Weighted Sum Rate Maximization for Hybrid Beamforming Design in Multi-Cell Massive MIMO OFDM Systems

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***Abstract*—This work deals with hybrid beamforming (HBF) for the MIMO Interfering Broadcast Channel (IBC), i.e. the Multi-Input Multi-Output (MIMO) Multi-User (MU) Multi-Cell downlink channel, in an orthogonal frequency-division multiplexing (OFDM) system. While most of the existing works on wideband hybrid systems focus on single-user systems and a few on multi-user single-cell systems, we consider HBF design for OFDM systems in the case of multi-cell. We optimize the weighted sum rate (WSR) using minorization and alternating optimization, the result of which is observed to converge fast to a local optimum. We furthermore propose a deterministic annealing based approach to avoid issues of local optima that plague phase shifter constrained analog beamformers. Simulation results indicate that the proposed deterministic annealing based approach performs significantly better than state of the art Weighted Sum Mean Squared Error (WSMSE) or WSR based solutions in a wideband OFDM setting. We also propose a closed form solution for the frequency flat analog BF in case the number of RF chains equals or exceeds the total number of multipath components and the antenna array responses are phasors.**

I. Introduction

In this paper, Tx may denote transmit/transmitter/transmission and Rx may denote receive/receiver/reception. Hybrid beamforming (HBF) is a two-stage architecture in which the BF is constructed by concatenation of a low-dimensional precoder (digital BF) and an analog BF, with the number of RF chains being less than the number of antennas. This technique was first introduced in [1], with the analog precoder implemented using phase shifters. Hybrid precoding designs for single user systems can be found in [2], [3]. The authors in [2] propose near-optimal solutions based on the formulation of sparse signal recovery for a single user mmWave system.

Hybrid beamforming designs for multi-user systems can be found in [4]–[9]. In [5], [10] the authors propose a WSMSE based approach for the joint design of digital and analog beamformers for multi-user MIMO system. In a very recent paper [8], analog beamformer is designed using average channel statistics and switches (designed using instantaneous channel knowledge) are used to select the analog beams.

Prior work for the design of HBF design for OFDM systems can be found in [11]–[14]. In [13], for a Multi-User system, an iterative algorithm is proposed for the hybrid beamformer in which the digital beamformer is derived using the WSMSE approach which is optimal. But for the analog beamformer a suboptimal method is introduced based on certain approximations for the WSR. The analog precoder design is based only on a quadratic sum of the channel matrices across all the subcarriers. In [11], again for a MU-MISO system, the hybrid beamforming design is based on the QR decomposition of the all-digital beamformer. The authors also derive the number of RF chains and phase shifting components required to realize a hybrid beamformer which is equivalent to an all-digital beamformer. In [15], a multi-beam transmission diversity scheme is proposed for an OFDM system. The analog beamformer is chosen on the basis of the beam steering angle which maximizes the sum rate.

The main issue with WSR/WSMSE optimization for a HBF hybrid design is the high non-convexity of the cost function. This implies that even if it is possible to show convergence to a local optimum [10], convergence to the global optimum cannot be guaranteed.

A. Contributions of this paper

- We first propose an HBF design based on the WSR criterion which is simplified using the minorization approach. The advantage compared to the WSMSE solution [10] is that the iterative algorithm converges faster (no ping-pong between Tx and Rx optimization, and direct power optimization). Compared to our previous work [16] which was for a narrowband system, we consider a wideband OFDM system in this paper and furthermore provide a convergence proof for the minorization algorithm.
- To achieve optimal fully digital performance, we show that the number of RF chains can be as small as the total number of multi-paths across all the users (thanks to the sparsity of the mmWave channels) and it doesn't depend on the number of antennas or the sub-carriers. Using this result, we explain why a frequency flat precoding for analog BF may be sufficient to guarantee optimal performance.
- Numerical results (provided for multi-cell systems also) suggest that the proposed deterministic annealing (DA) based HBF design allows to narrow the gap to optimal fully digital solutions [17], [18].

Notation: In the following, boldface lower-case and upper-case characters denote vectors and matrices respectively. the operators $\mathrm{E}(\cdot)$, $\mathrm{tr}\{\cdot\}$, $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^*$ represent expectation, trace, conjugate transpose, transpose and complex conjugate respectively. $\mathbf{V}_{max}(\mathbf{A},\mathbf{B})$ or $\mathbf{V}_{1:d_k}(\mathbf{A},\mathbf{B})$ represents (normalized) dominant generalized eigenvector or the matrix formed by the (normalized) d_k dominant generalized eigenvectors of $\mathbf{A}$ and $\mathbf{B}$. $\mathbf{\Sigma}_{1:d_k}(\mathbf{A},\mathbf{B})$ represent the diagonal matrix with d_k generalized eigen values. $\mathbf{x} = vec(\mathbf{X})$ represents the vector obtained by stacking each of the columns of $\mathbf{X}$ and $unvec(\mathbf{x})$ represents the inverse operation of $vec(.)$. $\mathbf{I}_N$ or $\mathbf{I}$ represents the identity matrix of size N or with appropriate dimensions. $\mathrm{diag}\,(\mathbf{x})$ represents the diagonal matrix obtained by the vector $\mathbf{x}$ as its entries.

II. Multi-User MIMO System Model

In this paper we shall consider a multi-stream approach with d_k streams for user k. So, consider an Interfering BroadCast Channel (IBC) (i.e. multi-cell MU downlink) OFDM system of C cells with a total of K users, with per-BS power constraint P_C and N_t^c transmit antennas in cell c. N_s represents the total number of subcarriers which is shared across all the users. User k is equipped with N_k antennas. $\mathbf{H}_{k,c}[n]$ represents the $N_k \times N_t^c$ MIMO channel between user k and BS c and we define $\mathrm{E}(\mathbf{H}_{k,c}^H[n]\mathbf{H}_{k,c}[n]) = \mathbf{\Theta}_k^c[n]$. n represents the subcarrier index throughout the paper. It is important to emphasize here that the analog precoder is assumed to be frequency flat (same for all subcarriers) and digital precoder to be frequency selective. Note that we consider the Rx to be a fully digital system since N_k is not very high at the UE. User k receives

$$\begin{aligned}\mathbf{y}_k[n] = &\mathbf{H}_{k,b_k}[n]\mathbf{V}^{b_k}\,\mathbf{G}_k[n]\,\mathbf{s}_k[n] + \\ &\sum_{i\neq k}\mathbf{H}_{k,b_i}[n]\mathbf{V}^{b_i}\,\mathbf{G}_i[n]\,\mathbf{s}_i[n] + \mathbf{v}_k[n],\end{aligned} \tag{1}$$

where $\mathbf{s}_k[n]$, of size $d_k \times 1$, is the intended signal stream vector (all entries are white, unit variance). b_i refers to the serving base station of user i. BS c serves U_c users and $K = \sum_{c=1}^{C} U_c$. We are considering a noise whitened signal representation so that we get for the noise $\mathbf{v}_k \sim \mathcal{CN}(0, \mathbf{I}_{N_k})$ (circularly complex Gaussian random vector). The analog beamformer $\mathbf{V}^c$ for base station c is of dimension $N_t^c \times M^c$ where M^c is the number of RF chains at BS c. The $M^c \times d_k$ digital beamformer is $\mathbf{G}_k[n]$, where $\mathbf{G}_k[n] = \left[\mathbf{g}_k^{(1)}[n] \;...\; \mathbf{g}_k^{(d_k)}[n]\right]$ and $\mathbf{g}_k^{(s)}[n]$ represents the beamformer for stream s of user k.

A. Channel Model

In this sub-section, we omit the user and cell indices for simplicity. We consider a geometric channel model for a mmWave propagation environment [19] with L_s scattering clusters and L_r scatterers or rays in each cluster. The delay-d MIMO channel can be written as,

$$\mathbf{H}_d = \sum_{s=1}^{L_s}\sum_{l=1}^{L_r} \alpha_{s,l}\mathbf{h}_r(\theta_{s,l})\mathbf{h}_t(\phi_{s,l})^H p(dT_s - \tau_s - \tau_{rl}) \tag{2}$$

Here $\theta_{s,l}, \phi_{s,l}$ represent the angle of arrival (AoA) and angle of departure (AoD) respectively for the l^{th} path in the s^{th} cluster. $\mathbf{h}_r(\cdot), \mathbf{h}_t(\cdot)$ represent the antenna array responses at Rx and Tx respectively. The complex path gain, $\alpha_{s,l} \sim \mathcal{CN}(0, \frac{N_t N_r}{L_s L_r})$ and $p(\tau)$ represents the band-limited pulse shaping filter response evaluated at τ seconds. Each cluster has a time delay $\tau_s \in \mathcal{R}$ and each ray $l = 1, ..L_r$ has a relative time delay τ_{rl}. Now, we write the channel in the subcarrier n as, $\mathbf{H}[n] = \sum_{d=1}^{D}\mathbf{H}_d e^{-j2\pi\frac{nd}{N_s}}$. In a more compact form, this can be represented as,

$$\mathbf{H}[n] = \mathbf{H}_r \sum_{d=1}^{D}\mathbf{A}_d[n]\mathbf{H}_t^H, \;\text{ where} \tag{3}$$

where $\mathbf{H}_r = [\mathbf{h}_r(\theta_{1,1}), ..., \mathbf{h}_r(\theta_{L_s,L_r})]$, $\mathbf{H}_t = [\mathbf{h}_t(\phi_{1,1}), ..., \mathbf{h}_t(\phi_{L_s,L_r})]$, $\mathbf{A}_d[n] = \mathrm{diag}\,(\alpha_{1,1}p(dTs - \tau_1 - \tau_{r1}), ..., \alpha_{L_s,L_r}p(dTs - \tau_{L_s} - \tau_{rL_r}))e^{-j2\pi\frac{nd}{N_s}}$. Note that our HBF design which follows, is applicable for general MIMO channel models and the channel model outlined here is utilized for the simulations in Section VI. Another remark here is that, even though for an HBF system, at the baseband we have access to only the low-dimensional effective channel, i.e. the propagation channel combined with the analog precoder, it is still possible to estimate the individual components in a pathwise channel model as we consider here, for e.g. [3], [20]. The solution of [3] is based on a hierarchical multi-resolution codebook and beamtraining.

III. WSR Maximization via Minorization and Alternating Optimization

Consider the optimization of the HBF design using WSR maximization of the Multi-cell MU-MIMO OFDM system:

$$\begin{aligned}[\mathbf{V}\;\;\mathbf{G}] &= \arg\max_{\mathbf{V},\mathbf{G}} WSR(\mathbf{G},\mathbf{V}) \\ &= \arg\max_{\mathbf{V},\mathbf{G}} \sum_{k=1}^{K} u_k \sum_{n=1}^{N_s} \ln\det(\mathbf{R}_{\overline{k}}[n]^{-1}\mathbf{R}_k[n]),\end{aligned} \tag{4}$$

where the u_k are the rate weights, $\mathbf{G}$ represents the collection of digital BFs $\mathbf{G}_k[n]$, $\mathbf{V}$ the collection of analog BFs $\mathbf{V}^{b_k}$. We define $\mathbf{Q}_i[n] = \mathbf{V}^{b_i}\mathbf{G}_i[n]\mathbf{G}_i[n]^H\mathbf{V}^{b_i\,H}$. From [17], [18], we can write,

$$\begin{aligned}\mathbf{R}_{\overline{k}}[n] &= \sum_{i=1, i\neq k}^{K}\mathbf{H}_{k,b_i}[n]\mathbf{Q}_i[n]\mathbf{H}_{k,b_i}^H[n] + \mathbf{I}_{N_k}, \\ \mathbf{R}_k[n] &= \sum_{i=1}^{K}\mathbf{H}_{k,b_i}[n]\mathbf{Q}_i[n]\mathbf{H}_{k,b_i}^H[n] + \mathbf{I}_{N_k},\end{aligned} \tag{5}$$

where $\mathbf{R}_{\overline{k}}[n]$ is the interference plus noise covariance matrix.With the definition of the Tx covariance matrices $\mathbf{Q}_i[n]$, the power constraints can be written as,

$$\sum_{k:b_k=c}\sum_{n=1}^{N_s}\mathrm{tr}\left\{\mathbf{Q}_k[n]\right\} \leq P_c\,. \tag{6}$$

The WSR problem is non-concave in the $\mathbf{Q}_k[n]$ due to the interference terms. Therefore finding the global optimum is challenging. In order to render a feasible solution, we consider the difference of convex functions (DC programming) approach as in [21] in which the WSR is written as the summation of a convex and a concave term. Consider the dependence of the WSR on $\mathbf{Q}_k[n]$ alone:

$$WSR(\mathbf{G},\mathbf{V}) = u_k \ln\det(\mathbf{R}_{\overline{k}}[n]^{-1}\mathbf{R}_k[n]) + WSR_{\overline{k}}[n] + \sum_{s=1,s\neq n}^{N_s}\sum_{k=1}^{K} u_k \ln\det(\mathbf{R}_{\overline{k}}[s]^{-1}\mathbf{R}_k[s]),$$
$$WSR_{\overline{k}}[n] = \sum_{i=1,\neq k}^{K} u_i \ln\det(\mathbf{R}_{\overline{i}}[n]^{-1}\mathbf{R}_i[n]), \tag{7}$$

where $\ln\det(\mathbf{R}_{\overline{k}}[n]^{-1}\mathbf{R}_k[n])$ is concave in $\mathbf{Q}_k[n]$, $WSR_{\overline{k}}[n]$ is convex in $\mathbf{Q}_k[n]$ and the third summation across subcarriers other than n is independent of $\mathbf{Q}_k[n]$. Since a linear function is simultaneously convex and concave, consider the first order Taylor series expansion of $WSR_{\overline{k}}[n]$ in $\mathbf{Q}_k[n]$ around $\widehat{\mathbf{Q}}[n]$ (i.e. all $\widehat{\mathbf{Q}}_i[n]$, $\widehat{\mathbf{R}}_{\overline{i}}[n]$, $\widehat{\mathbf{R}}_i[n]$ corresponds to $\widehat{\mathbf{Q}}_i[n]$).

$$\underline{WSR_{\overline{k}}}[n](\mathbf{Q}_k[n],\widehat{\mathbf{Q}}[n]) \approx WSR_{\overline{k}}[n](\widehat{\mathbf{Q}}_k[n],\widehat{\mathbf{Q}}[n]) - \text{tr}\left\{(\mathbf{Q}_k[n]-\widehat{\mathbf{Q}}_k[n])\widehat{\mathbf{A}}_k[n]\right\},$$
$$\widehat{\mathbf{A}}_k[n] = -\left.\frac{\partial WSR_{\overline{k}}[n](\mathbf{Q}_k[n],\widehat{\mathbf{Q}}[n])}{\partial \mathbf{Q}_k[n]}\right|_{\widehat{\mathbf{Q}}_k[n],\widehat{\mathbf{Q}}[n]}$$
$$= \sum_{i=1,\neq k}^{K} u_i\, \mathbf{H}_{i,b_k}^H[n]\left(\widehat{\mathbf{R}}_{\overline{i}}[n]^{-1} - \widehat{\mathbf{R}}_i[n]^{-1}\right)\mathbf{H}_{i,b_k}[n]\,. \tag{8}$$

Note that the linearized tangent expression $\underline{WSR_{\overline{k}}}[n]$ constitutes a (touching) lower bound for $WSR_{\overline{k}}[n]$ via $-\text{tr}\{\mathbf{R}^{-1}[n]\boldsymbol{\Delta}\} \leq -\ln\det(\mathbf{R}^{-1}[n](\mathbf{R}[n]+\boldsymbol{\Delta}))$ and $\mathbf{R}_k[n] \geq \mathbf{R}_{\overline{k}}[n]$. Hence the DC approach is also a minorization approach [22], regardless of the (re)parameterization of $\mathbf{Q}$. Now, dropping constant terms, reparameterizing the $\mathbf{Q}_k[n] = \mathbf{V}^{b_k}\mathbf{G}_k[n]\mathbf{G}_k[n]^H\mathbf{V}^{b_k\,H}$, performing this linearization for all users and across all subcarriers, we get the Lagrangian after including the Tx power constraints,

$$\mathcal{L}(\mathbf{G},\mathbf{V},\boldsymbol{\Lambda}) = \sum_{k=1}^{K}\sum_{n=1}^{N_s}[u_k \ln\det(\mathbf{I} + \mathbf{G}_k^H[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k[n])$$
$$-\text{tr}\{\mathbf{G}_k^H[n]\mathbf{V}^{b_k\,H}(\widehat{\mathbf{A}}_k[n]+\lambda_{b_k}\mathbf{I})\mathbf{V}^{b_k}\mathbf{G}_k[n]\}] + \sum_{j=1}^{C}\lambda_j P_j, \tag{9}$$

where $\widehat{\mathbf{B}}_k[n] = \mathbf{H}_{k,b_k}^H[n]\widehat{\mathbf{R}}_{\overline{k}}^{-1}[n]\mathbf{H}_{k,b_k}[n]$. $\boldsymbol{\Lambda}$ represents the set of Lagrange multipliers λ_c. It can be verified that the summation across all the subcarriers after the DC approximation is still a minorizer of the original WSR using the same argument as before for a single subcarrier. In what follows, we shall optimize the WSR with perfect CSIT by alternating optimization between digital and analog beamformers.

A. Digital BF Design

By Hadamard's inequality [23, p. 233], it can be seen that for the maximization problem above, $\mathbf{G}_k^H[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k[n]$ should be diagonal and thus maximizing w.r.t $\mathbf{G}_k$ leads to the following eigen vector condition. The gradient w.r.t. $\mathbf{G}_k[n]$ of (9) (which is still the same as that of (4)) leads to the solution as d_k dominant generalized eigenvectors,

$$\mathbf{G}_k^{'}[n] = \mathbf{V}_{1:d_k}(\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}, \mathbf{V}^{b_k\,H}(\widehat{\mathbf{A}}_k[n]+\lambda_{b_k}\mathbf{I})\mathbf{V}^{b_k}), \tag{10}$$

with associated generalized eigenvalues $\boldsymbol{\Sigma}_k[n] = \boldsymbol{\Sigma}_{1:d_k}(\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}, \mathbf{V}^{b_k\,H}(\widehat{\mathbf{A}}_k[n]+\lambda_{b_k}\mathbf{I})\mathbf{V}^{b_k})$. Let $\boldsymbol{\Sigma}_k^{(1)}[n] = \mathbf{G}_k^{'\,H}[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k^{'}[n]$ and $\boldsymbol{\Sigma}_k^{(2)}[n] = \mathbf{G}_k^{'\,H}[n]\mathbf{V}^{b_k\,H}\widehat{\mathbf{A}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k^{'}[n]$. Intuitively, (10) represents a compromise between increasing the signal part and reducing the interference. The advantage of formulation (9) is that it allows straightforward power adaptation: introducing stream powers in the diagonal matrices $\mathbf{P}_k[n] \geq 0$ and substituting $\mathbf{G}_k[n] = \mathbf{G}_k^{'}[n]\,\mathbf{P}_k^{\frac{1}{2}}[n]$ in (9) yields the following interference leakage aware water filling (WF) (jointly for the $\mathbf{P}_k[n]$ and λ_c)

$$\mathbf{P}_k[n] = (u_k(\boldsymbol{\Sigma}_k^{(2)}[n] + \lambda_{b_k}\mathbf{V}^{b_k\,H}\mathbf{V}^{b_k})^{-1} - \boldsymbol{\Sigma}_k^{-(1)}[n])^+, \tag{11}$$

where $(\mathbf{X})^+$ denotes the positive semi-definite part of Hermitian $\mathbf{X}$ and the Lagrange multipliers are adjusted to satisfy the power constraints. This can be done by bisection and gets executed per BS.

B. Design of Analog BF

At first we consider the case in which the analog BF is unconstrained. Hence the resulting design would also be applicable to more general two-stage BF design [24] in which the outer BF stage ($\mathbf{V}^c$) is in common to all users in a cell.

To optimize $\mathbf{V}^c$, we equate the gradient of (9) w.r.t. $\mathbf{V}^c$ to zero. Using $\partial \ln\det\mathbf{X} = \text{tr}(\mathbf{X}^{-1}\partial\mathbf{X})$ and $\det(\mathbf{I}_M + \mathbf{A}\mathbf{B}) = \det(\mathbf{I}_N + \mathbf{B}\mathbf{A})$ from [25], we get

$$\sum_{k:b_k=c}\sum_{n=1}^{N_s}(\widehat{\mathbf{B}}_k[n]\mathbf{V}^c\mathbf{G}_k[n]\boldsymbol{\zeta}_k[n]\mathbf{G}_k^H[n] - (\widehat{\mathbf{A}}_k[n]+\lambda_c\mathbf{I})\mathbf{V}^c\mathbf{G}_k[n]\mathbf{G}_k^H[n]) = 0,$$
$$\text{with } \boldsymbol{\zeta}_k[n] = u_k(\mathbf{I} + \mathbf{G}_k^H[n]\mathbf{V}^{b_k H}\widehat{\mathbf{B}}_k[n]\mathbf{V}^{b_k}\mathbf{G}_k[n])^{-1} \tag{12}$$

Now with $vec(\mathbf{A}\mathbf{X}\mathbf{B}) = (\mathbf{B}^T\otimes\mathbf{A})vec(\mathbf{X})$ [25], where $\otimes$ represents the Kronecker product between the two matrices, we get

$$\mathbf{V}^c = unvec(\mathbf{V}_{max}(\mathbf{B}_c[n],\mathbf{A}_c[n])),\ \text{with}$$
$$\mathbf{B}_c[n] = \sum_{k:b_k=c}\sum_{n=1}^{N_s}(\mathbf{G}_k[n]\boldsymbol{\zeta}_k[n]\mathbf{G}_k^H[n])^T\otimes\widehat{\mathbf{B}}_k[n],$$
$$\mathbf{A}_c[n] = \sum_{k:b_k=c}\sum_{n=1}^{N_s}(\mathbf{G}_k[n]\mathbf{G}_k^H[n])^T\otimes(\widehat{\mathbf{A}}_k[n]+\lambda_c\mathbf{I})\,. \tag{13}$$

We emphasize here that the extension to the partially connected HBF architecture is quite straightforward and we include the comparison of both in section VI.

C. Algorithm Convergence

The convergence proof follows in the same direction as in [26]. For the WSR cost function $WSR(\mathbf{Q})$ in (7) we construct the minorizer as in (8), (9) leading to

$$WSR(\mathbf{Q}) \geq \underline{WSR}(\mathbf{Q},\widehat{\mathbf{Q}}) = \sum_{k=1}^{K}\sum_{n=1}^{N_s}[u_k \ln\det(\mathbf{I}+\widehat{\mathbf{B}}_k[n]\mathbf{Q}_k[n]) - \text{tr}\{\widehat{\mathbf{A}}_k[n](\mathbf{Q}_k[n]-\widehat{\mathbf{Q}}_k[n])\}], \tag{14}$$

where $\underline{WSR}(\widehat{\mathbf{Q}},\widehat{\mathbf{Q}}) = WSR(\widehat{\mathbf{Q}})$. The minorizer, which is concave in $\mathbf{Q}$, still has the same gradient as $WSR(\widehat{\mathbf{Q}})$ and hence KKT conditions are not affected. Now reparameterizing $\mathbf{Q}$ in terms of $\mathbf{P},\mathbf{G}',\mathbf{V}$ as in (5) and adding the power constraints to the minorizer, we get the Lagrangian (9). Every alternating update of $\mathcal{L}$ w.r.t. $\mathbf{V}$, $\mathbf{G}'$, or $(\mathbf{P},\boldsymbol{\Lambda})$ leads to an increase of the WSR, ensuring convergence (within each of these 3 parameter goups, we further alternate between each user or BS). For the KKT conditions, at the convergence point, the gradients of $\mathcal{L}$ w.r.t. $\mathbf{V}$ or $\mathbf{G}'$ correspond to the gradients of the Lagrangian of the original WSR. For fixed $\mathbf{V}$ and $\mathbf{G}'$, $\mathcal{L}$ is concave in $\mathbf{P}$, hence we have strong duality for the saddle point $\max_{\mathbf{P}} \min_{\mathbf{\Lambda}} \mathcal{L}$. Also, at the convergence point the solution to $\min_{\mathbf{\Lambda}} \mathcal{L}(\mathbf{V}^o, \mathbf{G}'^o, \mathbf{P}^o, \mathbf{\Lambda})$ satisfies the gradient KKT condition for $\mathbf{P}$ and the complementary slackness conditions for $c = 1, \ldots, C$

$$\lambda_c^o \, (P^c - \sum_{k:b_k=c} \sum_{n=1}^{N_s} \text{tr}\{\mathbf{V}^{co}\mathbf{G}_k'^o[n]\mathbf{P}_k^o[n]\mathbf{G}_k'^{o\,H}[n]\mathbf{V}^{co\,H}\} = 0, \tag{15}$$

where all individual factors in the products are nonnegative. In the proposed approach, $g(\mathbf{\Lambda}|\mathbf{V}, \mathbf{G}') = \max_{\mathbf{P}} \mathcal{L}(\mathbf{V}, \mathbf{G}', \mathbf{P}, \mathbf{\Lambda})$.

IV. HBF with Fully Digital Performance

In this section we analyze to what extent a hybrid BF can achieve the same performance as a fully digital BF in a wideband OFDM system. In particular we shall see that this is possible for a sufficient number of RF chains and with the antenna array responses being phasors. For notational simplicity we shall consider a uniform $L = L_s L_r$ and $N_k = N_r, \forall k, N_t^c = N_t, M^c = M, \forall c$. Let the antenna array response for BS c be $\mathbf{h}_t^c(\phi)$ for Angle of Departure (AoD) ϕ. We assume that all entries of $\mathbf{h}_t^c(\phi)$ have the same magnitude. The concatenated antenna array response matrix to all users can be written as, $\overline{\mathbf{H}}_t^c = \left[\mathbf{H}_{t,1}^c \; \mathbf{H}_{t,2}^c \; ... \; \mathbf{H}_{t,K}^c\right]$, of dimension $N_t \times N_p$, where we denote the total number of paths $N_p = LK$. We define $\mathbf{A}_{d,k}^c[n]$ as the diagonal path amplitude matrix for the channel from BS c to user k for subcarrier n. Similarly we define $\overline{\mathbf{H}}_r^c$ and $\overline{\mathbf{A}}^c[n] =$ diag $(\sum_{d=1}^{D} \mathbf{A}_{d,1}^c[n], ..., \sum_{d=1}^{D} \mathbf{A}_{d,K}^c[n])$ for the concatenated Rx antenna array responses and complex path amplitudes. $\overline{\mathbf{A}}^c[n]$ is a $N_p \times N_p$ block diagonal matrix with blocks of size $L \times L$ and $\overline{\mathbf{H}}_r^c$ is a $KN_r \times N_p$ block diagonal matrix with blocks of size $N_r \times L$. Finally, we can write the $KN_r \times N_t$ MIMO channel from BS c to all a users as $\mathbf{H}^{c\,H}[n] = \overline{\mathbf{H}}_t^c \, \overline{\mathbf{A}}^{c\,H}[n] \, \overline{\mathbf{H}}_r^{c\,H}$.

Theorem 1. *For a multi-cell MU MIMO OFDM system with $M \geq N_p$ and phasor antenna responses, to achieve optimal all-digital precoding performance, the analog beamformer can be chosen as the Tx side concatenated antenna array response and thus frequency flat.*

Proof: From [18], the optimal all-digital beamformer for any subcarrier n is of the form

$$\begin{aligned}&(\mathbf{H}^{c\,H}[n]\mathbf{D}_1^c[n]\mathbf{H}^c[n] + \lambda_c\mathbf{I})^{-1}\mathbf{H}^{c\,H}[n]\mathbf{D}_2^c[n] \\ &= \mathbf{H}^{c\,H}\mathbf{B}^c = \overline{\mathbf{H}}_t^c \; \overline{\mathbf{A}}^{c\,H}[n] \, \overline{\mathbf{H}}_r^{c\,H} \, \mathbf{B}^c[n],\end{aligned} \tag{16}$$

where $\mathbf{B}^c[n] = \left(\lambda_c\mathbf{I} + \mathbf{D}_1^c[n]\mathbf{H}^c[n]\mathbf{H}^{c\,H}[n]\right)^{-1}\mathbf{D}_2^c[n]$, $\mathbf{D}_1^c[n]$, $\mathbf{D}_2^c[n]$ are block diagonal matrices and we used the identity $(\mathbf{I}+\mathbf{XY})^{-1}\mathbf{X} = \mathbf{X}(\mathbf{I}+\mathbf{YX})^{-1}$. Under the Theorem assumptions we can then separate the BFs as

$$\mathbf{V}^c = \overline{\mathbf{H}}_t^c, \;\; \mathbf{G}^c[n] = \overline{\mathbf{A}}^{c\,H}[n]\overline{\mathbf{H}}_r^{c\,H}\mathbf{B}^c[n]\,. \tag{17}$$

Hence $\mathbf{V}$ depends only on the Tx antenna array responses. □
Note that whereas the digital BF $\mathbf{G}$ in (17) is a function of the instantaneous CSIT, the analog BF $\mathbf{V}^c$ is only a function of AoDs, hence only of the slow fading channel components. So it is independent of the subcarrier number and this explains why it is optimal to consider a frequency flat design for analog BF. Also, while the spatial angles in antenna array responses may include a frequency dependency called as beam-squint in the literature [27], we don't consider this factor at the moment.

Further we consider the design of phase shifter constrained analog BF in the case when $M < N_p$. In order to avoid the local optima issues with alternating optimization of each of the phasor elements, we proposed DA for analog BF design for narrowband systems in [16] and we refer the reader for a more detailed discussion on this to our paper.

In the below table Algorithm 1, we describe in detail the HBF algorithm which combines minorization and DA.

Algorithm 1 Minorization and DA based HBF design

Given: $P_c, \mathbf{H}_{k,c}[n], u_k \, \forall k, c, n$, b is a constant < 1, say 0.9.
Initialization: $\mathbf{V}^c = e^{j\angle \mathbf{V}_{1:M^c}(\sum_{k:b_k=c} \mathbf{\Theta}_k^c[n], \sum_{i:b_i \neq c} \mathbf{\Theta}_i^c[n])}$,
The $\mathbf{G}_k^{(0)}[n]$ are taken as the ZF precoders for the effective channels $\mathbf{H}_{k,b_k}[n]\mathbf{V}^{b_k}$ with uniform powers. **Iteration** (j) :

1) Compute $\widehat{\mathbf{B}}_k[n], \widehat{\mathbf{A}}_k[n], \, \forall k, n$ from (8), (9).
2) Update $\mathbf{G}_k^{'(j)}[n]$ from (10), and $\mathbf{P}_k[n]$ from (11), $\forall k, n$.
3) Update $(\mathbf{V}_{p,q}^c)^{(j)}$, $\forall c, \forall (p,q)$, using DA (phasor constrained) or from (13) (unconstrained).
4) Check for convergence of the WSR: if not go to step 1).
5) Scale $\forall \, (i,j) : |\mathbf{V}_{i,j}^c| \leftarrow e^{b \ln |\mathbf{V}_{i,j}^c|}$ ($\mathbf{V}_{i,j}^c = |\mathbf{V}_{i,j}^c| e^{j\theta_{i,j}^c}$).
6) Reoptimize all $\theta_{i,j}^c$ and all digital BFs using 1)-4).
7) Update stream powers and Lagrange multipliers.
8) Go to 5) for a number of iterations.
9) Finally redo 6)-7) a last time with all $|\mathbf{V}_{i,j}^c| = 1$ in 5).

V. Simulation Results

Monte-Carlo simulations are carried out to validate the performance of the proposed HBF algorithms for a single cell and multi-cell system (Figure 2) with K single antenna users and for an OFDM system with $N_s = 32$ subcarriers. We use the pathwise channel model in (3). We consider a Uniform Linear Array (ULA) of antennas with $\mathbf{h}_{t,k}(\phi_{c,l})$, the AoD $\phi_{c,l}$ are assumed to be uniformly distributed in the interval $[0^o, 30^o]$. For the multi-cell case in Figure 2, the parameters used are the same for both the cells, i.e. $M^1 = M^2 = M, N_t^1 = N_t^2 = N_t, U_1 = U_2 = K/2, L_s = 1, L_r = 4, L = L_s L_r$. Furthermore, we consider the case in which the number of RF chains $M < LK$ (with local optima issues). Notations used in the figure: ABF refers to the analog BF. EV phasors refers to the matrix generated by the phases of the dominant eigen vectors of the sum of the channel covariance matrices of all users. We compare the performance of the proposed algorithms with the optimal fully digital BF [18] (referred to as "Optimal Fully Digital"), approximate WSR based hybrid design [13] (referred to as "HBF with ABF based on Channel Average". For the multi-cell version of [13], channel average with only the direct user channels in a cell are considered. "HBF with Alternating Optimization of Phasors" refers to our own algorithm in this paper where the analog phasors are updated as in Section III.C.

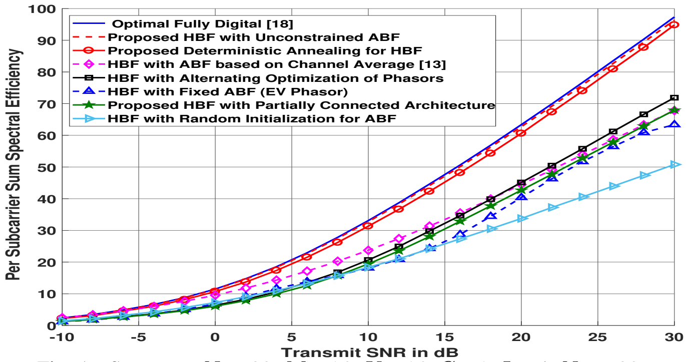

Fig. 1. Sum rate, $N_t = 32, M = 16, K = 16, C = 1, L = 4, N_s = 32$.

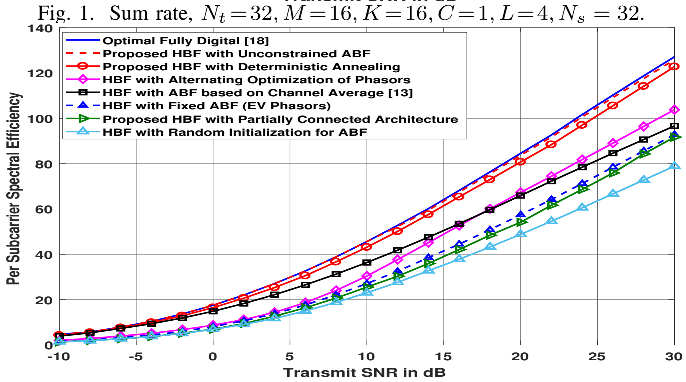

Fig. 2. Sum rate, $N_t = 64, M = 16, K = 16, C = 2, L = 4, N_s = 32$.

It is clear from Figure 1 and 2 that the proposed unconstrained HBF solution has the same performance as the fully digital solution. With phase shifter constrained analog precoder, the proposed DA based design narrows the gap to the fully digital performance and performs much better than state of the art solutions such as WSMSE which suffer from the issue of local optima for analog phasors. Also, it is evident that the performance degrades for a partially connected architecture compared to the fully connected system. One remark here is that the complexity of the proposed HBF design is $O(N_t^3 N_{it})$, where N_{it} is the number of iterations required to converge.

VI. Conclusion

In this paper, we derived and presented an optimal BF algorithm for the HBF scenario in a Multi-cell MU-MIMO OFDM system. An iterative algorithm is obtained which jointly optimizes both analog and digital beamformers. Convergence of the alternating minorization approach was shown and adding deterministic annealing allowed to attain the global optimum. Simulation results indicate that the resulting global optimum is much better than typical local optima and that the thus optimized HBF performance can be very close to the optimal fully digital performance.

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