Rate Balancing for Multiuser Multicell Downlink MIMO Systems

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***Abstract*—In this paper, we consider rate balancing problem for the Multiple-Input Multiple-Output (MIMO) Interfering Broadcast Channel (IBC), i.e. the multiuser multicell downlink (DL). We address the MIMO DL beamformer design and power allocation for maximizing the minimum weighted user rate with sum-power constraint with the weighting reflecting user priorities. The proposed solution is based on reformulating the max-min user rate optimization problem into a weighted Mean Squared Error (MSE) balancing problem. Employing MSE duality between DL channel and its equivalent Uplink (UL) channel, we propose an iterative algorithm to jointly design the transceiver filters and the power allocation. Simulation results verify the computational efficiency of the proposed algorithm and provide appreciable performance improvements as compared to optimizing the conventional unweighted per user MSE.**

***Index Terms*—rate balancing, max-min fairness, MSE duality, tranceiver optimization, multiuser multicell MIMO systems**

I. Introduction

We consider a wireless network in which a set of base stations serve their users and all nodes are equipped with multiple antennas. The goal is to maximize the minimum rate among all users in the system subject to a total sum power constraint, to achieve network-wide fairness [1].

Actually, several works in the literature have studied the max-min/min-max fairness problems w.r.t. given utility functions. For instance, the max-min signal-to-interference-plus-noise ratio (SINR) problem is of particular interest because it is directly related to common performance measures like system capacity and bit error rates [2]. Maximizing the minimum user SINR in the uplink can be done straightforwardly since the beamformers can be optimized individually and SINRs are only coupled by the users' transmit powers. In contrast, downlink optimization is generally a nontrivial task because the user SINRs depend on all optimization variables and have to be optimized jointly. Downlink transmitter optimization for single antenna receivers with a constraint on the total transmit power is comprehensively studied in [3] and [4] where algorithmic solutions for maximizing the minimal user SINR are proposed. This SINR balancing technique has been extended to underlay cognitive radio networks with transmit power and interference constraints in [5], [6].

Another utility of interest for fairness optimization problems is the mean squared error (MSE). In fact, the min-max MSE optimization is based on the stream-wise MSE duality where it has been shown that the same MSE values are achievable in the downlink and the uplink with the same transmit power constraint. This MSE duality has been exploited to solve various minimum MSE (MMSE) based optimization problems [7], [8]. In [9], three levels of MSE dualities have been established between MIMO BC and MIMO MAC with the same transmit power constraint and these dualities have been exploited to reduce the computational complexity of the sum-MSE and weighted sum-MSE minimization problems (with fixed weights) in a MIMO Broadcast Channel (BC).

In this work, we focus on user rate balancing in a way to maximize the minimum per user (weighted) rate in the network. This balancing problem is studied in [10] without providing an explicit precoder design. As in [11], we provide here a solution via the relation between user rate (summed over its streams) and a weighted sum MSE. But also another ingredient is required: the exploitation of scale factor that can be freely chosen in the weights for the weighted rate balancing. User-wise rate balancing outperforms user-wise MSE balancing or streamwise rate (or MSE/SINR) balancing when the streams of any MIMO user are quite unbalanced. In [11] the problem is handled for BC (single cell) and is transformed into weighted MSE balancing using non-diagonal weight matrices. Here we consider a multicell case and solve the user rate balancing problem using diagonal weight matrices by diagonalizing the user signal error covariance matrices, which allows to link the per stream and per user power allocation problems.

II. System Model

We have C cells. Each cell c has one base station (BS) of M_c transmit antennas serving K_c users of each N_{k_c} antennas, ($k_c = 1, ..., K_c$ is the users' index with $k_c \in \mathcal{K}$). The set of all users is defined as $\mathcal{K} = \{k_c | 1 \leq k_c \leq K_c, 1 \leq c \leq C\}$. The channel between the k_cth user in cell c and the BS in cell j is denoted by $\boldsymbol{H}^{\mathrm{H}}_{j,k_c} \in \mathbb{C}^{M_j \times N_{k_c}}$, and $\boldsymbol{H}^{\mathrm{H}}_{j,c} = [\boldsymbol{H}^{\mathrm{H}}_{j,1}, ..., \boldsymbol{H}^{\mathrm{H}}_{j,K_c}]$ is the overall channel matrix between transmitter in cell j and cell c. We consider zero-mean white Gaussian noise $\boldsymbol{n}_{k_c} \in \mathbb{C}^{N_{k_c} \times 1}$ with distribution $\mathcal{CN}(0, \sigma_n^2 \boldsymbol{I})$ at the k_cth user.

We assume independent unity-power transmit symbols $\mathbf{s}_c = [\mathbf{s}_1^{\mathrm{T}} \ldots \mathbf{s}_{K_c}^{\mathrm{T}}]^{\mathrm{T}}$, i.e., $\mathbb{E}[\mathbf{s}_c \mathbf{s}_c^{\mathrm{H}}] = \boldsymbol{I}$, where $\mathbf{s}_{k_c} \in \mathbb{C}^{d_{k_c} \times 1}$ is the data vector to be transmitted to the k_cth user, with d_{k_c} being the number of streams allowed by user k_c. The latter is transmitted using the transmit filtering matrix $\boldsymbol{\mathcal{G}}_c = \boldsymbol{G}_c \boldsymbol{P}_c^{1/2} \in \mathbb{C}^{M_c \times N_c}$, composed of the beamforming matrix $\boldsymbol{G}_c = [\boldsymbol{G}_1 \ldots \boldsymbol{G}_{K_c}] = [\boldsymbol{g}_1 \ldots \boldsymbol{g}_{N_c}]$ with normalized columns $\|\boldsymbol{g}_i\|_2 = 1$ and the diagonal non-negative downlink power allocation $\boldsymbol{P}_c^{1/2} = \mathrm{blkdiag}\{\boldsymbol{P}_1^{1/2}, \ldots, \boldsymbol{P}_{K_c}^{1/2}\}$ where $\mathrm{diag}(\boldsymbol{P}_{k_c}) \in \mathbb{R}_+^{d_{k_c} \times 1}$ contains

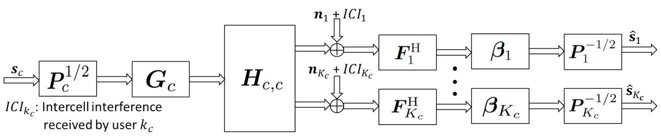

Fig. 1: Downlink channel in cell c.

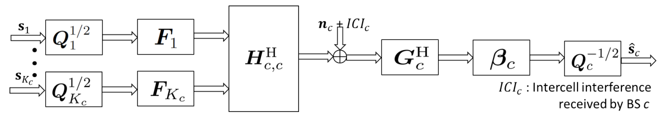

Fig. 2: Dual uplink channel in cell c.

the transmission powers and $N_c = \sum_{k_c=1}^{K_c} d_{k_c}$ is the total number of streams in cell c. Let $\boldsymbol{\mathcal{G}}_{k_c} = \boldsymbol{G}_{k_c}\boldsymbol{P}_{k_c}^{1/2} \in \mathbb{C}^{M_c\times d_{k_c}}$ and $\boldsymbol{P} = \text{blkdiag}\{\boldsymbol{P}_1\ldots\boldsymbol{P}_C\}$; the total transmit power is limitted, i.e., $\mathrm{tr}(\boldsymbol{P}) \le P_{\max}$.

Similarly, the receive filtering matrix for each user k_c is defined as $\boldsymbol{\mathcal{F}}_{k_c}^{\mathrm{H}} = \boldsymbol{P}_{k_c}^{-1/2}\boldsymbol{\beta}_{k_c}\boldsymbol{F}_{k_c}^{\mathrm{H}} \in \mathbb{C}^{d_{k_c}\times N_{k_c}}$, composed of beamforming matrix $\boldsymbol{F}_{k_c}^{\mathrm{H}} \in \mathbb{C}^{d_{k_c}\times N_{k_c}}$ and the diagonal matrices $\boldsymbol{\beta}_{k_c}$ contain scaling factors which ensure that the columns of $\boldsymbol{F}_{k_c}^{\mathrm{H}}$ have unit norm. We define $\boldsymbol{\beta}_c = \text{blkdiag}\{\boldsymbol{\beta}_1,\ldots,\boldsymbol{\beta}_{K_c}\} = \mathrm{diag}\{[\beta_1\ldots\beta_{N_c}]\}$, $\boldsymbol{F}_c = \text{blkdiag}\{\boldsymbol{F}_1,\ldots,\boldsymbol{F}_{K_c}\}$ and $\boldsymbol{F} = \text{blkdiag}\{\boldsymbol{F}_1,\ldots,\boldsymbol{F}_C\} = [\boldsymbol{f}_1\ldots\boldsymbol{f}_{N_s}]$ with normalized per-stream receivers, i.e., $\|\boldsymbol{f}_i\|_2 = 1$, ($N_s = \sum_{c=1}^{C} N_c$ being the total number of streams in the network), see Figure 1.

The MSE per stream is defined as $\varepsilon_{i_c}^{\mathrm{DL}}$ between the decision variable $\hat{s}_{i_c}$ and the transmit data symbol s_{i_c} in cell c as follows

$$\begin{aligned}\varepsilon_{i_c}^{\mathrm{DL}} &= \mathbb{E}\Big\{|\hat{s}_{i_c} - s_{i_c}|^2\Big\}\\ &= \beta_{i_c}^2/p_{i_c}\boldsymbol{f}_{i_c}^{\mathrm{H}}\Big(\sum_{j=1}^{C}\boldsymbol{H}_{j,c}\big(\sum_{n_j=1}^{N_j} p_{n_j}\boldsymbol{g}_{n_j}\boldsymbol{g}_{n_j}^{\mathrm{H}}\big)\boldsymbol{H}_{j,c}^{\mathrm{H}}\Big)\boldsymbol{f}_{i_c}\\ &\quad - 2\beta_{i_c}\mathrm{Re}\big\{\boldsymbol{f}_{i_c}^{\mathrm{H}}\boldsymbol{H}_{c,c}\boldsymbol{g}_{i_c}\big\} + \sigma_n^2\beta_{i_c}^2/p_{i_c} + 1, \forall i_c,\end{aligned} \tag{1}$$

where the index i_c refers to the ith stream in cell c with $\boldsymbol{f}_{i_c} = [\boldsymbol{F}_c]_{:,i}$.

III. Problem Formulation

In this work, we aim to solve the weighted user-rate max-min optimization problem under a total transmit power constraint, i.e., the user rate balancing problem expressed as follows

$$\begin{aligned}\max_{\{\boldsymbol{G}_c,\boldsymbol{P}_c,\boldsymbol{F}_c,\boldsymbol{\beta}_c\}}\ &\min_{k_c}\ r_{k_c}/r_{k_c}^{\circ}\\ &\text{s.t.}\ \ \mathrm{tr}(\boldsymbol{P}) \le P_{\max}\end{aligned} \tag{2}$$

where r_{k_c} is the k_cth user-rate

$$\begin{aligned}r_{k_c} = \log\det\Big(&\boldsymbol{I} + \boldsymbol{H}_{c,k_c}\boldsymbol{\mathcal{G}}_{k_c}\boldsymbol{\mathcal{G}}_{k_c}^{\mathrm{H}}\boldsymbol{H}_{c,k_c}^{\mathrm{H}}\big(\sigma_n^2\boldsymbol{I}+\\ &\sum_{l_j\in\{\mathcal{K}|k_c\}}\boldsymbol{H}_{j,k_c}\boldsymbol{\mathcal{G}}_{l_j}\boldsymbol{\mathcal{G}}_{l_j}^{\mathrm{H}}\boldsymbol{H}_{j,k_c}^{\mathrm{H}}\big)^{-1}\Big)\end{aligned} \tag{3}$$

and $r_{k_c}^{\circ}$ is the rate scaling factor for user k_c. However, the problem presented in (2) is complex and can not be solved directly.

Lemma 1. *The rate of user k_c in (3) can also be represented as [12]*

$$r_{k_c} = \max_{\boldsymbol{W}_{k_c},\boldsymbol{\mathcal{F}}_{k_c}}\big[\ln\det(\boldsymbol{W}_{k_c}) - \mathrm{tr}(\boldsymbol{W}_{k_c}\mathbf{E}_{k_c}^{\mathrm{DL}}) + d_{k_c}\big]. \tag{4}$$

where

$$\begin{aligned}\mathbf{E}_{k_c}^{\mathrm{DL}} &= \mathbb{E}\Big[(\hat{\mathbf{s}}_{k_c} - \mathbf{s}_{k_c})(\hat{\mathbf{s}}_{k_c} - \mathbf{s}_{k_c})^{\mathrm{H}}\Big]\\ &= (\boldsymbol{I} - \boldsymbol{\mathcal{F}}_{k_c}^{\mathrm{H}}\boldsymbol{H}_{c,k_c}\boldsymbol{\mathcal{G}}_{k_c})(\boldsymbol{I} - \boldsymbol{\mathcal{F}}_{k_c}^{\mathrm{H}}\boldsymbol{H}_{c,k_c}\boldsymbol{\mathcal{G}}_{k_c})^{\mathrm{H}}\\ &\quad + \sum_{l_j\in\{\mathcal{K}|k_c\}}\boldsymbol{\mathcal{F}}_{k_c}^{\mathrm{H}}\boldsymbol{H}_{j,k_c}\boldsymbol{\mathcal{G}}_{l_j}\boldsymbol{\mathcal{G}}_{l_j}^{\mathrm{H}}\boldsymbol{H}_{j,k_c}^{\mathrm{H}}\boldsymbol{\mathcal{F}}_{k_c} + \sigma_n^2\boldsymbol{\mathcal{F}}_{k_c}^{\mathrm{H}}\boldsymbol{\mathcal{F}}_{k_c}\end{aligned} \tag{5}$$

is the k_cth-user downlink MSE matrix between the decision variable $\hat{\mathbf{s}}_{k_c}$ and the transmit signal $\mathbf{s}_{k_c}$, and $\boldsymbol{W} = \{\boldsymbol{W}_{k_c}\}_{k_c\in\mathcal{K}}$ are auxiliary weight matrix variables with optimal solution $\boldsymbol{W}_{k_c}^{\mathrm{opt}} = \big[\mathbf{E}_{k_c}^{\mathrm{DL}}\big]^{-1}$ and

$$\boldsymbol{\mathcal{F}}_{k_c} = \big(\sigma_n^2\boldsymbol{I} + \sum_{l_j\in\{\mathcal{K}\}}\boldsymbol{H}_{j,k_c}\boldsymbol{\mathcal{G}}_{l_j}\boldsymbol{\mathcal{G}}_{l_j}^{\mathrm{H}}\boldsymbol{H}_{j,k_c}^{\mathrm{H}}\big)^{-1}\boldsymbol{H}_{c,k_c}\boldsymbol{\mathcal{G}}_{k_c}.$$

Now consider both (2) and (4), and let us introduce $\xi_{k_c} = \ln\det(\boldsymbol{W}_{k_c}) + d_{k_c} - r_{k_c}^{\triangle}$, the WMSE requirement, with target rate $r_{k_c}^{\triangle}$. Assume that we shall be able to concoct an optimization algorithm that ensures that at all times and for all users the matrix-weighted MSE (WMSE) satisfies $\epsilon_{w,k_c}^{\mathrm{DL}} = \mathrm{tr}(\boldsymbol{W}_{k_c}\mathbf{E}_{k_c}^{\mathrm{DL}}) \le d_{k_c}$ and $\ln\det(\boldsymbol{W}_{k_c}) \ge r_{k_c}^{\triangle}$ or hence $\xi_{k_c} \ge d_{k_c}$. This leads $\forall k_c$ to

$$\begin{aligned}&\frac{\epsilon_{w,k_c}^{\mathrm{DL}}}{\xi_{k_c}} \le 1\\ \Longleftrightarrow\ &\ln\det(\boldsymbol{W}_{k_c}) + d_{k_c} - \mathrm{tr}(\boldsymbol{W}_{k_c}\mathbf{E}_{k_c}^{\mathrm{DL}}) \ge r_{k_c}^{\triangle}\\ \overset{(a)}{\Longrightarrow}\ &r_{k_c}/r_{k_c}^{\triangle} \ge 1\end{aligned} \tag{6}$$

where (a) follows from (4). To get to (6), what we can exploit in (2) is a scale factor t that can be chosen freely in the rate weights $r_{k_c}^{\circ}$ in (2). We shall take $t = \min_k r_{k_c}/r_{k_c}^{\circ}$, which allows to transform the rate weights $r_{k_c}^{\circ}$ into target rates $r_{k_c}^{\triangle} = tr_{k_c}^{\circ}$, and at the same time allows to interpret the WMSE weights ξ_{k_c} as target WMSE values.

Doing so, the initial rate balancing optimization problem (2) can be transformed into a matrix-weighted MSE balancing problem expressed as follows

$$\begin{aligned}\min_{\{\boldsymbol{G}_c,\boldsymbol{P}_c,\boldsymbol{F}_c,\boldsymbol{\beta}_c\}}\ &\max_{k_c}\ \epsilon_{w,k_c}^{\mathrm{DL}}/\xi_{k_c}\\ &\text{s.t.}\ \ \mathrm{tr}(\boldsymbol{P}) \le P_{\max},\end{aligned} \tag{7}$$

which needs to be complemented with an outer loop in which $\boldsymbol{W}_{k_c} = (\mathbf{E}_{k_c}^{\mathrm{DL}})^{-1}$, $t = \min_{k_c} r_{k_c}/r_{k_c}^{\circ}$, $r_{k_c}^{\triangle} = tr_{k_c}^{\circ}$ and $\xi_{k_c} = d_{k_c} + r_{k_c} - r_{k_c}^{\triangle}$ get updated.

The problem in (7) is still difficult to be handled directly. In the next sections, we solve the problem via uplink and downlink MSE duality. To this aim, we model an equivalent uplink-downlink channel plus transceivers pair by separating the filters into two parts: a matrix with unity-norm columns and a scaling matrix [13]. Then, the uplink and downlink are proved to share the same MSE by swiching the role of the normalized filters in the uplink and downlink. Doing so, an algorithmic solution can be derived for the optimization problem (7).

IV. Dual uplink channel

In the equivalent uplink model, we switch between the role of the normalized transmit and receive filters. In fact, $\boldsymbol{F}_{k_c}\boldsymbol{Q}_{k_c}^{1/2}$ is the k_cth transmit filter and $\boldsymbol{Q}_c^{-1/2}\boldsymbol{\beta}_c\boldsymbol{G}_c^{\mathrm{H}}$ is a multiuser receive filter in cell c, where $\boldsymbol{Q}_c = \text{blkdiag}\{\boldsymbol{Q}_1,\ldots,\boldsymbol{Q}_{K_c}\}$ with $\mathrm{diag}(\boldsymbol{Q}_{k_c}) \in \mathbb{R}_+^{d_{k_c}\times 1}$ being the uplink power allocation in cell c, and $\boldsymbol{Q} = \text{blkdiag}\{\boldsymbol{Q}_1,\ldots,\boldsymbol{Q}_C\}$ collect the uplink power allocation of the network, see Figure 2.

Although the quantities $\boldsymbol{H}_{j,c}, \boldsymbol{G}_c, \boldsymbol{F}_c$ and $\boldsymbol{\beta}_c$ are the same, the uplink power allocation $\boldsymbol{q} = [q_1\ldots q_{N_s}]^{\mathrm{T}} = \mathrm{diag}(\boldsymbol{Q})$ may differ from the downlink allocation $\boldsymbol{p} = [p_1 \dots p_{N_s}]^{\mathrm{T}} = \mathrm{diag}(\boldsymbol{P})$, both verifying the same sum power constraint $\|\boldsymbol{p}\|_1 = \|\boldsymbol{q}\|_1 \le P_{\max}$.

The corresponding uplink per stream MSE $\varepsilon_{i_c}^{\mathrm{UP}}$ in cell c is given by

$$\varepsilon_{i_c}^{\mathrm{UL}} = \beta_{i_c}^2/q_{i_c}\boldsymbol{g}_{i_c}^{\mathrm{H}}\Big(\sum_{j=1}^{C}\boldsymbol{H}_{j,c}^{\mathrm{H}}\big(\sum_{n_j=1}^{N_j} q_{n_j}\boldsymbol{f}_{n_j}\boldsymbol{f}_{n_j}^{\mathrm{H}}\big)\boldsymbol{H}_{j,c}\Big)\boldsymbol{g}_{i_c} - 2\beta_{i_c}\mathrm{Re}\big\{\boldsymbol{g}_{i_c}^{\mathrm{H}}\boldsymbol{H}_{c,c}^{\mathrm{H}}\boldsymbol{f}_{i_c}\big\} + \sigma_n^2\beta_{i_c}^2/q_{i_c} + 1, \forall i_c \in \{1,...,N_c\}. \quad (8)$$

V. MSE Duality

With the equivalent downlink channel and its dual uplink, it has been shown that the same per stream MSE values are achieved in both links, i.e., $\boldsymbol{\varepsilon}^{\mathrm{UP/DL}} = \mathrm{diag}\big\{[\varepsilon_1^{\mathrm{UP/DL}} \dots \varepsilon_{N_s}^{\mathrm{UP/DL}}]\big\} = \mathrm{diag}\big\{[\varepsilon_1 \dots \varepsilon_{N_s}]\big\} = \boldsymbol{\varepsilon}$.

The uplink and downlink power allocation, obtained by solving the MSE expressions as in (8) for UL w.r.t. the powers, are given by

$$\boldsymbol{q} = \sigma_n^2(\boldsymbol{\varepsilon} - \boldsymbol{D} - \boldsymbol{\beta}^2\boldsymbol{\Psi})^{-1}\boldsymbol{\beta}^2\mathbf{1}_{N_s} \quad (9)$$

and

$$\boldsymbol{p} = \sigma_n^2(\boldsymbol{\varepsilon} - \boldsymbol{D} - \boldsymbol{\beta}^2\boldsymbol{\Psi}^T)^{-1}\boldsymbol{\beta}^2\mathbf{1}_{N_s} \quad (10)$$

respectively, where $\boldsymbol{D} = \mathrm{blkdiag}(\boldsymbol{D}_1, \dots, \boldsymbol{D}_C)$ with the diagonal matrices $\{\boldsymbol{D}_c\}_{1\le c\le C}$ defined as

$$[\boldsymbol{D}_c]_{ii} = \beta_{i_c}^2\boldsymbol{g}_{i_c}^{\mathrm{H}}\boldsymbol{H}_{c,c}^{\mathrm{H}}\boldsymbol{f}_{i_c}\boldsymbol{f}_{i_c}^{\mathrm{H}}\boldsymbol{H}_{c,c}\boldsymbol{g}_{i_c} - 2\beta_{i_c}\mathrm{Re}\{\boldsymbol{g}_{i_c}^{\mathrm{H}}\boldsymbol{H}_{c,c}^{\mathrm{H}}\boldsymbol{f}_{i_c}\} + 1$$

and

$$[\boldsymbol{\Psi}]_{ij} = \begin{cases} |\boldsymbol{g}_i^{\mathrm{H}}\boldsymbol{H}_n^{\mathrm{H}}\boldsymbol{f}_j|^2, & i \ne j \\ 0, & i = j. \end{cases}$$

where $H_n^{\mathrm{H}} = [H_{n,1}^{\mathrm{H}}, \dots, H_{n,C}^{\mathrm{H}}]$, n is the cell in where the per-stream $\boldsymbol{g}_i$ performs and $i = \sum_{c=1}^{n-1} N_c + l$, such that the ith stream of the network is the lth stream of cell n.

In fact, the MSE duality allows to optimize the transceiver design by switching between the virtual uplink and actual downlink channels. The optimal receive filtering matrices in both uplink and downlink are MMSE filters and are given by

$$\boldsymbol{G}_{k_c}\boldsymbol{\beta}_{k_c}\boldsymbol{Q}_{k_c}^{-1/2} = \big(\sum_{j=1}^{C}\boldsymbol{H}_{c,j}^{\mathrm{H}}\boldsymbol{F}_j\boldsymbol{Q}_j\boldsymbol{F}_j^{\mathrm{H}}\boldsymbol{H}_{c,j} + \sigma_n^2\boldsymbol{I}\big)^{-1}\boldsymbol{H}_{c,k_c}^{\mathrm{H}}\boldsymbol{F}_{k_c}\boldsymbol{Q}_{k_c}^{1/2}, \quad (11)$$

$$\boldsymbol{F}_{k_c}\boldsymbol{\beta}_{k_c}\boldsymbol{P}_{k_c}^{-1/2} = \big(\sum_{j=1}^{C}\boldsymbol{H}_{j,k_c}\boldsymbol{G}_j\boldsymbol{P}_j\boldsymbol{G}_j^{\mathrm{H}}\boldsymbol{H}_{j,k_c}^{\mathrm{H}} + \sigma_n^2\boldsymbol{I}\big)^{-1}\boldsymbol{H}_{c,k_c}\boldsymbol{G}_{k_c}\boldsymbol{P}_{k_c}^{1/2}. \quad (12)$$

VI. The matrix weighted User-MSE Optimization

In this section, the problem (7) with respect to the matrix weighted user-MSE is studied. First, we start by the uplink power allocation strategies. Then, the joint optimization will follow given the MSE duality.

In fact, the MSE duality opens up a way to obtain optimal downlink MMSE designs in (11) and (12), i.e., the downlink matrix weighted user-MSE optimization problems can be solved by optimizing the weighted MSE values of the dual uplink system. For given $\boldsymbol{F}_c, \boldsymbol{G}_c$ and $\boldsymbol{\beta}_c$, the latter can be formulated as

$$\min_{\{\boldsymbol{G}_c,\boldsymbol{F}_c,\boldsymbol{W}_c\}} \max_{k_c} \ \epsilon_{w,k_c}^{\mathrm{UL}}/\xi_{k_c} \quad \text{s.t. } \mathrm{tr}\big(\boldsymbol{Q}\big) \le P_{\max} \quad (13)$$

where $\epsilon_{w,k_c}^{\mathrm{UL}} = \mathrm{tr}\big(\boldsymbol{W}_{k_c}\mathbf{E}_{k_c}^{\mathrm{UL}}\big)$ and and

$$\begin{aligned}\mathbf{E}_{k_c}^{\mathrm{UL}} &= (\boldsymbol{I} - \boldsymbol{Q}_{k_c}^{-1/2}\boldsymbol{\beta}_{k_c}\boldsymbol{G}_{k_c}^{\mathrm{H}}\boldsymbol{H}_{c,k_c}^{\mathrm{H}}\boldsymbol{F}_{k_c}\boldsymbol{Q}_{k_c}^{1/2}) \\ &\times (\boldsymbol{I} - \boldsymbol{Q}_{k_c}^{-1/2}\boldsymbol{\beta}_{k_c}\boldsymbol{G}_{k_c}^{\mathrm{H}}\boldsymbol{H}_{c,k_c}^{\mathrm{H}}\boldsymbol{F}_{k_c}\boldsymbol{Q}_{k_c}^{1/2})^{\mathrm{H}} \\ &\sum_{l_j\ne k_c}\boldsymbol{Q}_{k_c}^{-1/2}\boldsymbol{\beta}_{k_c}\boldsymbol{G}_{k_c}^{\mathrm{H}}\boldsymbol{H}_{c,l_j}^{\mathrm{H}}\boldsymbol{F}_{l_j}\boldsymbol{Q}_{l_j}\boldsymbol{F}_{l_j}^{\mathrm{H}}\boldsymbol{H}_{c,l_j}\boldsymbol{G}_{k_c}\boldsymbol{\beta}_{k_c}\boldsymbol{Q}_{k_c}^{-1/2} + \\ &+ \sigma_n^2\boldsymbol{Q}_{k_c}^{-1/2}\boldsymbol{\beta}_{k_c}\boldsymbol{G}_{k_c}^{\mathrm{H}}\boldsymbol{G}_{k_c}\boldsymbol{\beta}_{k_c}\boldsymbol{Q}_{k_c}^{-1/2}. \end{aligned} \quad (14)$$

Now, define a modified transmit uplink filter as

$$\breve{\boldsymbol{F}}_{k_c}\breve{\boldsymbol{Q}}_{k_c}^{1/2} = \boldsymbol{F}_{k_c}\boldsymbol{Q}_{k_c}^{1/2}\boldsymbol{V}_{k_c}\ ,\ \breve{\mathbf{E}}_{k_c}^{\mathrm{UL}} = \boldsymbol{\Sigma}_{k_c}\ , \quad (15)$$

where $\boldsymbol{V}_{k_c}$ is given by the eigenvalue decomposition $\mathbf{E}_{k_c}^{\mathrm{UL}} = \boldsymbol{V}_{k_c}\boldsymbol{\Sigma}_{k_c}\boldsymbol{V}_{k_c}^{\mathrm{H}}$. This operation allows us to diagonalize $\{\mathbf{E}_{k_c}^{\mathrm{UL}}, \boldsymbol{W}_{k_c}\}$ and does not change the user rates [14], but changes the identity of the streams (layers) of a user and the power distribution over them.

The matrix weighted per user MSE can be expressed as follows, with $l_{k_c} = \sum_{i=1}^{k_c-1} d_{i_c} + 1$ (N_j and d_{i_c} being respectively the number of streams of cell j and user i_c)

$$\epsilon_{w,k_c}^{\mathrm{UL}} = \mathrm{tr}\big(\boldsymbol{W}_{k_c}\mathbf{E}_{k_c}^{\mathrm{UL}}\big) = \sum_{i=l_{k_c}}^{l_{k_c}+d_{k_c}} w_{i_c}\varepsilon_{i_c}^{\mathrm{UL}}, \forall k_c. \quad (16)$$

Collecting all layer MSEs in a vector, we get with $\boldsymbol{Q} = \mathrm{diag}\{\boldsymbol{q}\}$

$$\boldsymbol{\varepsilon}^{\mathrm{UL}} = \boldsymbol{Q}^{-1}\Big[(\boldsymbol{D} + \boldsymbol{\beta}^2\boldsymbol{\Psi})\boldsymbol{q} + \sigma_n^2\boldsymbol{\beta}^2\mathbf{1}_{N_s}\Big]. \quad (17)$$

Note that $\boldsymbol{Q}^{-1}\boldsymbol{D}\boldsymbol{q} = \boldsymbol{D}\mathbf{1}_{N_s}$. Now introduce the user powers $\tilde{\boldsymbol{q}} = [\tilde{q}_1 \dots \tilde{q}_{|\mathcal{K}|}]$, which relate to the stream powers as $\boldsymbol{Q}_{k_c} = \bar{\boldsymbol{Q}}_{k_c}\tilde{q}_{k_c}$ with $\mathrm{tr}\{\bar{\boldsymbol{Q}}_{k_c}\} = 1$, $\bar{\boldsymbol{Q}} = \mathrm{blkdiag}(\bar{\boldsymbol{Q}}_1, \dots, \bar{\boldsymbol{Q}}_C)$ and $\bar{\boldsymbol{Q}}_c = \mathrm{blkdiag}(\bar{\boldsymbol{Q}}_1, \dots, \bar{\boldsymbol{Q}}_{K_c})$. Also consider the case of diagonal weighting matrices $\boldsymbol{W}_{k_c}$ and the overall diagonal $\boldsymbol{W} = \mathrm{blkdiag}(\boldsymbol{W}_1, \dots, \boldsymbol{W}_{|\mathcal{K}|})$. Consider now the weighted user MSE $\epsilon_{w,k_c}^{\mathrm{UL}} = \mathrm{tr}\{\boldsymbol{W}_{k_c}\boldsymbol{E}_{k_c}^{\mathrm{UL}}\}$ where the diagonal of $\boldsymbol{E}^{\mathrm{UL}} = \mathrm{blkdiag}(\boldsymbol{E}_1^{\mathrm{UL}}, \dots, \boldsymbol{E}_{|\mathcal{K}|}^{\mathrm{UL}})$ is the set of stream MSEs $\boldsymbol{\varepsilon}^{\mathrm{UL}}$, and let $\boldsymbol{\epsilon}_w^{\mathrm{UL}} = \mathrm{diag}(\epsilon_{w,1}^{\mathrm{UL}}, \dots, \epsilon_{w,|\mathcal{K}|}^{\mathrm{UL}})$ be the set of user WMSEs. We shall also need the per user stream distribution matrix $\mathbb{1} = \mathrm{blkdiag}(\mathbf{1}_{d_1}, \dots, \mathbf{1}_{d_{|\mathcal{K}|}})$. Then we get from (17)

$$\boldsymbol{\epsilon}_w^{\mathrm{UL}}\mathbf{1}_{|\mathcal{K}|} = \mathbb{1}^H\boldsymbol{W}\boldsymbol{\varepsilon}^{\mathrm{UL}} = \mathbb{1}^H\boldsymbol{W}\boldsymbol{Q}^{-1}\Big[(\boldsymbol{D} + \boldsymbol{\beta}^2\boldsymbol{\Psi})\boldsymbol{q} + \sigma_n^2\boldsymbol{\beta}^2\mathbf{1}_{N_s}\Big]. \quad (18)$$

Note that $\boldsymbol{Q}\mathbf{1}_{N_s} = \boldsymbol{q} = \bar{\boldsymbol{Q}}\mathbb{1}\tilde{\boldsymbol{q}}$ and $\mathbf{1}_{N_s} = \mathbb{1}\,\mathbf{1}_{|\mathcal{K}|}$. By multiplying both sides of (18) with $\mathrm{diag}\{\tilde{\boldsymbol{q}}\}$, we get

$$\boldsymbol{\epsilon}_w^{\mathrm{UL}}\,\tilde{\boldsymbol{q}} = \boldsymbol{A}\,\tilde{\boldsymbol{q}} + \sigma_n^2\,\boldsymbol{C}\,\mathbf{1}_{|\mathcal{K}|} \quad \text{with} \quad (19)$$

$$\boldsymbol{A} = \mathbb{1}^H\boldsymbol{W}(\boldsymbol{D} + \boldsymbol{\beta}^2\bar{\boldsymbol{Q}}^{-1}\boldsymbol{\Psi}\bar{\boldsymbol{Q}})\mathbb{1} \quad \text{and} \quad \boldsymbol{C} = \mathbb{1}^H\boldsymbol{W}\boldsymbol{\beta}^2\bar{\boldsymbol{Q}}^{-1}\mathbb{1}.$$

Let $\boldsymbol{\xi} = \mathrm{diag}(\xi_1, \dots, \xi_{|\mathcal{K}|})$, then

$$\boldsymbol{\xi}^{-1}\boldsymbol{\epsilon}_w^{\mathrm{UL}}\,\tilde{\boldsymbol{q}} = \boldsymbol{\xi}^{-1}\boldsymbol{A}\,\tilde{\boldsymbol{q}} + \sigma_n^2\,\boldsymbol{\xi}^{-1}\boldsymbol{C}\,\mathbf{1}_K. \quad (20)$$

Actually, problem (7) always has a global minimizer $\tilde{\boldsymbol{q}}^{\mathrm{opt}}$ characterized by the equality $\boldsymbol{\xi}^{-1}\boldsymbol{\epsilon}_w^{\mathrm{UL}}(\boldsymbol{q}^{\mathrm{opt}}) = \Delta^{\mathrm{UL}}\boldsymbol{I}$, i.e.,

$$\Delta^{\mathrm{UL}}\tilde{\boldsymbol{q}}^{\mathrm{opt}} = \boldsymbol{\xi}^{-1}\boldsymbol{A}\tilde{\boldsymbol{q}}^{\mathrm{opt}} + \sigma_n^2\boldsymbol{\xi}^{-1}\boldsymbol{C}\mathbf{1}_{|\mathcal{K}|}. \quad (21)$$

On the other hand we have the power constraint $\mathbf{1}_{|\mathcal{K}|}^H\boldsymbol{q}^{\mathrm{opt}} = P_{\max}$, which allows us to write (21) as follows

$$\Delta^{\mathrm{UL}}\,\tilde{\boldsymbol{q}}^{\mathrm{opt}} = \Big[\boldsymbol{\xi}^{-1}\boldsymbol{A} + \frac{\sigma_n^2}{P_{\max}}\boldsymbol{\xi}^{-1}\boldsymbol{C}\,\mathbf{1}_{|\mathcal{K}|}\mathbf{1}_{|\mathcal{K}|}^{\mathrm{H}}\Big]\,\tilde{\boldsymbol{q}}^{\mathrm{opt}} \quad (22)$$

Having $\boldsymbol{\Lambda} = \boldsymbol{\xi}^{-1}\boldsymbol{A} + \frac{\sigma_n^2}{P_{\max}}\boldsymbol{\xi}^{-1}\boldsymbol{C}\,\mathbf{1}_{|\mathcal{K}|}\mathbf{1}_{|\mathcal{K}|}^{\mathrm{H}}$, the optimal power allocation $\tilde{\boldsymbol{q}}^{\mathrm{opt}}$ is the principal eigenvector of the matrix $\boldsymbol{\Lambda}$, i.e., the optimal power allocation can be found by solving

$$\boldsymbol{\Lambda}\tilde{\boldsymbol{q}} = \lambda_{\max}\tilde{\boldsymbol{q}}. \quad (23)$$

TABLE I: Pseudo code of the proposed algorithm

1. initialize: $\boldsymbol{F}_{k_c}^{\mathrm{H}(0,0)} = (\boldsymbol{I}_{d_{k_c}} : \boldsymbol{0})$, $\bar{\boldsymbol{Q}}^{(0,0)} = \frac{P_{\max}}{N_s}\boldsymbol{I}$, $m = n = 0$ and $n_{\max}, m_{\max}$ and fix $r_{k_c}^{\circ(0)}$
2. compute UL receive filter $\boldsymbol{G}_c^{(0,0)}$ and $\boldsymbol{\beta}_c^{(0,0)}$ with (11)
3. find optimal user power allocation $\tilde{\boldsymbol{q}}^{(0,0)}$ by solving (23) and compute $\boldsymbol{Q}_{k_c}^{(0,0)} = \tilde{q}_{k_c}^{(0,0)}\bar{\boldsymbol{Q}}_{k_c}^{(0,0)}$
4. update $\boldsymbol{F}_{k_c}^{(tmp,tmp)}$, $\boldsymbol{Q}_{k_c}^{(tmp,tmp)}$ with (15) and $\bar{\boldsymbol{Q}}_{k_c}^{(tmp,tmp)} = \boldsymbol{Q}_{k_c}^{(tmp,tmp)}/\mathrm{tr}\big(\boldsymbol{Q}_{k_c}^{(tmp,tmp)}\big)$
5. compute $\boldsymbol{W}_{k_c}^{(0)} = (\mathbf{E}_{k_c}^{\mathrm{UL}(0)})^{-1}$ and initialize $\xi_{k_c}^{(0)} = d_{k_c}$
6. **repeat**
   - 6.1. $m \leftarrow m+1$
   - 6.2. **repeat**
     $n \leftarrow n+1$
     uplink channel:
     - update $\boldsymbol{G}_c^{(n,m-1)}$ and $\boldsymbol{\beta}^{(tmp,tmp)}$ with (11)
     - compute the MSE values $\boldsymbol{\varepsilon}^{\mathrm{UL},(n)}$ with (8)

     downlink channel:
     - compute $\boldsymbol{P}^{(n,m-1)}$ with (10)
     - update $\boldsymbol{F}_c^{(n,m-1)}$ and $\boldsymbol{\beta}_c^{(n,m-1)}$ with (12)
     - compute the MSE values $\boldsymbol{\varepsilon}^{\mathrm{DL}(n)}$ with (1)

     uplink channel:
     - compute $\boldsymbol{Q}^{(n,m-1)}$ with (9) and $\bar{\boldsymbol{Q}}_{k_c}^{(n,m-1)} = \boldsymbol{Q}_{k_c}^{(n,m-1)}/\mathrm{tr}\big(\boldsymbol{Q}_{k_c}^{(n,m-1)}\big)$
     - find optimal user power allocation $\tilde{\boldsymbol{q}}^{(n,m)}$ by solving (23) and compute $\boldsymbol{Q}_{k_c}^{(n,m-1)} = \tilde{q}_{k_c}^{(n,m)}\bar{\boldsymbol{Q}}_{k_c}^{(n,m-1)}$
   - 6.3 **until** required accuracy is reached or $n \geq n_{\max}$
   - 6.4 update $\mathbf{E}_{k_c}^{\mathrm{UL}(m)}, \boldsymbol{F}_{k_c}^{(0,m)}, \boldsymbol{Q}_{k_c}^{(0,m)}$ with (15), $\bar{\boldsymbol{Q}}_{k_c}^{(0,m)} = \boldsymbol{Q}_{k_c}^{(0,m)}/\mathrm{tr}\big(\boldsymbol{Q}_{k_c}^{(0,m)}\big)$, $\boldsymbol{W}_{k_c}^{(m)} = (\mathbf{E}_{k_c}^{\mathrm{UL}(m)})^{-1}$ and do $t = \min_{k_c}\frac{r_{k_c}^{(m)}}{r_{k_c}^{\circ(m-1)}}$, $r_{k_c}^{\circ(m)} = t\, r_{k_c}^{\circ(m-1)}$, and $\xi_{k_c}^{(m)} = d_{k_c} + r_{k_c}^{(m)} - r_{k_c}^{\circ(m)}$
   - 6.5 do $n \leftarrow 0$ and set $(.)^{(n_{\max},m-1)} \rightarrow (.)^{(0,m)}$ in order to re-enter the inner loop
7. **until** required accuracy is reached or $m \geq m_{\max}$

VII. Algorithmic Solution and Simulations

A. Algorithm

The proposed optimization framework is summarized hereafter in Table I. Superscripts $(.)^{(n)}$ and $(.)^{(tmp)}$ denote the nth iteration and a temporary value, respectively. The algorithm is based on a double loop. The inner loop solves the WMSE balancing problem in (7) whereas the outer loop iteratively transforms the WMSE balancing problem into the original rate balancing problem in (2).

B. Proof of Convergence

In case the rate weights r_k° would not satisfy $r_k \geq r_k^\circ$, this issue will be rectified by the scale factor t after one iteration (of the outer loop). It can be shown that $t = \min_{k_c}\frac{r_{k_c}^{(m)}}{r_{k_c}^{\circ(m-1)}} \geq 1$. By contradiction, if this was not the case, it can be shown to lead to $\frac{\mathrm{tr}\left(\boldsymbol{W}_{k_c}^{(m-1)}\mathbf{E}_{k_c}^{(m)}\right)}{\xi_{k_c}^{(m-1)}} > 1,\ \forall k_c$ and hence $\Delta^{(m)} > 1$. But we have

$$\begin{aligned}\Delta^{(m)} &= \frac{\mathrm{tr}\left(\boldsymbol{W}_{k_c}^{(m-1)}\mathbf{E}_{k_c}^{(m)}\right)}{\xi_{k_c}^{(m-1)}},\ \forall k_c, = \max_{k_c}\frac{\mathrm{tr}\left(\boldsymbol{W}_{k_c}^{(m-1)}\mathbf{E}_{k_c}^{(m)}\right)}{\xi_{k_c}^{(m-1)}} \\ &\overset{(a)}{<} \max_{k_c}\frac{\mathrm{tr}\left(\boldsymbol{W}_{k_c}^{(m-1)}\mathbf{E}_{k_c}^{(m-1)}\right)}{\xi_{k_c}^{(m-1)}} = \max_{k_c}\frac{d_{k_c}}{\xi_{k_c}^{(m-1)}} \overset{(b)}{<} 1\,.\end{aligned} \tag{24}$$

Let $\mathbf{E} = \{\mathbf{E}_k, k = 1, ..., |\mathcal{K}|\}$ and $f^{(m)}(\mathbf{E}) = \max_{k_c}\frac{\mathrm{tr}\left(\boldsymbol{W}_{k_c}^{(m-1)}\mathbf{E}_{k_c}\right)}{\xi_{k_c}^{(m-1)}}$. Then (a) is due to the fact that the algorithm in fact performs alternating minimization of $f^{(m)}(\mathbf{E})$ w.r.t. $\{\boldsymbol{G}_c, \boldsymbol{F}_c\}$, $\tilde{\boldsymbol{q}}$ and hence will lead to $f^{(m)}(\mathbf{E}^{(m)}) < f^{(m)}(\mathbf{E}^{(m-1)})$. On the other hand, (b) is due to $\xi_{k_c}^{(m-1)} = d_{k_c} + r_{k_c}^{(m-1)} - r_{k_c}^{\circ(m-1)} > d_{k_c}$, for $m \geq 3$. Hence, $t \geq 1$. Of course, during the convergence $t > 1$. The increasing rate targets $\{r_{k_c}^{\circ(m)}\}$ constantly catch up with the increasing rates $\{r_{k_c}^{(m)}\}$. Now, the rates are upper bounded by the single user MIMO rates (using all power), and hence the rates will converge and the sequence t will converge to 1. That means that for at least one user k_c, $r_{k_c}^{(\infty)} = r_{k_c}^{\circ(\infty)}$. The question is whether this will be the case for all users, as is required for rate balancing. Now, the WMSE balancing leads at every outer iteration m to $\frac{\mathrm{tr}\left(\boldsymbol{W}_{k_c}^{(m-1)}\mathbf{E}_{k_c}^{\mathrm{DL}(m)}\right)}{\xi_{k_c}^{(m-1)}} = \Delta^{(m)}, \forall k_c$. At convergence, this becomes $\frac{d_{k_c}}{\xi_{k_c}^{(\infty)}} = \Delta^{(\infty)}$ where $\xi_{k_c}^{(\infty)} = d_{k_c} + r_{k_c}^{(\infty)} - r_{k_c}^{\circ(\infty)}$. Hence, if we have convergence because for one user k_{c_∞} we arrive at $r_{k_{c_\infty}}^{(\infty)} = r_{k_{c_\infty}}^{\circ(\infty)}$, then this implies $\Delta^{(\infty)} = 1$ which implies $r_{k_c}^{(\infty)} = r_{k_c}^{\circ(\infty)}, \forall k_c$. Hence, the rates will be maximized and balanced.

Remark 1. *In fact, the algorithm also converges with $n_{\max} = 1$, i.e., with only a single loop.*

C. Simulation results

In this section, we numerically evaluate the performance of the proposed algorithm. The simulations are obtained under a channel modeled as follows : $\boldsymbol{H}_{j,k_c}^{\mathrm{H}} = \boldsymbol{B}_{j,k_c}\boldsymbol{\mathcal{U}}_{k_c}\boldsymbol{A}_{k_c}$ where $\boldsymbol{B}_{j,k_c}, \boldsymbol{A}_{k_c}$ are of dimensions $(M_j \times N_{k_c})$ and $(N_{k_c} \times N_{k_c})$ respectively, and have i.i.d. elements distributed as $\mathcal{CN}(0,1)$; $\boldsymbol{\mathcal{U}}_{k_c} = \mu\boldsymbol{U}_{k_c}$, with the normalization parameter $\mu = \left(\mathrm{trace}\big(\boldsymbol{U}_{k_c}\big)\right)^{-1/2}$ and $\boldsymbol{U}_{k_c} = \mathrm{diag}\big\{1, \alpha, \alpha^2, \ldots, \alpha^{N_{k_c}-1}\big\}$ ($\alpha \in \mathbb{R}$ being a scalar parameter). This model allows to control the rank profile of the MIMO channels. For all simulations, we fix $\alpha = 0.3$ and take 1000 channel realisations and $n_{\max} = 20$. The algorithm converges after 4-5 (or 13-15) iterations of m at SNR = 15dB (or 30dB).

Figure 3 plots the minimum achieved per user rate using *i)* our max-min user rate approach with equal priorities and *ii)* the user MSE balancing approach [13] with respect to the Signal to Noise Ratio (SNR). We observe that our approch outperforms significantly the unweighted MSE balancing optimization, and the gap gets larger w.r.t. number of streams.

We observe, in Figure 4, the same behavior with the classical i.i.d. Gaussian channel, but with a smaller gap. Also, we can see that the balanced rate obtained using diagonal $\{\boldsymbol{W}_{k_c}\}$ outperforms the balanced rate derived with non-diagonal weight matrices [11].

In Figure 5, we illustrate how rate is distributed among users according to their priorities represented by the rate targets $r_{k_c}^\circ$. We can see that, using the min-max weighted MSE approach, the rate is equally distributed between the users with equal user priorities, i.e., $r_{k_c}^\circ = r_1^\circ\ \forall k_c$, whereas with different user priorities, the rate differs from one user to another accordingly.

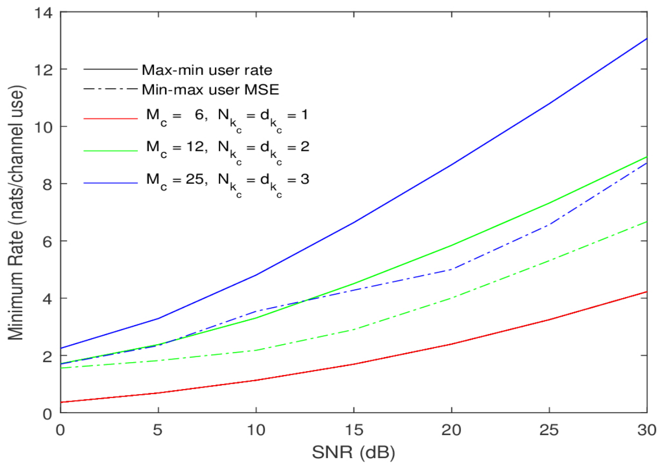

Fig. 3: Minimum rate in the system VS SNR: $C = 2, K_c = 3$.

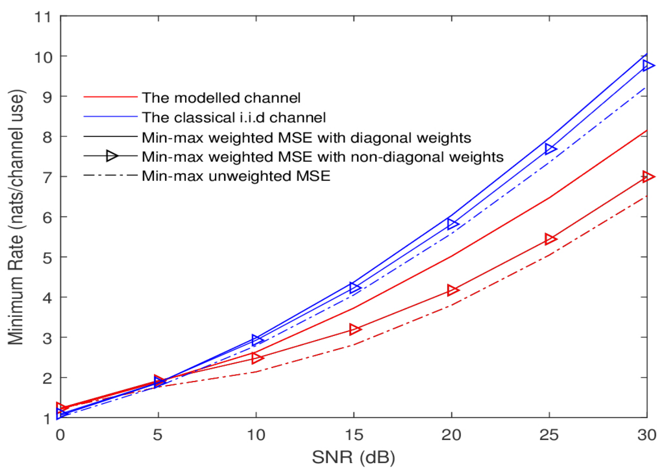

Fig. 4: Minimum rate in the system VS SNR: $C = 1, K_c = 3, M_c = 6, N_{k_c} = d_{k_c} = 2$.

VIII. Conclusions

In this work, we addressed the multiple streams per user case (MIMO links) for which we considered user rate balancing, not stream rate balancing, in multicell downlink channel. Actually, we optimized the rate distribution over the streams of a user, within the rate balancing of the users. In this regard, we proposed an iterative algorithm to balance the rate between the users in a MIMO system. The latter was derived by transforming the max-min rate optimization problem into a min-max weighted MSE optimization problem to enable MSE duality. We also provided comparison between our weighted MSE balancing approach and the min-max unweighted MSE optimization. Simulation results showed that our solution maximizes the minimum rate.

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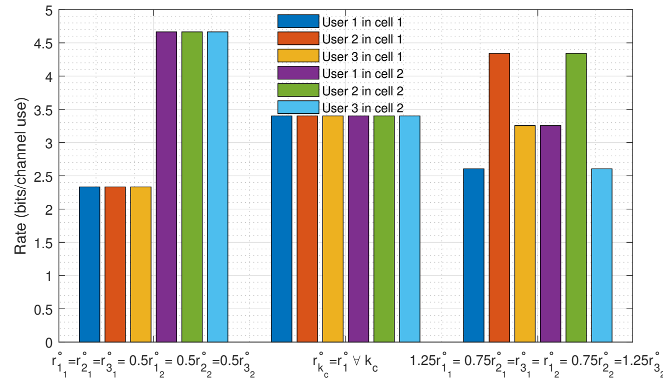

Fig. 5: Rate distribution among users: SNR= 10 dB, $C = 2, K_c = 3, M_c = 12, N_{k_c} = d_{k_c} = 2$.

References

[1] L. Zheng, D. W. H. Cai, and C. W. Tan, "Max-min Fairness Rate Control in Wireless Networks: Optimality and Algorithms by Perron-Frobenius Theory," *IEEE Transactions on Mobile Computing*, Jan. 2018.

[2] D. W. H. Cai, T. Q. S. Quek, C. W. Tan, and S. H. Low, "Max-Min SINR Coordinated Multipoint Downlink Transmission—Duality and Algorithms," *IEEE Trans. Signal Processing*, Oct. 2012.

[3] M. Schubert and H. Boche, "Solution of the Multiuser Downlink Beamforming Problem with Individual SINR Constraints," *IEEE Trans. Vehic. Tech.*, Jun 2004.

[4] W. Yu and T. Lan, "Transmitter Optimization for the Multi-antenna Downlink with Per-antenna Power Constraints," *IEEE Trans. Signal Processing*, June 2007.

[5] L. Zhang, R. Zhang, Y. C. Liang, Y. Xin, and H. V. Poor, "On Gaussian MIMO BC-MAC Duality with Multiple Transmit Covariance Constraints," *IEEE Trans. Inform. Theory*, Apr. 2012.

[6] K. Cumanan, L. Musavian, S. Lambotharan, and A. B. Gershman, "SINR Balancing Technique for Downlink Beamforming in Cognitive Radio Networks," *IEEE Signal Process. Lett.*, Feb. 2010.

[7] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE Transceiver Optimization for Multiuser MIMO Systems: Duality and sum-MSE Minimization," *IEEE Trans. Signal Process*, vol. 55, no. 11, Nov 2007.

[8] ——, "Capacity Balancing for Multiuser MIMO Systems," *Proc. IEEE ICASSP*, Apr. 2007.

[9] R. Hunger, M. Joham, and W. Utschick, "On the MSE-Duality of the Broadcast Channel and the Multiple Access Channel," *IEEE Trans. Signal Processing*, vol. 57, no. 2, p. 698–713, Feb 2009.

[10] M. Razaviyayn, M. Hong, and Z.-Q. Luo, "Linear Transceiver Design for a MIMO Interfering Broadcast Channel Achieving Max-min Fairness," *Proc. Asilomar Conf. Signals, Systems and Computers*, Nov. 2011.

[11] I. Ghamnia, D. Slock, and Y. Yuan-Wu, "Rate Balancing for Multiuser MIMO Systems," *IEEE 20th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Jul 2019.

[12] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, "An Iteratively Weighted MMSE Approach to Distributed Sum-Utility Maximization for a MIMO Interfering Broadcast Channel," *IEEE Trans. Signal Processing*, Sept. 2011.

[13] S. Shi, M. Schubert, and H. Boche, "Downlink MMSE Transceiver Optimization for Multiuser MIMO Systems: MMSE Balancing," *IEEE Trans. Signal Processing*, vol. 56, no. 8, pp. 606–619, Aug 2008.

[14] S. S. Christensen, R. Agarwal, E. de Carvalho, and J. M. Cioffi, "Weighted Sum-Rate Maximization using Weighted MMSE for MIMO-BC Beamforming Design," *IEEE Trans. Wireless Comm's*, Dec. 2008.