Massive MIMO mmWave Full Duplex Relay for IAB with Limited Dynamic Range

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Abstract—**In-band Full-Duplex (FD) is a promising wireless transmission technology that can revolutionize wireless communications. This paper proposes a novel hybrid beamforming design for a mmWave FD Integrated access and backhaul (IAB) scenario for weighted sum-rate (WSR) maximization. We consider non-ideal circuity for the FD relay modelled with the limited dynamic range (LDR) noise model. The problem under consideration is simplified into two sub-problem, and an alternating optimization mechanism is adopted to update the rate maximizing beamformers. Simulation results show significant performance gain compared to the traditional mmWave HD IAB scenario. However, the maximum achievable gain results to be strictly limited by the LDR noise variance.**

***Keywords*— Massive MIMO, Full Duplex relay, limited dynamic range, Integrated Access and backhaul**

I. INTRODUCTION

The mmWave band $30 - 300$ GHz offers much wider bandwidths than the traditional cellular networks and results to be a vital resource for future wireless communications. However, it is expected to accommodate the existing technologies and new emerging technologies like e-health, connected-vehicle-to-everything (V2X), Augmented/ Virtual reality and Internet-of-things (IoT). Full-Duplex (FD) communication in the mmWave can enable simultaneous transmission in the same frequency band to double the spectral efficiency and enable efficient management of the spectrum. Moreover, it can improve data security, reduce the air interface latency and delay issues [1].

Self-Interference (SI) is a major challenge to deal with to achieve FD operation, which could be around $90 - 110$ dB higher compared to the received signal of interest. However, continuous advancement in the SI cancellation (SIC) techniques has made the FD operation feasible by cancelling SI up to the noise floor. The performance of the SIC schemes is directly dictated by the limited dynamic range (LDR) of the circuitry in the transmit and receive chains [2]–[5], which can be modelled with the LDR noise model.

Communication based on FD relays in mmWave serves as an effective measure to ensure reliability, provide high throughput and extend network coverage. However, the mmWave systems have to rely on hybrid beamforming, which is cost-efficient and consists of large dimensional phasor processing in analog domain and lower dimensional digital processing. In [6], the authors proposed the first hybrid beamforming design for an amplify and forward FD relay for the mmWave backhaul link. In [7], a novel hybrid beamforming design for a FD relay assisted mmWave macrocell scenario is investigated. In [8], a robust hybrid design for an amplify and forward mmWave FD relay with imperfect channel state information (CSI) serving multiple single antenna users is studied. In [9], a joint beamforming, positioning and power control scheme for a mmWave FD UAV relay is presented. In [10], energy efficient precoding design for FD relay is proposed. The authors of [11] present a two-time scale hybrid beamforming for multiple FD relays scenario. In [12] a robust hybrid beamforming design for a network of FD relays under imperfect CSI is investigated. In [13], hybrid beamforming for a single stream two hop amplify and forward FD relay is proposed. Note that the available hybrid beamforming designs for mmWave are limited to one multi-antenna uplink and downlink user communicating with a FD relay or single antenna multiple downlink users served by a FD relay.

In this paper, we consider a scenario of one HD base station (BS) communicating with multiples multi-antenna users to be served with multiples streams. The users are assumed to be very far from the BS with no direct link and the communication takes place with a massive MIMO FD relay. The scenario under consideration represents an Integrated access and backhaul (IAB) framework. Compared to the state-of-the-art, we also assume that the transmit and receive chains of the FD relay have limited dynamic range (LDR) which dictates the overall noise level at the FD relay. Note that the LDR noise model takes into account the overall imperfections of the hardware in the transmit and recieve chains including mixers, power amplifier (PA), analog-to-digital converter etc. To the authors best knowledge, our work is first contribution to mmWave FD relay system with LDR range which serves multiple multi-antenna users with multiple streams. We present a novel hybrid beamforming and combining design to maximize weighted sum-rate (WSR) from BS to the users via FD relay. The overall optimization problem is separated into two sub-problems under the assumption that the self-interference (SI) at the FD relay is cancelled, for example based on the well known SI cancellation (SIC) techniques. Firstly, the WSR optimization from the BS to the FD relay with optimal hybrid beamformer and combiner is solved. Then, at the second stage, the WSR optimization from the relay to the multi-antenna users is solved. To solve the two sub-problems we adopt the alternating optimization approach based on the minorization maximization method

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[14].

Notation: Boldface lower and upper case characters denote vectors and matrices, respectively, and $E\{\cdot\}, \text{Tr}\{\cdot\}, (\cdot)^H, (\cdot)^T, \boldsymbol{I}$, and $\boldsymbol{D}_{1:d}$ represent expectation, trace, Hermitian, transpose, identity matrix and the d dominant vector selection matrix, respectively. diag$(\boldsymbol{X})$ and $\angle \boldsymbol{X}$ return the diagonal and unit modulus elements of $\boldsymbol{X}$. Cov$(\boldsymbol{x})$ denote the covariance of vector $\boldsymbol{x}$.

II. SYSTEM MODEL

We consider a mmWave multiuser FD system consisting of one massive MIMO HD base station, one massive MIMO FD relay serving K multi-antenna users. Let $\mathcal{K} = \{1, ..K\}$ denote the set containing the indices of the users. The BS is equipped with B_t transmit antennas and B_r RF chains, respectively. FD relay is assumed to be equipped with N receive and transmit antennas and M receive and transmit RF chains, respectively. We assume a narrow-band flat fading radio channel and with no direct link between the BS and the end users due to severe attenuation at the mmWave. Let $\boldsymbol{s}_k \in \mathbb{C}^{d_k \times 1}$ denote the white unitary data streams intended for user k. Let $\boldsymbol{F}_k \in \mathbb{C}^{B_r \times d_k}$ and $\boldsymbol{V} \in \mathbb{C}^{B_t \times B_r}$ denote the digital and the analog beamformer for the HD base station. The signal received at the relay can be written as

$$\boldsymbol{y}_r = \boldsymbol{H}_{r,b}\boldsymbol{V}\sum_{j\in\mathcal{K}} \boldsymbol{F}_j \boldsymbol{s}_j + \boldsymbol{n}_{SI} + \boldsymbol{n}_r + \boldsymbol{e}_r \tag{1}$$

where $\boldsymbol{H}_{r,b} \in \mathbb{C}^{M \times B_t}$ denote the channel from the BS to the relay, $\boldsymbol{e}_r \sim \mathcal{CN}(0, \sigma_r^2 \boldsymbol{I})$ denotes the thermal noise vector at the relay and $\boldsymbol{n}_{SI} \sim \mathcal{CN}(0, \sigma_{SI}^2 \boldsymbol{I})$ denotes the residual SI. The distortions due to LDR at the receiver side are denoted

$$\boldsymbol{n}_r \sim \mathcal{CN}(0, \beta \text{ diag}(\boldsymbol{\Phi}_r)) \tag{2}$$

where $\beta \ll 1$, $\boldsymbol{\Phi}_r = \text{Cov}(\boldsymbol{r})$, and $\boldsymbol{r} = \boldsymbol{y}_r - \boldsymbol{n}_r$ denotes the undistorted received signal. Let $\boldsymbol{U}_r \in \mathbb{C}^{M \times N}$ and $\boldsymbol{U}_t \in \mathbb{C}^{N \times M}$ denote the analog combiner and beamformer at the FD relay, respectively. Let $\boldsymbol{W}_k \in \mathbb{C}^{d_k \times M}$ and $\boldsymbol{E}_k \in \mathbb{C}^{M \times d_k}$ denote the digital combiner and beamformer for user k at the relay, respectively. We assume that there is no delay in processing and after the hybrid combining, the data streams of user k can be estimated as

$$\hat{\boldsymbol{s}}_k = \boldsymbol{W}_k \boldsymbol{U}_r \boldsymbol{y}_r. \tag{3}$$

Let $\boldsymbol{Q}_{j,r} = \boldsymbol{V}\boldsymbol{F}_j\boldsymbol{F}_j^H\boldsymbol{V}^H$ denote the transmit covariance matrix for user k transmitted to the FD relay. Let $\overline{k}$ denote all the indices in $\mathcal{K}$, without the index k. The received (signal plus) interference and noise covariance matrix for user k at the relay is denoted with $(\boldsymbol{R}_{k,r})$ $\boldsymbol{R}_{\overline{k},r}$ and after the hybrid combining are given by

$$\boldsymbol{\Phi}_r = \boldsymbol{H}_{r,b}\sum_{j\in\mathcal{K}, j\neq k} \boldsymbol{Q}_{j,r}\boldsymbol{H}_{r,b}^H + \sigma_{SI}^2\boldsymbol{I} + \sigma_r^2\boldsymbol{I} \tag{4}$$

$$\boldsymbol{R}_{\overline{k},r} = \boldsymbol{W}_k\boldsymbol{U}_r(\boldsymbol{\Phi}_r + \beta \text{diag}((\boldsymbol{\Phi}_r))\boldsymbol{U}_r^H\boldsymbol{W}_k^H \tag{5}$$

$$\boldsymbol{R}_{k,r} = \boldsymbol{W}_k\boldsymbol{U}_r\boldsymbol{H}_{r,b}\boldsymbol{Q}_{k,r}\boldsymbol{H}_{r,b}^H\boldsymbol{U}_r^H\boldsymbol{W}_k^H + \boldsymbol{R}_{\overline{k},r} \tag{6}$$

where $\boldsymbol{\Phi}_r$ denotes the undistorted received covariance matrix. The WSR maximization problem from the base station to the FD relay can be formally stated as

$$\max_{\substack{\boldsymbol{U}_r, \boldsymbol{W}_k \\ \boldsymbol{V}, \boldsymbol{F}_k}} \sum_{k\in\mathcal{K}} w_k \ln|\boldsymbol{R}_{\overline{k},r}^{-1}\boldsymbol{R}_{k,r}| \quad (P_1) \tag{7a}$$

$$\text{s.t.} \quad \text{Tr}\big(\sum_{k\in\mathcal{K}} \boldsymbol{Q}_{j,r}\big) \preceq p_b, \qquad (c_1) \tag{7b}$$

$$|\boldsymbol{U}_r(m,n)|^2 = 1, \quad \text{and} \quad |\boldsymbol{V}(m,n)|^2 = 1 \tag{7c}$$

where p_b denotes the total sum-power constraint, (7c) denote the unit modulus constraint on the analog part. Once the data streams are detected from the FD relay, they are then forwarded to the multi-antenna users with optimal power allocation based on the channel state information from the relay to users. The transmit signal from relay can be written as

$$\boldsymbol{y}_t = \boldsymbol{U}_t\sum_{j\in\mathcal{K}} \boldsymbol{E}_k\hat{\boldsymbol{s}}_j + \boldsymbol{n}_t \tag{8}$$

where $\boldsymbol{n}_t$ denotes the transmit LDR distortions, which can be modelled as

$$\boldsymbol{n}_t \sim \mathcal{CN}(0, k \text{ diag}(\boldsymbol{\Phi}_t)), \tag{9}$$

where $\boldsymbol{\Phi}_t = \text{Cov}(\boldsymbol{y}_0)$, where $\boldsymbol{y}_0 = \boldsymbol{y}_t - \boldsymbol{n}_t$ denotes the undistorted transmitted signal. Let $\boldsymbol{H}_k \in \mathbb{C}^{M_k \times N}$ denote the MIMO channel from the FD relay to the HD user k. The received signal at user k can be written as

$$y_k = \boldsymbol{H}_k(\boldsymbol{U}_t\sum_{j\in\mathcal{K}} \boldsymbol{E}_j\hat{s}_j + \boldsymbol{n}_t) + \boldsymbol{n}_k, \tag{10}$$

where $\boldsymbol{n}_k \sim \mathcal{CN}(0, \sigma_k^2\boldsymbol{I})$ denotes the noise vector at user k from the relay. Let $\boldsymbol{Q}_k = \boldsymbol{U}_t\boldsymbol{E}_k\boldsymbol{E}_k\boldsymbol{U}_t$ denote the transmit covariance for user k. The received (signal plus) interference and noise covariance at user k is denoted with $(\boldsymbol{R}_k)$ $\boldsymbol{R}_{\overline{k}}$ and can be written as

$$\boldsymbol{R}_{\overline{k}} = \boldsymbol{H}_k\sum_{j\in\mathcal{K}, j\neq k} \boldsymbol{Q}_j\boldsymbol{H}_k^H + k\ \boldsymbol{H}_k\text{diag}(\sum_{j\in\mathcal{K}} \boldsymbol{Q}_j)\boldsymbol{H}_k^H + \sigma_k^2\boldsymbol{I}, \tag{11}$$

$$\boldsymbol{R}_k = \boldsymbol{H}_k\boldsymbol{Q}_k\boldsymbol{H}_k^H + \boldsymbol{R}_{\overline{k}}. \tag{12}$$

The WSR maximization problem from FD relay to the HD multi-antenna users can be now formally stated as

$$\max_{\boldsymbol{U}_t, \boldsymbol{E}_k} \sum_{k\in\mathcal{K}} w_j \ln|\boldsymbol{R}_{\overline{k}}^{-1}\boldsymbol{R}_k| \quad (P_2) \tag{13a}$$

$$\text{s.t.} \quad \text{Tr}\big(\sum_{k\in\mathcal{K}} \boldsymbol{Q}_j\big) \preceq p_b, \quad \text{and} \quad |\boldsymbol{U}_t(m,n)|^2 = 1 \tag{13b}$$

where (13b) denote the total sum-power constraint and the unit modulus constraint.

A. Problem Simplification

The problem (7) and (13) are non-concave due to interference terms and finding a global optimum is challenging. Hence to render a feasible solution, we consider the minorization maximization approach. Let WSR^r and WSR denote the

$$G_{k,r} = \sum_{j\in\mathcal{K}, j\neq k} w_j (\boldsymbol{H}_{r,b}^H[\boldsymbol{U}_j^H \boldsymbol{W}_j^H(\boldsymbol{R}_{\bar{j},r}^{-1} - \boldsymbol{R}_{j,r}^{-1})\boldsymbol{W}_j\boldsymbol{U}_j - \beta\,\mathrm{diag}(\boldsymbol{U}_j^H\boldsymbol{W}_j^H(\boldsymbol{R}_{\bar{j},r}^{-1} - \boldsymbol{R}_{j,r}^{-1})\boldsymbol{W}_j\boldsymbol{U}_j)]\boldsymbol{H}_{r,b}), \tag{17}$$

$$\boldsymbol{G}_k = \sum_{j\in\mathcal{K}, j\neq k} w_j(\boldsymbol{H}_j^H[\boldsymbol{R}_{\bar{j}}^{-1} - \boldsymbol{R}_j^{-1}]\boldsymbol{H}_j + k\mathrm{diag}(\boldsymbol{H}_j^H[\boldsymbol{R}_{\bar{j}}^{-1} - \boldsymbol{R}_j^{-1}]\boldsymbol{H}_j) \tag{18}$$

WSR from the half-duplex base station to the relay and from the FD relay to the HD users, respectively. They can be written as the weighted rate (WR) of user k (WR_k or $WR_{k,r}$) and the WSR of users $\overline{k}$,

$$WSR_r = WR_{k,r} + WR_{\overline{k},r}, \quad WSR = WR_k + WR_{\overline{k}} \tag{14}$$

where WR_{k_r} is concave in $\boldsymbol{Q}_{k,r}$ and $WR_{\overline{k},r}$ is non-concave in $\boldsymbol{Q}_{k,r}$. Similarly, WR_k is concave in $\boldsymbol{Q}_k$ and $WR_{\overline{k}}$ is non-concave in $\boldsymbol{Q}_k$. Since a linear function is simultaneously convex and concave, difference of convex (DC) programming [15] introduces the first order Taylor series expansion of $WR_{\overline{k},r}$ around $\boldsymbol{Q}_{k,FD}$ and of $WR_{\overline{k}}$ around $\boldsymbol{Q}_k$. The new linearized tangent expression for the WSR can be written as

$$\underline{WR}_{k,r} = WR_{k,r}(\boldsymbol{Q}_{k,r}, \hat{\boldsymbol{Q}}) - tr((\boldsymbol{Q}_{k,r} - \hat{\boldsymbol{Q}})\boldsymbol{G}_{k,r}) \tag{15}$$

$$\underline{WR}_k(\boldsymbol{Q}_k, \hat{\boldsymbol{T}}) = WR_k(\boldsymbol{Q}_k, \hat{\boldsymbol{Q}}) - tr((\boldsymbol{Q}_k - \hat{\boldsymbol{Q}})\boldsymbol{G}_k) \tag{16}$$

where $\boldsymbol{G}_{k,r}$ and $\boldsymbol{G}_k$ (given at the top) denote the gradients of $WR_{\overline{k},r}$ and $WR_{\overline{k}}$ with respect to $\boldsymbol{Q}_{k,r}$ and $\boldsymbol{Q}_k$, respectively. Note that, the linearized tangent expression constitutes a touching lower bound for (7) and (13), hence DC programming is also a minorization-maximization approach [14], regardless of the reparameterization of WSR as a function of beamformers. Let λ_b and λ_r be the Lagrange multipliers associated with the sum-power at the base station and at the FD relay. Let

$$\boldsymbol{A} \triangleq \boldsymbol{V}^H\boldsymbol{H}_{r,b}^H\boldsymbol{U}_r^H\boldsymbol{W}_k^H\boldsymbol{R}_{\overline{k},FD}^{-1}\boldsymbol{W}_k\boldsymbol{U}_r\boldsymbol{H}_{r,b}\boldsymbol{V}, \tag{19a}$$

$$\boldsymbol{B} \triangleq \boldsymbol{V}^H(\boldsymbol{G}_k^r + \lambda_b\boldsymbol{I})\boldsymbol{V}, \tag{19b}$$

$$\boldsymbol{C} \triangleq \boldsymbol{U}_t^H\boldsymbol{H}_k^H\boldsymbol{R}_{\overline{k}}^{-1}\boldsymbol{H}_k\boldsymbol{U}_t, \quad \boldsymbol{D} \triangleq \boldsymbol{U}_t^H(\boldsymbol{G}_k + \lambda_r\boldsymbol{I})\boldsymbol{U}_t, \tag{19c}$$

and dropping the constant terms, reparameterizing back the covariance matrices, performing this linearization for all users, augmenting the WSR^r with WSR with their constraints, yields the following Lagrangians

$$\max_{\substack{\boldsymbol{U}_r,\boldsymbol{W}_k\\ \boldsymbol{V},\boldsymbol{F}_k}} \sum_{k\in\mathcal{K}} w_k\ \ln|\boldsymbol{I} + \boldsymbol{F}_k^H\boldsymbol{A}\boldsymbol{F}_k| - \mathrm{Tr}(\boldsymbol{F}_k^H\boldsymbol{B}\boldsymbol{F}_k) \tag{20a}$$

$$\max_{\boldsymbol{U}_t,\boldsymbol{E}_k} \sum_{k\in\mathcal{K}} w_k\ \ln|\boldsymbol{I} + \boldsymbol{E}_k^H\boldsymbol{C}\boldsymbol{E}_k| - \mathrm{Tr}(\boldsymbol{E}_k^H\boldsymbol{D}\boldsymbol{E}_k). \tag{20b}$$

III. HYBRID BEAMFORMING

This section present the solution for the problems (P_1) and (P_2) based on the problem simplification (20) and (20).

A. Digital Beamforming

To optimize the digital transmit beamformers for the base station and FD relay, we first take the derivative of (33) and (34) with respect to $\boldsymbol{F}_k$ and $\boldsymbol{E}_k$, which yield the following (Karush–Kuhn–Tucker) KKT conditions

$$\boldsymbol{A}\boldsymbol{F}_k(\boldsymbol{I} + \boldsymbol{F}_k^H\boldsymbol{A}\boldsymbol{F}_k)^{-1} - \boldsymbol{B}\boldsymbol{F}_k = 0, \tag{21}$$

$$\boldsymbol{C}\boldsymbol{E}_k(\boldsymbol{I} + \boldsymbol{E}_k^H\boldsymbol{C}\boldsymbol{E}_k)^{-1} - \boldsymbol{D}\boldsymbol{E}_k = 0, \tag{22}$$

Theorem 1. *The optimal digital beamformers $\boldsymbol{F}_k$ and $\boldsymbol{E}_k$ at each iteration are given by the dominant generalized eigenvector of the pairs*

$$\boldsymbol{F}_k = \boldsymbol{D}_{1:d_k}(\boldsymbol{A}, \boldsymbol{B}), \quad \boldsymbol{E}_k = \boldsymbol{D}_{1:d_k}(\boldsymbol{C}, \boldsymbol{D}) \tag{23}$$

Proof. The result follows directly from the proof available for Proposition 1 [15] with the covariance matrix (6). ☐

B. Analog Beamforming

To optimize the analog beamformer, we exploit the knowledge that the optimal fully digital beamformers $\boldsymbol{F}_{k,dig}$ and $\boldsymbol{E}_{k,dig}$ are known. Let $\boldsymbol{F}^{opt} \in \mathbb{C}^{B_t\times d_1+..+d_K}$ and $\boldsymbol{E}^{opt} \in \mathbb{C}^{N\times d_1+..+d_K}$ contain the fully digital beamformers at the base station and FD relay, respectively. Let $\boldsymbol{F}_{RF}^{opt} \in \mathbb{C}^{B_r\times d_1+..+d_K}$ $\boldsymbol{E}_{RF}^{opt} \in \mathbb{C}^{M\times d_1+..+d_K}$ contain all the digital beamforming vectors for the matched filter solution of the size of RF chains. Since the analog beamformer is common to all the users, we optimize the analog beamformer as the one which minimize the squared error between the fully digital and hybrid solution. Formally, the two unconstrainted problem can be stated as

$$\min_{\boldsymbol{V}}.||\boldsymbol{F}^{opt} - \boldsymbol{V}\boldsymbol{F}_{RF}^{opt}||, \quad \min_{\boldsymbol{U}_t}.||\boldsymbol{E}^{opt} - \boldsymbol{U}_t\boldsymbol{E}_{RF}^{dig}|| \tag{24}$$

and solving it yield the solution

$$\boldsymbol{V} = (\boldsymbol{F}^{opt}(\boldsymbol{F}_{RF}^{opt})^H)(\boldsymbol{F}_{RF}^{opt}(\boldsymbol{F}_{RF}^{opt})^H)^{-1}, \tag{25}$$

$$\boldsymbol{U}_t = (\boldsymbol{E}^{opt}(\boldsymbol{E}_{RF}^{dig})^H)(\boldsymbol{E}_{RF}^{dig}(\boldsymbol{E}_{RF}^{dig})^H)^{-1}. \tag{26}$$

from which we take the unit modulus solution, i.e. $\boldsymbol{V} = \angle\boldsymbol{V}$ and $\boldsymbol{U}_t = \angle\boldsymbol{U}_t$

C. Digital Combining

To further improve the rate and suppress the interference, we design the combiners in the minimum mean squared error (MMSE) fashion. Namely, let $\boldsymbol{e}_k = \boldsymbol{s}_k - \hat{\boldsymbol{s}}_k$ denote the error vector for user k and the error covariance matrix $\boldsymbol{E}_{rr}$ at the relay can be written as

$$\boldsymbol{E}_{rr} = \boldsymbol{R}_{k,FD} - \boldsymbol{I} + (\boldsymbol{W}_k\boldsymbol{U}_r\boldsymbol{H}_{r,b}\boldsymbol{V}\boldsymbol{F}_j + \boldsymbol{F}_k^H\boldsymbol{V}^H\boldsymbol{H}_{r,b}\boldsymbol{U}_r^H)\boldsymbol{W}_k^H \tag{27}$$

and minimizing its trace lead to the optimal digital combiners

$$\boldsymbol{W}_k = \boldsymbol{F}_k^H\boldsymbol{V}^H\boldsymbol{H}_{r,b}\boldsymbol{U}_r^h[\boldsymbol{\Phi}_r + \beta\mathrm{diag}(\boldsymbol{\Phi}_r)]^{-1}, \quad \forall k\in\mathcal{K}. \tag{28}$$

D. Analog Combining

As the analog combiner is common to all the users, we adopt the same mechanism as in (24) to optimize it. Let

$\boldsymbol{W}^{opt} \in \mathbb{C}^{d_1+..+d_K \times N}$ contain the combining vectors to the fully digital solution obtained from (28) with $\boldsymbol{U}_r = \boldsymbol{I}, \forall k \in \mathcal{K}$. Let $\boldsymbol{W}_{RF}^{opt} \in \mathbb{C}^{d_1+..+d_K \times M}$ contain the combining vectors of correct size with limited number of RF chains as a matched filter solution. The analog combiner can be obtained by solving the following optimization problem

$$\min_{\boldsymbol{U}_r} ||\boldsymbol{W}^{opt} - \boldsymbol{U}_r \boldsymbol{W}_{RF}^{opt}||, \tag{29}$$

which leads to the solution

$$\boldsymbol{U}_r = (\boldsymbol{W}^{opt}(\boldsymbol{W}_{RF}^{opt})^H)(\boldsymbol{W}_{RF}^{opt}(\boldsymbol{W}_{RF}^{opt})^H)^{-1}. \tag{30}$$

Note that the digital beamformers (23) provide the optimal beamforming directions, but not the optimal powers. Therefore, we normalize the columns of the digital beamformers $\boldsymbol{E}_k$ and $\boldsymbol{F}_k$ to be unit norm. Let

$$\boldsymbol{\Sigma}_{\boldsymbol{k}}^{\boldsymbol{1}} = \boldsymbol{F}_k^H \boldsymbol{A} \boldsymbol{F}_k, \quad \boldsymbol{\Sigma}_{\boldsymbol{k}}^{\boldsymbol{1}} = \boldsymbol{F}_k^H \boldsymbol{B} \boldsymbol{F}_k, \tag{31}$$

$$\boldsymbol{S}_{\boldsymbol{k}}^{\boldsymbol{1}} = \boldsymbol{E}_k^H \boldsymbol{C} \boldsymbol{E}_k, \quad \boldsymbol{S}_{\boldsymbol{k}}^{\boldsymbol{1}} = \boldsymbol{E}_k^H \boldsymbol{D} \boldsymbol{E}_k. \tag{32}$$

and assuming that the optimal digital beamformers are computed at the last based on the results in Theorem 1, (31) and (32) are diagonal. The optimal power allocation can be included by solving the following optimization problem

$$\max_{\boldsymbol{P}_{k,BS}} \sum_{k \in \mathcal{K}} w_k \ln|\boldsymbol{\Sigma}_{\boldsymbol{k}}^{\boldsymbol{1}} \boldsymbol{P}_{k,BS}| - \text{Tr}(\boldsymbol{\Sigma}_{\boldsymbol{k}}^{\boldsymbol{2}} \boldsymbol{P}_{k,BS}) \tag{33a}$$

$$\max_{\boldsymbol{P}_{k,FD}} \sum_{k \in \mathcal{K}} w_k \ln|\boldsymbol{I} + \boldsymbol{S}_{\boldsymbol{k}}^{\boldsymbol{1}} \boldsymbol{P}_{k,FD}| - \text{Tr}(\boldsymbol{S}_{\boldsymbol{k}}^{\boldsymbol{2}} \boldsymbol{P}_{k,FD}) \tag{34a}$$

where $\boldsymbol{P}_{k,BS}$ and $\boldsymbol{P}_{k,FD}$ denote the optimal power allocation for the BS and the FD relay. Solving it yield the optimal power allocation

$$\boldsymbol{P}_{k,BS} = (w_k(\boldsymbol{\Sigma}_{\boldsymbol{k}}^{\boldsymbol{2}})^{-1} - (\boldsymbol{\Sigma}_{\boldsymbol{k}}^{\boldsymbol{1}})^{-1})^+ \tag{35}$$

$$\boldsymbol{P}_{k,FD} = (w_k(\boldsymbol{S}_{\boldsymbol{k}}^{\boldsymbol{2}})^{-1} - (\boldsymbol{S}_{\boldsymbol{k}}^{\boldsymbol{1}})^{-1})^+ \tag{36}$$

where $(x)^+ = max(0, x)$. To search for the optimal Lagrange multipliers λ_b and λ_r satisfying the sum power constraints, we adopt the bisection method in the search range $[0, \lambda_{b,max}]$ and $[0, \lambda_{r,max}]$, respectively. To meet the unit modulus constraints, we normalize the analog beamformer and combiners with $\boldsymbol{X} = \angle \boldsymbol{X}$.

E. Convergence

The convergence can be proved by noticing that the KKT conditions of the simplified problem and the original problem are the same. Each Iteration leads to and increase in the WSR sum-rate which ensures convergence. A formal proof can be stated exactly as the one given Proposition 3 [15]. However, given the space limitation we omit it.

IV. SIMULATION RESULTS

In this Section, we evaluate the performance of our proposed hybrid beamforming and combining design.

Algorithm 1 Hybrid Beamforming for IAB FD

Given: The CSI and rate weights.
Initialize: the beamformers $\forall k \in \mathcal{K}$.
Iterate first with $\boldsymbol{U}_r, \boldsymbol{V}, \boldsymbol{U}_t = \boldsymbol{I}$ to get $\boldsymbol{F}^{opt}, \boldsymbol{E}^{opt}$ and $\boldsymbol{W}^{Dig}$
Set: $\angle \boldsymbol{V}, \angle \boldsymbol{U}_t, \angle \boldsymbol{U}_r$ with (25)-(26) and (30)
Repeat until convergence
for: $k = 1, .., K$.
  Compute $\boldsymbol{G}_{k,r}$ with (17).
  Compute $\boldsymbol{F}_k$ with (23) and normalize it.
  Set $\underline{\lambda_b} = 0$ and $\overline{\lambda_b} = \lambda_{b,max}$.
  Repeat until convergence
    set $\lambda_b = (\underline{\lambda_b} + \overline{\lambda_b})/2$.
      Compute $\boldsymbol{P}_{k,BS}$ with (35),
      If constraint for μ_i is violated,
       set $\underline{\lambda_b} = \lambda_b$, else $\overline{\lambda_b} = \lambda_b$,
  Set $\boldsymbol{Q}_{j,FD} = \boldsymbol{V} \boldsymbol{F}_j \boldsymbol{P}_{k,BS} \boldsymbol{F}_j^H \boldsymbol{V}^H$
  Update $\boldsymbol{W}_k$ with (28)
 Next k.
Repeat until convergence
for: $k = 1, .., K$.
  Compute $\boldsymbol{G}_k$ with (18).
  Compute $\boldsymbol{E}_k$ with (23) and normalize it.
  Set $\underline{\lambda_r} = 0$ and $\overline{\lambda_r} = \lambda_{r,max}$.
  Repeat until convergence
    set $\lambda_r = (\underline{\lambda_r} + \overline{\lambda_r})/2$.
      Compute $\boldsymbol{P}_{k,FD}$ with (36),
      If constraint for λ_r is violated,
       set $\underline{\lambda_r} = \lambda_r$, else $\overline{\lambda_r} = \lambda_r$,
  Set $\boldsymbol{Q}_{j,FD} = \boldsymbol{U}_t \boldsymbol{E}_k \boldsymbol{P}_{k,FD} \boldsymbol{E}_k^H \boldsymbol{U}_t^H$
 Next k.

The users channel and the channel from the BS to the relay are modelled with the path wise channel model, with each channel matrix modelled as

$$\boldsymbol{H} = \sqrt{\frac{M_0 N_0}{N_c N_p}} \sum_{n_c=1}^{N_c} \sum_{n_p=1}^{N_p} \alpha_k^{(n_p,n_c)} \boldsymbol{a}_r(\phi_k^{n_p,n_c}) \boldsymbol{a}_t^T(\theta_k^{n_p,n_c}), \tag{37}$$

where N_c and N_p denote the number of clusters and number of rays, respectively, $\alpha_k^{(n_p,n_c)} \sim \mathcal{CN}(0,1)$ is a complex Gaussian random variable with amplitudes and phases distributed according to the Rayleigh and uniform distribution, respectively, and $\boldsymbol{a}_r(\phi_k^{n_p,n_c})$ and $\boldsymbol{a}_t^T(\theta_k^{n_p,n_c})$ denote the receive and transmit antenna array response with angle of arrival (AoA) $\phi_k^{n_p,n_c}$ and angle of departure (AoD) $\theta_k^{n_p,n_c}$. The BS and relay are assumed to have uniform linear arrays with $B_t = N = 100$ and RF chains $B_r = M = 32$. The maximum transmit power $p_b = p_r = 23$ dBm and the noise variance is chosen to meet the requirement for the desired transmit SNR. We assume that thermal noise level and the residual SI variance to be the same, i.e., $\sigma_{SI}^2 = \sigma_r^2 = \sigma_k^2$. Note that with this we are assuming that the residual SI is cancelled up to the noise floor for the known part of the SI channel and by varying k and β we vary the overall noise level at the FD node, which indirectly

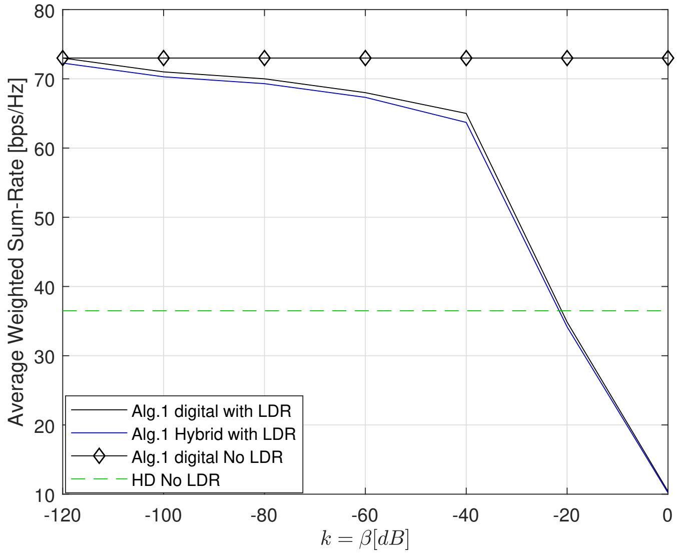

Fig. 1: Average WSR as function of $k = \beta$ dB, with $SNR = -5$ dB.

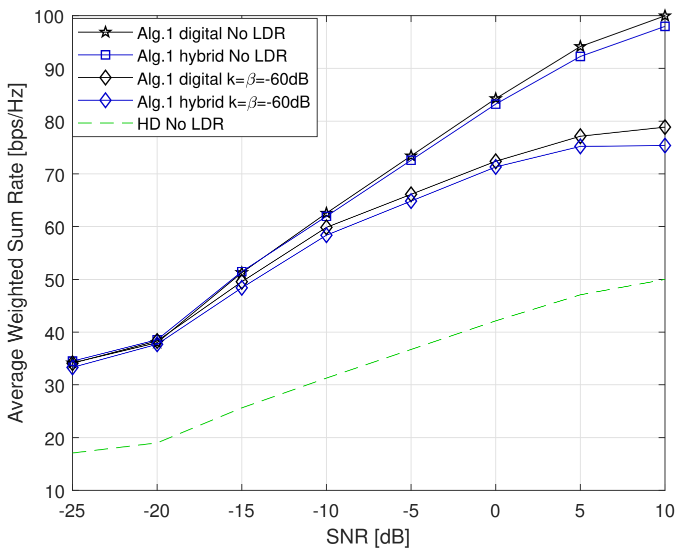

Fig. 2: Average WSR as function of SNR with different levels of LDR noise.

can be seen as a non-cancelled part of SI (like higher order non linear components due to power amplifier). Therefore, we are analyzing exactly how the LDR affects the overall maximum achievable performance. We consider a scenario of two downlink users with 5 antennas to be served with two data streams. For comparison purpose, we define a fully digital HD system which splits the resource in time and takes two time slots to transmit from BS to the users.

Figure 1 shows the average WSR as a function of the overall LDR noise level with $SNR = -5$ dB. It can be seen that when the LDR noise variance is below the overall noise level, the FD communication system achieves an additional gain of 98% compared to the HD system. However, as the overall LDR noise variance increases, the overall maximum achievable performance tends closer and closer to the HD system. Figure 2 show the achievable WSR as a function of SNR with $k = \beta = 0$ (in linear scale, no LDR noise case) and $k = \beta = -60$ dB. It can be seen that our proposed hybrid scheme performs closer to the fully digital one with 32 RF chains. Moreover, with $k = \beta = -60$ dB the maximum achievable performance results to be very far from the achievable performance of an ideal FD system. Increasing transmission power increases also the LDR noise variance. Thus it is desirable to have FD transceivers with limited LDR noise, such that the overall noise results to be closer to the thermal noise floor. Having very low cost circuitry leads to very high LDR noise, which can potentially limit the overall maximum achievable performance in a mmWave massive MIMO FD IAB system.

V. CONCLUSIONS

In this paper, we proposed the first ever IAB FD communication system via hybrid beamforming and combining for multi antenna users. The proposed beamforming design has a performance close to the fully digital case with limited number of RF chains. However, the maximum achievable performance results to be strictly limited by the LDR noise variance. Our results highlight a trade-off between the LDR noise variance and how much impact it has on the maximum achievable performance.

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