Queueing analysis of simple FEC schemes for IP Telephony

Eitan Altman, Chadi Barakat, Victor M. Ramos R.

Abstract—**In interactive voice applications, FEC schemes are necessary for the recovery from packet losses. These schemes need to be simple with a light coding and decoding overhead in order to not impact the interactivity. The objective of this paper is to study a well known simple FEC scheme that has been proposed and implemented [1], [2], in which for every packet n, some redundant information is added in some subsequent packet $n + \phi$. If packet n is lost, it will be reconstructed in case packet $n + \phi$ is well received. The quality of the reconstructed copy of packet n will depend on the amount of information on packet n we add to packet $n+\phi$. We propose a detailed queueing analysis based on a ballot theorem and obtain simple expressions for the audio quality as a function of the amount of redundancy and its relative position to the original information. The analysis shows that this FEC scheme does not scale well and that the quality will finish by deteriorating for any amount of FEC and for any offset ϕ.**

Keywords—**IP Telephony, ballot theorem, FEC, audio quality.**

I. Introduction

Real -time audio transmission is now widely used over the Internet and has become a very important application. Audio quality is still however an open problem due to the loss of audio packets and the variation of end-to-end delay (jitter). These two factors are a natural result of the simple best effort service provided by the current Internet. Indeed, the Internet provides a simple packet delivery service without any guarantee on bandwidth, delay or drop probability. The audio quality deteriorates (noise, poor interactivity) when packets cross a loaded part of the Internet. In the wait for some QoS facilities from the network side like resource reservation, call admission control, etc., the problem of audio quality must be studied and solved on an end-to-end basis. Some mechanisms must be introduced at the sender and/or at the receiver to compensate for packet losses and jitter. The jitter is often solved by some adaptive playout algorithms at the receiver. Adaptive playout mechanisms are treated in detail in [3], and more recently in [4]. In this paper we focus on the problem of recovery from audio packet losses.

Mechanisms for recovering from packet losses can be classified as open loop mechanisms, or closed loop mechanisms [5]. Closed loop mechanisms like ARQ (Automatic Repeat reQuest) are not adequate for real-time interactive applications since they increase considerably the end-to-end delay due to packet retransmission. Open loop mechanisms like FEC (Forward Error Correction) are better adapted to real-time applications given that packet losses are recovered without the need for a retransmission. Some redundant information is transmitted with the basic data flow. Once a packet is lost, the receiver uses (if possible) the redundant information to reconstruct the lost information. FEC schemes are recommended whenever the end-to-end delay is large so that a retransmission deteriorates the overall quality.

FEC has been often used for loss recovery in audio communication tools. It is a sender-based repair mechanism. An efficient FEC scheme is a one that is able to repair most of packet losses. Now, when FEC fails to recover from a loss, applications can resort to other receiver-based repair mechanisms like insertion, interpolation, or regeneration, using well known methods [5]. The FEC schemes proposed in the literature are often simple, so that the coding and the decoding of the redundancy can be quickly done without impacting the interactivity. In particular, the redundancy is computed over small blocks of audio packets. Well known audio tools as Rat [2], and Freephone [1], generally work by adding some redundant information on (i.e. a copy of) packet n to the next packet $n + 1$, so that if packet n is dropped in the network, it can be recovered and played out in case packet $n + 1$ is correctly received. The redundant information carried by a packet is generally obtained by coding the previous packet with a code of lower rate than that of the code used for coding the basic audio flow. Thus, if the reconstruction succeeds, the lost packet is played out with a lower quality. This has been shown to give better quality than playing nothing at the receiver. Fig. 1 depicts this simple FEC scheme.

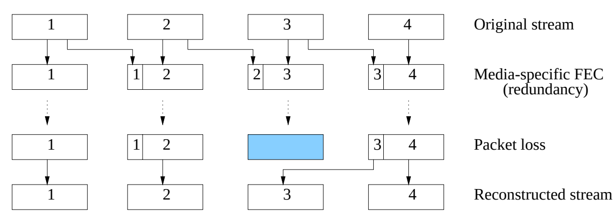

Fig. 1. The simple FEC mechanism where packet $n + 1$ carries redundant information on packet n.

In this paper we address the problem of audio quality under this FEC scheme. In all the paper when we talk about FEC in general, it is this scheme that we mean. We evaluate analytically the audio quality at the destination as a function of the parameters of the FEC scheme, of the basic audio flow and of the network. The performance of this FEC scheme has been evaluated via simulations [6], [7], and tools like Freephone and Rat have implemented it. In [8], the authors propose to increase the offset between the original packet and its redundancy. They claim that the loss process in the Internet is bursty and thus, increasing the offset could give better performance than having the redundancy placed in the packet following immediately the original one. However, the authors in [8] did not propose any analytical expression that permits to study the impact of this spacing on the audio quality.

In this paper we use probabilistic methods and a ballot the-

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orem [9] to find an explicit expression for the audio quality in the case of a general offset not necessarily equal to one. We use a simple function for the audio quality proportional to the volume of data we receive. Our results show that we always lose in quality with this simple FEC scheme. Similar negative results have been already obtained using analytical tools for more sophisticated FEC schemes, see [10], [11], [12].

The paper is organized as follows. In Section II we describe the general scenario for applications using FEC, and we define a quality function which we will use in the rest of the paper. In Section III we study the simple case when packet n carries redundant information on packet $n-1$ assuming an $M/M/1/K$ queueing model. In Section IV we solve the problem in the general case when packet n carries redundant information on packet $n-\phi$ with $\phi \geq 1$. We look in Section V at the quality in the case of infinite spacing $\phi \to \infty$. We present some concluding remarks in Section VI.

II. Analysis

In a large network as the Internet, a flow of packets crosses several routers before reaching the other end. Most of the losses from a flow occur in the router having the smallest available bandwidth in the chain of routers, so that one may model the whole chain by one single router called "*the bottleneck*." This assumption has both theoretical and experimental justification [13], [14]. We shall use the simple $M/M/1/K$ queue to model the network and thus the loss process of audio packets. In other words, we assume that audio packets arrive at the bottleneck according to a Poisson process of intensity λ, and we assume that the time required to process an audio packet at the bottleneck is distributed exponentially with parameter μ. The Poisson assumption on inter-arrival times could be justified by the random delay added to packets by routers located upstream the bottleneck. The service time represents the time between the beginning of the transmission of an audio packet on the bottleneck interface leading to the destination until the beginning of the transmission of the next packet from the same audio flow. Since the two packets may be spaced apart by a random number of packets from other applications, one may use the exponential distribution as a candidate for modeling the service time of audio packets at the bottleneck. The reason for choosing this simplistic model for the network is to be able to obtain simple mathematical formulas that give us some insights on the gain from using FEC.

Let $\rho = \lambda/\mu$ be the intensity of audio traffic. Assume that audio packets share alone the buffer K. This can be the case of a bottleneck crossed only by audio packets, or the case of a bottleneck router implementing a per-flow or a per-class queueing. Thus, for $\rho < 1$, the loss probability of an audio packet in steady state is given by [15]:

$$\pi(\rho) = \frac{1-\rho}{1-\rho^{K+1}}\rho^K, \tag{1}$$

and for $\rho = 1$ it is equal to

$$\pi(\rho) = \frac{1}{K+1}.$$

Now, we add redundancy to each packet in a way that if a packet is lost, it can be still "partially" retrieved if the packet containing its redundancy is not lost. The redundancy is located ϕ packets apart from the original packet. It consists in a low quality copy of the original packet. Let α be the ratio of the volume of the redundant information and the volume of the original packet. α is generally less than one. Along with the possibility to retrieve the lost information in the network, we should consider the negative impact of the addition of FEC on the loss probability. This addition has an impact on the service times since packets require now more time to be retransmitted at the output of the bottleneck. It may also have an impact on the buffering capacity at the bottleneck since each packet now contains more bits. We shall propose the following two possible negative impacts of FEC, in order to study later the tradeoff between the positive and negative impacts:

- **Impact of FEC on service time.** We assume that audio packets including redundancy require a longer service time which is exponentially distributed with parameter $\frac{\mu}{(1+\alpha)}$. This can be the case when our audio flow has an important share of the bottleneck bandwidth. If it is not the case, this assumption can hold when the exogenous traffic at the bottleneck (or at least an important part of it) is formed of audio flows that implement the same FEC scheme. Our assumption also holds when the bottleneck router implements a per-flow scheduling that accounts for the size of packets.
- **Impact of FEC on buffering.** The buffering capacity in the bottleneck router will be affected by the addition of FEC in one of two ways: (1) Since packets are now longer by a factor $(1+\alpha)$, we can consider that the amount of buffering is diminished by this quantity, or (2) We can assume that the queue capacity is not function of packet length, but rather of the number of packets. Hence, the queue capacity is not affected by the use of FEC. Let K_α denote the buffer size after the addition of FEC in terms of packets. It is equal to $K/(1+\alpha)$ if the buffer capacity is changed and to K otherwise. Thus, the loss probability in the presence of FEC takes the following form:
$$\pi_\rho(\alpha) = \frac{1-\rho(1+\alpha)}{1-\Big(\rho(1+\alpha)\Big)^{K_\alpha}}\Big(\rho(1+\alpha)\Big)^{K_\alpha}. \tag{2}$$

Before we define the quality of audio received at the destination, we introduce a random variable Y_n that indicates a successful arrival of a packet at the destination or not. Then,

$$\begin{aligned} Y_n &= 0, \quad \text{if packet } n \text{ is lost, and} \\ Y_n &= 1, \quad \text{if packet } n \text{ is correctly received.} \end{aligned}$$

Let $\phi \geq 1$ be the variable indicating the distance, or the offset, between the original packet and its redundancy. We make the simple assumption that the audio quality is proportional to the amount of information we receive. A quality equal to 1 indicates that we are receiving all the information (the basic audio flow). The quality we get after the reconstruction of an original packet from the redundancy is taken equal to α, where α is the ratio of redundancy volume and original packet volume. We thus define the quality function as,

$$\begin{aligned} &Q(\alpha) \\ &= P(Y_n = 1) + \alpha P(Y_n = 0) P(Y_{n+\phi} = 1 | Y_n = 0) \\ &= 1 - \pi_\rho(\alpha)(1 - \alpha P(Y_{n+\phi} = 1 | Y_n = 0)). \end{aligned} \tag{3}$$

This equation gives us the audio quality at the destination under a FEC scheme of rate $(1+\alpha)^{-1}$, and of distance ϕ between an original packet and its redundancy. For the case $\alpha = 0$, our definition for the quality coincides with the probability that a packet is correctly received. For the case $\alpha = 1$, it coincides with the probability that the information in an original packet is correctly received, either because it was not lost, or because it was fully retrieved from the redundancy. One may imagine to use another quality function that the one we chose. In particular, one can use a quality function that is not only a function of the amount of data correctly received but also of the coding algorithm used. Different algorithms have been used in [1], [2] for coding the original data and the redundancy. In the rest of the paper, we will use the following notation:

TABLE I

NOTATION USED IN THIS PAPER.

Expression	Definition
$Q(\alpha)$	The audio quality.
ϕ	The offset between the original packet and the packet including its redundancy.
K_α	The size of the queue.
X_j	The random variable which represents the number of packets in the queue just before the arrival of the j-th audio packet.
Z_j	The random variable which represents the number of services between the arrivals of the $j-1$-th and the j-th audio packets.[1]

We ask the following question: "*How does the audio quality vary as a function of α?*" That would permit us to evaluate the benefits from such a recovery mechanism and to find the appropriate amount of redundancy α that must be added to each packet. In the next sections we find the audio quality for different values of ϕ. The only missing parameter is the probability that the redundant information on a packet is correctly received given that the packet itself is lost. This is the function $P(Y_{n+\phi} = 1 | Y_n = 0)$ in (3). In the following sections we put ourselves in the stationary regime and we compute this probability.

III. SPACING BY $\phi = 1$

In this section we analyze the case when the redundant information on packet n is carried by packet $n+1$, i.e., $\phi = 1$. This mechanism is implemented in well known audio tools as Freephone [1] and Rat [2]. The probability that the redundancy is correctly received given that the original packet is lost, is no other than the probability that the next event after the loss of the original packet is a departure and not an arrival. This happens with probability,

$$P(Y_{n+1} = 1 | Y_n = 0) = \frac{1}{\rho(1+\alpha)+1}. \qquad (4)$$

Substituting (4) in (3), we obtain

$$Q_{\phi=1}(\alpha) = 1 - \pi_\rho(\alpha)\Big(1 - \frac{\alpha}{\rho(1+\alpha)+1}\Big).$$

To study the impact of FEC on the audio quality, we plot $Q_{\phi=1}(\alpha)$ as a function of α for different values of K_α and ρ. In Fig. 2, we show the results when the buffering capacity at the bottleneck is assumed to change with the amount of FEC ($K_\alpha = K/(1+\alpha)$), and in Fig. 3 we show the results for the case where the buffering capacity is not changed ($K_\alpha = K$). We see that, for both cases, audio quality deteriorates when α increases (when we add more redundancy), and this deterioration becomes more important when the traffic intensity increases and when the buffer size decreases. The main interpretation of such behavior is that the loss probability of an original packet increases with α faster than the gain in quality we got from retrieving the redundant information. This should not be surprising. Indeed, even in more sophisticated schemes in which a single redundant packet is added to protect a whole block of M packets, it is known that FEC often has an overall negative effect, see [10], [11], [12]. Yet in such schemes the negative effect of adding the redundancy is smaller than in our scheme, since the amount of added information per packet is smaller (i.e., a single packet protects a whole group of M packets). But, we know that for such schemes and in case of light traffic, the overall contribution of FEC is positive [11], [12]. This motivates us to analyze more precisely the impact of FEC in our simplistic scheme in case of light traffic.

Define the function $\Delta(\rho) = Q(1) - Q(0)$ and consider the case when the buffering capacity at the bottleneck is not affected by the amount of FEC. This is an optimistic scenario where it is very probable to see the gain brought by FEC, of course if this gain exists. We have,

$$\Delta(\rho) = -2(2\rho)^{\frac{K_\alpha}{2}}\Big(\frac{1+\rho}{2\rho+1}\Big)\Big(\frac{1-2\rho}{1-(2\rho)^{\frac{K_\alpha}{2}+1}}\Big) + \frac{1-\rho}{1-\rho^{K_\alpha+1}}\rho^{K_\alpha} \qquad (5)$$

Finding $\lim_{\rho\to 0}\Delta(\rho)$ would permit us to evaluate the audio quality for a very low traffic intensity. We took $K_\alpha = 2M$ in (5) and we expanded $\Delta(\rho)$ in a Taylor series. We found that all the first coefficients of the series $c_0, c_1, ..., c_{M-1}$ are equal to zero, and that the coefficient c_M is negative and equal to $-2(2\rho)^M$. c_i is the coefficient of ρ^i in the Taylor series of $\Delta(\rho)$ and can be computed by

$$c_j = \frac{d}{d\rho^j}\Delta^j(\rho)_{|\rho=0}.$$

Thus, for small ρ, $\Delta(\rho)$ can be written as $-2(2\rho)^M + o(\rho^M)$ and the gain from the addition of FEC can be seen to be negative. With this simple FEC scheme, we lose in audio quality when adding FEC even for a very low traffic intensity. This loss in quality decreases with the increase in buffer size.

[1] As is frequently done, we include in Z_j not only real services but also "potential services": these are services that occur while the system is empty; thus at the end of such a service no packet leaves.

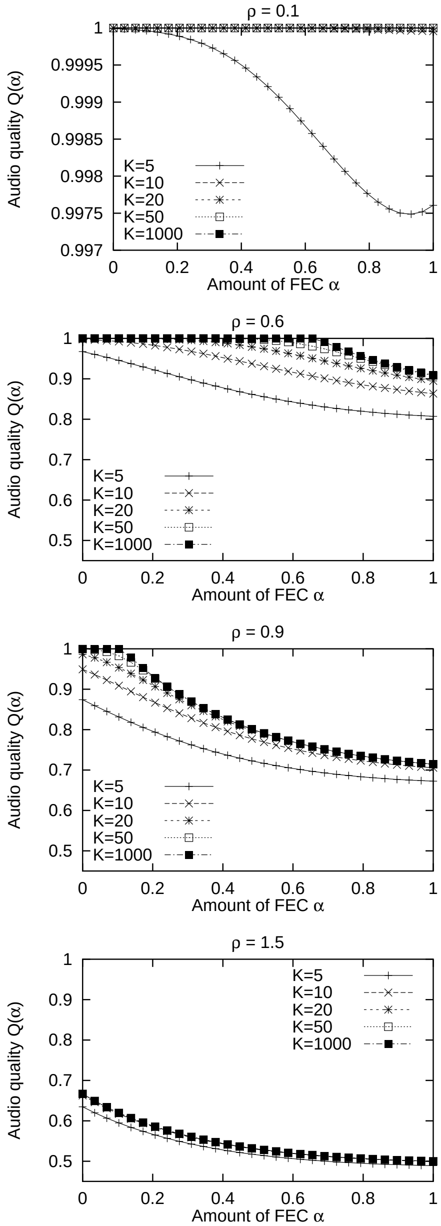

Fig. 2. $\phi = 1$ and the queue capacity is changed.

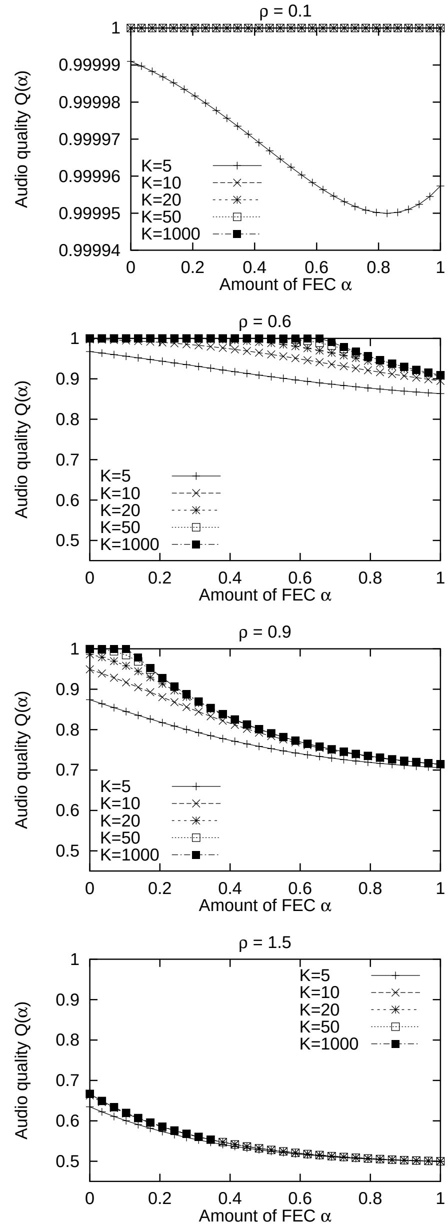

Fig. 3. $\phi = 1$ and the queue capacity is not changed.

IV. General case: Spacing by $\phi \geq 1$

Now, we consider the more general case when the spacing between the original packet and its redundancy is greater than 1. The idea behind this type of spacing is that losses in real networks tend to appear in bursts, and thus spacing the redundancy from the original packet by more than one improves the probability to retrieve the redundancy in case the original packet is lost. Indeed, a packet loss means that the queue is full and thus the probability of losing the next packet is higher than the steady state probability of losing a packet. The spacing gives the redundancy of a packet more chance to find a non full buffer at the bottleneck, and thus to be correctly received. We note that the phenomenon of the correlation between losses of packets was already modeled and studied in other papers: [10], [11], [12]. Measurements have also shown that most of the losses are correlated [16], [17], [18].

Here, we are interested in finding the probability that packet $n + \phi$ is lost given that packet n is also lost. This will give us $P(Y_{n+\phi} = 1|Y_n = 0)$ which in turn gives us the expression for the audio quality (as expressed in (3)). Since we assume that the system is in its steady state, we can omit the index n and substitute it by zero. We have $Y_0 = 0$ which means $X_0 = K_\alpha$. We are interested in the probability that $X_\phi = K_\alpha$. For the ease of calculation we consider the case $\phi \leq K_\alpha$. We believe that this is quite enough given that a large spacing between the original packet and the redundancy leads to an important jitter and a poor interactivity.

In order to obtain an explicit expression for the probability $P(X_\phi = K_\alpha|X_0 = K_\alpha)$, we first provide an explicit sample-path expression for the event of loss of the packet carrying the redundancy, given that the original packet itself was lost.

Theorem 1: Let $X_0 = K_\alpha$ and $1 \le \phi \le K_\alpha$. then:

Packet ϕ is not lost if and only if

$$X_\phi < K_\alpha \Leftrightarrow \begin{cases} Z_\phi - 1 & \ge 0 \\ & \text{or} \\ Z_\phi + Z_{\phi-1} - 2 & \ge 0 \\ & \text{or} \\ & \vdots \\ & \text{or} \\ Z_\phi + Z_{\phi-1} + \cdots + Z_1 - \phi & \ge 0 \end{cases}$$

or equivalently, packet ϕ is lost if and only if

$$X_\phi = K_\alpha \Leftrightarrow \begin{cases} Z_\phi - 1 & < 0 \\ & \text{and} \\ Z_\phi + Z_{\phi-1} - 2 & < 0 \\ & \text{and} \\ & \vdots \\ & \text{and} \\ Z_\phi + Z_{\phi-1} + \cdots + Z_1 - \phi & < 0 \end{cases} \quad (6)$$

Proof: We can express the number of packets that the $i+1$-th audio packet will find in the queue upon arrival as follows:

$$X_{i+1} = \Big((X_i + 1) \wedge K_\alpha - Z_{i+1}\Big) \vee 0 \quad \forall i \ge 0, \quad (7)$$

where $\wedge$ and $\vee$ are respectively the minimum and maximum operators. The rest of the proof goes in three steps that are summarized in Lemma 1, Lemma 2 and Corollary 1 below. ■

Now, we define

$$\tilde{X}_{i+1} \triangleq (\tilde{X}_i + 1) \wedge K_\alpha - Z_{i+1}. \quad (8)$$

This new variable corresponds to the number of packets that would be found in the queue upon the arrival of packet $i+1$ if the queue size could become negative. We next show that it can be used as a lower bound for X_{i+1}.

Lemma 1: If $\tilde{X}_0 \le X_0$ then $\tilde{X}_i \le X_i \quad \forall i \ge 0$.

Proof: We proceed for the proof by induction. This relation is valid for $i = 0$. Suppose that it is valid for $i \ge 0$. We show that it is valid for $i+1$,

$$\begin{aligned} \tilde{X}_{i+1} &\le \Big((\tilde{X}_i + 1) \wedge K_\alpha - Z_{i+1}\Big) \vee 0 \\ &\le \Big((X_i + 1) \wedge K_\alpha - Z_{i+1}\Big) \vee 0 \\ &= X_{i+1}. \end{aligned}$$

■

Lemma 2: Let $\tilde{X}_0 = K_\alpha$, then

$\tilde{X}_i = K_\alpha - \max_{1\le l\le i} \sum_{j=l}^{i} (Z_j - 1) - 1 \quad \forall i \ge 0.$

Proof:

$$\begin{aligned} \tilde{X}_0 &= K_\alpha \\ \tilde{X}_1 &= K_\alpha - Z_1 \\ \tilde{X}_2 &= (K_\alpha - Z_1 + 1) \wedge K_\alpha - Z_2 \\ \tilde{X}_3 &= \Big((K_\alpha - Z_1 + 1) \wedge K_\alpha - Z_2 + 1\Big) \wedge K_\alpha \\ &\quad - Z_3 \\ &= (K_\alpha - Z_1 - Z_2 + 2) \wedge (K_\alpha - Z_2 + 1) \wedge K_\alpha \\ &\quad - Z_3 \\ &= K_\alpha - (Z_1 + Z_2 - 2) \vee (Z_2 - 1) \vee 0 - Z_3 \\ &\ \ \vdots \\ \tilde{X}_i &= K_\alpha - \max_{1\le l<i} \Big\{0, \sum_{j=l}^{i-1} (Z_j - 1)\Big\} - Z_i \\ &= K_\alpha - \max_{1\le l<i} \Big\{0, \sum_{j=l}^{i-1} (Z_j - 1)\Big\} - Z_i \\ \Rightarrow \tilde{X}_i &= K_\alpha - \max_{1\le l\le i} \Big\{ \sum_{j=l}^{i} (Z_j - 1)\Big\} - 1. \end{aligned}$$

■

Corollary 1: Expression (6) holds if $X_0 = K_\alpha$ and $\phi \le K_\alpha$.

Proof: The right hand side in (6) is no other than $\max_{1\le l\le\phi} \Big\{ \sum_{j=l}^{\phi} (Z_j - 1)\Big\}$. Suppose first that $\max_{1\le l\le\phi} \Big\{ \sum_{j=l}^{\phi} (Z_j - 1)\Big\} < 0$. Using Lemma 2 then Lemma 1, we have $\tilde{X}_\phi \ge K_\alpha$ which gives $X_\phi \ge K_\alpha$. Thus, $X_\phi = K_\alpha$.

Now, we need to show that if $X_\phi = K_\alpha$ we get $\max_{1\le l\le\phi} \Big\{ \sum_{j=l}^{\phi} (Z_j - 1)\Big\} < 0$. We define:

$$\phi^* = \min\Big\{i \quad | \quad \tilde{X}_i < X_i\Big\} \quad (9)$$

According to (9), we distinguish between the two following cases:

- $\phi^* > \phi$, and
- $\phi^* \le \phi$.

Consider the first case. Using the definition of ϕ^* and Lemma 2, we write: $\phi^* > \phi \Rightarrow \tilde{X}_\phi = X_\phi \Rightarrow \tilde{X}_\phi = K_\alpha \Rightarrow \max_{1\le l\le\phi} \Big\{ \sum_{j=l}^{\phi} (Z_j - 1)\Big\} = -1 < 0$.

Now, suppose that $\phi^* \le \phi$, thus $\tilde{X}_{\phi^*} < 0$ and $X_{\phi^*} = 0$. We write,

$$X_\phi \le X_{\phi^*} + (\phi - \phi^*) = (\phi - \phi^*) < \phi \le K_\alpha,$$

if there were no service. Thus, we get in this case $X_\phi < K_\alpha$ which is in contradiction with our assumption that $X_\phi = K_\alpha$. The case $\phi^* \le \phi$ does not appear if ϕ is chosen less or equal to the buffering capacity. Thus, for $X_\phi = K_\alpha$ we have $\max_{1\le l\le\phi} \Big\{ \sum_{j=l}^{\phi} (Z_j - 1)\Big\} < 0$. This concludes the proof of Theorem 1. ■

According to Ballot's Theorem [9] (see the Appendix in Section VI for details), we have for $k < \phi$:

Lemma 3:

$$P\Big\{ \max_{1\le l\le\phi} \Big\{ \sum_{j=l}^{\phi}(Z_j-1)\Big\} < 0 \,|\, \sum_{l=1}^{\phi} Z_l = k\Big\} = 1-\frac{k}{\phi}$$

Let A be the event that $X_\phi = K_\alpha$ given that $X_0 = K_\alpha$. We sometimes write A^ϕ to stress the dependence on ϕ. We conclude from Theorem 1 that if packet 0 is lost, i.e. if packet 0 finds K_α packets in the system, then

$$A = \left\{ \max_{1\le l\le\phi} \Big\{ \sum_{j=l}^{\phi}(Z_j-1)\Big\} < 0 \right\}$$

Then, we can represent the probability that packet $n+\phi$ is lost given that packet n is lost as

$$P(Y_{n+\phi}=0|Y_n=0) = P(A)$$

$$= \sum_{k=0}^{\phi-1} P(A|Z_1+\cdots+Z_\phi=k)P(Z_1+\cdots+Z_\phi=k) \quad (10)$$

Once this probability is computed, the audio quality can be directly derived using (3).

Theorem 2: Consider $1 \le \phi \le K_\alpha$ and let $\rho_\alpha = \rho(1+\alpha)$. Given that packet n is lost, the probability that packet $n+\phi$ is also lost is given by

$$P(A) = \sum_{k=0}^{\phi-1}\Big(1-\frac{k}{\phi}\Big)\Big(\frac{\rho_\alpha}{\rho_\alpha+1}\Big)^{\phi}\Big(\frac{1}{\rho_\alpha+1}\Big)^{k}\binom{\phi+k-1}{\phi-1} \quad (11)$$

where $()$ denotes the binomial coefficient. The quality function can be calculated by substituting $P(A)$ in (3). Note that $P(Y_{n+\phi}=1|Y_n=0) = 1-P(A)$.

Proof: The second right hand term of (10) must be solved by combinatorial reasoning. For that purpose, we define the vector $\vec{Z}$ to be:

$$\vec{Z} = \begin{pmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_\phi \end{pmatrix}, \quad (12)$$

where $\sum_{l=1}^{\phi} Z_l = k$, and we define S be the set of the different sets that $\vec{Z}$ may acquire: $S = \{\vec{Z}\}$. We must sum over all the possible trajectories:

$$\begin{aligned} P(\sum_{l=1}^{\phi} Z_l = k) & \\ &= \sum_S P(Z_1=z_1)P(Z_2=z_2)\cdots P(Z_\phi=z_\phi) \\ &= \sum_S \Big(\frac{\lambda}{\lambda+\mu_\alpha}\Big)^{\phi}\Big(\frac{\mu_\alpha}{\lambda+\mu_\alpha}\Big)^{k} \\ &= \Big(\frac{\lambda}{\lambda+\mu_\alpha}\Big)^{\phi}\Big(\frac{\mu_\alpha}{\lambda+\mu_\alpha}\Big)^{k}\binom{\phi+k-1}{\phi-1} \quad (13) \end{aligned}$$

We define μ_α as being equal to $\mu/(1+\alpha)$. It's easy to see that the combinatorial part of (13) holds. To do that, we can see the problem to be the number of distinguishable arrangements of k indistinguishable objects (the packet audio departures from the bottleneck) in ϕ inter-arrival intervals, just as it's depicted in Fig. 4.

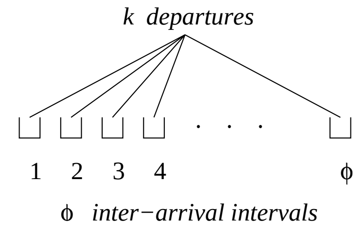

Fig. 4. Model to solve the combinatorial part.

Using (13) we get finally,

$$P(A) = \sum_{k=0}^{\phi-1}\Big(1-\frac{k}{\phi}\Big)\Big(\frac{\lambda}{\lambda+\mu_\alpha}\Big)^{\phi}\Big(\frac{\mu_\alpha}{\lambda+\mu_\alpha}\Big)^{k}\binom{\phi+k-1}{\phi-1}, \quad (14)$$

which yields (11) in terms of $\rho_\alpha = \rho(1+\alpha) = \lambda/\mu_\alpha$. The quality function can be obtained by substituting (11) in (3). The value of $\pi_\alpha(\rho)$ is given in (2). ■

We trace now plots of the audio quality as given by (3) and (11) for different values of K_α, ϕ and ρ. Fig. 5 depicts the behavior of $Q(\alpha)$ when the buffering capacity at the bottleneck is assumed to be divided by a factor $(1+\alpha)$, and Fig. 6 depicts this behavior when the buffering capacity is not changed.

We notice that, just as in the case of $\phi = 1$, we always lose in quality when we increase the amount of FEC even if we consider a large spacing. But, we also notice that for a given amount of FEC, the quality improves when spacing the redundancy from the original packet. This is the result of an improvement in the probability to retrieve the redundancy given that the original packet is lost. This monotonicity property holds, in fact, for any value of ϕ (not just for $\phi \le K_\alpha$). We show this theoretically in the next section.

A. *Monotone increase of the quality with the spacing*

The steady state probability of loss of a packet n does not depend on ϕ. It thus remains to check the behavior of $P(X_{n+\phi} = K_\alpha | X_n = K_\alpha)$ as a function of ϕ in order to decide on the quality variation (Eq. 3). The quality is a decreasing function of this probability. For $\phi \le K_\alpha$, the latter probability is equal to $P(A^\phi)$, and the monotonicity property can be seen directly from the fact that A^ϕ is a monotone decreasing set (since it requires for more summands to be smaller than zero, as ϕ increases, see Eq. 6).

Now, to see that $P(X_{n+\phi} = K_\alpha | X_n = K_\alpha)$ is monotone decreasing for any ϕ, we observe (7), which holds for any $i > 0$, and note that X_{i+1} is monotone increasing in X_i. Thus by iteration, we get that X_ϕ is monotone increasing in X_0. Now using this monotonicity, we have

$$\begin{aligned} P(X_{\phi+1} = K_\alpha | X_0 = K_\alpha) &= P(X_\phi = K_\alpha | X_{-1} = K_\alpha) \\ = \sum_{i=0}^{K_\alpha} & P(X_\phi = K_\alpha | X_0 = i, X_{-1} = K_\alpha) \times \\ & P(X_0 = i | X_{-1} = K_\alpha) \end{aligned}$$

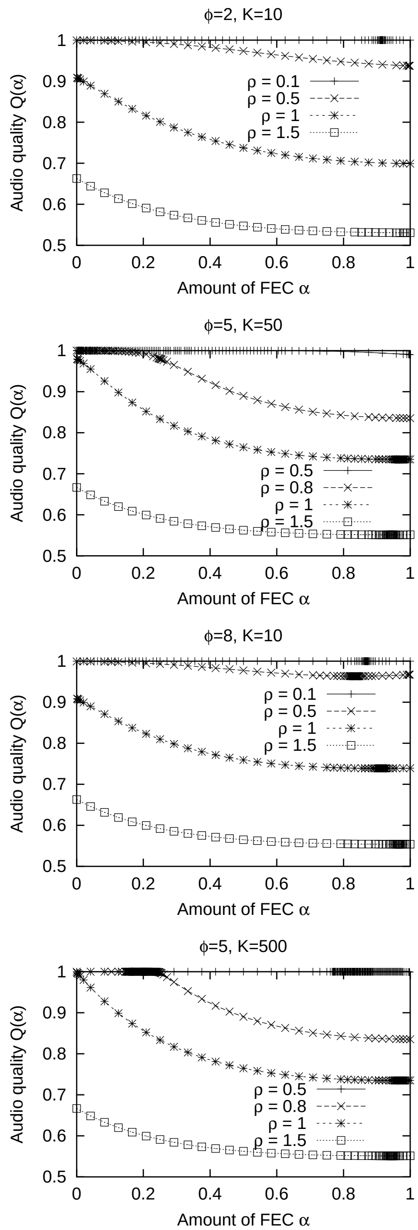

Fig. 5. Quality behavior in the presence of FEC and spacing $1 < \phi < K_\alpha$ assuming that queue size is changed.

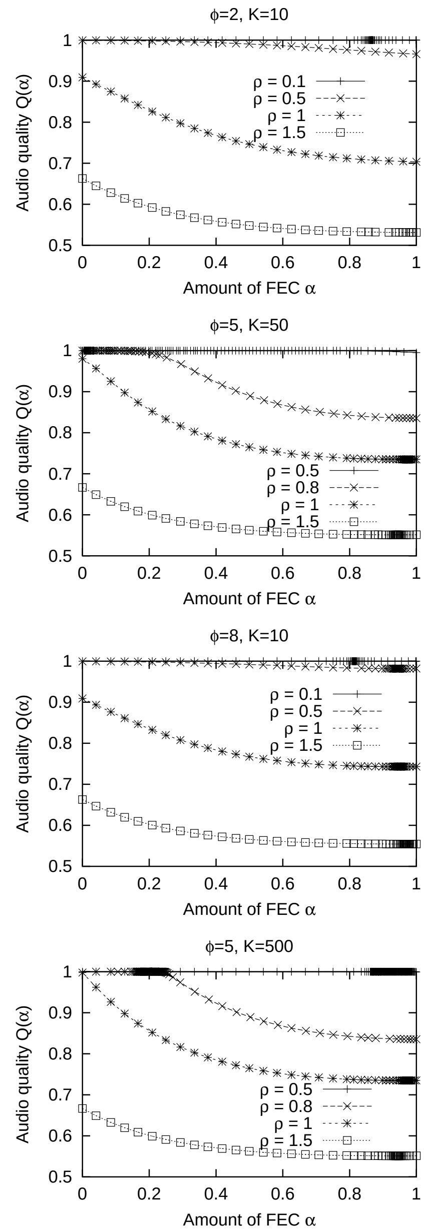

Fig. 6. Quality behavior in the presence of FEC and spacing $1 < \phi < K_\alpha$ assuming that queue size is not changed.

$$\begin{aligned}
&= \sum_{i=0}^{K_\alpha} P(X_\phi = K_\alpha | X_0 = i) P(X_0 = i | X_{-1} = K_\alpha) \\
&\leq \sum_{i=0}^{K_\alpha} P(X_\phi = K_\alpha | X_0 = K_\alpha) P(X_0 = i | X_{-1} = K_\alpha) \\
&= P(X_\phi = K_\alpha | X_0 = K_\alpha).
\end{aligned}$$

V. Limiting case: Spacing $\phi \to \infty$

The case of large ϕ is not of interest in interactive applications, since it means unacceptable delay. However, since we have found that the quality of the audio with FEC improves as the spacing grows, it is natural to study the limit ($\phi \to \infty$) in order to get an upper bound. Indeed, if we see that in this limiting case we do not improve the quality, it means that we lose by adding FEC according to our simple scheme for any finite offset ϕ.

When $\phi \to \infty$, the probability that the redundancy is dropped becomes equal to the steady state drop probability of a packet. Hence, (3) can be written as,

$$Q_{\phi\to\infty}(\alpha) = 1 - \pi_\rho(\alpha) + \alpha\pi_\rho(\alpha)(1 - \pi_\rho(\alpha)) \,. \qquad (15)$$

We plot (15) in Fig. 7 as a function of the amount of FEC for different values of K_α and ρ. We see well how, although we are in the most optimistic case, we lose in quality when adding FEC. That suggests that this class of FEC mechanisms are not adequate for real time transmission because it never improves the quality perceived at the receiver.

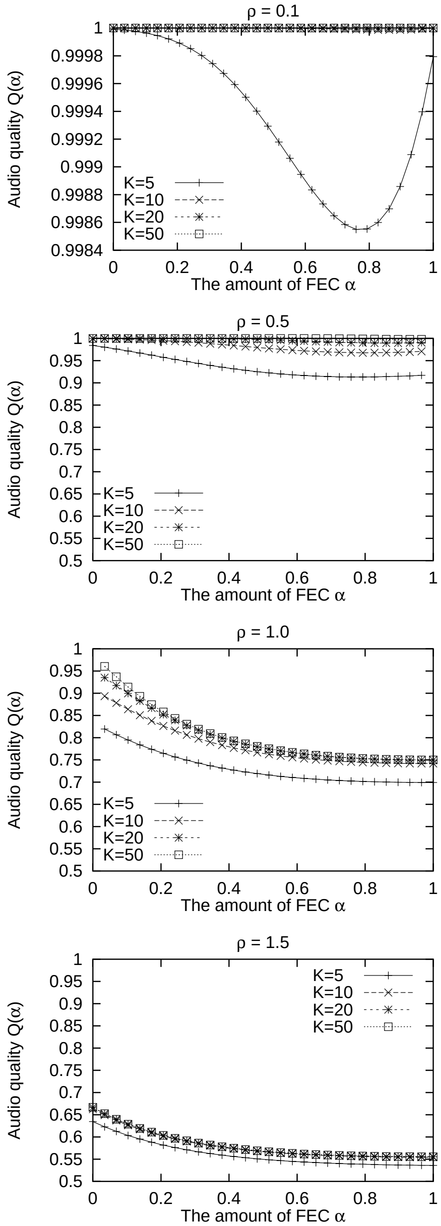

Fig. 7. Quality behavior in the presence of FEC and spacing $\phi \to \infty$.

VI. Conclusions

We have studied the effect that FEC schemes similar to the one used in [1] have on audio quality. This FEC scheme consists in adding a copy of an audio packet to a subsequent packet so that the copy can be used when the original packet is lost. We considered the different spacing strategies $\phi = 1$, $1 \leq \phi \leq K_\alpha$, and $\phi \to \infty$. Our simplistic $M/M/1/K$ queue shows that audio quality always deteriorates when applying this kind of FEC mechanism. It is therefore desirable to study other FEC methods that can provide a better quality. Recently, Ratton [19] found that media-independent FEC techniques using parity bits ([5]) perform better than media-specific FEC [16], [17], [5].

We can provide an intuitive explanation to the reason that the simplistic FEC studied here does not perform well. In this scheme, each added unit of redundancy protects only one unit of information that can be retrieved. There is only one possibility to retrieve a lost packet. We can define this as a protection gain of one unit. Other more sophisticated approaches allows a single unit of FEC to protect many packets (e.g. Reed Solomon coding that allows to retrieve up to n lost packets in any block of m packets to which n redundant packets are added). The protection gain of such more sophisticated mechanisms can thus be much higher. Even a simple XOR-based FEC, such as the one suggested in [19], has a high protection gain. We note however that the applicability of more sophisticated FEC mechanisms is still limited by delay constraints. Moreover, we note that even FEC mechanisms with high protection gain can suffer from deterioration of quality with respect to the case of no FEC, as was established in [11], [12].

We would like to give some comments on the validity of our results. Our analytical results are valid if our model for the network and the assumptions we made are correct. We believe that, due to traffic multiplexing, the $M/M/1/K$ model for the bottleneck is justified. But, this may not be sufficient since audio packets may be lost due to a transient congestion in another router. If this case is common, the FEC scheme may present better performance given that the total probability of the loss of packets does not increase so fast with the amount of FEC. The loss probability of packets in non-congested routers is supposed to not depend on α. Another problem that we think it limits our result is our assumption on the service time and the buffer space. As we noted, our assumptions hold when the bottleneck router implements a per-flow scheduling and a per-flow queueing. In case of a FIFO (First-In First-Out) buffer and a Drop Tail policy which is the most common case, our assumptions hold when the audio flow has a large share of the bottleneck bandwidth or when the other flows (or at least an important part) sharing the bottleneck with the audio flow implement the same FEC scheme. If it is not the case, we must wait for a better performance since the negative impact of the addition of FEC on the loss probability will be smaller. Probably, this is the reason for which experimentations have shown some gain with this FEC mechanism. The interpretation is so simple. If the exogenous traffic does not implement a similar FEC mechanism, the service time will be multiplied by a smaller factor than $(1+\alpha)$ and thus the increase in the load on the bottleneck will be smaller. This may result in a gain in performance. But, according to our results, this gain will disappear when the other flows start to implement FEC. Here appears the interest of our model since it indicates that the simple FEC scheme we studied in this paper is not a viable solution.

Appendix

I. Ballot theorems

In this appendix, we cite the ballot theorem that we have used to solve the problem for case $1 \leq \phi \leq K_\alpha$. The reader is referred to [9] for details.

Theorem 3: Suppose that an urn contains n cards marked with nonnegative integers $k_1, k_2, \ldots, k_n$, respectively, where $k_1 + k_2 + \cdots + k_n = k \leq n$. All the n cards are drawn without replacement from the urn. Denote by ν_r, $r = 1, 2, \ldots, n$, the number of the card drawn at the rth drawing. Then,

$$P\{\nu_1 + \cdots + \nu_r < r \quad \text{for} \quad r = 1, \ldots, n\} = 1 - \frac{k}{n}, \quad (16)$$

provided that all possible results are equally probable.

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