BEAMFORMING FOR BIDIRECTIONAL MIMO FULL DUPLEX UNDER THE JOINT SUM POWER AND PER ANTENNA POWER CONSTRAINTS

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ABSTRACT

This paper considers a bidirectional (BD) full-duplex (BD-FD) communication system design under the joint sum-power and per-antenna power constraints. The sum-power constraints are naturally imposed by regulation to limit the total transmit power, and the per-antenna power constraints consider the physical limits of the power amplifiers (PAs). We propose a novel beamforming design to maximize the weighted sum-rate (WSR) with alternating optimization under the limited dynamic range (LDR) noise model. At each iteration, we use minorization-maximization approach to optimize the beamformers and power allocation. Simulation results show significant performance gain compared to a half-duplex BD system or FD system with only sum-power constraints. However, the gains are limited by the maximum of the thermal noise variance or the LDR noise variance.

Index Terms— Full-Duplex, Beamforming, Weighted Sum Rate, Per-Antenna and Sum-Power constraints, LDR noise.

1. INTRODUCTION

Full-duplex (FD) has to potential to double the performance of a wireless communication system as it allows simultaneous transmission and reception in the same frequency band. It avoids using two independent channels for bi-directional communication, by allowing more flexibility in spectrum utilization, improving data security, and reducing the air interface latency and delay issues [1, 2]. Self-Interference (SI) is a significant challenge to deal with to achieve FD operation, which could be around 110 dB compared to the received signal of interest.

However, continuous advancement in the SI cancellation (SIC) techniques has made the FD operation feasible. SIC schemes split the workload into the passive, analog and digital domain. The most challenging SIC stage is the analog SIC stage, for which extra hardware is required [3]. The analogue-to-digital-converters (ADCs) have only limited dynamic range (LDR), and if the analog SIC stage fails to mitigate the SI sufficiently, it leads to saturation of the converters. Saturation noise is well-known to be the most challenging noise, which can significantly limit the performance of a FD system [4–7]. Also, the non-ideal circuity in the transmit and receive chains limit ideal SIC. Therefore, for correct performance analysis of a FD system, the effect of RF circuitry and ADCs by using the LDR model must be considered [8–12].

Bidirectional MIMO FD (BD-FD) communication has been widely studied in the literature [10, 13]. In [10], achievable rates under the LDR model are studied. In [13], the effect of SI and transmitter noise are analyzed in the asymptotic regime. In [14], the authors present a large system analysis for the rate regions. Weighted sum-rate (WSR) cost function can accommodate various types of traffic demands with correct rate weight selection. In particular, it has been widely studied for BD-FD combined with beamforming to optimize the performance. In [15], a low complexity beamforming design under the sum-power constraint is proposed. In [16], linear precoder and decoder design under the sum-power constraint is studied. In [17], the WSR maximization problem is studied for a MIMO interference channel with individual or system sum-power constraint. In [18], the authors propose novel hardware impairment aware linear precoder and decoder design under the sum power constraints. In [9], a hybrid beamforming design for BD-FD is presented.

In this paper, we consider digital beamformers' design for WSR maximization under the joint sum-power and per-antenna power constraint. The sum-power constraints at each terminal are imposed by the regulations, which limit the total transmit power. In practice, each transmit antenna is equipped with its power amplifier (PA) [19] and the per-antenna power constraints arise due to the power consumption limits imposed on the physical PAs. Traditionally, the joint sum-power and per-antenna power constraints take into account both the regulations and the physical limits to optimize the systems' performance. However, for FD communication, the joint constraints have much more to offer. If there is no saturation noise, the most dominant noise contribution comes for the PAs [6], which introduce additional non-linearities when operating in the non-linear region. Consequently, the residual SI power increases, limiting the maximum achievable gain for a FD system. With per-antenna power constraints, we can limit PAs' non-linear behaviour and improve the SI channel estimation while complying with the sum-power constraints naturally imposed by the regulations. Moreover, the transmit antennas nearest the receive array contribute the most to the line-of-sight (LoS) component of the SI signal. As the analog SIC stage has very high energy consumption, we can reduce it by restricting the per-antenna constraints on the transmit antennas nearest to the receive array. Note that, restricting a lot the per-antenna power constraints improves the uplink rate but can degrade the downlink rate. In practice, an optimal trade-off between the uplink and downlink rate must be investigated.

Notation: Boldface lower and upper case characters denote vectors and matrices, respectively, and $E\{\cdot\}, tr\{\cdot\}, (\cdot)^H, (\cdot)^T$, $(\cdot)^*$, $\boldsymbol{I}$, and $\boldsymbol{D}_{1:d}$ represent expectation, trace, conjugate transpose, transpose, complex conjugate, identity matrix and the d dominant vector selection matrix, respectively, and $diag(\cdot)$ denote a diagonal matrix.

2. SYSTEM MODEL

We consider a BD-FD communication system consisting of two MIMO FD node communicating with each other. Let $\mathcal{F} = \{1, 2\}$ contain the indices of the FD nodes. Let N_l and M_l denote the number of transmit and receive antenna at the FD node $l \in \mathcal{F}$, respectively. We consider a multi-stream approach and let $\boldsymbol{s}_l \in \mathbb{C}^{d_l \times 1}$ denote the d_l white and unitary variance data streams transmitted from node $l \in \mathcal{F}$. Let $\boldsymbol{V}_l \in \mathbb{C}^{N_l \times d_l}$ denote the digital beamformer at the node $l \in \mathcal{F}$. The signal received at the FD node l can be written as

$$\boldsymbol{y}_l = \boldsymbol{H}_{l,m}(\boldsymbol{V}_m \boldsymbol{s}_m + \boldsymbol{c}_m) + \boldsymbol{e}_l + \boldsymbol{n}_l + \boldsymbol{H}_{l,l}(\boldsymbol{V}_l \boldsymbol{s}_l + \boldsymbol{c}_l) \quad (1)$$

where l and $m \in \mathcal{F}$ and $l \neq m$. The channel between transmit array of node $m \in \mathcal{F}$ and receive array at node $l \in \mathcal{F}$, with $m \neq l$ is denoted with $\boldsymbol{H}_{l,m} \in \mathbb{C}^{M_l \times N_m}$ and the SI channel at the node l is denoted with $\boldsymbol{H}_{l,l} \in \mathbb{C}^{M_l \times N_l}, \forall l \in \mathcal{F}$. The vector $\boldsymbol{n}_l, \forall l \in \mathcal{F}$ denote the thermal noise vectors at the FD node l with variance $\sigma_l^2 \boldsymbol{I}_{M_l}$. Let $\boldsymbol{T}_l = \boldsymbol{V}_l \boldsymbol{V}_l^H$ denote the transmit covariance matrix of node $l \in \mathcal{F}$. The terms $\boldsymbol{c}_l$ and $\boldsymbol{c}_m$ are the transmitter and $\boldsymbol{e}_l$ and $\boldsymbol{e}_m$ are the receiver noise distortions due to LDR at the node l and m, respectively, with $l, m \in \mathcal{F}$ and $l \neq m$, and can be modelled as [10]

$$\boldsymbol{c}_l \sim \mathcal{CN}\big(\boldsymbol{0}_{N_l \times 1}, k_l \,\text{diag}(\boldsymbol{T}_l)\big), \quad \forall\, l \in \mathcal{F}, \quad (2)$$

$$\boldsymbol{e}_l \sim \mathcal{CN}\big(\boldsymbol{0}_{M_l \times 1}, \beta_l \,\text{diag}(\boldsymbol{\Phi}_l)\big), \quad \forall\, l \in \mathcal{F}, \quad (3)$$

where $k_l \ll 1$, $\beta_l \ll 1$ and $\boldsymbol{\Phi}_l = Cov(\boldsymbol{x}_l)$, where $\boldsymbol{x}_l$ denotes the undistorted received vector at node l, such that $\boldsymbol{x}_l = \boldsymbol{y}_l - \boldsymbol{e}_l, \forall l \in \mathcal{F}$. Let $\boldsymbol{X}_l$ be the received covariance matrix of the undistorted received signal at node l given by

$$\begin{aligned}\boldsymbol{X}_l = \boldsymbol{H}_{l,m}\boldsymbol{T}_m\boldsymbol{H}_{l,m}^H + \boldsymbol{H}_{l,m}k_m diag(\boldsymbol{T}_m)\boldsymbol{H}_{l,m}^H + \sigma_l^2\boldsymbol{I} + \\ \boldsymbol{H}_{l,l}(\boldsymbol{T}_l + k_l diag(\boldsymbol{T}_l))\boldsymbol{H}_{l,l}^H, \quad \forall l, m \in \mathcal{F} \text{ and } l \neq m,\end{aligned} \quad (4)$$

and let $\boldsymbol{K}_l \triangleq \boldsymbol{H}_{l,m}\boldsymbol{T}_m\boldsymbol{H}_{l,m}^H$ denote the useful signal part. The received (signal plus) interference and noise covariance matrices received at the FD node $l \in \mathcal{F}$ denoted with ($\boldsymbol{R}_l$) $\boldsymbol{R}_{\bar{l}}$ can be written as

$$\boldsymbol{R}_l \approx (\boldsymbol{X}_l + \beta_l \text{diag}(\boldsymbol{X}_l)), \quad \boldsymbol{R}_{\bar{l}} \approx \boldsymbol{R}_l - \boldsymbol{K}_l. \quad (5)$$

The WSR maximization problem for the bidirectional FD system under the joint sum-power and per antenna power for beamformers optimization can be stated as

$$\max_{\boldsymbol{V}_l} \sum_{m \in \mathcal{F}} w_m \ln\det\big(\boldsymbol{R}_{\overline{m}}^{-1}\boldsymbol{R}_m\big) \quad (6a)$$

$$\text{s.t.} \quad \text{diag}(\boldsymbol{V}_l\boldsymbol{V}_l^H) \preceq \boldsymbol{P}_l, \quad \forall l \in \mathcal{F}, \quad (6b)$$

$$tr(\boldsymbol{V}_l\boldsymbol{V}_l^H) \leq p_l, \quad \forall l \in \mathcal{F}, \quad (6c)$$

where w_m denote the rate weights, p_l and $\boldsymbol{P}_l$ (a diagonal matrix) denote the sum-power and per-antenna power constraints, respectively. The problem (6) is non-concave in $\boldsymbol{T}_l$ due to SI terms at both the FD nodes which leads to finding the global optimum solution very challenging.

3. BEAMFORMING

To find a feasible solution, we construct a minorizer of (6) using the minorization-maximization approach [20] to optimize the precoders $\boldsymbol{V}_l$ at each iteration of the alternating optimization process.

3.1. Digital Beamforming Design

The WSR can be written as a sum of weighted rate of nodes l and $m \in \mathcal{F}, m \neq l$, i.e. $WSR = WR_l + WR_m$. Note that WR_l is concave in $\boldsymbol{T}_m$ and non-concave in $\boldsymbol{T}_l$ and WR_m is concave in $\boldsymbol{T}_l$ and non-concave in $\boldsymbol{T}_m$ due to SI, $\forall l, m \in \mathcal{F}, m \neq l$. Since a linear function is simultaneously convex and concave, difference of convex (DC) programming [20] introduces the first order Taylor series expansion of WR_m in $\boldsymbol{T}_l$ and WR_l in $\boldsymbol{T}_m$ around $\boldsymbol{T}$ i.e., all $\boldsymbol{T}_i$, as

$$\underline{WR}_l(\boldsymbol{T}_m, \hat{\boldsymbol{T}}) = WR_l(\boldsymbol{T}_m, \hat{\boldsymbol{T}}) - tr((\boldsymbol{T}_m - \hat{\boldsymbol{T}})\boldsymbol{G}_m) \quad (7a)$$

$$\underline{WR}_m(\boldsymbol{T}_l, \hat{\boldsymbol{T}}) = WR_m(\boldsymbol{T}_l, \hat{\boldsymbol{T}}) - tr((\boldsymbol{T}_l - \hat{\boldsymbol{T}})\boldsymbol{G}_l) \quad (7b)$$

where $\boldsymbol{G}_m$ and $\boldsymbol{G}_l$ are the gradients of WR_m and WR_l with respect to $\hat{\boldsymbol{T}}_l$ and $\hat{\boldsymbol{T}}_m$, respectively, and given by $\forall l \in \mathcal{F}$ and $l \neq m$ as

$$\begin{aligned}\boldsymbol{G}_m = & w_m\big(\boldsymbol{H}_{m,m}^T(\boldsymbol{R}_{\overline{m}}^{-T} - \boldsymbol{R}_m^{-T} + \beta_l diag(\boldsymbol{R}_{\overline{m}}^{-T} - \boldsymbol{R}_m^{-T}))\\ & \boldsymbol{H}_{m,m}^* + k_m diag(\boldsymbol{H}_{m,m}^T(\boldsymbol{R}_{\overline{m}}^{-T} - \boldsymbol{R}_m^{-T})\boldsymbol{H}_{m,m}^*\big)\end{aligned} \quad (8)$$

which are derived by using the matrix differentiation properties defined in [21].Note that, the linearized tangent expression constitutes a touching lower bound for (6), hence DC programming is also a minorization approach, regardless of the reparameterization as a function of beamformers.

Let λ_l and $\boldsymbol{\Psi}_l = diag(\psi_1^l, ..., \psi_{N_l}^l)$ be the Lagrange multipliers associated with the sum-power and per-antenna power constraint at node $l \in \mathcal{F}$, respectively. Dropping the constant terms, reparameterizing back $\boldsymbol{T}_l$ as function of precoders, performing this linearization $\forall l \in \mathcal{F}$, augmenting the WSR cost function with the per-antenna and sum power constraints, yields the following Lagrangian

$$\begin{aligned}L = \sum_{m \in \mathcal{F}} \big(&\psi_m p_m + tr(\boldsymbol{\Psi}_m \boldsymbol{P}_m) + \ln\det(\boldsymbol{I} + \boldsymbol{V}_m^H\boldsymbol{H}_{l,m}^H\boldsymbol{R}_{\bar{l}}^{-1}\\ &\boldsymbol{H}_{l,m}\boldsymbol{V}_m) - tr(\boldsymbol{V}_m^H(\boldsymbol{G}_m + \lambda_m\boldsymbol{I} + \boldsymbol{\Psi}_m)\boldsymbol{V}_m)\big)\end{aligned} \quad (9)$$

Note that the powers are left out for now and will be included later. To optimize the precoder and decoders, we take the derivative of (9) with respect to $\boldsymbol{V}_l, \forall l \in \mathcal{F}$ and $l \neq m$, which yields the following Karush–Kuhn–Tucker (KKT) conditions $\forall l \in \mathcal{F}$

$$\begin{aligned} &\boldsymbol{H}_{l,m}^H \boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m} \boldsymbol{V}_l (\boldsymbol{I} + \boldsymbol{V}_m^H \boldsymbol{H}_{l,m}^H \boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m} \boldsymbol{V}_m)^{-1} \\ &\quad - (\boldsymbol{G}_m + \lambda_m \boldsymbol{I} + \boldsymbol{\Psi}_m) \boldsymbol{V}_m = 0. \end{aligned} \tag{10}$$

Theorem 1. *The optimal precoder $\boldsymbol{V}_m$, $\forall m \in \mathcal{F}$ is given by the generalized dominant eigen vector of the pairs*

$$\boldsymbol{V}_m \to \boldsymbol{D}_{1:d_l}\big(\boldsymbol{H}_{l,m}^H \boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m}, \boldsymbol{G}_m + \lambda_m \boldsymbol{I} + \boldsymbol{\Psi}_m\big) \tag{11}$$

Proof. The results follow directly by applying the proof available in [20] for Proposition 1, by substituting $H_{i,i}^H \boldsymbol{R}_i^{-1} H_{i,i}$ with $\boldsymbol{H}_{l,m}^H \boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m}$ and $\boldsymbol{A}_i + \mu_i \boldsymbol{I}$ with $\boldsymbol{G}_m + \lambda_m \boldsymbol{I} + \boldsymbol{\Psi}_m$ □

3.2. Optimal Power Allocation

Solution (11) diagonalize the matrices

$$\begin{aligned} \boldsymbol{V}_m^H \boldsymbol{H}_{l,m}^H \boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m} \boldsymbol{V}_m &= \Sigma_m^{(1)}, \\ \boldsymbol{V}_m^H (\boldsymbol{G}_m + \lambda_l \boldsymbol{I} + \boldsymbol{\Psi}_l) \boldsymbol{V}_m &= \Sigma_m^{(2)} \end{aligned} \tag{12}$$

at each iteration to maximize the weighted sum rate. Once the precoders are computed, the optimal power allocation can be included while searching for the multipliers, satisfying the constraints. Formally, the power optimization problem can be written as

$$\max_{\boldsymbol{P}_m} \quad w_l \text{lndet}(\boldsymbol{I} + \Sigma_m^{(1)} \boldsymbol{P}_m) - tr(\Sigma_m^{(2)} \boldsymbol{P}_m), \forall m \in \mathcal{F}. \tag{13}$$

with fixed multipliers and $\boldsymbol{V}_l$. Note that as $\boldsymbol{V}_l$ is a generalized dominant eigen vector solution of (9) and therefore by multiplying it by a diagonal matrix it still yields a generalized dominant eigen vector and Theorem 1 is still valid. The optimal power allocation at each FD node is obtained by solving (13), which yields

$$\begin{aligned} \boldsymbol{P}_m = \big(&w_l (\boldsymbol{V}_m^H (\boldsymbol{G}_m + \lambda_m \boldsymbol{I} + \boldsymbol{\Psi}_m) \boldsymbol{V}_m)^{-1} - \\ &\big(\boldsymbol{V}_m^H \boldsymbol{H}_{l,m}^H \boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m} \boldsymbol{V}_m\big)^{-1}\big)^+. \end{aligned} \tag{14}$$

where $(x)^+ = \max\{0, x\}$.

Now, in order to satisfy the per antenna and sum power constraints we consider the following Lagrange dual function

$$\min_{\lambda_l, \boldsymbol{\Psi}_l} \quad L(\lambda_l, \boldsymbol{\Psi}_l). \tag{15}$$

The dual function $L(\lambda_l, \boldsymbol{\Psi}_l)$ is the pointwise supremum of a family of functions of $\lambda_l, \boldsymbol{\Psi}_l$, it is convex [22] and the globally optimal value $\lambda_l, \boldsymbol{\Psi}_l$ and can be found by using any of the numerous convex optimization techniques. In this work, we adopt the bisection algorithm for the search of multipliers. Let $\mathcal{L}_l = \{\lambda_l, \psi_l, .., \psi_{N_l}\}$ contain the multipliers associated with the joint constraints at the node $l \in \mathcal{F}$. Let $\underline{\mu_i}$, and $\overline{\mu_i}$ denote the upper and lower bounds for the Lagrange multiplier $\mu_i \in \mathcal{L}_l$. The complete procedure to solve (6) is formally stated in Algorithm 1.

Algorithm 1 Beamforming for BD-FD

Given: The CSI and rate weights.
Initialize: $\boldsymbol{V}_l, \forall \in \mathcal{F}$.
Repeat until convergence
for: $\forall l \in \mathcal{F}, l \neq m$.
  Compute $\hat{\boldsymbol{G}}_m$ with (8).
  Compute $\boldsymbol{V}_m$ with (11) and normalize it.
  Set $\underline{\mu_i} = 0$ and $\overline{\mu_i} = \mu_{i_{max}}$ $\forall i \in \mathcal{L}_m$.
  for: $\forall \mu_i \in \mathcal{L}_m$
  Repeat until convergence
    set $\mu_i = (\underline{\mu_i} + \overline{\mu_i})/2$.
    Compute $\boldsymbol{P}_m$ with (13),
    If constraint for μ_i is violated,
    set $\underline{\mu_i} = \mu_i$, else $\overline{\mu_i} = \mu_i$,
  Set $\boldsymbol{T}_m = \boldsymbol{V}_m \boldsymbol{P}_m \boldsymbol{V}_m^H$
  Next l.

3.3. Convergence proof

To prove the convergence of Algorithm 1, the ingredients required are minorization [23], Lagrange duality, saddle point and KKT conditions [22]. Let $WSR(\boldsymbol{T})$ denote the cost function (6) as a function of transmit covariance marices and let $\underline{WSR}(\boldsymbol{T}, \hat{\boldsymbol{T}})$ leading to

$$\begin{aligned} WSR(\boldsymbol{T}) \geq \underline{WSR}(\boldsymbol{T}, \hat{\boldsymbol{T}}) = \sum_{l \in \mathcal{F}} w_l \text{lndet}\big(&\boldsymbol{I} + \boldsymbol{V}_m^H \boldsymbol{H}_{l,m}^H \\ &\boldsymbol{R}_{\bar{l}}^{-1} \boldsymbol{H}_{l,m} \boldsymbol{V}_m\big) - tr\big((\boldsymbol{T}_m - \hat{\boldsymbol{T}}) \boldsymbol{G}_m\big). \end{aligned} \tag{16}$$

and the minorizer which is now concave in $\boldsymbol{T}$ has the same gradient of (6), therefore the KKT conditions are not affected. Reparametrizing the transmit covariance matrices $\boldsymbol{T}$ as a function of the variables: powers $\boldsymbol{P}$ and digital beamformers $\boldsymbol{V}$, then adding the power sum-power and per-antenna power constraints yield the Lagrangian (9). During the alternating optimization process, every alternating update of (9) leads to an increase in the weighted sum rate, ensuring convergence for both the parameters. For the KKT conditions, at the convergence point, the gradients of (9) with respect to the digital beamformers yield the same gradients of the original cost function (6). For fixed digital beamformers, (9) is concave in $\boldsymbol{P}$, therefore we have strong duality for the saddle point $max_{\boldsymbol{P}}\ min_{\lambda,\Psi} L$. Moreover, at the convergence point the solution to $min_{\lambda,\Psi} L$ satisfies the complementary slackness condition i.e.,

$$\lambda_l (p_l - tr(\boldsymbol{V}_l \boldsymbol{V}_l^H)) = 0, \forall l \in \mathcal{F} \tag{17a}$$

$$tr(\Psi_l \boldsymbol{P}_l - diag(\boldsymbol{V}_l \boldsymbol{V}_l^H)) = 0, \forall l \in \mathcal{F}. \tag{18a}$$

4. SIMULATION RESULTS

This section presents simulation results for our novel digital beamforming design under the joint sum-power and per-

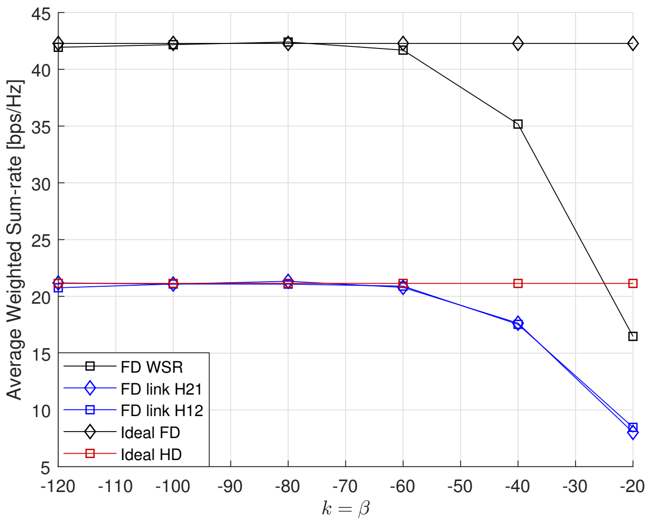

Fig. 1. Average WSR as function of LDR noise with $k = \beta$, $N_1 = N_2 = 10$ and $M_1 = M_2 = 5$ and SNR =30 dB.

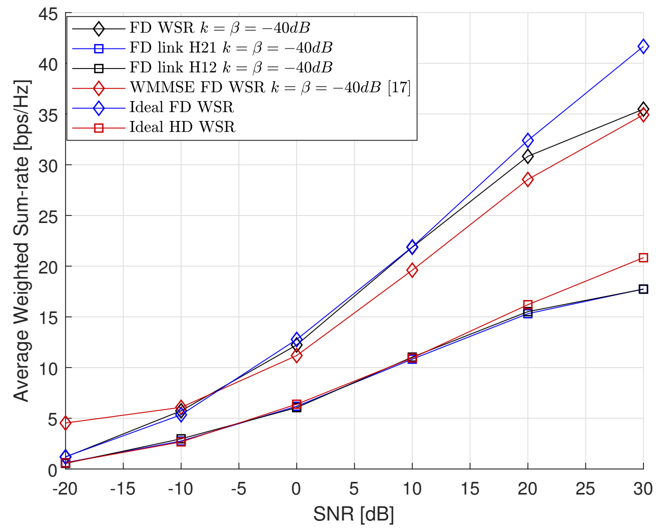

Fig. 2. Average WSR as a function of SNR with $k = \beta = -40$ dB, $N_1 = N_2 = 10$ and $M_1 = M_2 = 5$.

antenna power constraints. The SI channel is modelled with a Rician fading channel model. We assume a highly dominant line-of-sight (Los) component for the SI channel modelled with a Rician factor 10^5 dB [24]. The direct channels are modelled as $\mathcal{CN}(0, 1)$. The per-antenna power constraints at each FD node are the total sum-power divided by the number of transmit antennas. As we consider all the possible noise contributions due to LDR noise and σ_l^2, we label our design as a practical BD-FD (P-FD) design.

For comparison, we define the following benchmark schemes: 1) We define an ideal BD-FD (I-FD) communication system for which there is no LDR noise, and there is perfect knowledge of the SI channel, i.e. $k, \beta = 0$. 2) We define an ideal BD half-duplex (I-HD) system for which ideal circuitry is assumed in the transmit and receive chains, i.e. $k, \beta = 0$. and the resources are split equally in time to serve in uplink and downlink mode. Moreover, we compared our design with the WMSSE design proposed in [17]. Results are reported by averaging over 100 channel realizations.

Let $k = k_l = k_m$ and $\beta = \beta_l = \beta_m$, Figure 1 shows the performance of our proposed design as a function of the LDR noise, i.e., k and β. Note that the I-FD and I-HD system's performance is limited by σ_l^2, which set the maximum achievable performance gain. It is possible to decrease the LDR noise variance below σ_l^2. However, no performance improvement will be observed as the performance is limited by the maximum of the LDR noise variance or σ_l^2. On our P-FD design, the impact of LDR can be observed. It is visible that, as the amount of LDR noise variance increase, the performance of the P-FD decreases and tend towards the performance of an I-HD system. Figure 2 shows the performance as a function of SNR when with $k = \beta = -40$ dB. We can observe the impact of the LDR noise and how the maximum achievable gain deviates from the ideal FD system. We can also see, both the communication link ($H_{2,1}$ and $H_{1,2}$) achieves same performance on average. The most important result can be seen by comparing our design with the WMMSE design [15], which considers only the sum power constraints. It can be seen, when the transmit power is low, then the system with only sum-power constraints perform better as the LDR results to be below the noise level. When the transmit power increases, the LDR noise increases, being proportional to the total transmit power. However, by setting the per-antenna power constraint equal to sum-power divided by the number of antennas, equally distribute the total transmit power on each and minimize the impact of the LDR noise on each antenna. In a practical system, that would mean that the RF components are forced to operate in a highly linear region.

4.1. CONCLUSIONS

In this work, we studied the problem of WSR maximization for a BD-FD communication system under the joint-sum power and per-antenna power constraints. These constraints consider the hardware limitation and the maximum power transmission limits imposed by the regulations. The optimal digital beamformers are designed under the practical LDR model, which considers the effect of non-ideal circuitry. Simulation results show significant performance gain compared to a half-duplex BD communication system. It's observed that the achievable performance of a practical BD-FD system is limited by the maximum of the LDR noise variance or the thermal noise variance. Moreover, we can conclude that by designing the FD system under the joint constraints can be advantageous to limit the impact of the LDR noise.

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