indicates that the model is trained fully from distinct roundabouts data, and $\delta = 1.0$ indicates that the model is fully trained directly from the track data of the target roundabout.
- The y-axis shows the accuracy of the trained models, with 95% confidence intervals, in the following scenarios:
 1. *Context-aware model training*, i.e., the training data was completed with data from distinct roundabouts with a *similar* context, in blue colors.
 2. The training data was completed with data from distinct roundabouts with a *non-similar*, i.e., 'distant' context, in red colors.
 3. The training data was completed with data from all available distinct roundabouts, i.e., 'other' roundabouts, in green colors.

In all the considered roundabouts, context-aware exit probability model training significantly improved the accuracy of the trained model, specifically when a low proportion of training data is available from the target roundabout, i.e., $\delta \leq 0.2$. Specifically, when no training data is available for the target roundabout, using training data from roundabouts with a similar context yields a $80 \pm 5\%$ accuracy, i.e., a $8.5 \pm 4.8\%$ increase from using non-similar roundabouts.

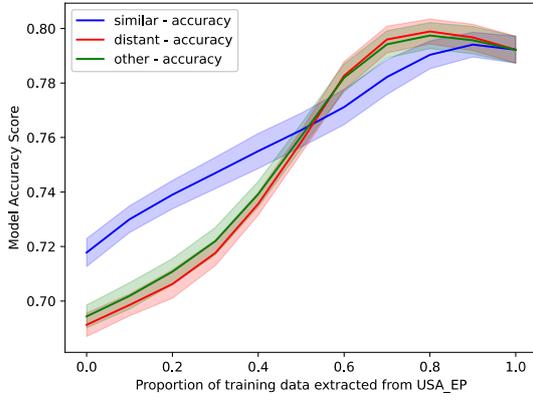
What is more, for DEU_OF, Round_0, and Round_2, near optimal accuracy is obtained from $\delta = 0$ and remains stable as the proportion of training data from the target roundabout increases. On the contrary, context-agnostic approaches produce model which reach optimal accuracy at higher values of δ , respectively, $\delta = 0.7$ for Round_0 as well as Round_2, and $\delta = 1.0$ for DEU_OF.

The obtained results demonstrate the impact and role of context on the accuracy of knowledge training in the case of roundabout exit probability knowledge, and opens perspectives on the potential generic impact of context-aware model training in vehicular, or transportation applications.

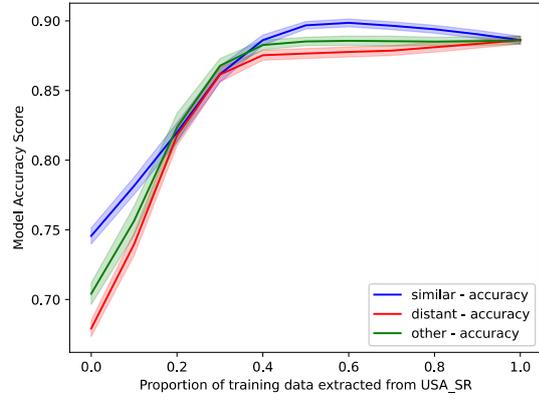
5.3.3 Impact & Discussion

In this section, we studied the role and impact of context on the accuracy of knowledge model training, in the applied case of roundabout exit probability knowledge. Namely, we consider the case in which a limited amount of training data is available for a roundabout. In turn, the results showed a significant increase in accuracy scores when the training data is completed from data collected in a *similar* context.

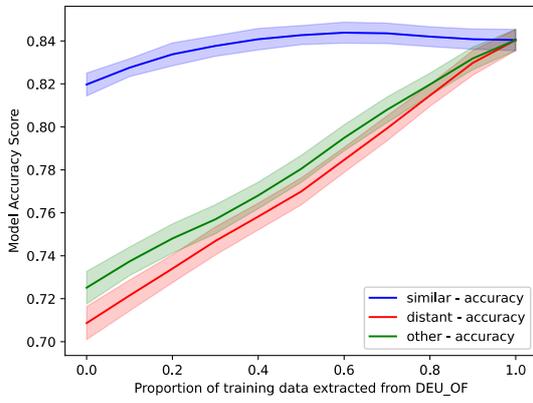
Generally, the obtained results open perspective on the potential impact of context networking in vehicular and transportation applications. Vehicles are increasingly involved in ML model training through distributed and cooperative training approaches such as Distributed ML [19] or Federated ML [20]. As the vehicular environment is highly dynamic, the driving environment of a vehicle is likely to evolve rapidly over time. As such, learning coordinators have optimized training client selection mechanisms based on physical characteristics such as channel link [21], computational power [22], energy usage [23], or mobility [24]. Yet, to the best of our knowledge, no mechanism has been defined to ensure that both:



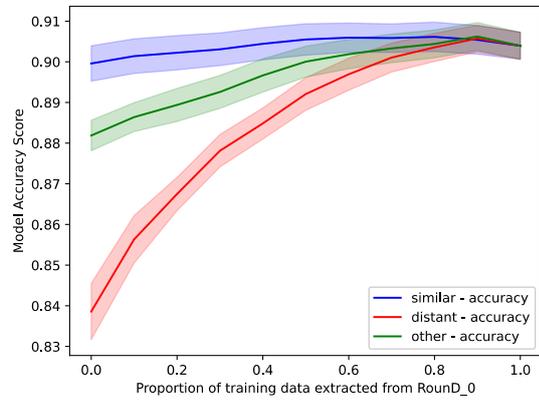
(a) USA_EP



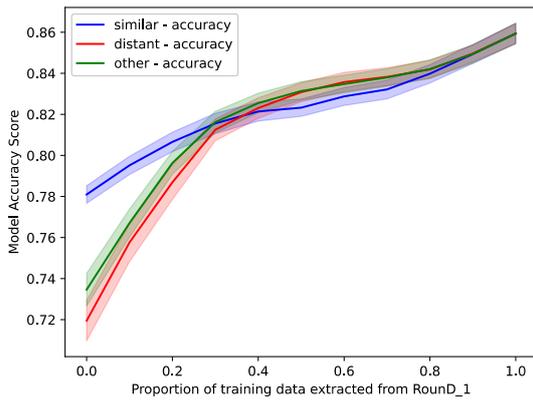
(b) USA_SR



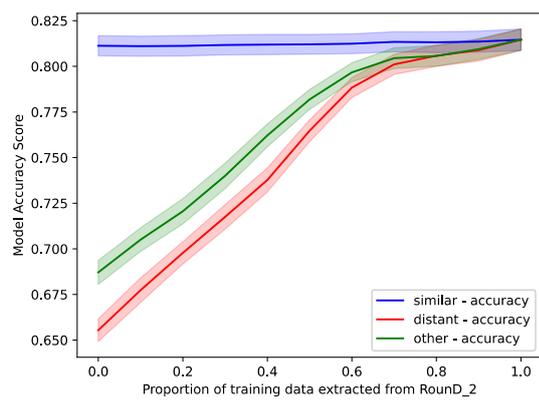
(c) DEU_OF



(d) Round_0



(e) Round_1



(f) Round_2

Figure 21: Accuracy of Ensemble Exit Probability Knowledge Transfer for Model Training using the 'Moderate Similarity' Condition

- The vehicle which is selected for training owns the right type of training data. For example, in the exit probability case, training vehicles must own the kinematics data of other sensed vehicles in a roundabout, which means they must have crossed a roundabout in recent history before maintenance mechanisms discard the training data.
- The context in which the training data was sensed matches the desired context of training of the model being trained. Namely, a key contribution of this paper is the significant role of context in knowledge training accuracy. According to the results of this section, contributing training data sensed in the wrong context, e.g., from a 7-entry roundabout, whereas the trained model targets a 4-entry roundabout, reduces the accuracy of the final model.

In turn, context-based knowledge training opens perspectives of (i) extending the amount of available training data by using training data sensed in the right context regardless of its source, while (ii) avoiding an accuracy loss related to selecting context-irrelevant data. Generally, the obtained results in terms of the accuracy of both context-aware knowledge application and training suggest a need for the definition and networking of context semantics for applications of ML which are highly relevant on the context. In turn, a generic *knowledge training as a service* could be defined for cooperative training in vehicular networks, which inherently feature heterogeneous driving and training contexts. Knowledge training as a service has the potential to match the semantic annotations describing a model with the context of driving of each vehicle. Then, appropriate training nodes can be selected to train knowledge with the right context and reach higher accuracy scores.

6 Conclusion

Roundabouts may be challenging intersections to cross for highly-automated vehicles. They are yield-type intersections which may require negotiation which other vehicles to be crossed safely and efficiently. Namely, in cases of congested traffic, understanding the intentions of vehicles which are already in the roundabout is necessary to take the decision of whether to enter the roundabout. If highly-automated vehicle waits until no potentially conflicting vehicle is in the roundabout, it may create long queues and dissatisfaction from human road actors. On the other hand, it is unsafe to take the decision to enter a roundabout without understanding the intentions of other vehicles on whether to exit or stay in the roundabout. In a previous work, we defined a machine learning model to estimate the probability of a vehicle to exit a roundabout based on its position relatively to the next exit. Yet, training such models are costly in terms of training data sensing, and so far requires flying drones over the roundabouts to extract vehicle track data. In this paper, we identify and evaluate the accuracy of a relevant semantic description to describe the context of training of an exit probability model, based on the roundabout it was trained on. We find that a low difference in the number of entry legs, as well as in the width and radius of two roundabouts leads to a higher similarity of their associated exit probability models, which translates to both (i) an increased accuracy when applying an exit probability model on the other roundabout, and (ii) when training a new exit probability model and completing the training data with data obtained in a roundabout with a similar context. The identified semantic description provides

a base to describe the context of usage and training of roundabout exit probability knowledge, and allows to efficiently provide exit probability knowledge on a large spectrum of roundabouts from only a small amount of trained models. Generally, it can be used to support smart knowledge networking operations in vehicular networks, which take the context of usage of models into account when describing, storing, and disseminating knowledge models.

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