Learned Chester

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I. Introduction

In this section, we outline the problem formulation for the mmWave channel estimation challenge. We consider a single user hybrid multi-carrier mmWave MIMO OFDM uplink with N_t transmit antennas at the user and N_r receive antennas at the base station. The hybrid architecture involves L_t RF chains at the transmit side and L_r RF chains at the receive side. The analog precoder at the transmit side is denoted by $\boldsymbol{F}_{RF} \in \mathcal{C}^{N_t \times L_t}$. The analog combiner at the receive side is $\mathbf{W}_{tr} \in \mathcal{C}^{N_r \times L_r}$. The number of streams to be transmitted is set as $N_s = 2$. The number of subcarriers are chosen as $K = 256$. The system operates with uniform linear arrays (ULAs) at both ends.

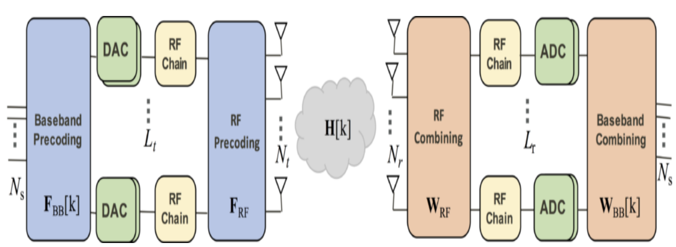

Fig. 1. Millimeter wave MIMO system based on a hybrid architecture. In the site-specific channel estimation challenge, the BS operates as receiver and the UE as transmitter.

We define a training pilot as an OFDM symbol known at both the Tx and the Rx. The received signal in the m^{th} training pilot and for the k^{th} subcarrier is written as

$$\mathbf{r}^{(m)}[k] = \mathbf{W}_{tr}^{(m)\,H}(\boldsymbol{H}[k]\boldsymbol{F}_{RF}^{(m)}\mathbf{q}^{(m)} + \mathbf{n}^{(m)}[k]), \tag{1}$$

where $\boldsymbol{H}[k] \in \mathcal{C}^{N_r \times N_t}$ represents the frequency domain MIMO channel matrix for subcarrier k. The noise vector $\mathbf{n}^{(m)}[k]$ is circularly symmetric complex Gaussian distributed with zero mean independent and identically distributed (i.i.d.) components of variance σ^2, denoted by $\mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_r})$. Each entry of the transmit pilot $\mathbf{q}^{(m)}$ is selected as $\frac{1}{\sqrt{2L_t}}(a + jb)$, where $a, b \in \{-1, 1\}$ and are uniformly distributed. Note that $\left\|\mathbf{q}^{(m)}\right\|^2 = 1$ and hence the transmit SNR is defined as $\rho = \frac{1}{\sigma_n^2}$. For this challenge, the transmit power is kept constant throughout. The provided training or test datasets are either from $\rho = -15$ dB, -10 dB or -5 dB, by changing the noise variance σ_n^2.

After vectorizing (1), using the result $vec(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})vec(\mathbf{X})$, we obtain

$$vec(\mathbf{r}^{(m)}[k]) = \underbrace{\left(\mathbf{q}^{(m)\,T}\boldsymbol{F}_{RF}^{(m)\,T} \otimes W_{tr}^{(m)\,H}\right)}_{\boldsymbol{\Phi}^{(m)}} vec(\boldsymbol{H}[k]) + W_{tr}^{(m)\,H}\mathbf{n}^{(m)}[k]. \tag{2}$$

Our goal is to estimate the channels $\boldsymbol{H}[k], k = 1, 2, \ldots, K$ using the above received pilot signals.

Notation: The operator $(\cdot)^H$ represents the conjugate transpose or conjugate for a matrix or a scalar respectively. The operator $\text{tr}(\cdot)$ represents trace of a matrix. The probability density function (pdf) of a complex Gaussian random variable x with mean μ and variance σ^2 is denoted by $\mathcal{CN}(x; \mu, \nu)$. $\mathbf{A}_{n,:}$ and $\mathbf{A}_n$ represent the n^{th} row and n^{th} column of $\mathbf{A}$, respectively. $\text{blkdiag}(\cdot)$ represents blockdiagonal part of a matrix. $\text{diag}(\mathbf{X})$ or $\text{diag}(\boldsymbol{x})$ represents a vector obtained by the diagonal elements of the matrix $\mathbf{X}$ or the diagonal matrix obtained with the elements of $\boldsymbol{x}$ in the diagonal respectively. Tx denotes the transmitter and Rx denotes the receiver.

A. Channel Model

The datasets provided for the challenge involve channels which are generated using the Raymobtime dataset available at https://www.lasse.ufpa.br/raymobtime/. The MIMO channel is assumed to be frequency selective, with delay tap length N_c in the time domain. The d^{th} delay tap of the channel is modeled as a pathwise channel model with L paths as follows:

$$\boldsymbol{H}_d = \sum_{l=1}^{L} \alpha_l p(dT_s - \tau_l) a_R(\theta_l) a_T(\phi_l)^H, \tag{3}$$

where α_l represents the complex path coefficient, θ_l is the angle of arrival (AoA) at the BS side, ϕ_l is the angle of departure (AoD) from the UE side and $p(\tau)$ represents the pulse shaping filter. τ_l denotes the delay of the l^{th} path and there are L multipaths. T_s denotes the sampling time. The channel $\boldsymbol{H}_d$ can be approximated using the virtual channel model [1]

$$\boldsymbol{H}_d = \mathbf{A}_R \boldsymbol{\Delta}_d \mathbf{A}_T^H. \tag{4}$$

The matrix $\mathbf{A}_R \in \mathcal{C}^{N_r \times G_r}$ contains the Tx side antenna array response vectors evaluated at a grid of size G_r for the AoA and $\mathbf{A}_T \in \mathcal{C}^{N_t \times G_t}$ contains the Rx side antenna array response vectors at a grid of size G_t for the AoD. $\boldsymbol{\Delta}_d \in \mathcal{C}^{G_r \times G_t}$ represents a sparse matrix with entries corresponding to the complex path coefficients $(\alpha_l p(dT_s - \tau_l))$ at the locations where a path with corresponding AoA and AoD exists. Further, the channel at subcarrier k can be written in terms of delay taps as follows.

$$\boldsymbol{H}[k] = \sum_{d=0}^{N_c-1} \boldsymbol{H}_d e^{-j2\pi\frac{kd}{K}} = \mathbf{A}_R \boldsymbol{\Delta}[k] \mathbf{A}_T^H, \tag{5}$$

where

$$\boldsymbol{\Delta}[k] = \sum_{d=1}^{N_c} \boldsymbol{\Delta}_d e^{-j2\pi\frac{kd}{K}}. \tag{6}$$

Vectorizing $\boldsymbol{H}[k]$

$$vec(\boldsymbol{H}[k]) = (\mathbf{A}_T^* \otimes \mathbf{A}_R)\, vec(\boldsymbol{\Delta}[k]). \tag{7}$$

Further, defining $\boldsymbol{\Psi} = \mathbf{A}_T^* \otimes \mathbf{A}_R$ and $\mathbf{h}[k] = vec(\boldsymbol{\Delta}[k])$ and substituting for $vec(\boldsymbol{H}[k])$ in (2), we get the received signal model as

$$vec(\boldsymbol{r}^{(m)}[k]) = \boldsymbol{\Phi}^{(m)} \boldsymbol{\Psi} \mathbf{h}[k] + W_{tr}^{(m)\,H} \mathbf{n}^{(m)}[k]. \tag{8}$$

II. Proposed Approach

We adopt a model based approach using a compressed sensing (CS) framework that integrates a greedy search procedure along with a statistical inference method to solve for $\mathbf{h}_k$ in (8). In the first step of our solution, we obtain an initial channel estimate using simultaneous weighted orthogonal matching pursuit (SW-OMP) algorithm [2]. As the sparsifying dictionary $\boldsymbol{\Psi}$ is unknown a priori, we use row-truncated discrete Fourier transform matrices of size $N_t \times G_t$ and $N_r \times G_r$ as the transmit and receive array steering matrices, respectively. We obtain coarse estimates of angles of departure (AoD) and angles of arrival (AoA) in this step. Subsequently, we use a multi-level beam search technique to find more accurate estimates of the AoD and AoA of the channel. In every level of this beam search, we form an equally spaced rectangular grid around the coarse AoD and AoA estimates obtained in the first step and find new AoD and AoA using a exhaustive search method. We repeat this step multiple times by narrowing the search space in subsequent iterations. At the end of this multi-level search procedure, we obtain the final channel estimate. The first step provides reasonable AoDs and AoA estimates, but the coarse sparsifying dictionary used may not be able to obtain the exact AoD and AoA that lie in the off grid regions of the dictionary. To combat this, we adopt a statistical inference procedure to obtain accurate AoDs and AoAs which improves the NMSE performance of our proposed algorithm. We model the off grid effects as follows:

$$\boldsymbol{H}[k] = \tilde{\mathbf{A}}_R \tilde{\boldsymbol{\Delta}}[k] \tilde{\mathbf{A}}_T^H, \tag{9}$$

where $\tilde{\mathbf{A}}_R$ and $\tilde{\mathbf{A}}_T$ are the receive and transmit array steering matrices corresponding to the initial AoD and AoA estimates obtained using SW-OMP, respectively, and $\tilde{\boldsymbol{\Delta}}[k]$ contains the channel estimates. In practice, the exact AoA and AoD may not match with the grid locations of the AoD and AoA steering matrices, in which case the channel estimate obtained in the first step may have high mean squared error. To illustrate this, let us suppose the original AoD and AoA for one path are θ_0 and ϕ_0, respectively, but the array steering vectors are positioned at θ and ϕ. We write $\theta_0 = \theta + \Delta\theta$ and $\phi_0 = \phi + \Delta\phi$, where $\Delta\theta$ and $\Delta\phi$ denote the offsets from the correct AoD and AoA, respectively. Using the structure of the array steering matrices, we write this as

$$\boldsymbol{H}[k] = \tilde{\mathbf{A}}_R \tilde{\mathbf{D}}_R \tilde{\boldsymbol{\Delta}}[k] \tilde{\mathbf{D}}_T^H \tilde{\mathbf{A}}_T^H, \tag{10}$$

where $\tilde{\mathbf{D}}_R$ and $\tilde{\mathbf{D}}_T$ are diagonal matrices which contain the offsets from the original AoD and AoA, respectively. Substituting this in (7), we get the channel model as

$$vec(\boldsymbol{H}[k]) = \left(\tilde{\mathbf{A}}_T^* \otimes \tilde{\mathbf{A}}_R\right) vec(\tilde{\mathbf{D}}_R \boldsymbol{\Delta}[k] \tilde{\mathbf{D}}_T^H). \tag{11}$$

Further, substituting in (8), we get

$$vec(\boldsymbol{r}^{(m)}[k]) = \boldsymbol{\Phi}^{(m)} \tilde{\boldsymbol{\Psi}} \mathbf{h}_s[k] + \boldsymbol{W}_{tr}^{(m)\,H} \mathbf{n}^{(m)}[k]. \;, \tag{12}$$

where $\mathbf{h}_s[k] = vec(\tilde{\mathbf{D}}_R \boldsymbol{\Delta}[k] \tilde{\mathbf{D}}_T^H)$ and $\tilde{\boldsymbol{\Psi}} = \tilde{\mathbf{A}}_T^* \otimes \tilde{\mathbf{A}}_R$. Now, we estimate $\mathbf{h}_s[k]$, $\forall k$ using sparse Bayesian learning (SBL) which is a type II maximum likelihood estimation procedure [3]. We impose a complex Gaussian prior on the channel and use expectation maximization procedure to obtain the posterior distribution of the channel. More details of SBL and type-II ML estimation can be found in [4]. Once we obtain the channel estimates, we estimate the support of the sparse vector and the channel coefficients using the hyperparameters obtained using SBL.

Once we obtain the frequency domain channel estimates using SBL, we exploit the lag domain sparsity of the channel to suppress the residual noise. We retain only the dominant lag domain channel taps and set the remaining entries to 0. We learn the number of channel taps and the thresholds for the hyperparameters using the training dataset.

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