Distributed Beamforming Design in Reduced-Rank MIMO Interference Channels and Application to Dynamic TDD

Amel Tibhirt[1,2], Dirk Slock[2] and Yi Yuan-Wu[1]
[1]Orange Labs, Châtillon, France,
[2]Communication Systems Department, EURECOM, France
{amel.tibhirt, yi.yuan}@orange.com, dirk.slock@eurecom.fr

***Abstract*—Employing dynamic Time Division Duplexing (DynTDD) can increase the system-wide spectral efficiency of scenarios with varying and unbalanced uplink (UL) and downlink (DL) data traffic requirements. However when using DynTDD, a different DL/UL slot configuration is likely to be selected by neighboring cells, leading to Cross Link Interference (CLI) between the Base Stations (BS), which is known as BS-to-BS or DL-to-UL interference, and between User Equipments (UE) which is known as UE-to-UE or UL-to-DL interference. Rank deficient channels are frequently encountered in Multi-Input Multi-Output (MIMO) networks, due to poor scattering and keyhole effects, or when using Massive MIMO and moving to mmWave. While the implications of rank deficient channels are well understood for the single user point to point setting, less is known for interference networks. In this paper we study the Degrees of Freedom (DoF) focusing on the UE-to-UE interference, by establishing necessary conditions, for centralized and distributed designs. We discuss the variation of these conditions for different scenarios involving rank deficient channels. In particular we observe reduced DoF gaps between distributed and centralized techniques compared to existing results.**

***Index Terms*—Dynamic TDD, MIMO, rank deficient, interference alignment**

I. Introduction

Dynamic Time Division Duplexing (DynTDD) is one promising way to improve the spectrum efficiency of the wireless communication networks since flexible traffic adaptation can be achieved by dynamically changing uplink (UL) or downlink (DL) transmission direction. DynTDD performance has been analyzed in the literature. [1] investigates the impact of synchronous DynTDD on the performance of the DL/UL in dense small cells networks (SCNs). The results show that DynTDD outperforms the static TDD in terms of average total Time Resource Allocation (TRA), and DL and UL Area Spectral Efficiency (ASE). However, DynTDD also brings some new challenges because of the introduction of cross-link interference (CLI), including DL-to-UL interference (e.g., gNB-to-gNB interference) and UL-to-DL interference (UE-to-UE interference). In this scope many techniques have been proposed for the cross-link interference mitigation. Some authors propose solutions based on an optimization problem, such as Mean Square Error (MSE) minimization with the constraint of the maximum transmit power for the DL BS and UL UE in [2], and the minimization of the total transmit sum power while satisfying a minimum signal-to-interference-plus-noise ratio (SINR) threshold for every UE in [3].

Degrees of Freedom (DoF) is one powerful metric, that allows an approximate characterization of rates at high Signal to Noise Ratio (SNR). In Multi-Input Multi-Output (MIMO) settings, with multiple antennas at transmitters (Tx) and receivers (Rx), DoF maximization requires (spatial) interference alignment (IA).

Some authors works on interference alignment (IA) and interference neutralization methods. [4] proposes an interference alignment based MIMO transmission scheme that effectively addresses the interference problem in DynTDD systems and improves the system capacity. The proposed technique in [5] is a distributed interference alignment (DIA) in a two-cell MIMO network with DynTDD mode under local channel state information (CSI) assumption, and [6] proposes an interference neutralization scheme to eliminate the inter user interference with the help of partial channel state information at the transmitter (CSIT). CLI cancellation methods have been studied in [7], where a joint user scheduling and transceiver design based CLI suppression scheme is investigated in multi-cell multi-user MIMO DynTDD systems to eliminate UL-to-DL interference. An algorithm is designed to avoid scheduling DL UEs which will be interfered by neighboring UL UEs. To suppress DL-to-UL interference, the DL-to-UL interference channel (IC) is divided into several interference sub-channels and a novel precoding and detection design is provided to make the wanted signal channel orthogonal to these interference sub-channels.

The feasibility conditions of IA have been analyzed in [5], [8], [9] [10] [11], [12] and [13]. [5] also mathematically characterizes the achievable DoF of their proposed DIA technique for a given number of antennas at BS/MS. In [8] the authors analyze the feasibility of linear IA for the MIMO Interfering Broadcast Channel (IBC) with constant coefficients. They pose and prove the necessary conditions of linear IA feasibility for general MIMO-IBC. Except for the proper condition, they find another necessary condition to ensure a kind of irreducible interference to be eliminated. They then prove the necessary and sufficient conditions for a special class of MIMO-IBC, where the numbers of antennas are divisible by the number of data streams per user. [9] established a necessary and sufficient condition on IA feasibility for MIMO Interfering Broadcast Multiple Access Channel (IBMAC), which characterizes the optimal sum DoF for various practical network configurations.

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[13] addresses (centralized) attainable DoF for general interference networks with general channel rank conditions. The multiple antennas give each node a certain zero-forcing (ZF) budget that for a given DoF distribution needs to be coordinated between all nodes to handle all interference. [14] gives also an approach to find the spatial filter matrices that offer the desired DoF scheduling and reduce the unwanted interference signal strength to close to zero (rather than absolute zero).

For the work reported in this paper, the starting point was the recently introduced simplified framework from [13] to analyze DoF feasibility (sufficient conditions) in interference networks. This ZF constraint accounting framework is particularly convenient to also analyze MIMO channels with reduced rank. However, it turns out that the presumed sufficient conditions in [13] are actually necessary conditions, and correspond to a role distribution in the well-known proper condition. We also make a correction to the conditions obtained in [13]. We furthermore introduce a second set of conditions based on the Sylvester rank inequality, which may allow to construct sufficient conditions. We apply our thus revised framework to the UL-to-DL MIMO interference scenario arising in DynTDD, which allows to characterize the DoF region as a function of number of users, antennas and channel ranks. We provide analytical expressions for SumDoF for a number of specific scenarios, including the uniform asymmetric scenario (antennas, ranks and user DoFs identical between users on one side, but different between UL and DL). We observe that the results differ (sometimes substantially) from the classical MIMO interference channel (IC) scenario, as here in DynTDD the useful signal links are to nodes outside of the UL-to-DL interfering group. Apart from the centralized design considered so far, we also introduce a distributed design, for which we had analyzed necessary and sufficient DoF conditions before [15]. We actually generalize the distributed design of [15] to a finer granularity of ZF role distribution between Tx and Rx, and we consider another distributed design which consists of unilateral ZF at either UL or DL side. The combination of these options lead to an optimized distributed solution of which the nature varies as a function of the channel ranks involved. It also leads to a reduced DoF gap to centralized designs and there exists a low rank region in which no DoF loss is observed. All this enhances the attractivity of a distributed design, which are in any case far more feasible to implement in practice since they only require local CSI.

II. Dyamic TDD System Model

We consider a MIMO system with two cells, one operating in DL and the other one in UL. Each cell has one BS of M antennas, with K_{ul} and K_{dl} interfering/interfered users in the UL and DL cell respectively. The kth DL UE and the lth UL UE have $N_{dl,k}$ and $N_{ul,l}$ antennas respectively. This scenario brings the two types of interference, the BS-to-BS interference, and the UE-to-UE interference between the UEs that are particularly on the edge of the two cells as shown in Fig 1. The channel between the lth user in the UL cell and the kth user in the DL cell is denoted as $\boldsymbol{H}_{k,l} \in \mathbb{C}^{N_{dl,k} \times N_{ul,l}}$ with $k \in [1, ..., K_{dl}]$ and $l \in [1, ..., K_{ul}]$. Denote $d_{dl,k}$ and $d_{ul,l}$ as the number of data stream from the DL BS to the kth DL UE and from the lth UL UE to the UL BS respectively. We denote the rank of the UE-to-UE interference channel (IC) as $r_{k,l}$. We have $r_{k,l}$ distinguishable significant paths contribute to $\boldsymbol{H}_{k,l}$. Then we can factorize $\boldsymbol{H}_{k,l}$ as:

$$\boldsymbol{H}_{k,l} = \boldsymbol{B}_{k,l} \boldsymbol{A}_{k,l}^H \tag{1}$$

with a full rank matrices $\boldsymbol{B}_{k,l} \in \mathbb{C}^{N_{dl,k} \times r_{k,l}}$ and $\boldsymbol{A}_{k,l} \in \mathbb{C}^{N_{ul,l} \times r_{k,l}}$. We have $\boldsymbol{r}_{k,l}$ distinguishable significant paths contribute to $\boldsymbol{H}_{k,l}$, where distinguishable means with linearly independent antenna array responses from other paths, at both the Tx side and the Rx side.

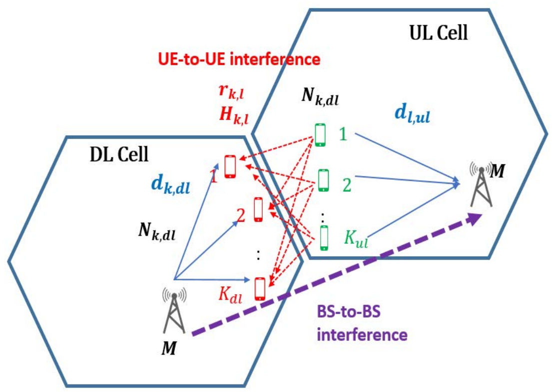

Fig. 1: DynTDD system Model

III. Some State of the Art

Let us start to analyze the UE-to-UE interference. Both the DL and UL UEs will contribute to cancel each link of interference between them. We consider $\boldsymbol{F}_k \in \mathbb{C}^{N_{dl,k} \times d_{dl,k}}$ and $\boldsymbol{G}_l \in \mathbb{C}^{N_{ul,l} \times d_{ul,l}}$ as the Rx/Tx beamforming (BF) matrices at the kth DL and the lth UL users respectively. ZF from UL UE l to the DL UE k requires:

$$\boldsymbol{F}_k^H \boldsymbol{H}_{k,l} \boldsymbol{G}_l = 0, \forall k \in \{1, ..., K_{dl}\}, \forall l \in \{1, ..., K_{ul}\}. \tag{2}$$

For the ZF conditions in interference networks [13] introduces binary variables $\mathbf{1}_{k,l}^T$ and $\mathbf{1}_{k,l}^R$ with the following definition:

$$\mathbf{1}_{k,l}^T = \begin{cases} 1 & \text{if Tx node } l \text{ is active for ZF from } l \text{ to } k, \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$\mathbf{1}_{k,l}^R = \begin{cases} 1 & \text{if Rx node } k \text{ is active for ZF from } l \text{ to } k, \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

We denote by $\boldsymbol{z}_{k,l}^R$ (resp. $\boldsymbol{z}_{k,l}^T$) the number of ZF constraints satisfied by the Rx (resp. the Tx). To cancel all interference from the UL UEs to the DL UEs, according to Theorem 1 in [13], the following conditions should be satisfied (here formulated for the UL2DL interference in the DynTDD problem considered):

$$\boldsymbol{z}_{k,l}^R \mathbf{1}_{k,l}^R + \boldsymbol{z}_{k,l}^T \mathbf{1}_{k,l}^T = \min(\mathbf{1}_{k,l}^R d_{ul,l} + \mathbf{1}_{k,l}^T d_{dl,k}, r_{k,l}) \tag{5a}$$

$$(\mathbf{1}_{k,l}^R, \mathbf{1}_{k,l}^T) \neq (0, 0) \tag{5b}$$

$$d_{dl,k} + \sum\nolimits_{l \in \boldsymbol{I}_{dl,k}} \boldsymbol{z}_{k,l}^R \leq N_{dl,k} \tag{5c}$$

$$d_{ul,l} + \sum\nolimits_{k \in \boldsymbol{I}_{ul,l}} \boldsymbol{z}_{k,l}^T \leq N_{ul,l} \tag{5d}$$

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where $\boldsymbol{I}_{dl,k}$ denotes the set of UL UEs for which the CLI is zero-forced at the kth DL UE, and $\boldsymbol{I}_{ul,l}$ denotes the set of DL UEs for which the CLI is zero-forced at the lth UL UE. Equation (5c) means that DL UE k has $N_{dl,k}$ antennas to receive $d_{dl,k}$ streams while performing ZF to the CLI coming from a certain number of UL UEs. And similarly for an UL UE in (5d). $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1,0)$ or $(0,1)$ or $(1,1)$ means that in this link, the ZF is performed by the Rx or the Tx or by both in a shared fashion. [13] believes that these conditions are sufficient (i.e. correspond to a feasible design of Tx/Rx filters) but actually they are necessary conditions. Furthermore, the first term on the RHS of (5a) is suboptimal in the case $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1,1)$, when the minimum corresponds to the first argument. The work in this paper is inspired by the ideas of [13], which at first sight appears to introduce an elegant and simplified approach to DoF analysis in general interference networks, furthermore applicable to MIMO channels with general rank conditions. However, [13] does not seem to be aware of the vast body of work on interference alignment, and believes that the state of the art corresponds to one-sided ZF. As a result, they e.g. believe that the DoF region in [13, Fig. 1] represents an improvement w.r.t. their assumed state of the art, but actually the point $(8,8)$ is also achievable in the DoF region of [13, Fig. 1]. But due to the suboptimality of (5a), [13] does not capture this. On the other hand, in other scenarios, some DoF distributions that are assumed to be feasible by [13] will in reality not be.

IV. Interference Alignment (IA) Conditions for the DynTDD UE-to-UE IC - Centralized Case

In this section we analyze the overall UL UE to DL UE interference, in the centralized case in which a central design unit disposes of the knowledge of all channels involved. Spatial linear interference alignment (IA) corresponds to joint ZF by TX and RX of all units involved, as indicated in (2).

A. Proper (IA) Conditions

We start by establishing the proper conditions, which were introduced in [15] for rank deficient MIMO channels. The proper conditions express that in order for a set of variables to be able to satisfy a set of non-linear (here bilinear) equations, the number of variables involved needs to equal at least the number of equations (constraints).

- The total number of variables in $\boldsymbol{G}_l$ is $d_{ul,l}(N_{ul,l} - d_{ul,l})$, since only the column space of $\boldsymbol{G}_l$ counts. Hence for ZF, $\boldsymbol{G}_l$ is determined up to a $d_{ul,l} \times d_{ul,l}$ mixture matrix.
- The total number of variables in $\boldsymbol{F}_k$ is $d_{dl,k}(N_{dl,k} - d_{dl,k})$, since again only the column space of $\boldsymbol{F}_k$ counts. $\boldsymbol{F}_k$ is determined up to a $d_{dl,k} \times d_{dl,k}$ mixture matrix.
- Equation (2) represents $\min(d_{dl,k}r_{k,l}, d_{ul,l}r_{k,l}, d_{ul,l}d_{dl,k})$ constraints for the cross (interfering) link from UL UE l to DL UE k [15].
- The total number of cross links is $K_{dl}K_{ul}$.

The global proper condition is then

$$\begin{aligned} &\sum_{l=1}^{K_{ul}} d_{ul,l}(N_{ul,l} - d_{ul,l}) + \sum_{k=1}^{K_{dl}} d_{dl,k}(N_{dl,k} - d_{dl,k}) \\ &\geq \sum_{l=1}^{K_{ul}} \sum_{k=1}^{K_{dl}} \min(r_{k,l}d_{dl,k}, r_{k,l}d_{ul,l}, d_{ul,l}d_{dl,k}) \,. \end{aligned} \tag{6}$$

The proper condition is a necessary condition for the feasibility of IA.

Now, inspired by the work in [13], we shall investigate localized instances of the proper condition, or stated differently, we shall consider a distribution of the roles of the Tx/Rx variables in satisfying the ZF conditions. Actually, we had already considered such a role distribution perspective in [12], which in general can go beyond the global proper condition. But in the scenario considered here, in which each DL UE receives interference from each UL UE (at least if all channel ranks are positive), the ensemble of local proper conditions add up to the single global proper condition. In any case, for the local version, consider first a somewhat simplified scenario in which the cross link ZF in (2) is either handled completely by the corresponding Rx $\boldsymbol{F}_k$ or completely by the corresponding Tx $\boldsymbol{G}_l$. For the links (k,l) handled by the Rx, i.e. $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1,0)$, then (2) represents a *linear* ZF equation in $\boldsymbol{F}_k$ which represents $z^R_{k,l} = \text{rank}(\boldsymbol{H}_{k,l}\boldsymbol{G}_l) = \min(d_{ul,l}, r_{k,l})$ constraints [13, Lemma 1]. These constraints can actually be interpreted as applying to each column of $\boldsymbol{F}_k$, which a Rx beamformer for the corresponding stream of user k. For the overall BF matrix $\boldsymbol{F}_k$, we account for the $d_{dl,k}$ streams and we get a total of $d_{dl,k}z^R_{k,l} = \min(d_{dl,k}d_{ul,l}, d_{dl,k}r_{k,l})$ ZF constraints. If we take into account that Rx $\boldsymbol{F}_k$ will handle the ZF for the links in $\boldsymbol{I}_{dl,k}$, then we get the following local proper condition for $\boldsymbol{F}_k$:

$$d_{dl,k}(N_{dl,k} - d_{dl,k}) \geq \sum_{l \in \boldsymbol{I}_{dl,k}} d_{dl,k}\boldsymbol{z}^R_{k,l} \tag{7}$$

$$\Rightarrow \quad N_{dl,k} - d_{dl,k} \geq \sum_{l \in \boldsymbol{I}_{dl,k}} \boldsymbol{z}^R_{k,l} \tag{8}$$

where the last equation corresponds exactly to (5c). It can be interpreted as a proper condition per stream, where the subtraction on the LHS makes sure that after the $N_{dl,k}$ antennas are used for ZF of interfering links, $d_{dl,k}$ dimensions are left for receiving that many streams.

A completely analogous reasoning can be made for the case $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (0,1)$ in which the ZF conditions are handled by the Tx side $\boldsymbol{G}_l$, which will lead to (5d). The remaining case is $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1,1)$, which is not handled correctly in [13] or in (5b). The correct treatment actually corresponds to a finer split between the ZF roles at Tx and Rx sides of a UE-to-UE link at stream level. The resulting correct local proper conditions are

$$\begin{array}{ll}\forall k,l: & \begin{cases} d_{dl,k}\boldsymbol{z}^R_{k,l} + d_{ul,l}\boldsymbol{z}^T_{k,l} = d_{dl,k}d_{ul,l}\,, \\ \quad \text{if } r_{k,l} \geq \max(d_{dl,k}\mathbf{1}_{\boldsymbol{z}^R_{k,l}}, d_{ul,l}\mathbf{1}_{\boldsymbol{z}^T_{k,l}}) \\ \boldsymbol{z}^T_{k,l} = \min(d_{dl,k}, r_{k,l} - z^R_{k,l})\,, \\ \quad \text{if } d_{dl,k} < r_{k,l} < d_{ul,l} \\ \boldsymbol{z}^R_{k,l} = \min(d_{ul,l}, r_{k,l} - z^T_{k,l})\,, \\ \quad \text{if } d_{ul,l} < r_{k,l} < d_{dl,k} \\ \boldsymbol{z}^R_{k,l} + \boldsymbol{z}^T_{k,l} = r_{k,l}\,, \\ \quad \text{otherwise} \end{cases} \\ \forall l: & N_{ul,l} - d_{ul,l} \geq \sum_{k\in\boldsymbol{I}_{ul,l}} \boldsymbol{z}^T_{k,l} \\ \forall k: & N_{dl,k} - d_{dl,k} \geq \sum_{l\in\boldsymbol{I}_{dl,k}} \boldsymbol{z}^R_{k,l} \end{array} \tag{9}$$

where $\mathbf{1}_x = 1$ if $x > 0$ and $\mathbf{1}_x = 0$ otherwise. One can check easily that the cases $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1,0),(0,1)$ discussed above can be recovered from (9). The proof of (9) appears in Appendix A. It turns out that summing up all local conditions in (9) leads to the global proper condition (6).

B. Sylvester Rank Inequality

Whereas the proper conditions may be necessary and sufficient for MIMO channels that are (close to) square, they do not capture the complete picture in general. Other conditions can be obtained by considering Sylvester's law of nullity [16], which states that the dimension of the null space of a product of two matrices cannot be larger than the sum of the null space dimensions of the two factors. This leads to a condition closely related to [11, (16) or (22)]. For singular (factorizable) matrices, one may consider an extension for the product of three matrices, which is provided by the Frobenius rank inequality [16]:

$$\text{rank}(\boldsymbol{ABC}) \geq \text{rank}(\boldsymbol{AB}) + \text{rank}(\boldsymbol{BC}) - \text{rank}(\boldsymbol{B}) \tag{10}$$

of which Sylvester's rank inequality is a special case with $\boldsymbol{B} = \boldsymbol{I}$.

Let $\boldsymbol{H}_{k,:} = [\boldsymbol{H}_{k,1} \cdots \boldsymbol{H}_{k,K_{ul}}]$, $\boldsymbol{G} = \text{blkdiag}\{\boldsymbol{G}_1, \ldots, \boldsymbol{G}_{K_{ul}}\}$. Also let $N_{ul} = \sum_{l=1}^{K_{ul}} N_{ul,l}$, $d_{ul} = \sum_{l=1}^{K_{ul}} d_{ul,l}$, $r_{k,:} = \sum_{l=1}^{K_{ul}} r_{k,l}$. Then putting together (2) for all Tx, we get

$$\boldsymbol{F}_k\, \boldsymbol{H}_{k,:}\, \boldsymbol{G} = 0\,. \tag{11}$$

For $\boldsymbol{F}_k$ to be able to perform this ZF while receiving $d_{dl,k}$ streams, we require

$$\text{rank}(\boldsymbol{H}_{k,:}\boldsymbol{G}) \leq N_{dl,k} - d_{dl,k}\,. \tag{12}$$

On the other hand Sylvester's rank inequality implies

$$\begin{aligned} \text{rank}(\boldsymbol{H}_{k,:}\boldsymbol{G}) &\geq \text{rank}(\boldsymbol{H}_{k,:}) + \text{rank}(\boldsymbol{G}) - N_{ul} \\ &= \min(N_{dl,k}, r_{k,:}, N_{ul}) + d_{ul} - N_{ul} \end{aligned} \tag{13}$$

where the statement about $\text{rank}(\boldsymbol{H}_{k,:})$ should be interpreted w.p. 1 for a sufficiently random channel distribution. Putting together (12) and (13), we get

$$N_{dl,k} - d_{dl,k} \geq \min(N_{dl,k}, r_{k,:}, N_{ul}) + d_{ul} - N_{ul}\,,\ \forall k\,. \tag{14}$$

By interchanging the roles or Tx and Rx (UL/DL duality), we get similarly

$$N_{ul,l} - d_{ul,l} \geq \min(N_{ul,l}, r_{:,l}, N_{dl}) + d_{dl} - N_{dl}\,,\ \forall l\,. \tag{15}$$

In the likely case that $\min(N_{dl,k}, r_{k,:}, N_{ul}) = N_{dl,k}$ and $\min(N_{ul,l}, r_{:,l}, N_{dl}) = N_{ul,l}$, (14) becomes

$$d_{ul} + d_{dl,k} \leq N_{ul}\,,\ \forall k \tag{16}$$

and (15) becomes

$$d_{dl} + d_{ul,l} \leq N_{dl}\,,\ \forall l \tag{17}$$

which would not be very constraining.

V. Interference Alignment (IA) Conditions for the DynTDD UE-to-UE IC - Distributed Case

The necessary conditions for the distributed solutions considered in this section are actually also sufficient conditions (the corresponding Tx/Rx designs are feasible) since they correspond to linear ZF equations.

A. Distributed Solutions Exploiting the Low Rank Channel Factorizations

As opposed to the centralized case, in the distributed case, each Tx/Rx disposes of at most local CSI, i.e. of the channels directly connected to it. In this case the global proper condition (6) reduces to

$$\begin{aligned} &\sum_{l=1}^{K_{ul}} d_{ul,l}(N_{ul,l} - d_{ul,l}) + \sum_{k=1}^{K_{dl}} d_{dl,k}(N_{dl,k} - d_{dl,k}) \\ &\geq \sum_{l=1}^{K_{ul}} \sum_{k=1}^{K_{dl}} r_{k,l}\, \min(d_{dl,k}, d_{ul,l}) \end{aligned} \tag{18}$$

which corresponds to decoupling between the designs for the $\boldsymbol{F}_k$ and the $\boldsymbol{G}_l$. The corresponding local proper conditions become

$$\begin{array}{ll} \forall k,l: & \boldsymbol{z}^R_{k,l} + \boldsymbol{z}^T_{k,l} = r_{k,l}\,, \\ \forall l: & N_{ul,l} - d_{ul,l} \geq \sum_{k\in\boldsymbol{I}_{ul,l}} \boldsymbol{z}^T_{k,l} \\ \forall k: & N_{dl,k} - d_{dl,k} \geq \sum_{l\in\boldsymbol{I}_{dl,k}} \boldsymbol{z}^R_{k,l} \end{array} \tag{19}$$

This represents a generalization of [15] in which we considered the special case in which either $\boldsymbol{z}^R_{k,l} = r_{k,l}$, $\boldsymbol{z}^T_{k,l} = 0$, or $\boldsymbol{z}^T_{k,l} = r_{k,l}$, $\boldsymbol{z}^R_{k,l} = 0$. In other words the interference of a particular UE-to-UE link is handled completely by either the Tx or the Rx. But the handling of all UE-to-UE links is still partitioned between UL and DL UEs. The ZF of any particular UE-to-UE link can also be shared between Tx and Rx, as considered here. The first line in (19) can be interpreted as

$$\begin{array}{l} \boldsymbol{B}^H_{k,l}\,\boldsymbol{F}_{k,l} \text{ has } \boldsymbol{z}^R_{k,l} \text{ rows of zeros} \\ \boldsymbol{A}^H_{k,l}\,\boldsymbol{G}_{k,l} \text{ has } \boldsymbol{z}^T_{k,l} \text{ rows of zeros} \end{array} \tag{20}$$

where the zero rows in the two factors are complementary so that (2) is satisfied.

B. Distributed Solutions Based on Fixed Tx/Rx Factors

When the selections $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1, 0)$ or $(0, 1)$ are applied to all links, then in general the design of Tx and Rx is coupled. This coupling can be broken if either $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1, 0)$ is applied to all links, or $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (0, 1)$ is applied to all links. In this case we get $\forall k, l$:

$$\begin{aligned} \text{either } \boldsymbol{z}^R_{k,l} &= \min(d_{ul,l}, r_{k,l}) \\ \text{or } \boldsymbol{z}^T_{k,l} &= \min(d_{dl,k}, r_{k,l}) \end{aligned} \tag{21}$$

where of course (5c), (5d) continue to apply. When the Rx handle all the ZF, then the Tx can be designed separately, e.g. based on the UE-BS channels, and vice versa.

VI. Uniform Scenarios

To have a simplified version of the conditions above we define a uniform asymmetric scenario, in which:

$d_{dl,k} = d_{dl}$, $\forall k \in [1, ..., K_{dl}]$
$d_{ul,l} = d_{ul}$, $\forall l \in [1, ..., K_{ul}]$
$N_{dl,k} = N_{dl}$, $\forall k \in [1, ..., K_{dl}]$
$N_{ul,l} = N_{ul}$, $\forall l \in [1, ..., K_{ul}]$
$r_{k,l} = r$, $\forall l \in [1, ..., K_{ul}]$, $\forall k \in [1, ..., K_{dl}]$

Then the centralized proper condition (6) becomes:

$$\begin{aligned} &K_{ul} d_{ul}(N_{ul} - d_{ul}) + K_{dl} d_{dl}(N_{dl} - d_{dl}) \\ &\geq K_{ul}\, K_{dl}\, \min(r d_{dl}, r d_{ul}, d_{ul} d_{dl}) \end{aligned} \tag{22}$$

We define also a uniform symmetric case when For $K_{dl} = K_{ul} = K$, $d_{dl} = d_{ul} = d$ and $N_{dl} = N_{ul} = N$, for which (22) becomes:

$$d \leq N - \frac{K}{2} \min(d, r) \tag{23}$$

Now, consider the local proper conditions and introduce $n_{F,k} = |\boldsymbol{I}_{dl,k}|$, $n_{G,l} = |\boldsymbol{I}_{ul,l}|$ Hence $n_{F,k}$ (resp.$n_{G,l}$) denote the number of UL (resp. DL) UEs for which the cross link interference is cancelled by the kth DL UE (resp. the lth UL UE). For this ZF role distribution to ensure the cancellation of all CLI, we require

$$\sum_{k=1}^{K_{dl}} n_{F,k} + \sum_{l=1}^{K_{ul}} n_{G,l} \geq K_{ul} K_{dl} \, . \tag{24}$$

In the uniform case, $n_{F,k} = n_F, \forall k \in [1, ..., K_{dl}]$ and $n_{G,l} = n_G, \forall l \in [1, ..., K_{ul}]$, equation (24) becomes:

$$K_{dl} n_F + K_{ul} n_G \geq K_{ul} K_{dl} \, . \tag{25}$$

The optimization of n_F, n_G depends on the desired point (d_{dl}, d_{ul}) in the DoF region. For a uniform asymmetric case, we get on the Rx side ($(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1, 0)$)

$$d_{dl} \leq N_{dl} - n_F min(d_{ul}, r) \tag{26}$$

and on the Tx side ($(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (0, 1)$).

$$d_{ul} \leq N_{ul} - n_G min(d_{dl}, r) \, . \tag{27}$$

Exploring the case $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1, 1)$ only leads to a finer granularity of ZF roles (at stream level instead of user level). For the symmetric case we have $n_F = n_G = \frac{K}{2}$ and we get back (23).

We can consider the distributed approaches with fixed Tx/Rx factors (symmetric uniform case):

- $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1, 0)$: applied to all links $n_F = K$:
$$d \leq N - K \min(d, r) \tag{28}$$
- $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (0, 1)$: applied to all links $n_G = K$:
$$d \leq N - K \min(d, r) \tag{29}$$
- $(\mathbf{1}^R_{k,l}, \mathbf{1}^T_{k,l}) = (1, 1)$: considering (5a) from [13], applied to all links, we can take $\boldsymbol{z}^R_{k,l} = \boldsymbol{z}^T_{k,l} = \min(2d, r)$ leading to
$$d \leq N - \frac{K}{2} \min(2d, r) \tag{30}$$

Which all three yield worse DoF than (23).

If we consider the distributed solution based on low rank channel factorizations, then for the uniform asymmetric case we obtain:

$$d_{dl} \leq N_{dl} - n_F r \tag{31a}$$

$$d_{ul} \leq N_{ul} - n_G r \tag{31b}$$

As in the centralized case, to ensure the cancellation of all the cross link interference equation (25) should be satisfied. For the uniform symmetric case, we get $n_F = n_G = \frac{K}{2}$, and (31a), (31b) become:

$$d \leq N - \frac{K}{2} r \, . \tag{32}$$

VII. Discussion

From the proper conditions established in the previous sections for the DynTDD UE-to-UE IC, we notice for the uniform asymmetric scenario that:

- When $\min(d_{ul}, r) = \min(d_{dl}, r) = r$, the conditions in the centralized case ((26) and (27)) meet the conditions in the distributed case ((31a) and (31b)).
- At a stream level the bilateral solution $(1^R_{k,l}, 1^T_{k,l}) = (1, 1)$ leads to a distributed design for $r \leq \min(d_{dl}, d_{ul})$ (and a half distributed solution when r is in between the two d's, see Appendix A),

In Table I we evaluate the DoF for a number of dimensions considering the uniform symmetric case. The first line corresponds to the global proper (necessary) condition, which in this case also yield feasible DoF. The other approaches lead to reduced DoF. The distributed solution does not exhibit suboptimality w.r.t. the centralized optimal solution for a rank up to $r = 5$. From the analysis of the table and considering a distributed design we can conclude:

- For the uniform symmetric case the best distributed solution is given by the distributed approach (32) for $r < d$, and for $r > d$ it is given by the unilateral cases (ZF at only Rx (28) or only Tx (29)).

VIII. Conclusions

In this work, we addressed the interference between users arising in DynTDD systems. Considering the centralized and distributed cases, we studied the proper and sufficient conditions for the interference cancellation. This analysis highlights the gap between centralized and distributed schemes, but

rank r	0	1	2	3	4	5	6	7	8	9	10	15
Centralized proper(23)	15	13	11	9	7	5	5	5	5	5	5	5
Combined unilateral ZF (26), (27)	15	13	11	9	7	5	5	5	5	5	5	5
suboptimal ZF at Rx and Tx(30)	15	13	11	9	7	5	3	3	3	3	3	3
Unilateral ZF at Rx(28) or Tx(29)	15	11	7	3	3	3	3	3	3	3	3	3
Distributed (32)	15	13	11	9	7	5	3	1	0	0	0	0

TABLE I: DoF per user as a function of the rank of any cross link channel for a uniform symmetric scenario with $N = 15$, $K_{ul} = K_{dl} = 4$.

shows a relatively limited DoF loss for the more realistic distributed schemes. We illustrated the DoF in a symmetric uniform scenario for a range of Tx/Rx design schemes, which have varying coordination and CSI needs.

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APPENDIX A

Here we prove the constraints on $\boldsymbol{z}^R_{k,l}$, $\boldsymbol{z}^T_{k,l}$ appearing in (9), esp. for the case $(1^R_{k,l}, 1^T_{k,l}) = (1,1)$

Case $r \leq \min(d_{dl}, d_{ul})$

(2) can be rewritten as

$$\boldsymbol{F}_k^H \boldsymbol{B}_{k,l} \boldsymbol{A}_{k,l}^H \boldsymbol{G}_l = 0\,. \tag{33}$$

An application of Sylvester's rank inequality to (33) yields

$$\text{rank}(\boldsymbol{B}_k^H \boldsymbol{F}_{k,l}) + \text{rank}(\boldsymbol{A}_{k,l}^H \boldsymbol{G}_l) \leq r_{k,l}\,. \tag{34}$$

We can choose $\boldsymbol{B}_k^H \boldsymbol{F}_{k,l}$ to have $\boldsymbol{z}^R_{k,l}$ zero rows and $\boldsymbol{A}_k^H \boldsymbol{G}_{k,l}$ to have $\boldsymbol{z}^t_{k,l}$ zero rows so that $\boldsymbol{z}^R_{k,l} + \boldsymbol{z}^T_{k,l} = r_{k,l}$. The optimized values for $\boldsymbol{z}^R_{k,l}$, $\boldsymbol{z}^T_{k,l}$ depend on the other variables d_{dl}, d_{ul}, N_{ul}, N_{dl}, K_{ul} and K_{dl}. We see that when $r \leq \min(d_{dl}, d_{dl})$, the case $(1^R_{k,l}, 1^T_{k,l}) = (1,1)$ leads to a distributed design: the design of $\boldsymbol{F}_k$ depends only on the factor $\boldsymbol{B}_{k,l}$ in $\boldsymbol{H}_{k,l}$ and not on $\boldsymbol{G}_l$, and similarly $\boldsymbol{G}_l$ only depends on $\boldsymbol{A}_{k,l}$.

Case $r \geq \max(d_{dl}, d_{ul})$

In this case $\boldsymbol{F}_k^H \boldsymbol{H}_{k,l} \boldsymbol{G}_l$ is a priori full rank, in the case of arbitrary Tx/Rx. Here we assume a uniformity of the number of zeros produced by the columns (beamformers) in $\boldsymbol{F}_k$ and $\boldsymbol{G}_l$. Let $\boldsymbol{F}_k$ now produce $\boldsymbol{z}^R_{k,l}$ zeros in each row of the matrix product $\boldsymbol{F}_k^H \boldsymbol{H}_{k,l} \boldsymbol{G}_l$, i.e. in total it produces $d_{dl,k} \boldsymbol{z}^R_{k,l}$ zeros (the position of the zeros in each row may be different so that the number of non-zeros per column is also equal between all columns). Then let $\boldsymbol{G}_l$ produce $\boldsymbol{z}^T_{k,l}$ zeros in each column, with the constraint that we produce a total of $d_{dl,k} \boldsymbol{z}^R_{k,l} + d_{ul,l} \boldsymbol{z}^T_{k,l} = d_{dl,k} d_{ul,l}$ zeros. In this case, the design of $\boldsymbol{F}_k$ and $\boldsymbol{G}_l$ is clearly coupled.

Now, in the combination of the two cases above, care has to be taken with the limiting cases $\boldsymbol{z}^R_{k,l} = 0$ or $\boldsymbol{z}^T_{k,l} = 0$, corresponding to one-sided ZF. In that case we have linear ZF equations representing a number of ZF constraints equal to the rank of the matrix of coefficients, as mentioned in the discussion of the cases $(\mathbf{1}_{\boldsymbol{z}^R_{k,l}}, \mathbf{1}_{\boldsymbol{z}^T_{k,l}}) = (1^R_{k,l}, 1^T_{k,l}) = (1,0)$ or $(0,1)$. This results in the first condition appearing in (9).

Case $d_{dl} < r < d_{ul}$ or $d_{ul} < r < d_{dl}$

Consider w.l.o.g. the case $d_{dl} < r < d_{ul}$. In this case let $\boldsymbol{F}_k$ produce $\boldsymbol{z}^R_{k,l}$ rows of zeros in $\boldsymbol{B}^H_{k,l} \boldsymbol{F}_k$ as in the first case. Then for $\boldsymbol{G}_l$ to produce $\boldsymbol{F}_k^H \boldsymbol{H}_{k,l} \boldsymbol{G}_l = 0$ imposes on it a number of ZF constraints of $\boldsymbol{z}^T_{k,l} = \text{rank}(\boldsymbol{F}_k^H \boldsymbol{H}_{k,l}) = \min(d_{dl,k}, r_{k,l} - \boldsymbol{z}^R_{k,l})$, [13, Lemma 1]. In this case $\boldsymbol{F}_k$ is decoupled from $\boldsymbol{G}_l$ but $\boldsymbol{G}_l$ is coupled to $\boldsymbol{F}_k$.

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