2D DDoA-based Self-Positioning for Mobile Devices

Hoang. M. Le[1,2], Dirk Slock[1], Jean-Pierre Rossi[2]

[1]Communication Systems Departement, EURECOM - Sophia Antipolis, France
[2]Orange Labs - Sophia Antipolis, France
Email: hoangminh.le@orange.com, leho@eurecom.fr

***Abstract*—Multi-antenna techniques enable Direction of Arrival (DoA) estimation and lead to highly accurate results. In DoA-based positioning, the angles values must be predefined in an interval of 2π-length, and the prior knowledge of x-direction is required. However, this prerequisite is impractical for a self-positioning task at a mobile device because its orientation is unknown. In this paper, we propose an algorithm using the differences among the DoAs to overcome such a difficulty. The algorithm relies on the estimation of the Direction Difference of Arrival (DDoA), which is linked to the DoAs. In our procedure, the definition of DoA utilizes the atan2 function, which has the 2π-long codomain to map the DoA. An iterative Maximum Likelihood (ML) estimator for position estimation is presented. Noisy measurements with values near the edges of this codomain can lead to drastic position estimation errors, making the convergence of iterative procedures more challenging. Therefore, a phase correction scheme is proposed to robustify the estimation considerably. Simulation results show substantial improvement in performances compared to the methods without correction.**

***Index Terms* - DDoA, Direction Difference of Arrival, 2D localization, Maximum Likelihood.**

I. Introduction

Positioning methods can be divided into 2 main types, based on where the mobile position estimate is computed [1].

- **Network-based:** The network of base stations (BSs) computes the coordinates of the mobile device from the signal(s) sent by the mobile device.
- **Mobile-based:** The mobile device itself computes its coordinates by using signals from the network of BSs.

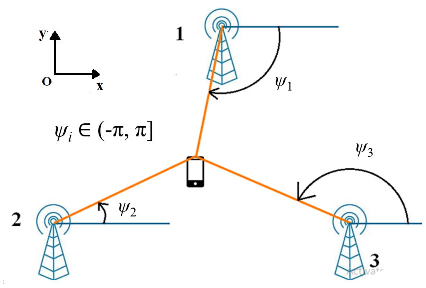
Mobile-based self-positioning, where the mobile device is able to determine its position by itself, is a vital problem in wireless communications. There are various positioning techniques: Time of Arrival (ToA), Time Difference of Arrival (TDoA), Received Signal Strength (RSS), and Direction of Arrival (DoA) (in some documents, it is also called Angle of Arrival - AOA) [1]. ToA-based [3-5] and TDoA-based [6-7] positioning require highly accurate clock synchronization among all BSs and mobile device. DoA-based systems do not require such synchronization. Instead, the resolution of DoA measurements is limited by the SNR, the number of sensors in the array, and the separation between these sensors. DoA estimation schemes are usually thought of as computationally expensive. However, recent developments propose computationally simple DoA estimation schemes that allow small antenna arrays with a reduced number of elements [2].

DoA-based localization computes the coordinates of the mobile device based on the direction of incident waves to base stations. The numerical expression of this direction is the trigonometric angle between the x-direction and the signal wave (Fig. 1a). To avoid confusion in measuring angles, all the angles' values must be defined in an interval whose length is 2π. Furthermore, at the boundaries of the interval, the DoA is very sensitive to noise. For instance, on the condition that an angle's set of definition is $[0; 2\pi)$, when the true value of the angle is ε_1, a small noise can make the angle's value $-\varepsilon_2$ (ε_1 and ε_2 are very small positive value). However, as the set of definition is $[0; 2\pi)$, the estimated value of this angle becomes $2\pi - \varepsilon_2$, which is very different from the true value.

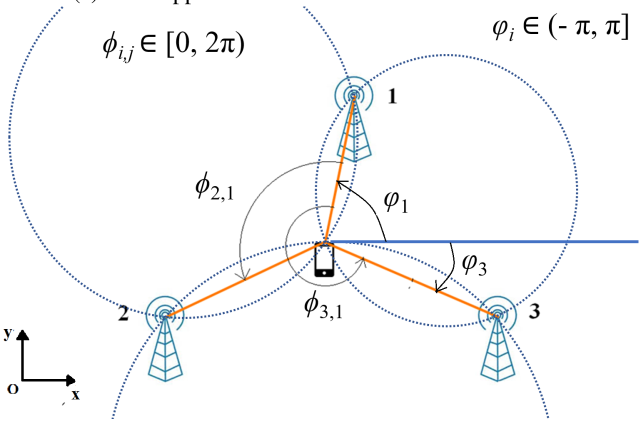
DoA-based positioning is only feasible for **Network-based** localization because the orientation of each base station is fixed and known. However, as for **Mobile-based** Self-positioning, since the orientation of a mobile device is unknown, it cannot refer to the x-direction to calculate the angle of arrival. Consequently, Direction Difference of Arrival (DDoA) is proposed. In this technique, only the difference in directions of arrival of incident waves from a pair of base stations is required (Fig. 1b). Mathematically, a DDoA is calculated by subtracting the 2 DoAs concerned. Recent achievements in DoA estimations at mobile devices [8-11] make DDoA-based positioning algorithms promising.

In [12], the very first ideas about DDoA are introduced. A DoA-based positioning method for mobile devices, in which the prior knowledge of the x-direction is not required, is also presented in [13]. Nevertheless, the authors use arctan function in the definition of DoA, which cannot cover all the possible values of an angle. In [14] and [15] the authors also demonstrate DDoA in different ways of explication. In general, they do not consider the sensitivity to noise of an angle's measured value.

This paper gives a clear analysis of localization based on the DDoAs of the incident wave from the mobile device to the network of base stations. In the definition of DoA, the atan2 function is utilized instead of arctan function. Subtraction of two DoAs returns a value in the range of $(-2\pi; 2\pi)$, so a modulo operation with a divisor of 2π is applied. For this reason, the codomain of DDoA computations is $[0; 2\pi)$. This interval is also the set of definition of DDoA measured values. Furthermore, an additional correction is added to the subtraction of two DoAs to avoid possible huge computing errors caused by small mistakes in practical measurements. Compared to [12], we formulate a Maximum Likelihood



(a) DoA approach for network-based localization.



(b) DDoA approach for mobile-based localization

Fig. 1: DoA vs DDoA approaches for positioning, in noiseless scenario.

estimator with the fitting criterion at the level of DDoAs instead of their tangents, allowing the original introduction of phase corrections (to offset the modulo operations).

Notation: **mod**(x, a) denotes x modulo a; **diag**$(a_1, a_2, \ldots, a_n)$ is the diagonal matrix whose diagonal elements are a_1, a_2, …, a_n resp.; *[a;b)* denotes an interval from a to b which includes a but excludes b, $[\mathrm{A}]_{i,j}$ is the element at i-th row and j-column of matrix A. **atan2** means 2-argument which is defined as: $\varphi = \mathrm{atan2(y,\ x)} \iff \mathrm{x + jy = r}e^{\mathrm{j}\varphi}$ with $\mathrm{r} = \sqrt{\mathrm{x}^2 + \mathrm{y}^2}$, $\varphi \in (-\pi; \pi]$ and j is the imaginary unit. The standard arctangent function **arctan** has values in $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Let

$$\mathrm{sign}(x) = \begin{cases} 1 & , x \geq 0 \\ -1 & , x < 0\,. \end{cases} \tag{1}$$

Then for $(x, y) \neq (0, 0)$, we have

$$\mathrm{atan2}(x, y) = \arctan\left(\frac{y}{x}\right) - (\mathrm{sign}(x) - 1)\,\mathrm{sign}(y)\,\frac{\pi}{2}\,. \tag{2}$$

II. DDoA-based Positioning Algorithm

A. Direction of Arrival

We define φ_i to be the **trigonometric angle** between the x axis and the signal ray received at the mobile station. Let (x, y) be the coordinates of the mobile station and (x_i, y_i) be the coordinates of the i-th base station. We then have the real DoA of the signal from the i-th base station:

$$\varphi_i = \mathrm{atan2}(y_i - y, x_i - x)\,. \tag{3}$$

In the presence of estimation errors, the measured value of i-th DoA shall be:

$$\hat{\varphi}_i = \varphi_i + n_{\mathrm{DoA},i} = \mathrm{atan2}(y_i - y, x_i - x) + n_{\mathrm{DoA},i} \tag{4}$$

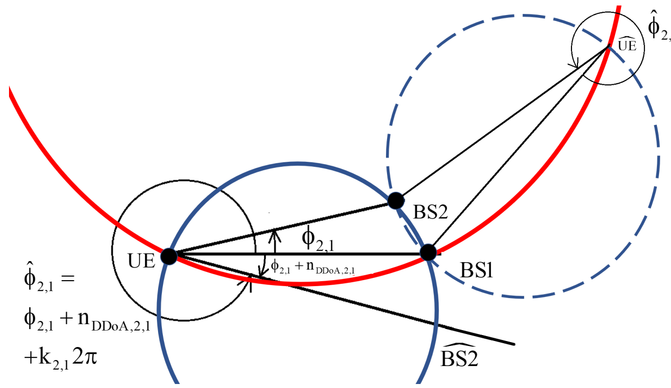


Fig. 2: Sensitivity to DDoA estimation error induced modulo switches.

where $n_{\mathrm{DoA},i}$ is the error in DoA estimation. The authors of [16] illustrates that when there is Gaussian noise in received signal, the error in estimation is asymptotically Gaussian distributed with zero-mean. As a result, we can assume that $n_{\mathrm{DoA},i}$ is Gaussian distributed with zero-mean and variance of σ_i^2.

B. Direction Difference of Arrival

As $\varphi_i \in (-\pi; \pi]$, we have the difference $d\varphi_{i,j} = \varphi_i - \varphi_j \in (-2\pi; 2\pi)$. However, the value of an angle must be predefined in a 2π-long range. Therefore, we state the Direction Difference of Arrival (DDoA) between signal ray from i-th base station and signal ray from j-th base station (where i from 1 to N, j from 1 to N, $i \neq j$, N is the number of base stations) as follows:

$$\begin{aligned} \phi_{i,j} &= \mathrm{mod}\left(\varphi_i - \varphi_j, 2\pi\right) = \mathrm{mod}(d\varphi_{i,j}, 2\pi) \\ &= \mathrm{mod}\left(\mathrm{atan2}(y_i - y, x_i - x) - \mathrm{atan2}(y_j - y, x_j - x), 2\pi\right) \end{aligned} \tag{5}$$

where for the last equality and below we assume the absence of errors. In terms of the arctan function, we get with (2)

$$\begin{aligned} \phi_{i,j} &= \arctan\frac{y_i - y}{x_i - x} - \arctan\frac{y_j - y}{x_j - x} + m_1\pi \\ &= \arctan\frac{(y_i - y)(x_j - x) - (x_j - x)(y_j - y)}{(y_i - y)(y_j - y) + (x_i - x)(x_j - x)} + m_2\pi \end{aligned} \tag{6}$$

where m_1 and m_2 are integers. Hence we get

$$\tan\phi_{i,j} = \frac{(y_i - y)(x_j - x) - (x_i - x)(y_j - y)}{(y_i - y)(y_j - y) + (x_i - x)(x_j - x)}\,. \tag{7}$$

$$\begin{aligned} &x[-y_i + y_j + (x_i + x_j)\tan(\phi_{i,j})] \\ &+y[x_i - x_j + (y_i + y_j)\tan(\phi_{i,j})] \\ &-(x^2 + y^2)\tan(\phi_{i,j}) \\ &= -x_j y_i + x_i y_j + (x_i x_j + y_i y_j)\tan(\phi_{i,j})\,. \end{aligned} \tag{8}$$

In the presence of errors, the DDoA measurements, which are the practical estimated values of the DDoAs, also in range of $[0; 2\pi)$ and denoted by $\{\hat{\phi}_{i,j}\}$, are modeled as

$$\hat{\phi}_{i,j} = \mathrm{mod}\left(\hat{\varphi}_i - \hat{\varphi}_j, 2\pi\right) = \phi_{i,j} + k_{i,j}2\pi + n_{\mathrm{DDoA},i,j} \tag{9}$$

where $n_{\mathrm{DDoA},i,j} = n_{\mathrm{DoA},i} - n_{\mathrm{DoA},j} \sim \mathcal{N}(0, \sigma_i^2 + \sigma_j^2)$ and the modulo induced noise term k_i is defined as:

$$k_{i,j}=\begin{cases}1 & ,\begin{cases}d\varphi_{i,j}\geq 0 \text{ and } d\varphi_{i,j}+n_{\text{DDoA},i,j}<0\\ \textbf{or } d\varphi_{i,j}+n_{\text{DDoA},i,j}<-2\pi\end{cases}\\ -1 & ,\begin{cases}d\varphi_{i,j}<0 \text{ and } d\varphi_{i,j}+n_{\text{DDoA},i,j}\geq 0\\ \textbf{or } d\varphi_{i,j}+n_{\text{DDoA},i,j}\geq 2\pi\end{cases}\\ 0 & \text{otherwise.}\end{cases} \tag{10}$$

C. Position estimation by the Least-Squares (LS) method

Regardless of $k_{i,j}$, for small enough $n_{,i,j}$ we get

$$\tan\hat{\phi}_{i,j}=\tan(\phi_{i,j}+k_{i,j}2\pi+n_{,i,j})\approx\tan\phi_{i,j}+n_{,i,j}\,. \tag{11}$$

(8) is a 2-variable quadratic equation, which is satisfied by (x_i, y_i), (x_j, y_j) and (x, y). In other words, (8) is the equation of a circle passing through the positions of i-th BS, j-th BS and the mobile. With 2 base station positions and a DDoA, the locus of all the possible positions of the mobile device is a circle to which the DDoA is an inscribed angle. When $\phi_{i,j}$ is close to 0, the error term $n_{\text{DDoA},i,j}$ can lead to a circle flip as illustrated in Fig. 2. With two base stations BS1 and BS2, the solid blue circle is the locus of UE positions with correct DDoA $\phi_{2,1}$. The UE position is at the intersection with another DDoA circle, the red one. Now, with a small error on $\phi_{2,1}$, which changes it from a small positive to a small negative value, the modulo operation kicks in, changing $\hat{\phi}_{2,1}$ to a value near 2π. As a result, given the positions of BS1 and BS2, the circle corresponding to the DDoA $\hat{\phi}_{2,1}$ now becomes the dashed blue circle and the estimated UE position jumps to its intersection with the red circle, which is very far from its true position (UE).

Now, with N BSs, we get $N(N-1)/2$ circles. In matrix notation, we define $\boldsymbol{\omega}=[x\;\;y\;\;x^2+y^2]^T$. In addition, $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{b}}$ are defined by (12) and (13) respectively, which are illustrated in the beginning of the following page.

We have

$$\min_{\boldsymbol{\omega}}\|\hat{\boldsymbol{A}}\boldsymbol{\omega}-\hat{\boldsymbol{b}}\|^2 \tag{14}$$

leading to the estimate of $\boldsymbol{\omega}$ being calculated by

$$\hat{\boldsymbol{\omega}}=\hat{\boldsymbol{A}}^{\dagger}\hat{\boldsymbol{b}} \tag{15}$$

where $\boldsymbol{A}^{\dagger}=(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T$ is the Moore-Penrose pseudo inverse of matrix $\boldsymbol{A}$. The estimated coordinates of the mobile device are the two first element of $\hat{\boldsymbol{\omega}}$:

$$\hat{\boldsymbol{x}}=\begin{bmatrix}[\hat{\boldsymbol{\omega}}]_1 & [\hat{\boldsymbol{\omega}}]_2\end{bmatrix}^T \tag{16}$$

To further optimize the estimation of the mobile's position, this can be taken as initialization of an iterative procedure discussed next.

D. Iterative Maximum Likelihood (ML) Procedure

To optimize $\hat{\boldsymbol{x}}$ obtained in (16), an iterative Maximum Likelihood estimator is applied. In vector form, we denote

$$\hat{\boldsymbol{\phi}}=\begin{bmatrix}\hat{\phi}_{2,1} & \hat{\phi}_{3,1} & \dots\hat{\phi}_{N,1}\end{bmatrix}^T \tag{17}$$

$$\boldsymbol{f}(\boldsymbol{x},\boldsymbol{k})=\begin{bmatrix}\phi_{2,1}(\boldsymbol{x})+k_{2,1}2\pi\\ \phi_{3,1}(\boldsymbol{x})+k_{3,1}2\pi\\ \dots\\ \phi_{N,1}(\boldsymbol{x})+k_{N,1}2\pi\end{bmatrix} \tag{18}$$

where $\boldsymbol{k}=[k_{2,1}\;\;k_{3,1}\cdots k_{N,1}]^T$, $\boldsymbol{x}=[x\quad y]^T$ and $\phi_{i,1}(\boldsymbol{x})$ is the estimated DDoA between the 1st and the i-th incident waves ($i\geq 2$) and computed by

$$\phi_{i,j}(\boldsymbol{x})=\text{mod}\left(\text{atan2}(y_i-y,x_i-x)-\text{atan2}(y_1-y,x_1-x),2\pi\right) \tag{19}$$

Let the noise vector $\boldsymbol{n}$

$$\boldsymbol{n}=\begin{bmatrix}n_{\text{DDoA},2,1} & n_{\text{DDoA},3,1} & \dots & n_{\text{DDoA},N,1}\end{bmatrix} \tag{20}$$

The covariance matrix of all the additive errors is

$$\boldsymbol{C}=\text{E}(\boldsymbol{n}\boldsymbol{n}^T)=\sigma_1^2\,\mathbf{1}\,\mathbf{1}^T+\text{diag}(\sigma_2^2,\sigma_3^2,\dots,\sigma_N^2) \tag{21}$$

where $\mathbf{1}=\begin{bmatrix}1 & 1 & \dots & 1\end{bmatrix}^T$ is the all-ones vector. Note that whereas for the LS method, exploiting all $N(N-1)/2$ circles is useful, for the ML method there are only $N-1$ linearly independent DDoAs among the $N(N-1)/2$ possible ones.

Treating the phase shift vector $\boldsymbol{k}$ as unknown parameters and ignoring their dependence on the noise, the measurement vector $\hat{\boldsymbol{\phi}}$ is Gaussian with mean vector of $\boldsymbol{f}$ and covariance matrix $\boldsymbol{C}$, we have the probability density function (pdf) [17]:

$$\text{p}(\hat{\boldsymbol{\phi}}|\boldsymbol{x},\boldsymbol{k})=\frac{(2\pi)^{-\frac{N}{2}}}{|\boldsymbol{C}|^{\frac{1}{2}}}\exp\left[\frac{-1}{2}(\hat{\boldsymbol{\phi}}-\boldsymbol{f})^T\boldsymbol{C}^{-1}(\hat{\boldsymbol{\phi}}-\boldsymbol{f})\right] \tag{22}$$

Maximizing the pdf in (22) is equivalent to finding

$$\hat{\boldsymbol{x}},\hat{\boldsymbol{k}}=\arg\min_{\boldsymbol{x},\boldsymbol{k}}(\hat{\boldsymbol{\phi}}-\boldsymbol{f}(\boldsymbol{x},\boldsymbol{k}))^T\boldsymbol{C}^{-1}(\hat{\boldsymbol{\phi}}-\boldsymbol{f}(\boldsymbol{x},\boldsymbol{k})) \tag{23}$$

which we shall perform alternatingly. We consider Gauss Newton [18] for $\boldsymbol{x}$. At iteration $(u+1)$

$$\hat{\boldsymbol{x}}^{(u+1)}=\hat{\boldsymbol{x}}^{(u)}+(\boldsymbol{G}^T\boldsymbol{C}^{-1}\boldsymbol{G})^{-1}\boldsymbol{G}^T\boldsymbol{C}^{-1}(\hat{\boldsymbol{\phi}}-\boldsymbol{f}(\hat{\boldsymbol{x}}^{(u)},\boldsymbol{k}^{(u+1)})) \tag{24}$$

where $\boldsymbol{G}$ is the Jacobian matrix of $\boldsymbol{f}(\boldsymbol{x})$

$$\boldsymbol{G}=\boldsymbol{G}(\hat{\boldsymbol{x}}^{(u)},\boldsymbol{k}^{(u+1)})\,,\;\boldsymbol{G}(\boldsymbol{x},\boldsymbol{k})=\frac{\partial\boldsymbol{f}(\boldsymbol{x},\boldsymbol{k})}{\partial\boldsymbol{x}^T}\,. \tag{25}$$

At this point, it is important to determine the value of $\boldsymbol{k}$. As we do not know the additive noise in each DoA measurement, $k_{i,1}$ cannot be determined by equation (10). From (9), we have

$$|n_{\text{DDoA},i,1}|=|\hat{\phi}_{i,1}-\phi_{i,1}-k_{i,1}2\pi| \tag{26}$$

We assume that $n_{\text{DDoA},i,1}$ is small enough, so $|n_{\text{DDoA},i,1}|<\pi$ with the probability almost 1. Thus $\hat{k}_{i,1}$ can be estimated by

$$\hat{k}_{i,1}^{(u+1)}=\arg\min_{k_{i,1}\in\{0;\pm 1\}}|\hat{\phi}_{i,1}-\phi_{i,1}(\hat{\boldsymbol{x}}^{(u)})-k_{i,1}2\pi| \tag{27}$$

where $\hat{\phi}_{i,1}(\hat{\boldsymbol{x}}^{(u)})$ is the estimated value of $\phi_{i,1}$ at the u-th iteration. A procedure is expected to terminate when $\|\hat{\boldsymbol{x}}^{(u)}-\hat{\boldsymbol{x}}^{(u-1)}\|_2<\varepsilon$, for the stopping value ε sufficiently small. Then, the final position of the procedure is considered to be the coordinates of the mobile device in the xy plane.

However, iterative procedures do not always converge. There are **three** possible outcomes for an iterative procedure:

- **Convergence:** The procedure quickly meets the stopping criterion and reaches the finite values.
- **Divergence:** The procedure reaches infinite values, and then it is forced to stop.
- **Oscillation:** The procedure oscillates between 2 or more repeated finite values. It does not diverge, but it is not able to converge. Experiments show that convergence or

$$\hat{\boldsymbol{A}} = \begin{bmatrix} -y_2 + y_1 + (x_2 + x_1) & x_2 - x_1 - (y_2 + y_1) & -\tan(\hat{\phi}_{2,1}) \\ -y_3 + y_1 + (x_3 + x_1) & x_3 - x_1 - (y_3 + y_1) & -\tan(\hat{\phi}_{3,1}) \\ \dots & & \\ -y_N + y_{N-1} + (x_N + x_{N-1}) & x_N - x_{N-1} - (y_N + y_{N-1}) & -\tan(\hat{\phi}_{N,N-1}) \end{bmatrix} \tag{12}$$

$$\hat{\boldsymbol{b}} = \begin{bmatrix} -x_1 y_2 + x_2 y_1 + (x_2 x_1 + y_2 y_1)\tan(\hat{\phi}_{2,1}) \\ -x_1 y_3 + x_3 y_1 + (x_3 x_1 + y_3 y_1)\tan(\hat{\phi}_{3,1}) \\ \dots \\ -x_{N-1} y_N + x_N y_{N-1} + (x_N x_{N-1} + y_N y_{N-1})\tan(\hat{\phi}_{N,N-1}) \end{bmatrix} \tag{13}$$

divergence appears in tens of iterations. Therefore, we set up the maximum number of iterations for each procedure is 1000. If at 1001st value, the stopping criterion is not met, the iterative procedure will be considered as an oscillating procedure and then forced to stop.

We take the final position of a converging procedure as the estimated position for the mobile device. As for a diverging procedure or an oscillating procedure, the initialization is selected as estimate.

In a nutshell, we propose the Algorithm 1, a Gauss-Newton iterative solution for the Maximum Likelihood estimator.

Algorithm 1: Proposed Maximum Likelihood estimator with phase correction $\widehat{\boldsymbol{k}}$

Take the measured DDoA $\hat{\phi}_{i,j}$ as the trigonometric angle of the incident wave from i-th base station and the incident wave from j-th base station.
Assign $u = 1$ and ε sufficiently small.
Compute the estimation $\hat{\boldsymbol{\omega}}$ by (15), then assign $\hat{\boldsymbol{x}}^{(1)} = \begin{bmatrix}[\hat{\boldsymbol{\omega}}]_1 & [\hat{\boldsymbol{\omega}}]_2\end{bmatrix}^T$ as the first estimated coordinates of the mobile device.
repeat
    Compute the estimated Direction Difference of Arrival $\hat{\phi}_{i,1}$ by (19).
    if $|\hat{\phi}_{i,1}(\hat{\boldsymbol{x}}^{(u)}) - \hat{\phi}_{i,1} \geq \pi$ **then**
        $\hat{k}_{i,1}$ = sign($\hat{\phi}_{i,1}(\hat{\boldsymbol{x}}^{(u)}) - \hat{\phi}_{i,1}$)
    else
        $\hat{k}_{i,1} = 0$;
    Compute $\hat{\boldsymbol{x}}^{(u+1)}$ by (24). $u = u + 1$;
until $\|\hat{\boldsymbol{x}}^{(u)} - \hat{\boldsymbol{x}}^{(u-1)}\|_2 < \varepsilon$ *or* $u > 1000$ *or* $\|\hat{\boldsymbol{x}}^{(u)}\|_2 = \pm\infty$;
if $u > 1000$ *or* $\|\hat{\boldsymbol{x}}^{(u)}\|_2 = \pm\infty$ **then**
    $\hat{\boldsymbol{x}}^{(1)}$ is the estimated position of the mobile device;
else
    $\hat{\boldsymbol{x}}^{(u)}$ is the estimated position of the mobile device;

E. Cramer-Rao Bound (CRB)

To evaluate the quality of the algorithm based on DDoA, we compare to the Cramer-Rao Bound (CRB), via the Fisher Information Matrix (FIM):

$$\boldsymbol{I}(\boldsymbol{x}) = \boldsymbol{G}^T(\boldsymbol{x})\boldsymbol{C}^{-1}\boldsymbol{G}(\boldsymbol{x})\,. \tag{28}$$

The CRB is the sum of all the diagonal elements of the inverse of FIM:

$$\text{CRB} = [\boldsymbol{I}^{-1}]_{1,1} + [\boldsymbol{I}^{-1}]_{2,2} \tag{29}$$

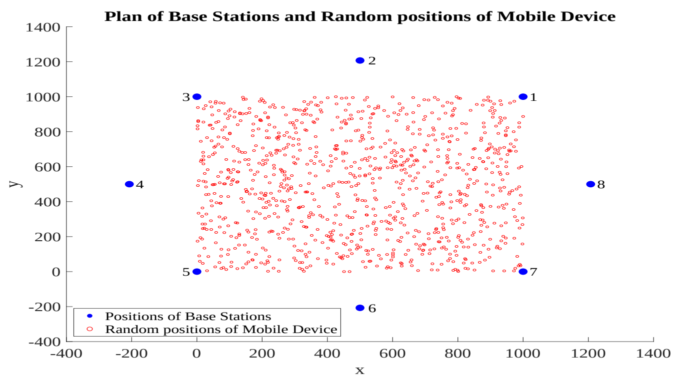


Fig. 3: Map of base stations and random mobile device positions.

III. Simulation Results

To compare the quality among of algorithms and CRB, we use Root Mean Square Position Error (RMSE) which is defined by

$$\text{RMSE} = \sqrt{\text{E}\{\|\hat{\boldsymbol{x}} - \bar{\boldsymbol{x}}\|^2\}} \tag{30}$$

where $\bar{\boldsymbol{x}}$ is the true position of the mobile device and $\hat{X}$ is its estimate. In the xy plane, RMSE averaging is over 1000 mobile positions picked randomly in a square of 1000m x 1000m centered in the circle of BSs. The network of 8 Base stations (numbered from 1 to 8) forms the circumscribed circle of this square (Fig. 3). The value for ε in the stopping criterion of the ML estimator is 0.01.

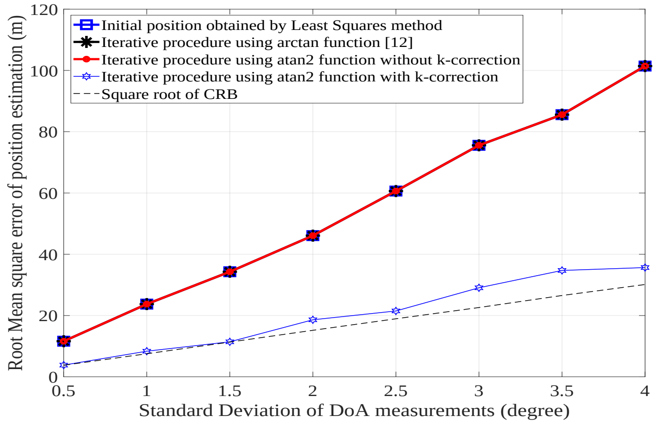
A. Results

In the figures, initial point" refers to the position found by the Least-Squares method in section II-C, whereas iterative procedure" refers to the Maximum Likelihood algorithm.

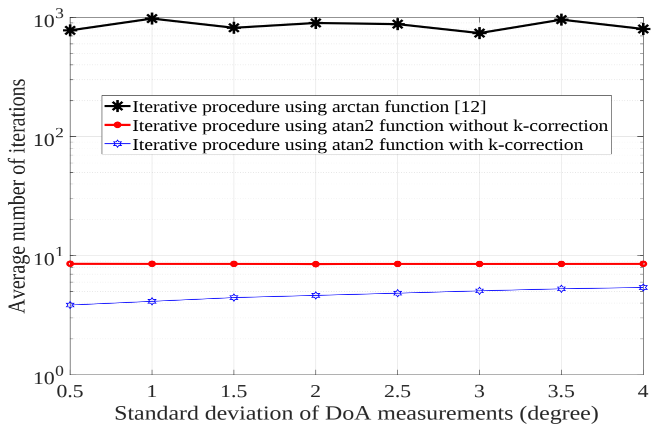
Instead of comparing the MSEs to the CRB, we compare their square roots: The Root Mean Square Error (RMSE) = $\sqrt{\text{MSE}}$ and square root of CRB ($\sqrt{\text{CRB}}$). In the simulations, all the DoA estimations are assumed to have the same standard deviation: $\sigma_1 = \sigma_2 = \cdots = \sigma_N = \sigma$.

Fig. 4 illustrates the results when the common standard deviation of DoA estimations (σ) varies from $0.5°$ to $4°$. More comprehensively, Fig. 4a compares the RMSEs of the 4 algorithms:

(a) The initial point obtained by Least Squares method shown in section II-C.



(a) RMSEs and $\sqrt{\text{CRB}}$.



(b) Average number of iterations.

Fig. 4: Algorithm comparison for $N = 8$ BSs, standard deviation of DOA measurements varies from 0.5^o to 4^o.

(b) Gauss-Newton iterative procedures using arctan function [12], by replacing atan2 by arctan in (19) (this corresponds to (4.7), (4.12) and (4.20) in [12]).

(c) Gauss-Newton iterative procedures using atan2 function **without** k-correction.

(d) Gauss-Newton iterative procedures using atan2 function **with** k-correction.

The $\sqrt{\text{CRB}}$ is also added to validate their performances. Fig. 4b compares the average number of iterations of the three algorithms (b), (c) and (d).

In Fig. 4a, there is no difference among the RMSEs of (a), (b) and (c). Undoubtedly, all the iterative procedures with algorithm (b) and (c) diverge or oscillate so that the ML estimator has no effect. On the otherhand, the RMSE of (d) is much smaller than the common RMSE of (a), (b) and (c), which means ML estimator plays an important role in position optimization. The RMSE of (d) is still higher than the $\sqrt{\text{CRB}}$, which demonstrates that this estimator is unbiased.

In Fig. 4b, the average number of iterations of (b) is about 900, which means that 90% of the procedures oscillate and only 10% of them diverge. As for (c), this average number is smaller than 10, because most of the procedures diverge. Our proposed algorithm (d) has the smallest average number of iterations, so it cam give a remarkable reduction of the time delay for localization processes.

IV. Conclusions

This paper analyzes DDoA-based positioning algorithms using the atan2 function with a phase correction to overcome possible phase jumps caused by errors in angle measurements. The simulations demonstrate the superior performance of the proposed algorithm: more accurate results and the lower computation time compared to some existing approaches. However, the problems in multipath environments are not taken into account. In addition, an extension to 3D localization should be implemented to make the proposed positioning algorithm more realistic.

Acknowledgment

EURECOMs research is partially supported by its industrial members: ORANGE, BMW, Symantec, SAP, Monaco Telecom, iABG. The Research of EURECOM and ORANGE was also supported in part by the French FUI project GEOLOC.

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