FAVORABLE PROPAGATION AND LINEAR MULTIUSER DETECTION FOR DISTRIBUTED ANTENNA SYSTEMS

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ABSTRACT

Cell-free MIMO, employing distributed antenna systems (DAS), is a promising approach to deal with the capacity crunch of next generation wireless communications. In this paper, we consider a wireless network with transmit and receive antennas distributed according to homogeneous point processes. The received signals are jointly processed at a central processing unit. We study if the *favorable propagation* properties, which enable almost optimal low complexity detection via matched filtering in massive MIMO systems, hold for DAS with line of sight (LoS) channels and general attenuation exponent. Making use of Euclidean random matrices (ERM) and their moments, we show that the analytical conditions for favorable propagation are not satisfied. Hence, we propose multistage detectors, of which the matched filter represents the initial stage. We show that polynomial expansion detectors and multistage Wiener filters coincide in DAS and substantially outperform matched filtering. Simulation results are presented which validate the analytical results.

Index Terms— Distributed antenna systems, large system analysis, favorable propagation, linear multiuser detection, cell-free

1. INTRODUCTION

In recent years, distributed antenna systems (DASs) have emerged as a promising candidate for future wireless communications thanks to their open architecture and flexible resource management [1, 2]. A DAS involves the use of a large number of antennas, allowing the accommodation of more users, higher data rates, and effective mitigation of fading. Extensive studies indicate that besides lower path loss effects to improve the coverage, a DAS has many attractive advantages over its centralized counterpart such as macro-diversity gain and higher power efficiency [3, 4]. Users' energy consumption is reduced and transmission quality is improved by reducing the access distance between users and geographically distributed access points (APs). DASs have been extensively studied in downlink, see, e.g., [5, 6] and references therein. In uplink, results on the sum capacity of DAS can be found in [7–9]. In [8, 9], a mathematical framework based on Euclidean random matrices was proposed to analyze the fundamental limits of DASs in terms of capacity per unit area in the large scale regime.

The concept of DASs has recently reappeared under the name cell-free (CF) massive MIMO [10, 11]. The new terminology is used for networks consisting of a massive number of geographically distributed single-antenna APs, which jointly serve a much smaller number of users distributed over a wide area. CF massive MIMO should combine the mentioned benefits of DAS with the advantages of massive MIMO. In principle, an optimal utilization of a DAS requires joint multiuser detection at a central unit. However, optimum detection schemes such as maximum likelihood are prohibitively complex to be implemented for a large system and low complexity linear multiuser detectors become appealing. Interestingly, in massive MIMO systems, as the number of antennas at the centralized base station increases the channels of different users with the base station tend to become pairwise orthogonal and the low complexity matched filters become asymptotically optimum detectors [12]. This appealing phenomenon is referred to as *favorable propagation* [13]. Under the assumption that a similar property holds in DAS, CF massive MIMO systems have been studied in [14] adopting matched filters at the central processing unit.

In this paper, our system model includes as special case CF massive MIMO systems when the intensity of receivers is much higher than the intensity of transmitters. We investigate the properties of channels in DAS through an analysis of the MIMO channel eigenvalue moments and analytically show that favorable propagation is limited also asymptotically as the AP's intensity tends to infinity while the users' intensity is kept constant. In this case, matched filtering is not almost optimum and the use of linear multiuser detectors capable to combat multiuser interference at an affordable computational cost becomes really appealing. Thus, we analyze the performance of multistage detectors that can be implemented with low complexity at the expense of a certain performance degradation. We consider both polynomial expansion detectors [15] and [16] and show their equivalence in DAS. Additionally, their performance analysis confirms that even low complexity multiuser detectors outperform considerably matched filtering.

The rest of paper is organized as follows. Section 2 describes the system and channel model. A recursive expression to obtain the eigenvalue moments of the channel covariance matrix of DASs is presented in Section 3. In Section 4, we analyze the conditions of favorable propagation and the performance of multistage detectors for DASs. Simulation results are illustrated in Section 5. Finally, Section 6 draws some conclusions.

Notation: Throughout the paper, $\mathbf{i} = \sqrt{-1}$, superscript ^{T} and ^{H} represent the transpose and Hermitian transpose operator, respectively. Uppercase and lowercase bold symbols are utilized to denote matrices and vectors, respectively. The expectation and the Euclidean norm operators are denoted by $\mathbb{E}(.)$ and $|\,.\,|$, respectively. $\mathrm{tr}(.)$ and $\mathrm{diag}(.)$ denote the trace and the squared diagonal matrix consisting of the diagonal elements of matrix argument, respectively.

2. SYSTEM MODEL

We consider a DAS in uplink consisting of N_T users and N_R APs in the Euclidean space $\mathbb{R}$. Each user and AP are equipped with a single antenna and are independently and uniformly distributed over $\mathcal{A}_L = \left[-\frac{L}{2}, +\frac{L}{2}\right)$, a segment of length L. All the APs are connected to and controlled by a central processing unit through a backhaul network such that detection and decoding are performed jointly.

We denote the channel coefficient between the j-th user and i-th AP by $h(r_i, t_j)$, where r_i and t_j denote the Euclidean coordinates of AP i and user j, respectively. Furthermore, we assume line of sight (LoS) and large scale fading such that the channel coefficient $h(r_i, t_j)$ is modeled as

$$h(r_i, t_j) = \begin{cases} \dfrac{d_0^\alpha}{|r_i - t_j|^\alpha} \mathrm{e}^{-\dfrac{\mathbf{i}2\pi|r_i - t_j|}{\lambda}} & \text{if } |r_i - t_j| > d_0 \\ 1 & \text{otherwise} \end{cases} \quad (1)$$

where d_0 is a reference distance, α is the path loss factor, and λ is the radio signal wavelength. Note the $|r_i - t_j|$ is the Euclidean distance between the j-th user and i-th AP denoted in the following as $d_{i,j}$ and $h(r_i, t_j)$ depends on r_i and t_j only via their distance. Then, when convenient, we denote $h(r, t)$ as $h(d)$. In (1), the phase rotation depends on the distance $d_{i,j}$ and is given by $\exp(-\mathbf{i}2\pi\lambda^{-1}d_{i,j})$. In (1), we ignore the shadowing effect and model the large scale fading as pure pathloss $d_{i,j}^{-\alpha}$. It is well-known that the function $d^{-\alpha}$ models properly a LoS channel for a large range of distances when the plane wave approximation holds, i.e., d is sufficiently large. At small distances, this decaying model introduces a clear artifact: for $d < d_0$, the transmit signal is amplified beyond the transmit signal level and the amplification presents a vertical asymptote for $d \rightarrow 0$. In order to remove this artifact while keeping the model simple [1], we assume negligible the signal attenuation in a close neighborhood of a transmitter and we fix the attenuation equal to 1.

The transmitting users do not have any knowledge of the channel and transmit with equal power P. The receivers are impaired by additive white Gaussian noise (AWGN) with variance σ^2. The received signal vector at the central processor and discrete time instant m is given by

$$\mathbf{y}(m) = \sqrt{P}\mathbf{H}\mathbf{x}(m) + \mathbf{n}(m), \quad (2)$$

where $\mathbf{x}(m) = (x_1(m), x_2(m), \dots x_{N_T}(m))^T$, $x_j(m)$ is the unitary energy symbol transmitted by user j, $\mathbf{H}$ is an $N_R \times$

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[1]It is worth noticing that in contrast to the approach in [8,9], the analysis proposed in this paper can be applied to any LoS model which admits a Fourier transform and it is not restricted to (1).

N_T channel matrix with element (i, j) equal to $h(r_i, t_j)$, and $\mathbf{n}(m)$ is the additive white Gaussian noise vector whose i-th component is the noise at AP i.

For the sake of analytical treatability, as in [9], we assume that users and APs are located on a grid in $\mathcal{A}_L$. Let $\tau > 0$ be an arbitrary small real such that $L = \theta\,\tau$ with θ positive, even integer. We denote by $\mathcal{A}_L^\#$ the set of points regularly spaced in $\mathcal{A}_L$ by τ, i.e., $\mathcal{A}_L^\# \equiv \left\{w \mid w = (-\theta + 2k)\tau/2, k = 0, 1 \dots \theta - 1\right\}$. We model the distributed users and APs as homogeneous point processes $\Phi_\mathcal{T}$ and $\Phi_\mathcal{R}$ in $\mathcal{A}_L^\#$ characterized by the parameters $\beta_T = \rho_T\tau$ and $\beta_R = \rho_R\tau$, where ρ_T and ρ_R are the intensities, i.e., the number per unit area, of transmitters and receivers, respectively. Observe that $N_T = \rho_T L = \beta_T\theta$ and $N_R = \rho_R L = \beta_R\theta$.

3. PRELIMINARY MATHEMATICAL TOOLS

In this section, we introduce mathematical tools for the analysis and design of DAS. Communication systems modeled by random channel matrices can be efficiently studied via their covariance eigenvalue spectrum [17, 18]. Then, in order to analyze DASs, we characterize the spectrum of the channel covariance matrix $\mathbf{C} = \mathbf{H}^H\mathbf{H}$ in terms of its eigenvalue moments

$$m_\mathbf{C}^{(n)} = \int \mu^n f_\mathbf{C}(\mu)\mathrm{d}\mu = \mathbb{E}\left\{\frac{1}{N_T}tr(\mathbf{C}^n)\right\} \quad (3)$$

where μ and $f_\mathbf{C}(\mu)$ denote the eigenvalue and eigenvalue distribution of the matrix $\mathbf{C}$, respectively. The expectation is with respect to the two homogeneous point processes $\Phi_\mathcal{T}$ and $\Phi_\mathcal{R}$.

Following the approach in [8, 9], we decompose $\mathbf{H}$ as follows

$$\mathbf{H} = \mathbf{\Psi}_R\mathbf{T}\mathbf{\Psi}_T^H \quad (4)$$

where $\mathbf{T}$ is a $\theta \times \theta$ matrix depending only on the function $h(d)$, $\mathbf{\Psi}_R$ and $\mathbf{\Psi}_T$ are an $N_R \times \theta$ and $N_T \times \theta$ random matrices depending only on random AP's and user's location, respectively. In order to define the matrices $\mathbf{\Psi}_T$, $\mathbf{\Psi}_R$, and $\mathbf{T}$, we consider the $\theta \times \theta$ channel matrix $\mathcal{H}$ of a system with θ transmit and receive antennas regularly spaced in $\mathcal{A}_L^\#$. It is easy to recognize that $\mathcal{H}$ is a band Toeplitz matrix and, asymptotically, for $\theta \rightarrow \infty$, it admits an eigenvalue decomposition based on a $\theta \times \theta$ Fourier matrix $\mathbf{F}$ [19]. Then, we consider the decomposition $\mathcal{H} = \mathbf{F}\mathbf{T}\mathbf{F}^H$, where the matrix $\mathbf{T}$ is a deterministic, asymptotically diagonal matrix depending on the function $h(d)$ via its discrete time Fourier transform. The random matrices $\mathbf{\Psi}_T$ and $\mathbf{\Psi}_R$ are obtained by extracting independently and uniformly at random N_T and N_R rows of $\mathbf{F}$. For the sake of conciseness, we omit here a detailed analytical definition of the three matrices since not required for further studies and refer the interested reader to [8, 9] for their detailed definition.

Further analysis requires $m_\mathbf{T}^{(n)}$, the n-order eigenvalue moment of $\mathbf{T}$ as $L, \theta \rightarrow +\infty$. Let us consider the sequence $\{h(k\tau)\}|_{k\in\mathbb{Z}}$ obtained by sampling the function $h(d)$ with period τ. Asymptotically, for $L, \theta \rightarrow +\infty$, the eigenvalues of the matrix $\mathbf{T}$ are given by $H(\omega)$, with $\omega \in [-\pi, +\pi]$, the discrete-time Fourier transform of the sequence $\{h(k\tau)\}|_{k\in\mathbb{Z}}$ [19] and $m_\mathbf{T}^{(n)} = \int_{-\pi}^{+\pi} H^n(\omega)\mathrm{d}\omega$.

In order to obtain the moments $m_{\mathbf{C}}^{(l)}$, we follow the approach in [9, 20] and approximate the random matrices $\mathbf{\Psi}_R$ and $\mathbf{\Psi}_T$ by the independent matrices $\mathbf{\Phi}_R$ and $\mathbf{\Phi}_T$, respectively, consisting of i.i.d zero mean Gaussian elements with variance θ^{-1}. This approximation enables the application of classical techniques from random matrix theory and free probability. In the following we introduce an algorithm for the recursive computation of $m_{\tilde{\mathbf{C}}}^{(n)}$, the n-order eigenvalue moment of the channel covariance matrix $\tilde{\mathbf{C}} = \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} = \mathbf{\Phi}_T \mathbf{T} \mathbf{\Phi}_R^H \mathbf{\Phi}_R \mathbf{T} \mathbf{\Phi}_T^H$ and $\tilde{C}_{kk}^{(n)}$, the diagonal elements of $\tilde{\mathbf{C}}^n$. The derivation and proof are based on techniques similar to the ones utilized in [21, 22] and it is omitted due to space constraints.

The algorithm holds asymptotically for $\theta, N_R, N_T \to +\infty$ with $N_T/\theta \to \beta_T$ and $N_R/\theta \to \beta_R$ and it is based on the relations between the matrix $\tilde{\mathbf{C}}^l$ and the matrices $\mathbf{T}, \mathbf{D} = \tilde{\mathbf{H}}\tilde{\mathbf{H}}^H$, $\mathbf{\Gamma}^{(l)} = \mathbf{T}\mathbf{\Phi}_R^H \mathbf{D}^{l-1}\mathbf{\Phi}_R \mathbf{T}$, and $\mathbf{\Delta}^{(l)} = \mathbf{T}\mathbf{\Phi}_T^H \tilde{\mathbf{C}}^{l-1}\mathbf{\Phi}_T \mathbf{T}$. It determines $m_{\tilde{\mathbf{C}}}^{(l)}$ and $\tilde{C}_{kk}^{(l)}$, the k-th diagonal element of the matrix $\tilde{\mathbf{C}}^l$, by a recurrent expression of the eigenvalue moments of the matrices $\mathbf{T}, \mathbf{D}, \mathbf{\Gamma}^{(l)}$, and $\mathbf{\Delta}^{(l)}$ and their diagonal elements. The eigenvalue moment of order l of the matrix $\mathbf{D}$ is denoted by $m_{\mathbf{D}}^{(l)}$. $m_{\mathbf{\Gamma}}^{(l)}$ and $m_{\mathbf{\Delta}}^{(l)}$ are the eigenvalue moments of the matrices $\mathbf{\Gamma}^{(l)}$ and $\mathbf{\Delta}^{(l)}$, respectively. Similarly, $D_{kk}^{(l)}$, $\Gamma_{kk}^{(l)}$, and $\Delta_{kk}^{(l)}$ are their respective diagonal elements.

Algorithm

Initial step: Set $m_{\tilde{\mathbf{C}}}^{(0)} = m_{\mathbf{D}}^{(0)} = \tilde{C}_{kk}^{(0)} = D_{kk}^{(0)} = 1$, $\Gamma_{kk}^{(1)} = \beta_R T_{kk}^2$, $\Delta_{kk}^{(1)} = \beta_T T_{kk}^2$, $m_{\mathbf{\Gamma}}^{(1)} = \tilde{C}_{kk}^{(1)} = \beta_R m_{\mathbf{T}}^{(2)}$, $m_{\mathbf{\Delta}}^{(1)} = D_{kk}^{(1)} = \beta_T m_{\mathbf{T}}^{(2)}$.

Step l: Compute

$$\Gamma_{kk}^{(l)} = \beta_R \beta_T T_{kk}^2 \sum_{s=0}^{s+r=0,1,\dots l-2} \sum_{r=0} m_{\tilde{\mathbf{C}}}^{(s)} m_{\mathbf{D}}^{(r)} \Gamma_{kk}^{(l-(s+r)-1)} +$$
$$+ \beta_R m_{\mathbf{D}}^{(l-1)} T_{kk}^2 \qquad\qquad l \geq 2$$

$$m_{\mathbf{\Gamma}}^{(l)} = \beta_R \beta_T \sum_{s=0}^{s+r=0,1,\dots,l-2} \sum_{r=0} m_{\tilde{\mathbf{C}}}^{(s)} m_{\mathbf{D}}^{(r)} \mathbb{E}\left(\frac{T_{kk}^2}{\theta}\Gamma_{kk}^{(l-(s+r)-1)}\right)$$
$$+ \beta_R m_{\mathbf{D}}^{(l-1)} m_{\mathbf{T}}^{(2)} \qquad\qquad l \geq 2$$

$$\Delta_{kk}^{(l)} = \beta_R \beta_T T_{kk}^2 \sum_{s=0}^{s+r=0,1,\dots l-2} \sum_{r=0} m_{\tilde{\mathbf{C}}}^{(s)} m_{\mathbf{D}}^{(r)} \Delta_{kk}^{(l-(s+r)-1)} +$$
$$+ \beta_T m_{\tilde{\mathbf{C}}}^{(l-1)} T_{kk}^2 \qquad\qquad l \geq 2$$

$$m_{\mathbf{\Delta}}^{(l)} = \beta_R \beta_T \sum_{s=0}^{s+r=0,1,\dots l-2} \sum_{r=0} m_{\tilde{\mathbf{C}}}^{(s)} m_{\mathbf{D}}^{(r)} \mathbb{E}\left(\frac{T_{kk}^2}{\theta}\Delta_{kk}^{(l-(s+r)-1)}\right)$$
$$+ \beta_T m_{\tilde{\mathbf{C}}}^{(l-1)} m_{\mathbf{T}}^{(2)} \qquad\qquad l \geq 2$$

$$D_{kk}^{(l)} = \sum_{n=0}^{l-1} m_{\mathbf{\Delta}}^{(l-n)} D_{kk}^{(n)} \qquad\qquad l \geq 1$$

$$m_{\mathbf{D}}^{(l)} = \sum_{n=0}^{l-1} m_{\mathbf{\Delta}}^{(l-n)} m_{\mathbf{D}}^{(n)} \qquad\qquad l \geq 1$$

$$\tilde{C}_{kk}^{(l)} = \sum_{n=0}^{l-1} m_{\mathbf{\Gamma}}^{(l-n)} \tilde{C}_{kk}^{(n)} \qquad\qquad l \geq 1$$

$$m_{\tilde{\mathbf{C}}}^{(l)} = \sum_{n=0}^{l-1} m_{\mathbf{\Gamma}}^{(l-n)} m_{\tilde{\mathbf{C}}}^{(n)} \qquad\qquad l \geq 1$$

Remarks
- *In order to compute $m_{\tilde{\mathbf{C}}}^{(l)}$, it is necessary to determine $m_{\mathbf{\Gamma}}^{(l)}$ and $m_{\mathbf{\Delta}}^{(l-1)}$.*
- *In the previous algorithm, at step $l = 1$ only the expressions defined for $l \geq 1$ are computed.*
- *It is easy to verify that the diagonal elements $\tilde{C}_{kk}^{(l)}$ and $D_{kk}^{(l)}$ are independent of the index k and all equal.*
- *For $l = 1$, $\tilde{C}_{kk} = m_{\tilde{\mathbf{C}}}^{(1)} = \beta_R m_{\mathbf{T}}^{(2)}$.*

By applying the previous algorithm we obtain the following eigenvalue moments.

$$m_{\tilde{\mathbf{C}}}^{(1)} = \beta_R m_{\mathbf{T}}^{(2)},$$
$$m_{\tilde{\mathbf{C}}}^{(2)} = \beta_R^2 \beta_T m_{\mathbf{T}}^{(4)} + \beta_R(\beta_R + \beta_T)(m_{\mathbf{T}}^{(2)})^2,$$
$$m_{\tilde{\mathbf{C}}}^{(3)} = \beta_R^3 \beta_T^2 m_{\mathbf{T}}^{(6)} + 3\beta_R^2 \beta_T (\beta_R + \beta_T) m_{\mathbf{T}}^{(2)} m_{\mathbf{T}}^{(4)}$$
$$+ \left[\beta_R \beta_T (3\beta_R + \beta_T) + \beta_R^3\right](m_{\mathbf{T}}^{(2)})^3.$$

4. FAVORABLE PROPAGATION AND MULTIUSER DETECTION IN DAS

In this section, we analyze the property of favorable propagation in DAS through the characteristics of their channel eigenvalue moments. In a favorable propagation environment, when the users have almost orthogonal channels, the channel covariance matrix $\mathbf{R}$ satisfies the following properties

$$\frac{m_{\mathbf{R}}^{(l)}}{\text{tr}[(\text{diag}(\mathbf{R}))^l]} \approx 1 \qquad \forall l \in \mathbb{N}^+ \qquad (5)$$

where $m_{\mathbf{R}}^{(l)}$ denotes the l-order eigenvalue moment of matrix $\mathbf{R}$. These properties are asymptotically satisfied for centralized massive MIMO systems, in rich scattering environments, when the number of users stays finite while the number of antennas at the central base station tends to infinity.

By making use of the observation that in large DAS, as $L \to \infty$, $\tilde{C}_{kk} = \beta_R m_{\mathbf{T}}^{(2)}$, we obtain that $\text{tr}[(\text{diag}(\tilde{\mathbf{C}}))^l] = \beta_R^l (m_{\mathbf{T}}^{(2)})^l$ such that (5) specializes for DAS and $l = 2, 3$ as follows

$$\frac{m_{\tilde{\mathbf{C}}}^{(2)}}{\beta_R^2 (m_{\mathbf{T}}^{(2)})^2} = 1 + \frac{\beta_T}{\beta_R} + \beta_T \frac{m_{\mathbf{T}}^{(4)}}{(m_{\mathbf{T}}^{(2)})^2}$$

$$\frac{m_{\tilde{\mathbf{C}}}^{(3)}}{\beta_R^3 (m_{\mathbf{T}}^{(2)})^3} = 1 + 3\frac{\beta_T}{\beta_R} + \frac{\beta_T^2}{\beta_R^2} + 3\beta_T\left(1 + \frac{\beta_T}{\beta_R}\right)\frac{m_{\mathbf{T}}^{(4)}}{(m_{\mathbf{T}}^{(2)})^2} +$$
$$+ \beta_T^2 \frac{m_{\mathbf{T}}^{(6)}}{(m_{\mathbf{T}}^{(2)})^3}$$

As β_R goes to infinity while β_T is kept constant, i.e., for $\beta_T/\beta_R \to 0$ and $\beta_T > 0$, $\dfrac{m_{\tilde{\mathbf{C}}}^{(2)}}{\beta_R^2(m_{\mathbf{T}}^{(2)})^2} \to 1 + \beta_T \dfrac{m_{\mathbf{T}}^{(4)}}{(m_{\mathbf{T}}^{(2)})^2}$ and
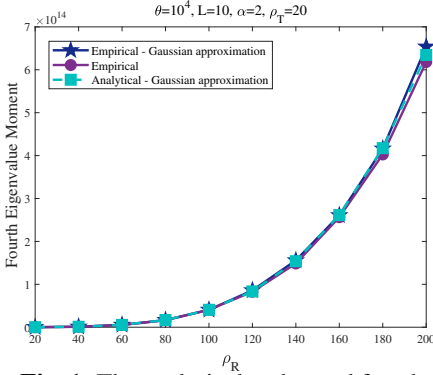
Fig. 1. The analytical and actual fourth eigenvalue moment of LoS channel versus intensity of receive antennas.
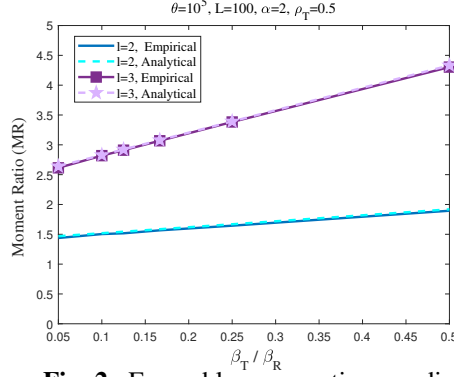

Fig. 2. Favorable propagation conditions MR= $m_{\tilde{\mathbf{C}}}^{(l)}/\mathrm{tr}\left[\left(diag(\tilde{\mathbf{C}})\right)^l\right]$ versus β_T/β_R for $l = 2, 3$.
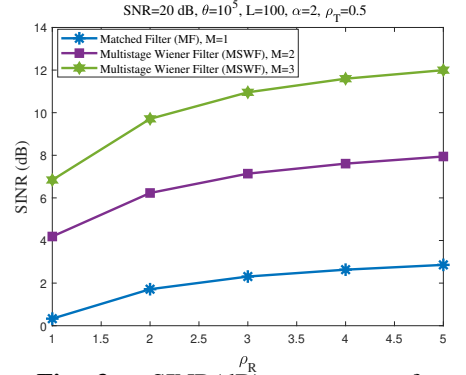

Fig. 3. SINR(dB) versus ρ_R for matched filter ($M = 1$) and multistage detectors ($M = 2, 3$).

$$\frac{m_{\tilde{\mathbf{C}}}^{(3)}}{\beta_R^3 (m_{\mathbf{T}}^{(2)})^3} \to 1 + 3\beta_T \frac{m_{\mathbf{T}}^{(4)}}{(m_{\mathbf{T}}^{(2)})^2} + \beta_T^2 \frac{m_{\mathbf{T}}^{(6)}}{(m_{\mathbf{T}}^{(2)})^3}$$ and conditions (5) are not satisfied.

Systems with favorable propagation can efficiently utilize the low complexity matched filter at the central processing unit since it achieves almost optimal performance in such environments. However, when conditions (5) are not satisfied, even linear multiuser detectors are expected to provide substantial gains compared to the matched filter. In the following, we consider low complexity multistage detectors including both polynomial expansion detectors, e.g., [15], and multistage Wiener filters [16] and we analyze their performance in terms of their signal to interference and noise ratio (SINR) by applying the unified framework proposed in [21,23]. In [21], it is shown that both design and analysis of multistage detectors with M stages can be described by a matrix $\mathbf{S}(X)$ defined as

$$\mathbf{S}(X) = \begin{pmatrix} X^{(2)} + \sigma^2 X^{(1)} & \cdots & X^{(M+1)} + \sigma^2 X^{(M)} \\ X^{(3)} + \sigma^2 X^{(2)} & \cdots & X^{(M+2)} + \sigma^2 X^{(M+1)} \\ \vdots & \ddots & \vdots \\ X^{(M+1)} + \sigma^2 X^{(M)} & \cdots & X^{(2M)} + \sigma^2 X^{(2M-1)} \end{pmatrix}$$

and a vector $\mathbf{s}(X) = (X^{(1)}, X^{(2)}, ..., X^{(M)})^T$ where $X = m_{\tilde{\mathbf{C}}}$ for polynomial expansion detectors and $X = \tilde{C}_{kk}$ for multistage Wiener filters. From the asymptotic property that $\tilde{C}_{kk}^{(l)} = m_{\tilde{\mathbf{C}}}^{(l)}$ for any k and l, we can conclude that multistage Wiener filters and polynomial expansion detectors are equivalent in DAS. Additionally, we can determine the performance of a centralized processor implementing multistage detectors by applying the following expression [21]

$$\mathrm{SINR} = \frac{\mathbf{s}^T(m_{\tilde{\mathbf{C}}})\mathbf{S}^{-1}(m_{\tilde{\mathbf{C}}})\mathbf{s}(m_{\tilde{\mathbf{C}}})}{1 - \mathbf{s}^T(m_{\tilde{\mathbf{C}}})\mathbf{S}^{-1}(m_{\tilde{\mathbf{C}}})\mathbf{s}(m_{\tilde{\mathbf{C}}})}. \quad (6)$$

It is worth noting that for $M = 1$, a multistage detector reduces to a matched filter and (6) can be applied also for the performance analysis of matched filters.

5. SIMULATION RESULTS

In this section, we validate the analytical results in Section 3 and 4. We consider systems with pathloss factor $\alpha = 2$ and

$d_0 = 1$. For Fig. 1, we consider a system with transmitters homogeneously distributed with intensity $\rho_T = 20$ over a segment of length L while the receivers' intensity varies in the range $\rho_R = [20, 200]$. Fig. 1 compares the fourth eigenvalue moments of LoS channels obtained analytically for $L \to \infty$ by the algorithm in Section 3 and the fourth eigenvalue moments of systems with L finite and with and without Gaussian approximation. The comparison shows that the asymptotic approximation matches very well practical systems. For Fig. 2 and 3, we assume $L = 100$, $\rho_T = 0.5$, and $\rho_R = [1,5]$. Fig. 2 shows the ratio $m_{\tilde{\mathbf{C}}}^{(l)}/\mathrm{tr}\left[\left(diag(\tilde{\mathbf{C}})\right)^l\right]$ versus β_T/β_R for $l = 2, 3$ to corroborate the analytical result that the conditions for favorable propagation are not satisfied. In fact, the curves do not tend to 1 for small ratios β_T/β_R. Finally, we consider a system with average signal to noise ratio (SNR) at the transmitters equal to 20dB and show the usefulness of multiuser detection in DAS. More specifically, Fig. 3 shows the SINR(dB) of matched filters ($M = 1$) and multistage detectors with two and three stages versus the intensity of receive antennas. For increasing values of ρ_R, the performance gap between the matched filter and the multistage detector is substantial and does not tend to vanish.

6. CONCLUSIONS

In this paper, we considered a system consisting of randomly distributed transmit and receive antennas and investigated to which extent the phenomenon of favorable propagation, widely exploited in massive MIMO systems, is present and can be utilized in DAS. The properties of DASs were analyzed using channel eigenvalue moments. We showed analytically that the conditions of favorable propagation are not satisfied. A final comparison between the performance of multistage detectors and matched filters corroborates the usefulness of multiuser detection in DAS.

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