Hybrid Beamforming for Bidirectional Massive MIMO Full Duplex Under Practical Considerations

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***Abstract*—In-band Full-Duplex (FD) is a promising wireless transmission technology allowing to increase data rates by up to a factor of two, via simultaneous transmission and reception, but with a potential to increase system throughput even much more in cognitive radio and random access systems thanks to simultaneous transmission and sensing. In this work, we consider a practical hybrid beamforming design for a bidirectional massive MIMO FD system under the joint per-antenna and sum-power constraints. Moreover, we consider non-ideal circuitry in the transmit and receive chains, which is modelled with the limited dynamic range (LDR) noise model. The per-antenna power constraints take into account the actual physical limits of the power amplifiers and the sum-power constraints are imposed to limit the total transmit power. The precoders are optimized with alternating optimization by using the minorization-maximization approach. Simulation results show significant performance improvement compared to a traditional bidirectional half-duplex system.**

***Keywords*— Massive MIMO, Full Duplex, Hybrid Beamforming, per antenna power constraints, limited dynamic range**

I. INTRODUCTION

Full-duplex (FD) has to potential to double the performance of a wireless communication system as it allows simultaneous transmission and reception in the same frequency band. It avoids using two independent channels for bi-directional communication, by allowing more flexibility in spectrum utilization, improving data security, and reducing the air interface latency and delay issues [1]. To achieve FD, Self-Interference (SI) is a major challenge to deal with to achieve FD operation, which could be around 110 dB compared to the received signal of interest.

However, continuous advancement in the SI cancellation (SIC) techniques has made the FD operation feasible. SIC schemes split the workload into the passive, analog and digital domain. The most challenging SIC stage is the analog SIC stage, for which extra hardware is required [2]. The analog-to-digital-converters (ADCs) have only limited dynamic range (LDR), and if analog SIC stage fails to mitigate the SI sufficiently, it leads to saturation of the converters. Saturation noise is well-known to be the most challenging noise, which can significantly limit the performance of a FD system [3], [4]. Also, the non-ideal circuity in the transmit and receive chains limit ideal SIC. Therefore, for correct performance analysis of a FD system, the effect of RF circuitry and ADCs by using the LDR model must be considered [5]–[7]. The LDR noise model models the noise with variance equal to the total signal power multiplying a very small scalar and considers all the possible noise contributions, e.g. phase noise, quantization noise, and non-linear distortions of the power amplifiers, etc. Therefore, the FD transceiver with highly non-ideal circuitry is modelled with a large scalar, and the quasi ideal transceiver can be modelled with an extremely small scalar.

Hybrid beamforming for a bidirectional MIMO FD (BD-FD) communication has been widely studied in the literature [7], [8]. In [7], achievable rates under the LDR model are studied. In [9], a novel hardware impairment aware linear precoder and decoder design under the sum power constraints is proposed. In [10], a hybrid beamforming design for FD millimeter wave (mmWave) point-to-point communication is proposed. In [11], a hybrid beamforming design for a FD MIMO relay system is studied. In [12], a learning-based hybrid beamformer optimization for a one-way FD mmWave relay system is proposed. In [6], hybrid beamforming optimization for a FD OFDM backhaul system is studied.

This paper considers the digital and analog beamformer design for the weighted sum rate (WSR) maximization under the joint sum-power and per-antenna power constraints. The LDR noise model is also assumed, reflecting the non-ideal circuitry in the transmit and receive chains. This is the first-ever FD communication design for Massive MIMO bidirectional full-duplex system under the joint constraints to the authors' best knowledge. The sum-power constraints at each terminal are imposed by the regulations, which limit the total transmit power. In practice, each transmit antenna is equipped with its PA [13] and the per-antenna power constraints arise due to the power consumption limits imposed on the physical PAs. Traditionally, the joint sum-power and per-antenna power constraints take into account both the regulations and the physical limits to optimize the systems' performance. However, for FD communication the joint constraints have much more to offer. If there is no saturation noise, the most dominant noise contribution comes for the PAs [3], which introduce additional non-linearities when operating in the non-linear region. Consequently, the residual SI power increases, limiting the maximum achievable gain for a FD system. With the per-antenna power constraints, we can limit PAs' non-linear behaviour and improve the SI channel estimation while complying with the sum-power constraints naturally imposed by the regulations.

The WSR maximization problem is solved by adopting the method of minorization-maximization [14], and the beamformers and powers are jointly optimized with alternating optimization. A novel power allocation design is also proposed to include the optimal powers at each iteration. Simulations results show significant performance improvement compared to the traditional bidirectional half-duplex system. However, the maximum achievable performance is strictly limited by the LDR noise.[1]

II. System Model

We consider a BD-FD communication system consisting of two MIMO FD node having massive number of transmit antenna elements and equipped with beamformers at the digital and at the analog stage. Let $\mathcal{F} = \{1, 2\}$ contain the indices of the FD nodes. Let N_l and M_l denote the number of transmit and receive antenna at the FD node $l \in \mathcal{F}$, respectively. Let N_l^r denote the number of radio frequency (RF) chains at the node $l \in \mathcal{F}$. We consider a multi-stream approach and let $\boldsymbol{s}_l \in \mathbb{C}^{d_l \times 1}$ denote the d_l white and unitary variance data streams transmitted from node $l \in \mathcal{F}$. Let $\boldsymbol{V}_l \in \mathbb{C}^{N_l^r \times d_l}$ and $\boldsymbol{G}_l \in \mathbb{C}^{N_t \times N_t^r}$ denote the digital and analog beamformer at node l, respectively, and $\boldsymbol{G}_l$ is assumed to be common to all the antennas (fully connected case). The signal received at the FD node l can be written as

$$\boldsymbol{y}_l = \boldsymbol{H}_{m,l}\bigg(\boldsymbol{G}_m\boldsymbol{V}_m\boldsymbol{s}_m + \boldsymbol{c}_m\bigg) + \boldsymbol{e}_l + \boldsymbol{n}_l + \boldsymbol{H}_l\bigg(\boldsymbol{G}_l\boldsymbol{V}_l\boldsymbol{s}_l + \boldsymbol{c}_l\bigg) \tag{1}$$

where l and $m \in \mathcal{F}$ and $l \neq m$. The channel between transmit array of node $m \in \mathcal{F}$ and receive array at node $l \in \mathcal{F}$, with $m \neq l$ is denoted with $\boldsymbol{H}_{m,l} \in \mathbb{C}^{M_l \times N_m}$ and the SI channel at the node l is denoted with $\boldsymbol{H}_l \in \mathbb{C}^{M_l \times N_l}, \forall l \in \mathcal{F}$. The vector $\boldsymbol{n}_l, \forall l \in \mathcal{F}$ denote the thermal noise vectors at the FD node l with variance $\sigma_l^2 \boldsymbol{I}_{M_l}$. Let $\boldsymbol{T}_l = \boldsymbol{G}_l\boldsymbol{V}_l\boldsymbol{V}_l^H\boldsymbol{G}_l^H$ denote the transmit covariance matrix of node $l \in \mathcal{F}$. The terms $\boldsymbol{c}_l$ and $\boldsymbol{c}_m$ are the transmitter and $\boldsymbol{e}_l$ and $\boldsymbol{e}_m$ are the receiver noise distortions due to LDR at the node l and m, respectively, with $l, m \in \mathcal{F}$ and $l \neq m$, and can be modelled as [7]

$$\boldsymbol{c}_l \sim \mathcal{CN}\bigg(\boldsymbol{0}_{N_l \times 1}, k_l \,\text{diag}\big(\boldsymbol{T}_l\big)\bigg), \quad \forall\, l \in \mathcal{F}, \tag{2}$$

$$\boldsymbol{e}_l \sim \mathcal{CN}\bigg(\boldsymbol{0}_{M_l \times 1}, \beta_l \,\text{diag}\big(\boldsymbol{\Phi}_l\big)\bigg), \quad \forall\, l \in \mathcal{F}, \tag{3}$$

where $k_l \ll 1$, $\beta_l \ll 1$ and $\boldsymbol{\Phi}_l = Cov(\boldsymbol{x}_l)$, where $\boldsymbol{x}_l$ denotes the undistorted received vector at node l, such that $\boldsymbol{x}_l = \boldsymbol{y}_l - \boldsymbol{e}_l, \forall l \in \mathcal{F}$. Let $\boldsymbol{X}_l$ be the received covariance matrix of the undistorted received signal at node l, transmitted from node m

$$\boldsymbol{X}_l = \boldsymbol{H}_{m,l}\boldsymbol{T}_m\boldsymbol{H}_{m,l}^H + \boldsymbol{H}_{m,l}k_m diag\big(\boldsymbol{T}_m\big)\boldsymbol{H}_{m,l}^H + \sigma_l^2\boldsymbol{I} + \boldsymbol{H}_l(\boldsymbol{T}_l + k_l diag\big(\boldsymbol{T}_l\big))\boldsymbol{H}_l^H, \ \forall l, m \in \mathcal{F} \text{ and } l \neq m, \tag{4}$$

[1]**Notation:** Boldface lower and upper case characters denote vectors and matrices, respectively, and $E\{\cdot\}, tr\{\cdot\}, (\cdot)^H, (\cdot)^T, (\cdot)^*, \boldsymbol{I}$, and $\boldsymbol{D}_{1:d}$ represent expectation, trace, conjugate transpose, transpose, complex conjugate, identity matrix and the d dominant vector selection matrix, respectively, and diag$(\cdot)$ denote a diagonal matrix. The operator $vec(\boldsymbol{X})$ stacks the column of $\boldsymbol{X}$ into a vector and $\angle\boldsymbol{X}$ returns the phases of matrix $\boldsymbol{X}$.

and let $\boldsymbol{K}_l \triangleq \boldsymbol{H}_{m,l}\boldsymbol{T}_m\boldsymbol{H}_{m,l}^H$ denote the useful signal part. The received (signal plus) interference and noise covariance matrices at the FD node $l \in \mathcal{F}$ denoted with $(\boldsymbol{R}_l)$ $\boldsymbol{R}_{\bar{l}}$ can be written as

$$\boldsymbol{R}_l \approx \bigg(\boldsymbol{X}_l + \beta_l \text{diag}(\boldsymbol{X}_l)\bigg), \quad \boldsymbol{R}_{\bar{l}} \approx \boldsymbol{R}_l - \boldsymbol{K}_l. \tag{5}$$

We consider the joint optimization of the digital and the analog beamformer. The WSR maximization problem for the bidirectional FD system under the joint sum-power and per-antenna constraints power for precoder optimization can be stated as

$$\max_{\boldsymbol{V}_l, \boldsymbol{G}_l} \sum_{l \in \mathcal{F}} w_l \ln\det\bigg(\boldsymbol{R}_{\bar{l}}^{-1}\boldsymbol{R}_l\bigg) \tag{6a}$$

$$\text{s.t.} \quad \text{diag}\bigg(\boldsymbol{V}_l\boldsymbol{V}_l^H\bigg) \preceq \boldsymbol{P}_l, \forall l \in \mathcal{F}, \tag{6b}$$

$$tr\bigg(\boldsymbol{G}_l\boldsymbol{V}_l\boldsymbol{V}_l^H\boldsymbol{G}_l^H\bigg) \leq p_l, \forall l \in \mathcal{F}, \tag{6c}$$

which is non-concave due to the (self-)interference terms.

III. Hybrid Beamforming

To find a feasible solution of (6), we use the minorization-maximization approach [14] for the precoders optimization $\boldsymbol{V}_l$ and $\boldsymbol{G}_l$ at each iteration of the alternating optimization process. The optimal powers are also included separately to meet the imposed constraints

A. Beamformer Design

The WSR can be written as a sum of weighted rate of nodes l and $m \in \mathcal{F}, m \neq l$, i.e. $WSR = WR_l + WR_m$. Note that WR_l is concave in $\boldsymbol{T}_m$ and non-concave in $\boldsymbol{T}_l$ and WR_m is concave in $\boldsymbol{T}_l$ and non-concave in $\boldsymbol{T}_m$, $\forall l, m \in \mathcal{F}, m \neq l$. Since a linear function is simultaneously convex and concave, difference of convex (DC) programming introduces the first order Taylor series expansion of WR_m in $\boldsymbol{T}_l$ and WR_l in $\boldsymbol{T}_m$ around $\boldsymbol{T}$ i.e., all $\boldsymbol{T}_i$, as

$$\underline{WR}_l(\boldsymbol{T}_m, \hat{\boldsymbol{T}}) = WR_l(\boldsymbol{T}_m, \hat{\boldsymbol{T}}) - tr((\boldsymbol{T}_m - \hat{\boldsymbol{T}})\boldsymbol{A}_l) \tag{7a}$$

$$\underline{WR}_m(\boldsymbol{T}_l, \hat{\boldsymbol{T}}) = WR_m(\boldsymbol{T}_l, \hat{\boldsymbol{T}}) - tr((\boldsymbol{T}_l - \hat{\boldsymbol{T}})\boldsymbol{A}_m) \tag{7b}$$

where $\boldsymbol{A}_l$ and $\boldsymbol{A}_m$ are the gradients of WR_m and WR_l with respect to $\hat{\boldsymbol{T}}_l$ and $\hat{\boldsymbol{T}}_m$, respectively. The gradients can be computed using the the matrix differentiation properties defined in [15] and by applying the commutative property of the Hadamard product for the diagonal terms due to the LDR noise model. The gradients expression for $\forall l \in \mathcal{F}$ and $l \neq m$ are given by

$$\begin{aligned}\boldsymbol{A}_l = & w_m\big(\boldsymbol{H}_{m,m}^T\big(\boldsymbol{R}_m^{-T} + \beta_m diag(\boldsymbol{R}_m^{-T} - \boldsymbol{R}_{\bar{l}}^{-T})\big)\boldsymbol{H}_{m,m}^* \\ & + k_m diag\big(\boldsymbol{H}_{m,m}^T(\boldsymbol{R}_m^{-T} - \boldsymbol{R}_{\bar{m}}^{-T})\boldsymbol{H}_{m,m}^*\big)\big).\end{aligned} \tag{8}$$

Note that, the linearized tangent expression (7a) and (7b) constitutes a touching lower bound for the weighted sum rate (6), hence DC programming results to be also a minorization approach, regardless of the reparameterization of the transmit covariance matrices as a function of beamformers.

Note that the gradients are fixed during the current iteration and computed using the results from the previous iteration. Let $\boldsymbol{S}_l = \boldsymbol{H}_{l,m}^H \boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m}$, then (6) at each iteration of the alternating optimization process can be restated under (6b)-(6c) as

$$\max_{\boldsymbol{V}_l, \boldsymbol{G}_l} \sum_{l \in \mathcal{F}} w_l \text{lndet}\big(\boldsymbol{I}+\boldsymbol{V}_l{}^H \boldsymbol{G}_l^H \boldsymbol{S}_l \boldsymbol{G}_l \boldsymbol{V}_l\big) - tr\big(\boldsymbol{V}_l{}^H \boldsymbol{G}_l^H \boldsymbol{A}_m \boldsymbol{G}_l \boldsymbol{V}_l\big). \tag{9}$$

Let λ_l and $\boldsymbol{\Psi}_l = diag(\psi_1^l, ..., \psi_{N_l}^l)$ be the Lagrange multipliers associated with the sum-power and per-antenna power constraint at node $l \in \mathcal{F}$, respectively. Dropping the constant terms, reparameterizing back $\boldsymbol{T}_l$ as function of precoders, performing this linearization $\forall l \in \mathcal{F}$, augmenting the WSR cost function with the joint constraints, yields the following Lagrangian

$$\begin{aligned} L = \sum_{l \in \mathcal{F}} \lambda_l p_l + tr(\boldsymbol{\Psi}_l \boldsymbol{P}_l) + \ln\, \det\big(\boldsymbol{I} + \boldsymbol{V}_l{}^H \boldsymbol{G}_l^H \boldsymbol{S}_l \boldsymbol{G}_l \boldsymbol{V}_l\big) \\ - tr\big(\boldsymbol{V}_l{}^H (\boldsymbol{G}_l^H (\boldsymbol{A}_m + \lambda_l \boldsymbol{I}) \boldsymbol{G}_l + \boldsymbol{\Psi}_l) \boldsymbol{V}_l\big). \end{aligned} \tag{10}$$

Note that the powers are left out for now and will be included later. To optimize the digital and analog beamformer, we take the derivative of (10) with respect to $\boldsymbol{V}_l$ and $\boldsymbol{G}_l, \forall l, m \in \mathcal{F}$ and $l \neq m$, which yields the following Karush–Kuhn–Tucker (KKT) conditions

$$\begin{aligned} \boldsymbol{G}_l^H \boldsymbol{S}_l \boldsymbol{G}_l \boldsymbol{V}_l (\boldsymbol{I} + \boldsymbol{V}_l{}^H \boldsymbol{G}_l^H \boldsymbol{S}_l \boldsymbol{G}_l \boldsymbol{V}_l)^{-1} \\ - (\boldsymbol{G}_l^H (\boldsymbol{A}_m + \lambda_l \boldsymbol{I}) \boldsymbol{G}_l + \boldsymbol{\Psi}_l) \boldsymbol{V}_l = 0. \end{aligned} \tag{11}$$

$$\begin{aligned} \boldsymbol{S}_j \boldsymbol{G}_0 \boldsymbol{V}_j \boldsymbol{V}_j^H \big(\boldsymbol{I} + \boldsymbol{V}_j \boldsymbol{V}_j^H \boldsymbol{G}_l^H \boldsymbol{S}_j \boldsymbol{G}_0\big)^{-1} \\ - \big(\boldsymbol{A}_m + \lambda_l \boldsymbol{I}\big) \boldsymbol{G}_0 \boldsymbol{V}_j \boldsymbol{V}_j^H = 0. \end{aligned} \tag{12}$$

Theorem 1. *The optimal digital beamformers $\boldsymbol{V}_l$ and the analog beamformer $\boldsymbol{G}_l$, $\forall l \in \mathcal{F}$ at each iteration is given by the dominant generalized eigen vector (GEV) solution of the pairs*

$$\boldsymbol{V}_l \rightarrow \boldsymbol{D}_{1:d_l}\Big(\big(\boldsymbol{G}_l^H \boldsymbol{S}_m \boldsymbol{G}_l, \boldsymbol{G}_l (\boldsymbol{A}_m + \lambda_l \boldsymbol{I}) \boldsymbol{G}_l + \boldsymbol{\Psi}_l\big)\Big) \tag{13}$$

$$\begin{aligned} vec(\boldsymbol{G}_l) \rightarrow \boldsymbol{D}_{1:1}\big((\boldsymbol{V}_l (\boldsymbol{I}+\boldsymbol{G}_l^H \boldsymbol{S}_l \boldsymbol{G}_l \boldsymbol{V}_l \boldsymbol{V}_l^H)^{-1} \boldsymbol{V}_l^H)^T \otimes \boldsymbol{S}_l, \\ (\boldsymbol{V}_l \boldsymbol{V}_l^H)^T \otimes (\boldsymbol{A}_m + \boldsymbol{\Psi}_l + \lambda_l \boldsymbol{I})\big)\big) \end{aligned} \tag{14}$$

Proof. We first consider the analog beamformer fixed and prove the result for the digital beamformers. The proof relies on simplifying (9) with fixed analog beamformer until the Hadamard's inequality applies. The Cholesky decomposition of the matrix $(\boldsymbol{G}_l^H (\boldsymbol{A}_m + \lambda_l \boldsymbol{I}) \boldsymbol{G}_l + \boldsymbol{\Psi}_l)$ is written as $\boldsymbol{L}_l \boldsymbol{L}_l^H$ where $\boldsymbol{L}_l$ is a lower-triangular Cholesky factor. Upon defining $\tilde{\boldsymbol{V}}_l = \boldsymbol{L}_l^H \boldsymbol{V}_l$ and by using the result provided in Proposition 1 [16], it follows immediately that the solution is the GEV. We now consider the digital beamformer fixed and prove for the analog beamformer. Similar proof follows also for the analog beamformers. However, the KKT conditions do not satisfy the GEV equation ($\boldsymbol{AX} = \boldsymbol{DBX}$, where $\boldsymbol{D}$ is a digonal matrix). Therefore, to shape it in a correct form, the following property $vec(\boldsymbol{AXB}) = (\boldsymbol{B}^T \otimes \boldsymbol{A})\boldsymbol{X}$, with $\boldsymbol{X}$ as the analog beamformer. Having the KKT conditions in the correct form (with diagonal on the left), the proof for the analog beamformer follows directly as for the digital beamformers based on the result provided in Proposition 1 [16]. □

As $\boldsymbol{G}_l$ is an unconstrainted vector, we need to reshape into a matrix and get $\angle \boldsymbol{G}_l$ to meet the unit modulus constraint.

B. Optimal Power Allocation

The solution (13), being a GEV diagonalize the matrices

$$\boldsymbol{V}_l{}^H \boldsymbol{G}_l^H \boldsymbol{S}_m \boldsymbol{G}_l \boldsymbol{V}_l = \boldsymbol{\Sigma_l^{(1)}} \tag{15}$$

$$\boldsymbol{V}_l{}^H (\boldsymbol{G}_l^H (\boldsymbol{A}_m + \lambda_l \boldsymbol{I}) \boldsymbol{G}_l + \boldsymbol{\Psi}_l) \boldsymbol{V}_l = \boldsymbol{\Sigma_l^{(2)}} \tag{16}$$

at each iteration. After computing the beamformers and normalizing its columns, the optimal powers can be included while searching for the multipliers, satisfying the constraints. Formally, power optimization problem can be stated as

$$\max_{\boldsymbol{P}_l} \quad w_l\ \text{lndet}\big(\boldsymbol{I} + \boldsymbol{\Sigma_l^{(1)}} \boldsymbol{P}_l\big) - tr\big(\boldsymbol{\Sigma_l^{(2)}} \boldsymbol{P}_l\big), \forall l \in \mathcal{F}. \tag{17}$$

with fixed multipliers and $\boldsymbol{V}_l$. Note that as the beamformers are given by the dominat GEV solution of (9), by multiplying it by a diagonal matrix it still yields a generalized dominant GEV solution and Theorem 1 is still valid. The optimal power allocation at each FD node is obtained by solving (17), which yields

$$\boldsymbol{P}_l = (w_l (\boldsymbol{\Sigma_l^{(2)}}))^{-1} - \big(\boldsymbol{\Sigma_l^{(2)}})\big)^{-1})^+. \tag{18}$$

where $(x)^+ = \max\{0, x\}$.

Now, in order to satisfy the per-antenna and sum power constraints we consider the following Lagrange dual function

$$\min_{\lambda_l, \boldsymbol{\Psi}_l} \quad L\big(\lambda_l, \boldsymbol{\Psi}_l\big). \tag{19}$$

The dual function $L(\lambda_l, \boldsymbol{\Psi}_l)$ is the pointwise supremum of a family of functions of $\lambda_l, \boldsymbol{\Psi}_l$, it is convex [17] and the globally optimal value $\lambda_l, \boldsymbol{\Psi}_l$ and can be found by using any of the numerous convex optimization techniques. In this work, we adopt the bisection algorithm for the search of multipliers. Let $\mathcal{L}_l = \{\lambda_l, \psi_l, .., \psi_{N_l^r}\}$ contain the multipliers associated with the joint constraints at the node $l \in \mathcal{F}$. Let $\underline{\mu_i}$, and $\overline{\mu_i}$ denote the upper and lower bounds for searching the Lagrange multiplier $\mu_i \in \mathcal{L}_l$. The complete procedure to solve (6) is formally stated in Algorithm 1.

1) Convergence proof: To prove the convergence of Algorithm 1, the ingredients required are minorization [14], Lagrange duality, saddle point and KKT conditions [17]. Let $WSR(\boldsymbol{T})$ denote the cost function (6) as a function of transmit covariance marices and let $\underline{WSR}(\boldsymbol{T}, \hat{\boldsymbol{T}})$ minorizer leading to

$$\begin{aligned} & WSR(\boldsymbol{T}) \geq \underline{WSR}(\boldsymbol{T}, \hat{\boldsymbol{T}}) = \\ & \sum_{l \in \mathcal{F}} w_l \text{lndet}\big(\boldsymbol{I} + \boldsymbol{V}_l{}^H \boldsymbol{G}_l^H \boldsymbol{S}_l \boldsymbol{G}_l \boldsymbol{V}_l\big) - tr\big((\boldsymbol{T}_l - \hat{\boldsymbol{T}}) \boldsymbol{A}_m\big). \end{aligned} \tag{20}$$

and the minorizer which is now concave in $\boldsymbol{T}$ has the same gradient of (6), therefore the KKT conditions are not affected.

Algorithm 1 Practical Hybrid Beamforming Design

Given: The CSI and rate weights.
Initialize:$\boldsymbol{V}_l$ and $\boldsymbol{G}_l, \forall \in \mathcal{F}$.
Repeat until convergence
for:$\forall l \in \mathcal{F}$.
  Compute $\boldsymbol{A}_m$ with (8).
  Compute $\boldsymbol{G}_l$ with (14) and get $\angle \boldsymbol{G}_l$.
  Compute $\boldsymbol{V}_l$ with (13) and normalize it.
  Set $\underline{\mu_i} = 0$ and $\overline{\mu_i} = \mu_{i_{max}}$ $\forall i \in \mathcal{L}_l$.
  for: $\forall \mu_i \in \mathcal{L}_l$
  Repeat until convergence
    set $\mu_i = (\underline{\mu_i} + \overline{\mu_i})/2$.
      Compute $\boldsymbol{P}_l$ with (17),
      If constraint for μ_i is violated,
       set $\underline{\mu_i} = \mu_i$, else $\overline{\mu_i} = \mu_i$,
  Set $\boldsymbol{T}_l = \boldsymbol{G}_l \boldsymbol{V}_l \boldsymbol{P}_l \boldsymbol{V}_l^H \boldsymbol{G}_l^H$
 Next l.

Reparametrizing the transmit covariance matrices $\boldsymbol{T}$ as a function of the variables: powers $\boldsymbol{P}_l$, digital beamformers $\boldsymbol{V}_l$ and the analog beamformers $\boldsymbol{G}_l$ and adding the power sum-power and per-antenna power constraints yield the Lagrangian (10). During the alternating optimization process, every alternating update of (10) leads to an increase in the weighted sum rate, ensuring convergence for all of the 3 parameters. For the KKT conditions, at the convergence point, the gradients of (10) with respect to the analog and digital beamformers yield the same gradients of the original cost function (6). For fixed analog and digital beamformers, (10) is concave in $\boldsymbol{P}_l$, therefore we have strong duality for the saddle point $max_{\boldsymbol{P}_l}\ min_{\lambda,\Psi} L$. Moreover, at the convergence point the solution to $min_{\lambda,\Psi} L$ satisfies the complementary slackness condition i.e.,

$$\lambda_l(p_l - tr(\boldsymbol{G}_l \boldsymbol{V}_l \boldsymbol{V}_l^H \boldsymbol{G}_l^H)) = 0, \forall l \in \mathcal{F} \tag{21a}$$

$$tr(\Psi_l \boldsymbol{P}_l - diag(\boldsymbol{V}_l \boldsymbol{V}_l^H)) = 0, \forall l \in \mathcal{F}. \tag{22a}$$

IV. SIMULATION RESULTS

In this section, we present simulation results for our novel digital and analog beamformer design under the practical sum-power and per-antenna power constraints.

We assume that the proposed bidirectional beamforming design operates in the millimeter-wave band, at which the channels can be modelled with the path-wise channel model such that

$$\boldsymbol{H}_{m,l} = \sqrt{\frac{N_l N_m}{N_c N_p}} \sum_{n_c=1}^{N_c} \sum_{n_p=1}^{N_p} \alpha_k^{(n_p,n_c)} \boldsymbol{a}_l(\phi_k^{n_p,n_c}) \boldsymbol{a}_m^T(\theta_k^{n_p,n_c}), \tag{23}$$

where N_c and N_p denote the number of clusters and number of rays, respectively, $\alpha_k^{(n_p,n_c)} \sim \mathcal{CN}(0,1)$ is a complex Gaussian random variable with amplitudes and phases distributed according to the Rayleigh and uniform distribution, respectively, and $\boldsymbol{a}_r(\phi_k^{n_p,n_c})$ and $\boldsymbol{a}_t^T(\theta_k^{n_p,n_c})$ denote the receive and transmit antenna array response with angle of arrival (AoA) $\phi_k^{n_p,n_c}$ and angle of departure (AoD) $\theta_k^{n_p,n_c}$, respectively. We assume uniform linear arrays for both transmission and reception at the FD nodes. The SI channel can be modelled as

$$\boldsymbol{H}_l = \sqrt{\frac{\kappa}{\kappa+1}} \boldsymbol{H}_{LoS} + \sqrt{\frac{1}{\kappa+1}} \boldsymbol{H}_{ref}, \tag{24}$$

where κ, $\boldsymbol{H}_{LoS}$ and $\boldsymbol{H}_{ref}$ denote the Rician factor, the line-of-sight (LoS) and reflected contributions of the SI signal, respectively. We assume that the SI contribution to be extremely dominated by its line-of-sight component, and thus the Rician factor is set to $\kappa = 10^5$ dB. The matrices $\boldsymbol{H}_{LoS}$ is set to be a matrix of all ones and $\boldsymbol{H}_{ref} \sim \mathcal{CN}(0,1)$. The sum-power $p_l, \forall \in \mathcal{F}$ is fixed to be 23 dBm and the per-antenna power constraints at each FD node are fixed to be 23 dBm divided by the total number of transmit antennas. The rate weights are fixed to be 1. It is assumed that the FD nodes are equipped with 100 transmit antennas, 32 transmit RF chains, $M_l = M_m = 20$ receive antenna and 2 data streams are transmitted from both the nodes. The number of clusters and the number of paths is set to be $N_c = 3$ and $N_p = 6$, respectively, the AoA and AoD are unifomly distributed in $\mathcal{U} \sim [0, 30°]$. The antennas are assumed to be placed at half-wavelength and the array response for the uniform linear array is simulated.

As we are considering all the possible noise contributions due to LDR noise and σ_l^2, we label our design as a practical BD-FD hybrid beamforming (P-FD-HYB) design. As this work is the first one to consider a BD-FD hybrid communication system design under the joint sum-power and per-antenna power constraints, we do not compare it with the state-of-the-art. For comparison, we define the following benchmark schemes: 1) We define a practical BD-FD digital communication system for which number of RF chains is equal to the number of antennas i.e. $N_l^r = N_l, \forall l \in \mathcal{F}$. This scheme set an upper bound for the maximum achievable gain with the hybrid beamforming design. The optimal beamformers for such a design can be found by setting the analog beamformer equal to identity and following the same steps of Algorithm 1 for the digital beamformers. 2) As a lower bound we define a BD half-duplex system which splits its resources in time to serve the uplink and the downlink users which satisfies the joint sum-power and per-antenna power constraints. The beamformer can be obtained with the generalized eigen vector method and results to be a special case of our FD desing. Results are reported by averaging over 100 channel realizations.

We assume that both the FD node have the same circuitry in the transmit and receive chains and therefore we set $k_l = k_m = k$ and $\beta_l = \beta_m = \beta$. Figure 1 shows the average weighted sum-rate as a function of the signal to noise ratio (SNR) with, $k = \beta = -110$dB. It can be seen that the proposed scheme achieves significant performance improvement compared to the BD half-duplex system. It is also shown that the rate achieved for both the communicating links results to be the same. The performance of the Hybrid design is limited, compared to the fully digital case, due to the unit modulus

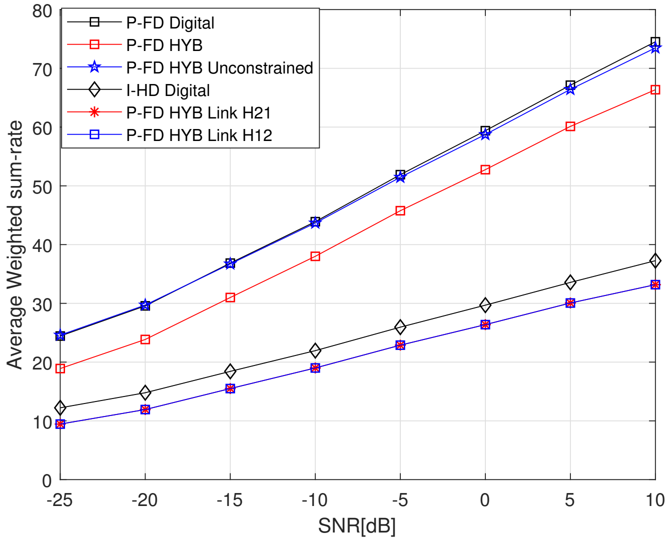

Fig. 1: Average WSR of a BD-FD system as function of SNR with , $k = \beta = -110$ dB, with $N_1 = N_2 = 100$, $N_1^r = N_2^r = 32$ and $M_1 = M_2 = 20$.

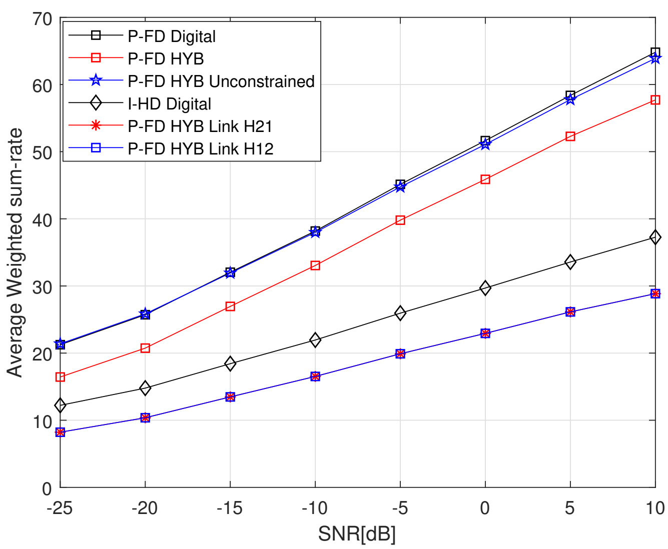

Fig. 2: Average WSR of a BD-FD system as function of SNR with , $k = \beta = -60$ dB, with $N_1 = N_2 = 100$, $N_1^r = N_2^r = 32$ and $M_1 = M_2 = 20$.

constraint. However, if amplitude manipulation is allowed for $\boldsymbol{G}_l$, the unconstrained beamformer has perforamance close to the fully digital case. Given an extremely small $k = \beta$, the achievable performance is strictly limited by the thermal noise variance.

Figure 2 shows the average weighted sum-rate when $k = \beta = -50$ dB, which represent moderately noisy circuitry in the transmit and receive chains. It must be noticed that the maximum achievable gain, also for the fully digital case, results to be less compared to the performance shown in Figure 1. Therefore, we can conclude that in general, in a practical BD-FD system the maximum achievable performance is limited by the maximum of the LDR noise, due to non ideal circuitry, or the thermal noise variance, if ideal-circuitry is deployed.

V. Conclusions

In this work, we studied the problem of WSR maximization for a BD-FD communication system under the joint-sum power and per-antenna power constraints. These constraints consider the hardware limitation and the maximum power transmission limits imposed by the regulations. The optimal analog and digital beamformers are designed under the practical LDR model, which considers the effect of non-ideal circuitry in the transmit and receive chains. Simulation results show significant performance gain compared to a half-duplex BD communication system. Moreover, it's observed that the achievable performance of a practical BD-FD system is limited by the maximum of the LDR noise variance or the noise variance σ_l^2.

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