On Achivable Rates in a Multi-Antenna Gaussian Broadcast Channel

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Abstract — **A Gaussian broadcast channel with r single-antenna receivers and t antennas at the transmitter is considered. Both transmitter and receivers have perfect knowledge of the channel. Despite apparent simplicity, this model is in general a non-degraded broadcast channel, for which the capacity region is not fully known. We propose a novel transmission scheme based on "ranked known interference" (RKI). In brief, the transmitter decomposes the channel into an ordered (or ranked) set of interference channels for which the interference signal of the i-th channel is generated as a linear combination of the signals transmitted in channels $j < i$. In this way, known techniques of coding for non-causally known interference can be applied to make the interference in each channel harmless without further power penalty. We show that the proposed scheme is throughputwise asymptotically optimal for both low and high SNR. We provide a modification of the basic RKI scheme which achieves optimal throughput for all SNRs in the special case of 2 antenna and 2 users. For independent Rayleigh fading closed-form throughput expressions are obtained in various cases of interest and numerical examples are provided for the infinite-dimensional Rayleigh channel.**

I. Problem statement

The $t \times 1 \dots r$ Gaussian boradcast channel (GBC) is described by $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$ where the notation $t \times 1 \dots r$ designates that the r receivers cannot cooperate. Here, $\mathbf{x} \in \mathbb{C}^t$ denotes the vector of symbols transmitted in parallel from the t antennas in any channel use, $\mathbf{H} \in \mathbb{C}^{r \times t}$ denotes the channel matrix, whose i,j-th element is the complex channel gain from antenna j to antenna i, $\mathbf{y} \in \mathbb{C}^r$ denotes the vector of received signals at the r receivers and $\mathbf{z} \in \mathbb{C}^r$ is the noise vector, $\sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, N_0\mathbf{I})$. The channel matrix $\mathbf{H}$ is known to the transmitter and to all receivers and the input is constrained to satisfy $\text{trace}(E[\mathbf{x}\mathbf{x}^H]) \leq \mathcal{E}$ where $\mathcal{E}$ is the maximum allowed total transmit energy per channel use.

We are interested in the channel throughput R^{gbc} (or rate-sum), defined as the sum of all achievable individual user rates. We shall consider the following scenarios: i) $\mathbf{H}$ is deterministic and fixed; ii) $\mathbf{H}$ is fixed during the transmission of each code word, but it is randomly selected according to a given probability distribution (composite channel).

II. The "ranked known interference" scheme

Let $\mathbf{H} = \mathbf{G}\mathbf{Q}$ be a QR-type decomposition of $\mathbf{H}$ where $\mathbf{G} \in \mathbb{C}^{r \times m}$ is lower triangular and $\mathbf{Q} \in \mathbb{C}^{m \times t}$ has orthonormal rows (we define $m \triangleq \min\{r, t\}$). The transmitted signal is obtained as $\mathbf{x} = \mathbf{Q}^H\mathbf{u}$. Let $g_{i,j}$ denote the (i,j)-th element of $\mathbf{G}$ and let $d_i = |g_{i,i}|^2$. The original channel is turned into the set of interference channels $y_i = g_{i,i}u_i + \sum_{j<i} g_{i,j}u_j + z_i$, for $i = 1, \dots, m$, while no information is sent to users $m+1, \dots, r$. Since $\mathbf{Q}\mathbf{Q}^H = \mathbf{I}$, $\text{trace}(E[\mathbf{u}\mathbf{u}^H]) = \text{trace}(E[\mathbf{x}\mathbf{x}^H])$. The signals u_i are all generated by the transmitter: for each i-th channel u_i is the wanted signal and $\sum_{j<i} g_{i,j}u_j$ is an interference signal non-causally known to the transmitter. With some care [5], we can apply the results of [2] or (equivalently) the more recent results of [3], and show that by using an appropriate coding strategy, the capacity of each i-th channel is equal to that of the virtual interference-free channel $y_i = u_i + z_i$:

Proposition 1. The maximum achievable throughput of the RKI scheme is given by $R^{\text{rki}-\max} = \sum_{i=1}^m [\log(\xi d_i)]_+$ where ξ is the solution of $\sum_{i=1} [\xi - 1/d_i]_+ =\triangleq \mathcal{E}/N_0$. □

III. Results

We denote by $R^{\text{zf}-\max}$ and by $R^{\text{coop}-\max}$ the maximum achievable throughputs of a conventional system based on *zero-forcing beamforming* [4] and of a system where the receivers are allowed to cooperate (equivalent to the single-user multiple-antenna case [1]). For $\mathbf{H}$ given and constant we have the following results: 1) For any channel matrix $\mathbf{H}$, $R^{\text{rki}-\max} \geq R^{\text{zf}-\max}$; 2) For any channel matrix $\mathbf{H}$ with full row-rank r, $\lim_{A\to\infty} \left(R^{\text{coop}-\max} - R^{\text{rki}-\max}\right) = 0$; 3) For any channel matrix $\mathbf{H}$, $\lim_{A\to 0} \frac{R^{\text{rki}-\max}}{R^{\text{zf}-\max}} = 1$. □

Notice that for $\mathbf{H}$ of rank r the RKI is asymptotically throughputwise optimal for large SNR, since obviously $R^{\text{coop}-\max} \geq R^{\text{gbc}}$.

A generalized RKI strategy where $\mathbf{x} = \mathbf{Q}^H\mathbf{R}\mathbf{u}$, where $\mathbf{R}$ is an optimized upper triangular matrix is shown to achieve the maximum throughput of the $2 \times 1 \dots 2$ GBC, for any SNR and any matrix $\mathbf{H}$ [5].

In the case of random $\mathbf{H}$ with i.i.d. Rayleigh fading, different RKI strategies with equal-power and constant-rate active users are comparatively investigated for a large array (i.e., for $r, t \to \infty$ and r/t fixed). It is demonstrated that the equal-power scheme performs very close to the waterfilling RKI and approaches the upper bound of cooperative rate for moderate and high SNRs [5]. The results essentially hold when $\mathbf{H}$ is available to the transmitter only [5].

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. on Telecomm. ETT,* Vol. 10, No. 6, pp. 585-596, Nov. 1999.

[2] M. Costa, "Writing on dirty paper," *IEEE Trans. on Inform. Theory,* Vol. 29, No. 3, pp. 439-441, May 1983.

[3] U. Erez, S. Shamai and R. Zamir, "Capacity and lattice-strategies for cancelling known interference," *ISITA 2000,* Honolulu, Hawaii (USA), Nov. 2000.

[4] P. Baier, M. Meurer, T. Weber and H. Troeger, "Joint transmission (JT), an alternative rationale for the downlink of time division CDMA using multi-element transmit antennas," *IEEE 6th Int. Symp. on Spread-Spectrum Tech. & Appl. (ISSSTA),* NJIT, N.J. (USA), Sept. 2000.

[5] G. Caire and S. Shamai, "On achievable rates in a multi-antenna Gaussian broadcast channel," submitted to *IEEE Trans. on Inform. Theory,* April 2001.