Joint Angle and Delay Estimation (JADE) by Partial Relaxation

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Contents

1. Introduction
2. System Model
3. JADE By Partial Relaxation
4. Computer Simulations
5. Conclusions
6. References

Contents

1. Introduction
2. System Model
3. JADE By Partial Relaxation
4. Computer Simulations
5. Conclusions
6. References

Localization by JADE

- Node positioning in a wireless system requires the gathering of location information from radio signals traveling between the target node and one, or multiple, reference anchors.
- The JADE (Joint Angle and Delay Estimation) approach measures delays/angles between an intended node and anchors to estimate the location of the former, with the help of the position of the latter.
- High accurarcy (in terms of MSE) is needed to reliably estimate location parameters for sub-meter accuracy.

Contents

1. Introduction
2. System Model
3. JADE By Partial Relaxation
4. Computer Simulations
5. Conclusions
6. References

System model

Model

$$\mathbf{x}(\ell) = \mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau})\boldsymbol{\gamma}(\ell) + \mathbf{n}(\ell) \tag{1}$$

$$\mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau}) = \begin{bmatrix} h(\theta_1, \tau_1) & \dots & h(\theta_q, \tau_q) \end{bmatrix} \tag{2}$$

$$\boldsymbol{\gamma}(\ell) = \begin{bmatrix} \gamma_1(\ell) & \dots & \gamma_q(\ell) \end{bmatrix} \tag{3}$$

$$\mathbf{x}(\ell) = \begin{bmatrix} \mathbf{x}_1^\top(\ell) & \dots & \mathbf{x}_N^\top(\ell) \end{bmatrix}^\top \tag{4}$$

$$\mathbf{x}_n(\ell) = \begin{bmatrix} x_{n,1}(\ell) & \dots & x_{n,M}(\ell) \end{bmatrix}^\top \tag{5}$$

- $\mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau})$ contains the multipath spatiotemporal signatures.
- $\boldsymbol{\gamma}(\ell)$ denotes multipath complex gains in ℓ^{th} frame.
- $x_{n,m}(\ell)$ denotes data on m^{th} sub-carrier received by n^{th} antenna in ℓ^{th} frame.

Data Collection

$$\mathbf{X} = \mathbf{H}(\boldsymbol{\theta}, \boldsymbol{\tau})\mathbf{G} + \mathbf{N} \tag{6}$$

Contents

1. Introduction
2. System Model
3. JADE By Partial Relaxation
4. Computer Simulations
5. Conclusions
6. References

JADE by Partial Relaxation (PR)

Objective

Solve

$$(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\tau}}) = \arg\min_{\boldsymbol{\theta},\boldsymbol{\tau}} \operatorname{tr}\left\{\boldsymbol{\mathcal{P}}^{\perp}_{\mathbf{H}(\boldsymbol{\theta},\boldsymbol{\tau})}\hat{\mathbf{R}}\right\} \tag{7}$$

PR Approach

Reformulate the problem as

$$(\hat{\boldsymbol{\theta}}_{PR}, \hat{\boldsymbol{\tau}}_{PR}) = \arg\min_{\theta,\tau,\mathbf{B}} \operatorname{tr}\left\{\boldsymbol{\mathcal{P}}^{\perp}_{[\mathbf{h}(\theta,\tau)\ \mathbf{B}]}\hat{\mathbf{R}}\right\} \tag{8}$$

Algo 1: DML

Unstructured Signatures Optimization Problem

Projection decomposition:

$$\mathcal{P}_{\mathbf{H}}^{\perp} = \mathbf{I} - \mathcal{P}_{[\mathbf{h}\,\mathbf{B}]} = \mathbf{I} - (\mathcal{P}_{\mathbf{h}} + \mathcal{P}_{\mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{B}}) = \mathcal{P}_{\mathbf{h}}^{\perp} - \mathcal{P}_{\mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{B}}$$

Focus on optimization of unstructured JADE signatures $\mathbf{B}$:

$$\mathcal{P}_{\mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{B}} = \mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{B}(\mathbf{B}^H\mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{B})^{-1}\mathbf{B}^H\mathcal{P}_{\mathbf{h}}^{\perp} = \mathbf{U}\,\mathbf{U}^H,\ \mathbf{U}^H\mathbf{U} = \mathbf{I},\ \mathbf{U}^H\mathbf{h} = 0$$

Now

$$\begin{aligned}&\mathrm{tr}\{\mathcal{P}_{\mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{B}}\hat{\mathbf{R}}\} = \mathrm{tr}\{\mathbf{U}\mathbf{U}^H\hat{\mathbf{R}}\} = \mathrm{tr}\{\mathbf{U}^H\hat{\mathbf{R}}\mathbf{U}\}\\ &= \mathrm{tr}\{\mathbf{U}^H(\mathcal{P}_{\mathbf{h}} + \mathcal{P}_{\mathbf{h}}^{\perp})\hat{\mathbf{R}}(\mathcal{P}_{\mathbf{h}} + \mathcal{P}_{\mathbf{h}}^{\perp})\mathbf{U}\} = \mathrm{tr}\{\mathbf{U}^H\mathcal{P}_{\mathbf{h}}^{\perp}\hat{\mathbf{R}}\mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{U}\}\end{aligned}$$

Then

$$\max_{\mathbf{U}^H\mathbf{U}=\mathbf{I}} \mathrm{tr}\{\mathbf{U}^H\mathcal{P}_{\mathbf{h}}^{\perp}\hat{\mathbf{R}}\mathcal{P}_{\mathbf{h}}^{\perp}\mathbf{U}\} = \sum_{k=1}^{q-1}\lambda_k\left(\mathcal{P}_{\mathbf{h}}^{\perp}\hat{\mathbf{R}}\mathcal{P}_{\mathbf{h}}^{\perp}\right)$$

Algo 1: DML

DML Optimization Problem

Combining, we get

$$\min_{\mathbf{B}} \mathrm{tr}\left\{\boldsymbol{\mathcal{P}}^{\perp}_{[\mathbf{h}(\theta,\tau)\ \mathbf{B}]}\hat{\mathbf{R}}\right\} = \mathrm{tr}\left\{\boldsymbol{\mathcal{P}}^{\perp}_{\mathbf{h}}\hat{\mathbf{R}}\right\} - \sum_{k=1}^{q-1}\lambda_k\left(\boldsymbol{\mathcal{P}}^{\perp}_{\mathbf{h}}\hat{\mathbf{R}}\right)$$

Solution is to search the peaks of the following 2D spectrum

$$f_{\mathrm{DML}}(\theta,\tau) = \frac{1}{\sum_{k=q}^{NM}\lambda_k\left(\boldsymbol{\mathcal{P}}^{\perp}_{\mathbf{h}(\theta,\tau)}\hat{\mathbf{R}}\right)} \tag{9}$$

Algo 2: Weighted Subspace Fitting (WSF)

WSF Optimization Problem

Weigh the covariance signal subspace by an appropriate matrix $\mathbf{W}$,

$$\underset{\mathbf{U},\theta,\tau}{\text{maximize}} \quad \text{tr}\left\{\mathbf{U}^H \boldsymbol{\mathcal{P}}^{\perp}_{\mathbf{h}(\theta,\tau)} \hat{\mathbf{U}}_s \mathbf{W} \hat{\mathbf{U}}_s^{\mathsf{H}} \mathbf{U}\right\} \tag{10a}$$

$$\text{subject to} \quad \mathbf{U}^H \mathbf{U} = \mathbf{I}, \tag{10b}$$

Solution is to search the peaks of the following 2D spectrum

$$f_{\text{WSF}}(\theta,\tau) = \frac{1}{\sum\limits_{k=q}^{NM} \lambda_k \left(\boldsymbol{\mathcal{P}}^{\perp}_{\mathbf{h}(\theta,\tau)} \hat{\mathbf{U}}_s \mathbf{W} \hat{\mathbf{U}}_s^{\mathsf{H}} \right)} \tag{11}$$

Algo 3: Covariance Fitting (CF)

CF Optimization Problem

Fit the covariance of the data according to the model

$$\mathbf{R} = \sigma_k^2 \mathbf{h}(\theta_k, \tau_k)\mathbf{h}^{\mathsf{H}}(\theta_k, \tau_k) + \mathbf{J}\mathbf{J}^{\mathsf{H}} \tag{12}$$

$$\begin{aligned} \underset{\theta_k, \tau_k, \sigma_k^2, \mathbf{J}}{\text{minimize}} \quad & \left\| \hat{\mathbf{R}} - \sigma_k^2 \mathbf{h}(\theta_k, \tau_k)\mathbf{h}^{\mathsf{H}}(\theta_k, \tau_k) - \mathbf{J}\mathbf{J}^{\mathsf{H}} \right\|^2 \\ \text{subject to} \quad & \hat{\mathbf{R}} - \sigma_k^2 \mathbf{h}(\theta, \tau)\mathbf{h}^{\mathsf{H}}(\theta, \tau) \succcurlyeq \mathbf{0} \end{aligned} \tag{13}$$

Solution is to search the peaks of the following 2D spectrum

$$f_{\text{CF}}(\theta, \tau) = \frac{1}{\sum_{k=q}^{NM} \lambda_k^2 \left(\hat{\mathbf{R}} - \frac{\mathbf{h}(\theta,\tau)\mathbf{h}^{\mathsf{H}}(\theta,\tau)}{\mathbf{h}^{\mathsf{H}}(\theta,\tau)\hat{\mathbf{R}}^{-1}\mathbf{h}(\theta,\tau)} \right)} \tag{14}$$

Contents

1. Introduction
2. System Model
3. JADE By Partial Relaxation
4. Computer Simulations
5. Conclusions
6. References

Simulation Parameters

- $q = 2$ sources (multipath components)
- $\theta_1 = 6^\circ$ and $\theta_2 = 66^\circ$
- $\tau_1 = 5$ nsec and $\tau_2 = 10$
- number of subcarriers $M = 32$, number of antennas $N = 2$
- Source covariance

$$\mathbf{P} = \begin{bmatrix} 1 & 0.2e^{-j\frac{\pi}{6}} \\ 0.2e^{j\frac{\pi}{6}} & 1 \end{bmatrix}$$

Computer Simulations

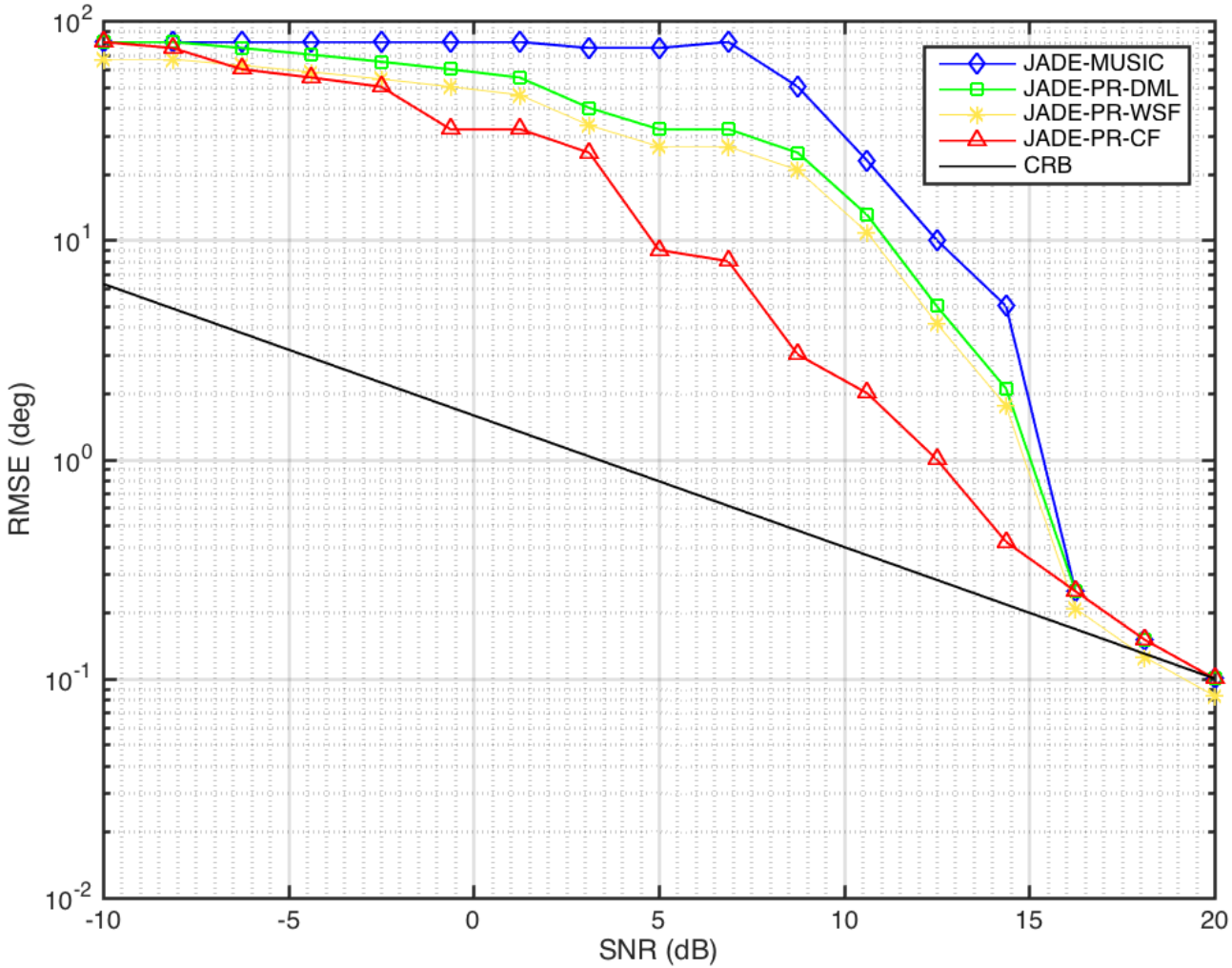


Figure: The RMSE of AoA estimates $\hat{\theta}_k$ per method vs SNR and the CRB.

Computer Simulations

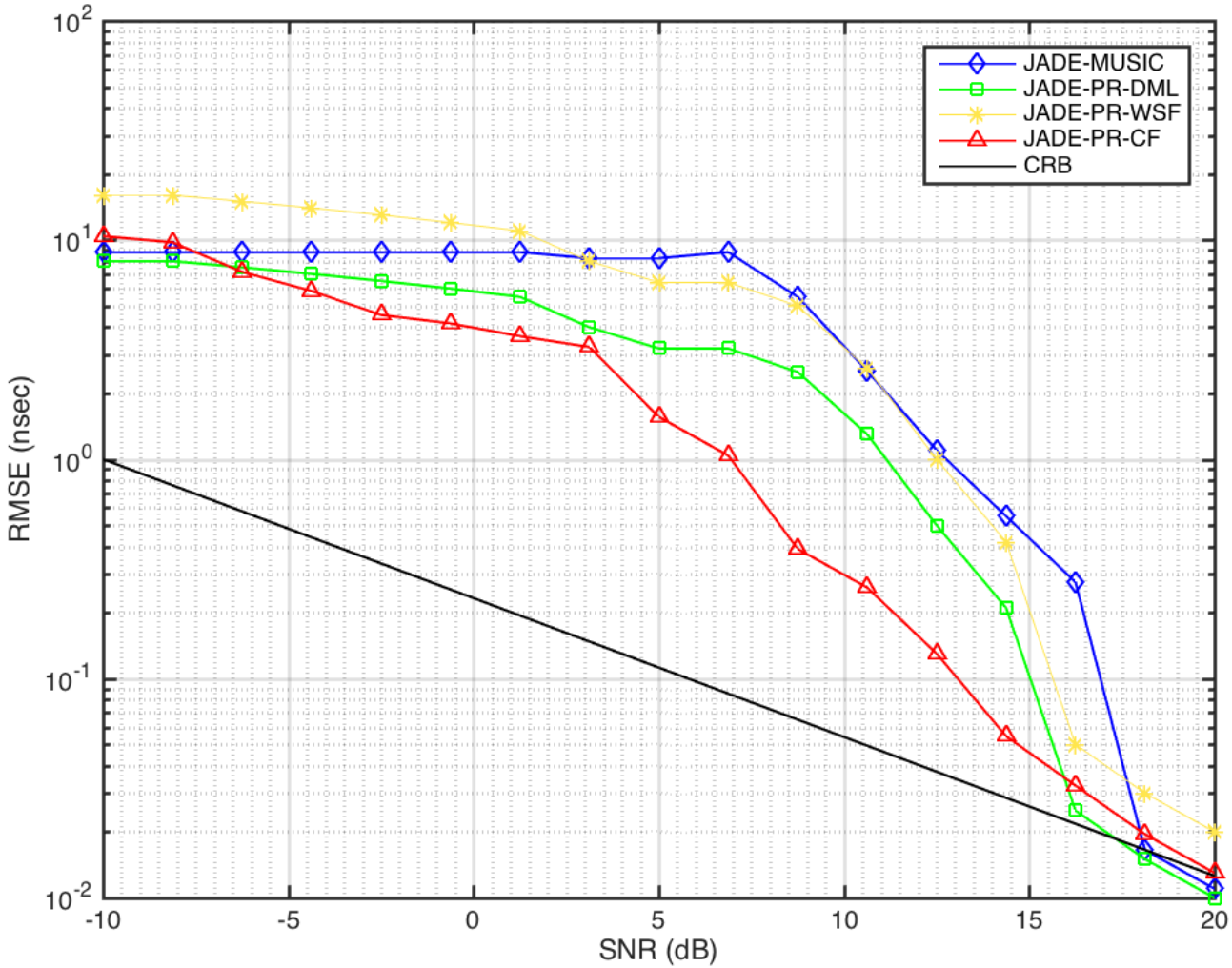


Figure: The RMSE of ToA estimates $\hat{\tau}_k$ per method vs SNR and the CRB.

Computer Simulations

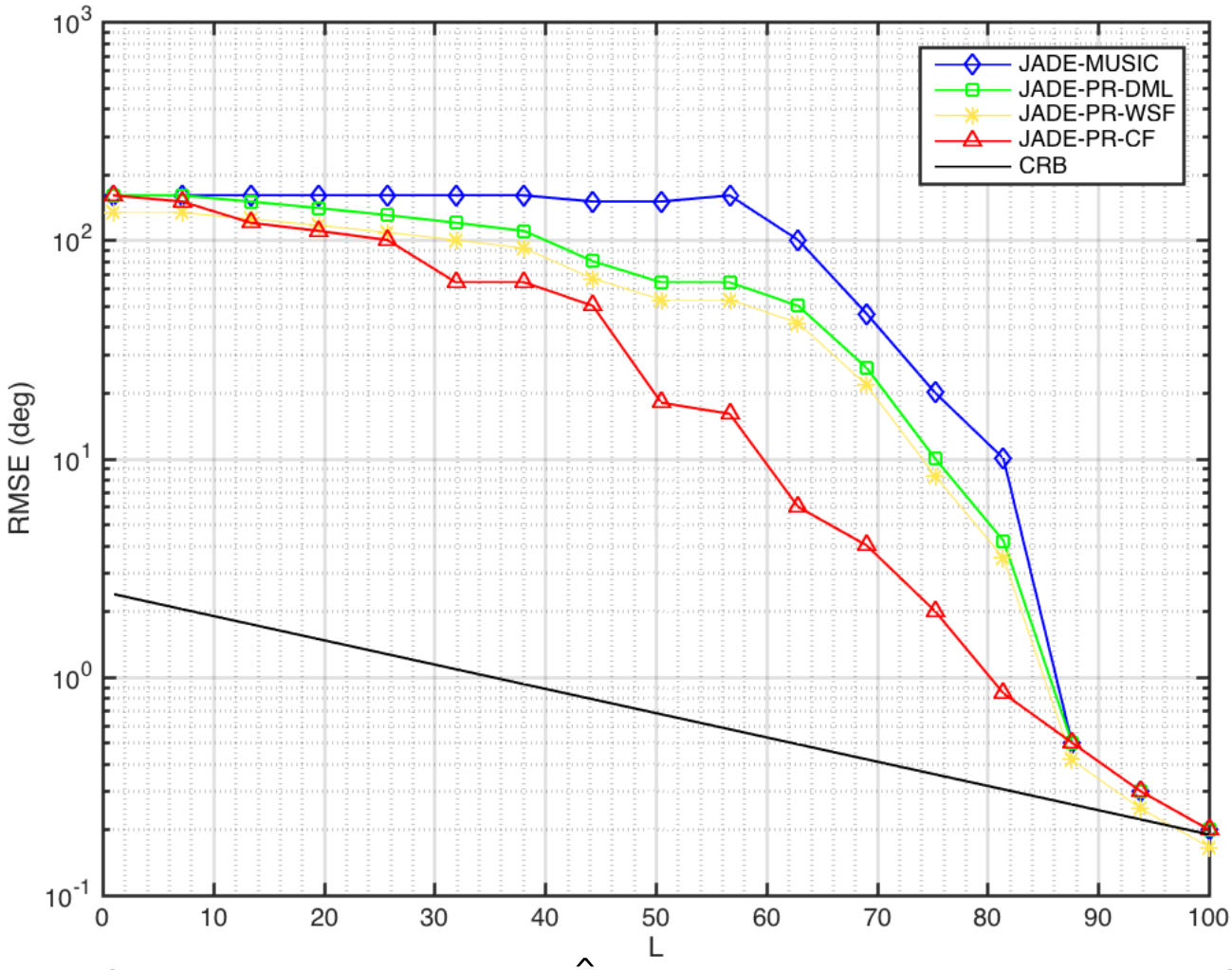


Figure: The RMSE of AoA estimates $\hat{\theta}_k$ per method vs Number of Snapshots and the CRB.

Computer Simulations

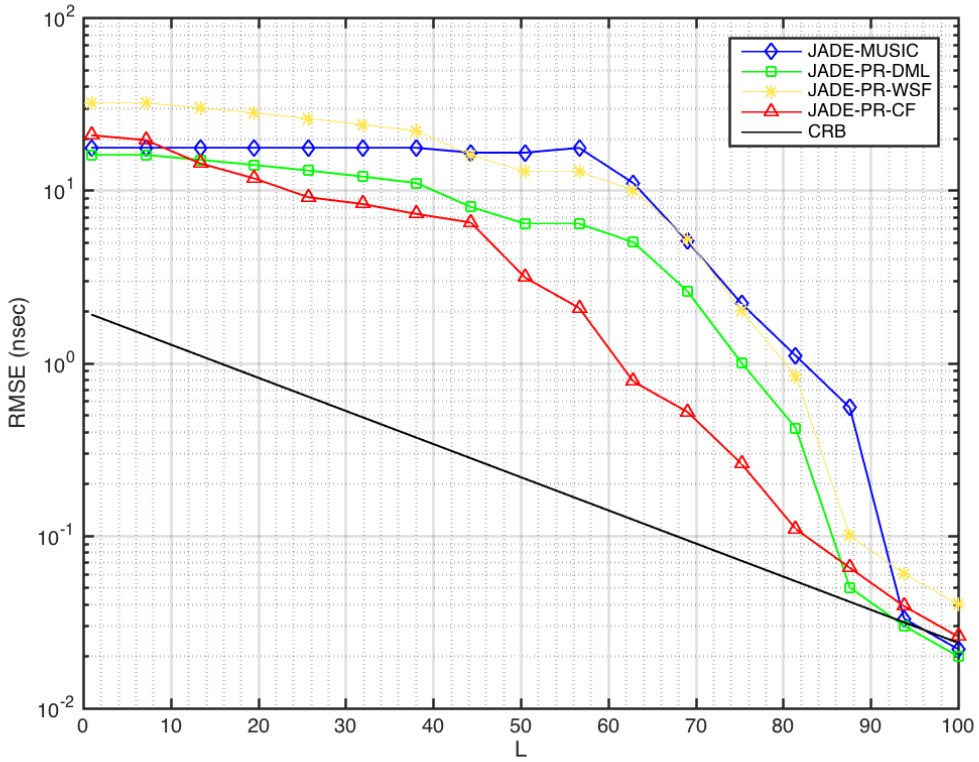


Figure: The RMSE of ToA estimates $\hat{\tau}_k$ per method vs Number of Snapshots and the CRB.

Contents

1. Introduction
2. System Model
3. JADE By Partial Relaxation
4. Computer Simulations
5. Conclusions
6. References

Conclusions

- We have introduced three JADE estimators to the partially relaxed model: DML, WSF, CF.
- Simulation results demonstrate the Mean-Squared-Error convergence towards the Cramér-Rao Bound of each of these methods, either in SNR or in number of snapshots.

Contents

1. Introduction
2. System Model
3. JADE By Partial Relaxation
4. Computer Simulations
5. Conclusions
6. References

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