DoF Region of the Decentralized MIMO Broadcast Channel—How many informed antennas do we need?

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***Abstract*—In this work, we study the impact of imperfect sharing of the Channel State Information (CSI) available at the transmitters on a Network MIMO setting in which a set of M transmit antennas, possibly not co-located, jointly serve two multi-antenna users endowed with N_1 and N_2 antennas, respectively. We consider the case where only a subset of k transmit antennas have access to perfect CSI, whereas the other $M-k$ transmit antennas have only access to finite precision CSI. The analysis of this configuration aims to answer the question of how much an extra informed antenna can help. We model this scenario as a Decentralized MIMO Broadcast Channel (BC) and characterize the Degrees-of-Freedom (DoF) region, showing that only $k = \max(N_1, N_2)$ antennas with perfect CSI are needed to achieve the DoF of the conventional BC with ubiquitous perfect CSI.**

I. INTRODUCTION

The availability of CSI at the Transmitters (CSIT) is one of the fundamental requirements for managing interference in MIMO and multi-user cooperative settings. On account of the infeasibility of acquiring perfect CSIT in many practical scenarios, there has been a significant interest in characterizing the impact of non-perfect CSIT on the system performance. The assumption of non-perfect CSIT has been analyzed from many different perspectives, considering e.g. the cases of noisy instantaneous CSIT [1], perfect delayed [2], partial [3], hybrid [4], or alternating CSIT [5], [6]. However, it is normally assumed that the CSIT is centralized, i.e., *perfectly shared* among the transmitters. Although this belief arises naturally in MIMO settings with one multi-antenna transmitter, it is unattainable in many practical settings with cooperating nodes. Such settings are expected to burgeon due to the increased heterogeneity and densification of the wireless networks.

Motivated by the foregoing, we aim to understand the impact of *imperfectly shared* CSIT, i.e., the case in which each transmitter may have a different CSI. This configuration, coined *Distributed CSIT* setting, has been previously studied in the Interference Channel [7] or the Network MISO setting [8], [9]. In this work, we focus on the Network MIMO setting. Note that a Network MIMO setting in which the transmitters perfectly share the user data but not the CSIT can be modeled as a MIMO BC setting with antenna-dependent CSIT, and consequently we denote this setting as the Decentralized MIMO BC.

Therefore, we consider the 2-user MIMO BC where the users have N_1 and N_2 antennas, respectively. The DoF metric of this setting has been analyzed for multiple heterogeneous, yet centralized, CSI configurations, e.g. the cases where the CSIT for each user can be Perfect, Delayed, or Not-available [4], [5], [10]. However, this work is to our knowledge the first to consider distributed CSIT. In particular, we assume that only k of the M transmit antennas have access to perfect CSI, whereas the other transmit antennas have only access to finite precision CSI.

This model, in which some transmit antennas are provided with global CSI (also from the other non-informed transmit antennas), arises in the context of FDD heterogeneous networks where the users feed back the global CSI to a main base station, which is in turn helped by secondary nodes or remote radio-heads with a limited backhaul. The availability of the user data at all transmit antennas is feasible at the same time thanks to caching and Cloud/Fog-RAN technologies [9].

Our main contributions are as follows: i) We present an outer bound for the DoF region of the 2-user MIMO BC when only k transmit antennas have access to perfect CSI; ii) we show that having perfect CSIT at $k = \max(N_1, N_2)$ antennas is enough to achieve the DoF region of the conventional MIMO BC with perfect CSIT at every antenna; and iii) we develop an achievable scheme that attains the DoF region for $k \geq \min(N_1, N_2)$ and partially closes the gap for $k < \min(N_1, N_2)$.

Notations: $[n]$ is defined as $[n] \triangleq \{1, \dots, n\}$ and, in any variable X, the superscript $^{[n]}$ stands for $\{X(i)\}_{i\in[n]}$. The joint entropy of a set of variables $\mathcal{S}$ is denoted as $H\big(\cap_{\mathcal{S}_i \in \mathcal{S}} \mathcal{S}_i\big)$.

II. SYSTEM MODEL

A. MIMO Broadcast Channel

We analyze a setting where M transmit antennas (TXs) jointly serve 2 users (RXs) of N_1 and N_2 antennas, considering w.l.o.g. that $N_1 \leq N_2$. The received signal at RX i is given by

$$\mathbf{Y}_i(t) \triangleq \sqrt{P}\, \mathbf{H}_i(t)\mathbf{X}(t) + \mathbf{N}_i(t), \tag{1}$$

where $\mathbf{Y}_i(t) \triangleq [Y_{i,1}(t), \dots, Y_{i,N_i}(t)]^{\mathrm{T}}$, $\mathbf{H}_i \in \mathbb{R}^{N_i \times M}$ denotes the matrix of channel coefficients for RX i, and P denotes the nominal SNR parameter. We define the global channel matrix as $\mathbf{H}^{\mathrm{T}} \triangleq \big[\mathbf{H}_1^{\mathrm{T}},\ \mathbf{H}_2^{\mathrm{T}}\big]$, and the channel vector between TX j and RX i as $\mathbf{H}_{i,j}$. The vector $\mathbf{X}(t) \triangleq [X_1(t), \dots, X_M(t)]^{\mathrm{T}}$ is the transmit signal vector, which satisfies a unitary power constraint, and $\mathbf{N}_i(t)$ denotes the AWGN noise at RX i. RX i wants to receive a message W_i, which is available at all the TXs. The definitions of achievable rates $R_i(P)$ and capacity region $\mathcal{C}(P)$ are standard [11]. The DoF for RX i is defined as $d_i \triangleq \lim_{P\to\infty} \frac{R_i(P)}{\log \bar{P}}$, where $\bar{P} \triangleq \sqrt{P}$. The closure of achievable DoF tuples (d_1, d_2) is called the DoF region $\mathcal{D}$.

We assume that the channel coefficients are bounded away from 0 and infinity and that are drawn from distributions that satisfy the bounded density assumption, which is presented below.

Definition 1 ([3, Def. 4] Bounded Density)**.** *Let $\mathcal{G}$ be a set of real-valued random variables, which satisfies both of the following conditions: i) The magnitudes of all the random variables in $\mathcal{G}$ are bounded away from infinity, i.e., there exists a constant $\Delta < \infty$ such that for all $g \in \mathcal{G}$ we have $|g| \leq \Delta$; ii) there exists a finite positive constant $f_{\max}$, such that for all finite cardinality disjoint subsets $\mathcal{G}_1, \mathcal{G}_2$ of $\mathcal{G}$, the joint probability density function of all random variables in $\mathcal{G}_1$, conditioned on all random variables in $\mathcal{G}_2$, exists and is bounded above by $f_{\max}^{|\mathcal{G}_1|}$.*

B. Distributed CSIT

We consider a Distributed CSIT setting [9] where the first k TXs are provided with perfect global CSI, such that they know the whole multi-user channel matrix $\mathbf{H}$, whilst the other $M-k$ TXs have only finite precision CSI. Hereinafter, we will denote this setting as the (M, N_1, N_2, k) MIMO BC.

Remark 1. *The notation "TX" refers to a single transmit antenna. The transmit antennas can be distributed among an arbitrary number of physical transmitters. Thus, there can be e.g. M single-antenna transmitters or two $\frac{M}{2}$-antenna transmitters.*

We split the set of transmit antennas in two different groups. Let us denote the i-th transmit antenna as TX_i, $i \in [M]$. Hence,

- $\mathbf{TX}_\star \triangleq [\mathrm{TX}_1, \ldots, \mathrm{TX}_k]$ denotes the k TXs that have access to perfect CSI, i.e., which know $\mathbf{H}$ instantaneously.
- $\mathbf{TX}_\varnothing \triangleq [\mathrm{TX}_{k+1}, \ldots, \mathrm{TX}_M]$ stands for the $M-k$ TXs with finite precision CSI. This implies that, for any TX in $\mathbf{TX}_\varnothing$, the channel coefficients satisfy the bounded density assumption of Definition 1 [3], [12].

The transmit signals from $\mathbf{TX}_\varnothing$, $\mathbf{TX}_\star$ are denoted as $\mathbf{X}_\varnothing$, $\mathbf{X}_\star$.

Remark 2. *Although considering both perfect and finite CSIT may resemble the conventional BC with Hybrid CSIT in which there exists perfect CSIT for one RX and no CSIT for the other RX (the so-called 'PN' setting) [3], [4], [13], the CSI model here considered is substantially different: In the mentioned 'PN' setting,* all *the TXs share the same CSI, i.e., all of them have access to perfect CSI for one RX and* no *TX has access to CSI of the other RX. However, in our setting, a* subset *of TXs has access to perfect* global *CSI (for both RXs), whereas the other subset has access only to finite precision CSI of the* global *CSI. Further discussion about this CSIT setting can be found in [9].*

III. DoF Region of the (M, N_1, N_2, k) MIMO BC

We analyze the DoF region of the MIMO BC as a function of the number of TXs with perfect CSIT (k). Therefore, we can measure the gain (in terms of DoF) that is obtained by providing an extra TX with perfect CSIT, which would require either backhaul of feedback resources. We first present an outer bound.

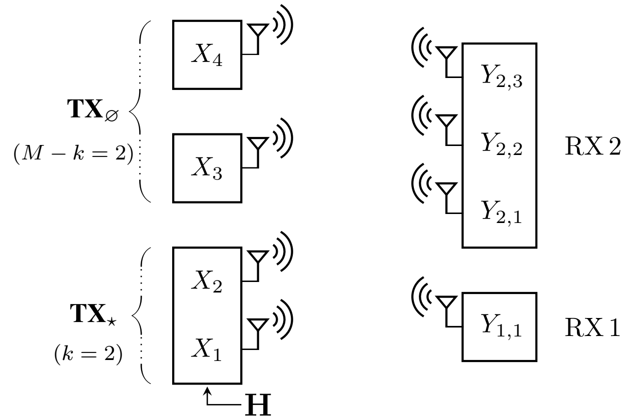

Figure 1: System model for $(M, N_1, N_2, k) = (4, 1, 3, 2)$. The transmit antennas can belong to non-colocated transmitters.

Theorem 1. *Let us consider the (M, N_1, N_2, k) MIMO BC. If $k < N_2$ and $M > N_2$, the DoF region ($\mathcal{D}$) is enclosed in*

$$(d_1, d_2) \in \begin{cases} d_1 \leq \min(M,\ N_1) & \text{(2a)} \\ d_2 \leq \min(M,\ N_2) & \text{(2b)} \\ d_1 + d_2 \leq \min(M,\ N_1 + N_2) & \text{(2c)} \\ \frac{d_1}{\min(M, N_1+N_2)-k} + \frac{d_2 - k}{\min(N_2, M)-k} \leq 1 & \text{(2d)} \end{cases}$$

Otherwise (i.e., if $k \geq N_2$ or $M \leq N_2$), $\mathcal{D}$ is enclosed in

$$(d_1, d_2) \in \begin{cases} d_1 \leq \min(M,\ N_1) & \text{(3a)} \\ d_2 \leq \min(M,\ N_2) & \text{(3b)} \\ d_1 + d_2 \leq \min(M,\ N_1 + N_2) & \text{(3c)} \end{cases}$$

Proof: The proof is relegated to Section V. ■

The DoF region in (3) matches the DoF region of the MIMO BC with perfect CSIT. Moreover, the bound (2d) holds for any value of M. However, note that, if $M \leq N_2$, (2d) becomes (3c), and hence we recover (3). Let us consider now the sum DoF.

Lemma 1. *The sum DoF of the (M, N_1, N_2, k) MIMO BC, defined as $d_\Sigma \triangleq \max_{(d_1,d_2)\in\mathcal{D}}(d_1 + d_2)$, is upper-bounded by*

$$d_\Sigma \leq \min\left(N_1 + N_2,\ M,\ N_2 + \frac{N_1 \min(N_1, M - N_2)}{\min(N_1 + N_2, M) - k}\right).$$

Lemma 1 is a direct aftermath of Theorem 1. Thus, the sum DoF upper bound is strictly smaller than the DoF of the BC with perfect CSIT [4] for the regime of (2), and matches it for the regime of (3). Next, we introduce the achievability results.

Theorem 2. *The DoF region outer bound of Theorem 1 is achievable for $k \geq \min(N_1, N_2)$.*

Proof: The proof follows from a novel transmission scheme introduced in Section VI, which shows that the sum DoF of Lemma 1 is achievable. The DoF region can be obtained then by time-sharing. The transmission scheme achieving Theorem 2 is based on the Active-Passive Zero-Forcing precoding (AP-ZF) introduced in [9] and the fact that exploiting the unavoidable interference as side information is beneficial. ■

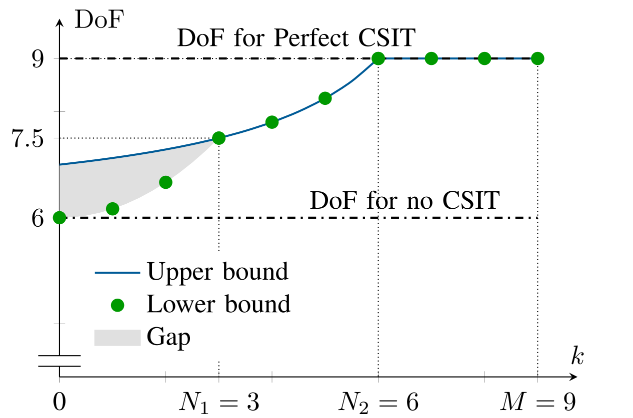

Figure 2: Sum DoF as a function of the number of TXs with perfect CSIT (k) for the case $(M, N_1, N_2) = (9, 6, 3)$.

Hence, for the case where $M = N_1 + N_2$, it follows that

$$d_\Sigma = \begin{cases} N_2 + N_1 & \text{if } k \geq N_2 \\ N_2 + N_1 \frac{N_1}{N_1+N_2-k} & \text{if } N_1 \leq k < N_2. \end{cases} \quad (4)$$

Unfortunately, besides particular cases, no tight general bound is known for the regime $k < N_1$. Nevertheless, we can extend the proposed scheme to obtain a general lower bound, whose proof is relegated to the extended version of this work [14].

Proposition 1. *Let us assume that $k < N_1$ and define $m_{N_2}^{M-k}$ as $m_{N_2}^{M-k} \triangleq \min(N_2,\ M-k)$. Then, the sum DoF of the (M, N_1, N_2, k) MIMO BC is lower-bounded by*

$$d_\Sigma \geq \max\Big(\min(N_2,\ M),\ m_{N_2}^{M-k} + \frac{k^2}{m_{N_2}^{M-k}}\Big). \quad (5)$$

IV. Discussion

The sum DoF of the 2-user MIMO BC with perfect CSIT is $\text{DoF}^\star = \min(M, N_2+N_1)$ [4]. Hence, Theorem 1 implies that we only need perfect CSI at $k = N_2$ to recover the maximum DoF. This aftermath extends the results of previous works on the MISO setting [8], [9], where it was shown that having the most accurate CSI at only a subset of TXs is (sometimes) enough to recover the DoF achieved with perfect CSI sharing.

Fig. 2 represents the sum DoF as a function of k. We observe how for $k \geq N_2$ the DoF obtained with centralized perfect CSIT is attained, and that for $N_1 \leq k \leq N_2$ the bound is tight. For the case $k < N_1$, there exists a gap between the upper and the lower bound. We can infer that the upper bound is loose from the fact that for $k = 0$ we obtain that $\text{DoF} = N_2 + 1$, but the DoF in this case is known to be $\text{DoF} = N_2$ [3]. It is noteworthy that, the closer k is to the number of antennas of any of the RXs, the more the DoF increases from k to $k+1$. In Fig. 3, we present the DoF region for the case $(4, 1, 3, k)$. Interestingly, a single informed antenna can considerably increase the performance, specially for RX 1.

Besides this, the DoF obtained for this decentralized setting has an appreciable similarity with the DoF of the centralized MIMO BC in which the transmitter has perfect CSI for RX 1 and delayed CSI for RX 2, also known as the 'PD' setting,

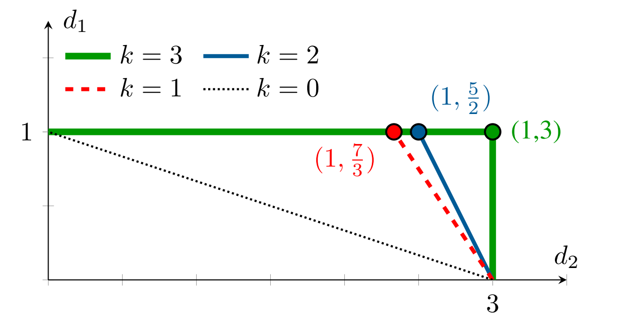

Figure 3: DoF region for the $(M, N_1, N_2, k) = (4, 1, 3, k)$ MIMO BC for $k \in \{0, 1, 2, 3\}$.

whose DoF region was derived in [10]. By way of example, consider the scenario with $M = N_1 + N_2$ and $N_1 \leq k < N_2$, such that the particular bound of (2d) is active. If we compare these two settings with the perfect-CSIT MISO BC, we can observe that there exists an analogy between both settings:

1) In the 'PD' setting, the loss of DoF due to having delayed CSIT for RX 2 instead of perfect CSIT is $-N_2 \frac{N_1}{N_1+N_2}$.
2) In our decentralized setting, the loss of DoF due to having perfect CSIT *only* at k antennas is $-(N_2 - k)\frac{N_1}{N_1+(N_2-k)}$.

Therefore, the (M, N_1, N_2, k) setting seems analogous to a 'PD' case where only $N_2 - k$ antennas suffer from having delayed CSI. An intuition behind this result is that we can apply a change of basis at RX 2 so that the TXs with perfect CSI are only listened by k antennas of RX 2. Hence, even if those TXs have perfect CSI for the other $N_2 - k$ antennas, these antennas receive only information from the TXs with finite precision CSI.

V. Converse of Theorem 1

We prove Theorem 1 for real channels. The extension to complex variables is intuitive but cumbersome, and hence we omit it for sake of conciseness. First, let us consider a genie-aided setting with perfect CSIT available at every transmit antenna. This genie-aided scenario corresponds to the well-known conventional MIMO BC with perfect CSIT [4], whose DoF region coincides with (3). Since providing with additional CSI can not hurt, we obtain that (3) is an outer bound for the (M, N_1, N_2, k) MISO BC. Hence, it remains to prove that the bound (2d), i.e.,

$$\frac{d_1}{\min(M, N_1 + N_2) - k} + \frac{d_2 - k}{N_2 - k} \leq 1, \quad (6)$$

holds when $M > N_2$ and $k < N_2$. We present here the proof for the key regime $N_2 < M \leq N_1 + N_2$. The outer bound for the regime $M > N_1 + N_2$ follows from invertible transformations at the nodes and is relegated to the extended version [14].

1) Deterministic Channel Model: We start similarly as in [1], [3], [15] by discretizing the channel, what leads to a deterministic channel model first introduced in [16]. The discretized model is such that the input signals $\bar{X}_j(t) \in \mathbb{Z}$ and output signals $\bar{Y}_i(t) \in \mathbb{Z}$ are given by

$$\bar{X}_j(t) \in \{0, 1, \ldots, \lceil \bar{P} \rceil\}, \quad \forall j \in [M], \quad (7)$$

$$\bar{\mathbf{Y}}_i(t) \triangleq \sum_{j=1}^{M} \lfloor \mathbf{H}_{i,j} \bar{X}_j(t) \rfloor, \quad \forall i \in \{1, 2\}. \quad (8)$$

In the following, we obtain an outer bound for this channel model. From [1, Lemma 1], this DoF outer bound is also an outer bound for the channel model that we have considered.

2) Weighted sum rate: We obtain (6) by means of bounding the weighted sum rate $n(N_2-k)R_1 + n(M-k)R_2$. First of all, we present an instrumental lemma.

Lemma 2. *Let the number of transmit antennas with perfect CSIT satisfy that $k < N_2$. Then, it holds that*

$$\begin{aligned}(N_2-k)H(\bar{\mathbf{Y}}_1^{[n]}|\mathbf{H}^{[n]},W_2)&\\ -(M-k)H(\bar{\mathbf{Y}}_2^{[n]}|\mathbf{H}^{[n]},W_2)&\leq o(\log\bar{P}).\end{aligned}\tag{9}$$

Proof: The proof is relegated to Section V-A. ■

We start from Fano's inequality to obtain that

$$\begin{aligned}n(R_1+R_2)&\leq I(W_1;\bar{\mathbf{Y}}_1^{[n]}|\mathbf{H}^{[n]},W_2)+I(W_2;\bar{\mathbf{Y}}_2^{[n]}\mid\mathbf{H}^{[n]})\\ &=H(\bar{\mathbf{Y}}_2^{[n]}\mid\mathbf{H}^{[n]})-H(\bar{\mathbf{Y}}_2^{[n]}\mid\mathbf{H}^{[n]},W_2)\\ &\quad+H(\bar{\mathbf{Y}}_1^{[n]}\mid\mathbf{H}^{[n]},W_2)+o(n).\end{aligned}\tag{10}$$

The entropy of a random variable is bounded by its support, i.e., $H(\bar{\mathbf{Y}}_2^{[n]})\leq N_2\, n\log\bar{P}$. This fact and Lemma 2 yield

$$\begin{aligned}&n(N_2-k)R_1+n(M-k)R_2\\ &\leq(M-k)\big(H(\bar{\mathbf{Y}}_2^{[n]}|\mathbf{H}^{[n]})-H(\bar{\mathbf{Y}}_2^{[n]}|\mathbf{H}^{[n]},W_2)\big)\\ &\quad+(N_2-k)H(\bar{\mathbf{Y}}_1^{[n]}|\mathbf{H}^{[n]},W_2)+o(n)\\ &\leq n\,(M-k)N_2\log\bar{P}+n\,o(\log\bar{P})+o(n).\end{aligned}\tag{11}$$

We can divide (11) by $(M-k)(N_2-k)$ to write

$$\frac{nR_1}{M-k}+\frac{nR_2}{N_2-k}\leq\frac{nN_2\log\bar{P}}{N_2-k}+n\,o(\log\bar{P})+o(n).\tag{12}$$

From the definition of DoF, it follows that

$$\frac{d_1}{M-k}+\frac{d_2}{N_2-k}\leq\frac{N_2}{N_2-k}\ \Rightarrow\ \frac{d_1}{M-k}+\frac{d_2-k}{N_2-k}\leq 1,$$

what concludes the proof of (2d) for $N_2 < M \leq N_1+N_2$. □

A. Proof of Lemma 2

Next, we prove Lemma 2. We omit some intermediate steps for space constraints, while a more detailed step-by-step proof can be found in [14]. We first recall a key definition from [12].

Definition 2 ([12, Def. 4])**.** *For real numbers $x_1,x_2,\ldots,x_K$, define the notations $L_j^b(x_i,\ i\in[K])$, and $L_j(x_i, i\in[K])$, as*

$$L_j^b(x_1,x_2,\ldots,x_k)\triangleq\sum\nolimits_{i\in[K]}\lfloor g_{j,i}x_i\rfloor\tag{13}$$

$$L_j(x_1,x_2,\ldots,x_k)\triangleq\sum\nolimits_{i\in[K]}\lfloor h_{j,i}x_i\rfloor\tag{14}$$

for distinct random variables $g_{j,i}\in\mathcal{G}$ with bounded density, and for real valued and finite constants $h_{j,i}\in\mathcal{H}$, $|h_{j,i}|\leq\delta_z<\infty$. Subscript j is used to distinguish among multiple sums.

We recall that $\bar{\mathbf{Y}}_i^{[n]}\triangleq[\bar{Y}_{i,1}^{[n]},\ldots,\bar{Y}_{i,N_i}^{[n]}]$. Moreover, it follows from Definition 2 that $\bar{Y}_{i,j}(t)=L_{i,j}(t)(\bar{X}_1(t),\ldots,\bar{X}_M(t))$. Note that the signals $\bar{X}_1^{[n]},\ldots,\bar{X}_k^{[n]}$ may be a function of the messages *and the channel*, but $\{\bar{X}_{k+1}^{[n]},\ldots,\bar{X}_M^{[n]}\}\triangleq\bar{\mathbf{X}}_{\varnothing}^{[n]}$ are independent of the channel. We can apply a rotation matrix at RX 2 such that the k first TXs ($\mathbf{TX}_\star$) are only heard by the first k antennas of RX 2. Hence, for any $k<j\leq N_2$, we have that

$$\bar{Y}_{2,j}^{[n]}=L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})\tag{15}$$

since $\mathbf{TX}_\varnothing$ has only finite precision CSI. We omit hereinafter that $j\leq N_2$ for ease of readability. From (15), it follows that

$$H\big(\bigcap_{j>k}\bar{Y}_{2,j}^{[n]}|\mathbf{H}^{[n]},W_2\big)=H\big(\bigcap_{j>k}L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})|\mathbf{H}^{[n]},W_2\big).\tag{16}$$

From the fact that $H(A,B)\geq H(A)$, it holds that

$$\begin{aligned}&(N_2-k)H(\bar{\mathbf{Y}}_1^{[n]}|\mathbf{H}^{[n]},W_2)-(M-k)H(\bar{\mathbf{Y}}_2^{[n]}|\mathbf{H}^{[n]},W_2)\\ &\leq(N_2-k)\big(H(\bar{\mathbf{Y}}_1^{[n]}|\mathbf{H}^{[n]},W_2)-H(\bar{\mathbf{Y}}_2^{[n]}|\mathbf{H}^{[n]},W_2)\big)\\ &\quad-(M-N_2)H\big(\bigcap_{j>k}\bar{Y}_{2,j}^{[n]}|\mathbf{H}^{[n]},W_2\big).\end{aligned}\tag{17}$$

Let us first describe the intuition behind the proof before deriving the result. In (17), there are N_2-k negative entropy terms, each one of N_2 variables, and another $M-N_2$ negative entropy terms, each one of N_2-k variables. All the variables are linear combinations of the M transmit signals ($\bar{X}_i$). Our goal is to show that all those negative terms can be reordered so as to create N_2-k terms of M independent linear combinations. If this statement is true, from the fact that $H(A)-H(B)\leq H(A|B)$, we can remove the contribution of the N_2-k positive terms $H(\bar{\mathbf{Y}}_1^{[n]}|\mathbf{H}^{[n]},W_2)$, since we can decode the M signals with high probability from M independent linear combinations. In the following we show rigorously that the previous idea is indeed applicable. First, let us note that

$$\bar{\mathbf{Y}}_2^{[n]}\triangleq\big\{\bigcap\nolimits_{m\leq k}\bar{Y}_{2,m}^{[n]},\ \bigcap\nolimits_{j>k}L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})\big\},\tag{18}$$

and let us present a useful lemma that follows directly from [1].

Lemma 3. *Consider $\beta>0$ and random variables $F_j^{[n]}$, $G_j^{[n]}$, $j\in[J]$ that satisfy the bounded density assumption. Let $\bar{X}_j^{[n]}$ be independent of $F_j^{[n]},G_j^{[n]}$, $\forall j\in[J]$. Then, it holds that*

$$H\big(\sum_{j=1}^{J}\lceil\bar{P}^{\beta}F_j^{[n]}\bar{X}_j^{[n]}\rceil\big)\leq H\big(\sum_{j=1}^{J}\lceil\bar{P}^{\beta}G_j^{[n]}\bar{X}_j^{[n]}\rceil\big)+o(\log\bar{P}).$$

Hereinafter, we omit the $o(\log\bar{P})$ terms for ease of notation and because they are irrelevant for the DoF metric. Lemma 3 and the fact that $H(L(X_i))\leq H(L^b(X_i))$ [12], [15] yield

$$\begin{aligned}&H\big(\bigcap\nolimits_{j>k}L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})\mid\mathbf{H}^{[n]},W_2\big)\\ &\geq H\big(L^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}),\ \bigcap\nolimits_{j>k+1}L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})\mid\mathbf{H}^{[n]},W_2\big).\end{aligned}\tag{19}$$

Until now, we have presented the preliminary steps. Next, we bound $H\big(\bar{\mathbf{Y}}_2^{[n]}|\mathbf{H}^{[n]},W_2\big)+H\big(\bigcap_{j>k}\bar{Y}_{2,j}^{[n]}|\mathbf{H}^{[n]},W_2\big)$ as a key intermediate step. To conclude, we will show how we can repeat this step so as to obtain Lemma 2. It holds that

$$\begin{aligned}&H\big(\bar{\mathbf{Y}}_2^{[n]}\mid\mathbf{H}^{[n]},W_2\big)+H\big(\bigcap\nolimits_{j>k}\bar{Y}_{2,j}^{[n]}\mid\mathbf{H}^{[n]},W_2\big)\\ &\overset{(a)}{\geq}H\big(\bigcap\nolimits_{m\leq k}\bar{Y}_{2,m}^{[n]},\bigcap\nolimits_{j>k}L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})\mid\mathbf{H}^{[n]},W_2\big)\\ &\quad+H\big(\bigcap\nolimits_{j>k+1}L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}),L^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})\mid\mathbf{H}^{[n]},W_2\big)\end{aligned}\tag{20}$$

$$\overset{(b)}{\geq} H\big(\bar{\mathbf{Y}}_2^{[n]}, L^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big) + H\big(\bigcap_{j>k+1} L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big), \quad (21)$$

where (a) comes from (15), (18), and (19), and (b) comes from (18) and the sub-modularity property, which states that $H(A,B)+H(B,C) \geq H(A,B,C)+H(B)$ [17, Th. 1].

Recovering (17), we focus on its negative terms. Let us introduce $\mathbf{L}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \triangleq \{L_1^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}), \cdots, L_{M-N_2}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]})\}$, which is composed of $M-N_2$ independent linear combinations of $\bar{\mathbf{X}}_{\varnothing}^{[n]}$. Therefore, handily repeating (21) yields

$$\begin{aligned}
&(N_2-k)H\big(\bar{\mathbf{Y}}_2^{[n]} \mid \mathbf{H}^{[n]}, W_2\big) \\
&\quad + (M-N_2)H\big(\bigcap_{j>k} L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big) \\
&\overset{(a)}{\geq} (N_2-k-1)H\big(\bar{\mathbf{Y}}_2^{[n]} \mid \mathbf{H}^{[n]}, W_2\big) \\
&\quad + H\big(\bar{\mathbf{Y}}_2^{[n]},\ \mathbf{L}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big) \\
&\quad + (M-N_2)H\big(\bigcap_{j>k+1} L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big) \\
&\overset{(b)}{\geq} (N_2-k)H\big(\bar{\mathbf{Y}}_2^{[n]},\ \mathbf{L}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big)
\end{aligned} \quad (22)$$

where (a) comes from repeating (21) for each of the $H\big(\bigcap_{j>k} L_{\bar{Y},j}^{b[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big)$ terms that appear in (17) (which sum up $M-N_2$ terms), and (b) follows after repeating (a) up to N_2-k times for $j = k+1, k+2, \ldots, N_2$. Note that the entropy terms $H\big(\bar{\mathbf{Y}}_2^{[n]}, \mathbf{L}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big)$ are composed of M independent linear combinations of the transmitted signals $\{\bar{X}_i^{[n]}\}_{i\in[M]}$, such that it follows that

$$\begin{aligned}
&H\big(\bar{\mathbf{Y}}_1^{[n]} \mid \mathbf{H}^{[n]}, W_2\big) - H\big(\bar{\mathbf{Y}}_2^{[n]}, \mathbf{L}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}) \mid \mathbf{H}^{[n]}, W_2\big) \\
&\quad \leq H\big(\bar{\mathbf{Y}}_1^{[n]} \mid \bar{\mathbf{Y}}_2^{[n]}, \mathbf{L}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}), \mathbf{H}^{[n]}, W_2\big) \\
&\quad \leq o(n).
\end{aligned} \quad (23)$$

From (22) and (23), it holds that

$$\begin{aligned}
&(N_2-k)H(\bar{\mathbf{Y}}_1^{[n]}|\mathbf{H}^{[n]}, W_2) - (M-k)H(\bar{\mathbf{Y}}_2^{[n]}|\mathbf{H}^{[n]}, W_2) \\
&\quad \leq (N_2-k)H(\bar{\mathbf{Y}}_1^{[n]} \mid \bar{\mathbf{Y}}_2^{[n]}, \mathbf{L}^{[n]}(\bar{\mathbf{X}}_{\varnothing}^{[n]}), \mathbf{H}^{[n]}, W_2) \\
&\quad \leq o(n),
\end{aligned} \quad (24)$$

what concludes the proof of Lemma 2. □

VI. Achievability Results

The transmission scheme exploits the unavoidable interference as side information, in a similar way as in [10] for the centralized 'PD' setting. At the same time, the proposed scheme also exploits the instantaneous CSI available at $\mathbf{TX}_\star$ by means of the AP-ZF precoding scheme that was introduced in [9]. The key of the use of AP-ZF is the following lemma (cf. [9]).

Lemma 4 ([9]). *Consider k TXs with perfect CSI and $M-k$ TXs with finite precision CSI. By precoding with AP-ZF the interference can be canceled at k different receive antennas.*

We refer to [9] for more details about AP-ZF. We present in the following the DoF-optimal transmission scheme for $N_1 \leq k < N_2$, i.e., the proof of Theorem 2. The achievable scheme for the case $M \leq N_2$ ($\mathrm{DoF} = M$) is trivial and thus we omit it for sake of conciseness. Given that the DoF does not increase for M bigger than $M = N_1 + N_2$, we consider that $N_2 < M \leq N_1 + N_2$.

We transmit a set $\mathcal{S}_i$ of $S_i \triangleq |\mathcal{S}_i|$ symbols to RX i, $i \in \{1,2\}$. In particular, we send a total of $S_1 = (M-k)N_1$ symbols to RX 1 and $S_2 = N_2(M-k-N_1) + kN_1$ symbols to RX 2 in a transmission spanning $M-k$ Time Slots (TS). The scheme is composed of two phases, the first one lasting N_1 TS and the second one lasting $M-k-N_1$ TS. Specifically, at each one of the N_1 TS of the first phase, we transmit:

- $M-N_1$ independent linear combinations (i.l.c.) of the symbols in $\mathcal{S}_2$ that are canceled at RX 1 through AP-ZF precoding (which is possible because $k \geq N_1$).
- N_1 i.l.c. of the symbols in $\mathcal{S}_1$, which are canceled at k antennas of RX 2 using AP-ZF precoding (see Lemma 4).

Then, at the end of the first phase, RX 1 has N_1^2 i.l.c. of its S_1 symbols. Thus, RX 1 needs another $(M-k-N_1)N_1$ i.l.c. to decode all the symbols in $\mathcal{S}_1$. On the other hand, RX 2 has N_2N_1 i.l.c. of S_2 desired symbols and $(N_2-k)N_1$ interference variables, since the symbols for RX 1 can be canceled only at k of the N_2 antennas. Let us denote the set of interference terms received at RX 2 during the first phase as $\mathcal{I}_2$, $|\mathcal{I}_2| = (N_2-k)N_1$. At $\mathbf{TX}_\star$, we can reconstruct the set $\mathcal{I}_2$ thanks to the perfect CSI available. Hence, $\mathbf{TX}_\star$ can create $(M-k-N_1)N_1$ i.l.c. of $|\mathcal{I}_2|$ interference terms, which are functions of the symbols of RX 1, because $M-N_1 \leq N_2$.

In the second phase, which lasts $M-k-N_1$ TS, we send at each TS:

- N_1 of the $(M-k-N_1)N_1$ i.l.c. of $\mathcal{I}_2$ from $\mathbf{TX}_\star$.
- N_2-N_1 i.l.c. of the symbols in $\mathcal{S}_2$, which are canceled at RX 1 through AP-ZF precoding.

Consequently, at the end of phase 2, RX 1 has obtained $N_1^2 + (M-k-N_1)N_1 = S_1$ i.l.c. of its S_1 symbols. Hence, RX 1 can decode all its symbols. Further, RX 2 has $N_2N_1 + N_2(M-k-N_1) = N_2(M-k)$ i.l.c. of S_2 desired symbols and $(N_2-k)N_1$ interference variables, what amounts to $N_2(M-k)$ variables. Thus, RX 2 can decode its intended symbols.

Hence, at the end of the communication we have successfully delivered a total of $S_1 + S_2 = (M-N_2)N_1 + N_2(M-k)$ symbols over $M-k$ TS, what leads to a sum DoF of

$$d_\Sigma = N_2 + N_1 \frac{M-N_2}{M-k}, \quad (25)$$

what concludes the proof of Theorem 2. □

VII. Conclusion

We have analyzed the 2-user MIMO BC setting in which only k transmit antennas have access to perfect CSI, whereas the other $M-k$ transmit antennas have access only to finite precision CSI. We have derived an outer bound for the DoF region that is tight for $k \geq \min(N_1, N_2)$, characterizing the loss of DoF obtained from reducing the number of informed antennas. On this basis, we have shown that it is not necessary to have perfect CSI at every transmit antenna, but only at $\max(N_1, N_2)$ antennas. We have also presented an achievable scheme that adapts to the distributed CSI setting so as to boost the DoF with respect to the use of conventional centralized schemes.

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