Multiple-Accessing over Frequency-Selective Fading Channels

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Abstract-This work considers the transmission of information from many independent sources to a common receiver over a channel impaired by multipath propagation. In cellular radio communications this is the case of the uplink.

We start by examining the achievable rate region of the multiuser frequency-selective fading channel without knowledge of the channel on the transmission end. It has been shown that SSMA (*Spread Spectrum Multiple Access*) is theoretically capable of higher data rates than FDMA (*Frequency Division Multiple Access*) or *slow frequency-hopping*[1]. When the average received power for all the users is equal, which corresponds to a perfect slow power control, we show that the maximum spectral efficiency of SSMA exceeds that of FDMA or slow frequency-hopping by .5772 *nats/s/Hz* for many users and Rayleigh fading.

Also, we formulate the optimal multiple-access scheme when all the channels are known to the transmitters. In turns out that only one user should transmit at any given frequency. Moreover, the input power spectra for the transmitters are water-filling formulae both in frequency and time. It is shown that the spectral efficiency for the optimal scheme is significantly higher than both those of SSMA and FDMA.

I INTRODUCTION

By far the greatest challenge for mobile radio communication engineers is the design of robust and efficient systems which are transparent to the hostile effects of multipath propagation. Because of the enormous growth of cellular communications in recent years, this has become even more important. The focus of this work will be on fundamental performance limits of different multiple-access methods in multipath fading channels.

Recent papers dealing with the information capacity of time-varying channels which are unknown to the transmitters include [2], in which the capacity vs. outage characteristics of two-path Rayleigh fading channels are addressed. This work handled the TDMA (*Time Division Multiple Access)* case and is essentially a single-user result. Comparisons are made for varying degrees of path delays and different diversity techniques. The multiuser case is addressed in [1] where the achievable rate regions of SSMA (*Spread-Spectrum Multiple-Access*) and FDMA (*Frequency Division Multiple-Access*) are compared for arbitrary frequency-selective fading channels. It is shown that the achievable rate region of FDMA is contained within that of SSMA and consequently higher data rates are theoretically possible with SSMA. The achievable rate region of FDMA is also that of a slow *frequency-hopping* system such as the *Global System for Mobile Communications* (GSM) (see [3, Chap 8]).

In [4] the single-user frequency-flat fading channel is examined from the point of view of power control, which is simply a form of channel feedback. It turns out that a power control law which is water-filling in time can increase spectral efficiency for low average signal-to-noise ratio. In [5] the optimal power control laws for a multiuser system with channel feedback in frequency-flat fading are considered. They are optimum in the sense that they maximize the total sum-of-rates, which is a good fundamental measure of the performance of a multiuser system when the average received power of the users are equal. This requires perfect control of the slowly-varying or average statistics of the channels. In the optimal system, only one user is permitted to transmit at any given time. The total sum-of-rates for this scheme is significantly greater than the non-fading Gaussian channel when the number of users is large.

In this work we first discuss the model of the frequency-selective fading channel followed by the achievable rate region of the multiuser channel. When the channels are known to the receivers, but not to the transmitters, we show that the total sum of rates for SSMA, with perfect slow power control, exceeds that of FDMA or slow frequency-hopping by.5772 *nats/s/Hz* in Rayleigh fading.

We then consider the case of channels known to the transmitters. Using the total sum of rates as a figure of merit we derive the optimal multiple-access scheme when the basestation can allocate the users' transmits powers arbitrarily in the spectrum. We show that as the number of users grows, the optimal scheme has a significantly higher spectral efficiency than SSMA and FDMA.

II MULTIUSER FADING CHANNEL MODEL

In the uplink of a single-cell multiuser communication system we have K users sharing a fixed bandwidth KW. Each user signal, $u_i(t)$, is transmitted over a different channel with time-varying impulse response $h_i(\tau, t)$. This is the response at time t to an impulse at time $t-\tau$. We have the following composite complex baseband signal at the basestation,

$$r(t) = \sum_{i=0}^{K-1} \int u_i(t-\tau) h_i(\tau, t) d\tau + z(t) , \qquad (1)$$

where $z(t)$ is complex gaussian noise with power spectral density N_0. The impulse response is the result of multipath propagation, and can be expressed as

$$h_i(\tau, t) = \sum_j a_{ij}(t) \delta(\tau - \tau_{ij}(t)) , \qquad (2)$$

where $a_{ij}(t)$ and $\tau_{ij}(t)$ are the zero-mean complex signal strength and time delay for the j^{th} path of channel i respectively. We ignore any slow variations in received signal level due to path loss and shadowing [6]. This is done since we will be concerned later with the average signal-to-noise ratio (SNR) which can incorporate any slowly-varying or constant terms, provided they are perfectly controlled. From this point onward, we refer to this as perfect slow power control. Furthermore, the multipath components are scaled such that $\overline{\sum_j |a_{ij}(t)|^2} = 1$ and are assumed to be uncorrelated.

In the following section, we will assume that the channels are time-invariant for blocks of length T_b. In addition, we place a guard-time T_G between such blocks upon transmission which is at least as long as the multipath delay spread. This assures that the received signal in any block is independent of the information from previous blocks. The channel responses for blocks $[n(T_b + T_G), (n+1)(T_b + T_G)]$ are denoted by $h_i(\tau, n)$. These restrictions will allow for frequency-domain representations for the average mutual information functionals.

III ACHIEVABLE RATE REGION (NO CHANNEL KNOWLEDGE)

In this section we consider the achievable rate region when the users have no complete knowledge of the channels. We assume, however, that all the channels are known to the receiver. The multiaccess coding theorem [7][8] states that the achievable information rates over an observation interval $[0, T]$, $R_i(T)$, $i = 0, \ldots, K-1$, are bounded by

$$\sum_{i \in A} R_i(T) < \frac{1}{T} \mathrm{I}(R(t); \{U_i(t), i \in A\} \,|\, \{U_i(t), i \notin A\}, \tag{3}$$

$$\{h_j(t, \tau), j = 0, \ldots, K-1\})$$

$$\forall A \subseteq \{0, 1, \ldots, K-1\}$$

where $\mathrm{I}(X(t); Y(t))$ is the average mutual information functional for the processes $x(t)$ and $y(t)$. Using similar arguments to those for the single-user case in [2] and the stationary block assumption from Section II, the RHS of the inequalities in (3) can be expressed as

$$\begin{aligned} &\sum_{i \in A} R_i(T) < \frac{1}{N(T_b + T_G)} \cdot \\ &\sum_{n=0}^{N-1} \mathrm{I}(R(t); U_i(t), i \in A \,|\, \{U_i(t), i \notin A\}, \\ &\{h_j(t, n), j = 0, \ldots, K-1\}, \\ &t \in [n(T_b + T_G), (n+1)(T_b + T_G)]) \\ &\forall A \subseteq \{0, 1, \ldots, K-1\}, \end{aligned} \tag{4}$$

which can be inner-bounded by

$$\sum_{\in A} R_i < \frac{1}{N} \sum_{n=1}^{N} KW \int_{-1/2}^{1/2} \ln\left[1 + \sum_{i \in A} \frac{S_i(f, n)|H_i(f, n)|^2}{KWN_0}\right] df, \tag{5}$$

$\forall A \subseteq \{0, 1, \ldots, K-1\}$.

The information processes are chosen to be Gaussian in order to maximize the mutual information functional, and $\ _i(f, n)$ and $\ _i(f, n)$ are respectively the power spectrum of the i^{th} users' signal and the frequency response of the i^{th} channel if the n^{th} block were infinite in duration. The channel responses are taken to be ergodic random processes, which is valid given our assumption of perfect slow power control, so that reliable communication is possible if

$$\sum_{\in A} R_i < KW \int_{-1/2}^{1/2} \mathrm{E}\left[\ln\left(1 + \sum_{i \in A} \frac{S_i(f, n)|H_i(f, n)|^2}{KWN_0}\right)\right] df, \tag{6}$$

$\forall A \subseteq \{0, 1, \ldots, K-1\}$.

We also assume that the average transmit power (not including slow power control) is constrained to

$$\lim_{\to \infty} \frac{KW}{N} \sum_{n=0}^{N-1} \int_{-1/2}^{1/2} S_i(f, n) df = KW \int_{-1/2}^{1/2} \mathrm{E}[S_i(f, n)] df \le S \tag{7}$$

Without channel knowledge on the transmission end, the optimal input spectrum is flat over the entire bandwidth (i.e. $S_i(f, n) = S_i/(KW)$). This follows from the fact that the statistics of the channel responses are independent of frequency. In order to achieve these rates, long observation times are required, or equivalently long interleaving depths, in order to average over many possible channel realizations.

For the case of equal average received SNR, the inequality in (6) which interests us the most will be the one corresponding to $A = \{0, 1, \ldots, K-1\}$, which is the total sum of rates. This is because, in the symmetric case, the equal-rate line $R_0 = R_1 = \ldots = R_{M-1}$ intersects the portion of the achievable rate region defined by this inequality. This intersection point determines the maximum rate at which all the users can transmit reliably. This is shown in Fig. 1 for a two-user system, where R denotes the maximum achievable rate for a single-user (which is the same for both users in the symmetric case) and $R_{\max}$ denotes the sum of rates.

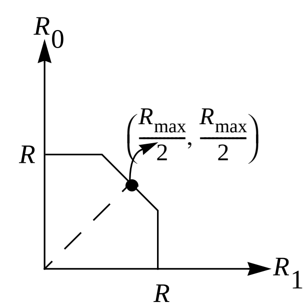

Figure 1: The Achievable Sum of Rates

The capacity region in (6) is essentially that of an SSMA system, since all the users transmit over the entire available bandwidth. For the sake of comparison, consider an FDMA system where each user occupies one of K equal size sub-bands. In this case, the achievable rate region is given by

$$\begin{aligned} \sum_{\in A} R_i &< W \sum_{i \in A} \mathrm{E}\left[\int_{-1/2}^{1/2} \ln\left(1 + \frac{S_i|H_i(f, n)|^2}{WN_0}\right) df\right] \\ &= W|A|\,\mathrm{E}\left[\ln\left(1 + \frac{S_i|H_i(f, n)|^2}{WN_0}\right)\right] \\ &\forall A \subseteq \{0, 1, \ldots, K-1\}, \end{aligned} \tag{8}$$

It is shown in [1] that this rate region is strictly enclosed by the region of an SSMA system s. Moreover, a slow frequency-hopping system is also bounded by these inequalities. This implies that higher data rates are theoretically possible with SSMA than with systems which use slow frequency-hopping.

Consider the case when $S_i = S$, which corresponds to the case when the average powers are equalized. The achievable sum of rates for an SSMA system is given by

$$C_{\mathrm{SSMA}} = \mathrm{E}\left[KW\ln\left[1 + \frac{S}{KWN_0}\sum_{i=0}^{K-1}|H_i(f,n)|^2\right]\right] \text{ (nats/s)} \quad (9)$$

Similarly, for FDMA or slow frequency-hopping we have

$$C_{\mathrm{FDMA}} = KW\mathrm{E}\left[\ln\left(1 + \frac{S|H_i(f,n)|^2}{WN_0}\right)\right] \text{ (nats/s)} . \quad (10)$$

It is clear that $C_{\mathrm{FDMA}} \le C_{\mathrm{SSMA}}$ by the convexity of the logarithm [1]. Furthermore, for large K and S, it can be shown that

$$C_{\mathrm{SSMA}} - C_{\mathrm{FDMA}} \approx -KW\mathrm{E}[\ln(|H_i(f,n)|^2)] \text{(nats/s)} . \quad (11)$$

If we assume a Rayleigh fading model, $|H_i(f,n)|^2$ is exponentially distributed with unit mean so that

$$C_{\mathrm{SSMA}} \approx C_{\mathrm{FDMA}} + \gamma_{\mathrm{Euler}} KW \text{ (nats/s)}, \quad (12)$$

where $\gamma_{\mathrm{Euler}} = .5772$ is Euler's constant.

IV Optimal Power Allocation Using Channel Knowledge

In the previous section we considered the case when the channels were not known to the users, aside from the average received SNR. Now we assume the channels are known completely to the users and they can choose to transmit their signals in arbitrary parts of the available bandwidth. The DECT system employs a technique along these lines. In this system, the available bandwidth is divided into several channels, which we assume to be frequency flat. The users measure the strength of each channel, and choose the best available channel on which to transmit. We show that under certain conditions the optimal scheme is a generalization of such techniques.

In many situations, the tap weights change slowly enough (with respect to the data rate) to be estimated accurately. Any *RAKE* receiver (see [9]), for instance, must perform this type of estimation. Assuming this is possible, consider a situation where the basestation estimates the channel response for each channel and instructs the users (via the downlink) to transmit with power spectrum, $(f|H_0(f,n), H_1(f,n), \ldots, H_{M-1}(f,n))$. Channel state information of this sort is a generalization of conventional power control which is normally frequency independent. Moreover, the instantaneous power spectrum for a given user depends on the channel responses of all the users in the system, which is not the case in conventional power control.

We would like to choose the power controllers ${}_i(f,n)$ (we have dropped explicit mention of the channel responses from the power controllers to simplify notation) to maximize a useful performance measure under the power constraint in (7). Assuming perfect slow power control and equal average received powers, the most logical choice for the performance measure is the total sum of rates. Since we have complete channel knowledge at the transmission end and equal average received powers, the maximization will yield the equal rate point on the capacity region. For a more detailed discussion see [10].

Using the Kuhn-Tucker theorem to solve this maximization problem, we obtain the following familiar *water-filling* formulae for the input power spectra (power control laws)

$$ {}_i(f,n) = \begin{cases} \left[\dfrac{1}{\lambda} - \dfrac{KWN_0}{|H_i(f,n)|^2}\right]^+ & |H_i(f,n)|^2 > |H_j(f,n)|^2 \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

where λ is a Lagrange multiplier chosen to satisfy (7). It is a water-filling solution since ${}_i(f,n) + KWN_0/|H_i(f,n)|^2$ is constant when user i is transmitting. This effect is similar to the single-user case [4,11] with a two added features. Firstly, the fact that there are multiple users and that the basestation can allocate different parts of the bandwidth arbitrarily, only one user is permitted to transmit at any given frequency. This user has the strongest instantaneous response at that frequency. Secondly, it is also water-filling in time; this is because more power is allocated when the channel is good and less when it is bad, which changes dynamically in time. The frequency-flat case [5] is a special case where we have a water-filling effect in time only, and only one user transmits at any given time over the entire bandwidth.

The total sum of rates when the channels are known, C_{KC}, can be shown to be

$$C_{\mathrm{KC}} = KW\sum_{i=1}^{K}(-1)^{i-1}\binom{K}{i}\mathrm{Ei}\left(\frac{i\lambda K}{\gamma_s}\right) \text{ (nats/s)} , \quad (14)$$

where $\mathrm{Ei}(\cdot)$ is the first order exponential integral and $\gamma_s = S/WN_0$. Both the Lagrange multiplier λ and the capacity are independent of the frequency-selectivity of the channel. We have, therefore, that the sum of rates is the same as for the frequency-flat channel [5]. The important difference between the frequency-flat and frequency-selective cases is that in the latter several users can share the entire band. This means that in spite of the fact that frequency-selectivity has no advantage in terms of average capacity, users may have to wait less time to access the channel. For the same total bandwidth, however, the instantaneous data rate for a given user will be lower than in the frequency-flat case.

We should also mention that in Rayleigh fading with high SNR, the optimal power adjustment (i.e. water-filling) does not yield a significant improvement in terms of spectral efficiency over the case when the transmit power is kept constant, but dynamic time/frequency allocation is still performed. Dynamic allocation is the key factor and can be interpreted as exploiting an inherent diversity in multiuser channels with fading.

We now compare the spectral efficiencies of FDMA, SSMA and dynamic allocation for Rayleigh fading. We first note that (10) can be expressed as [2]

$$C_{\mathrm{FDMA}} = KWe^{-\frac{WN_0}{S}}\mathrm{Ei}\left(\frac{WN_0}{S}\right), \quad (15)$$

and (9) can be computed numerically. The three per user spectral efficiencies (expressed now in bits/s/Hz) are

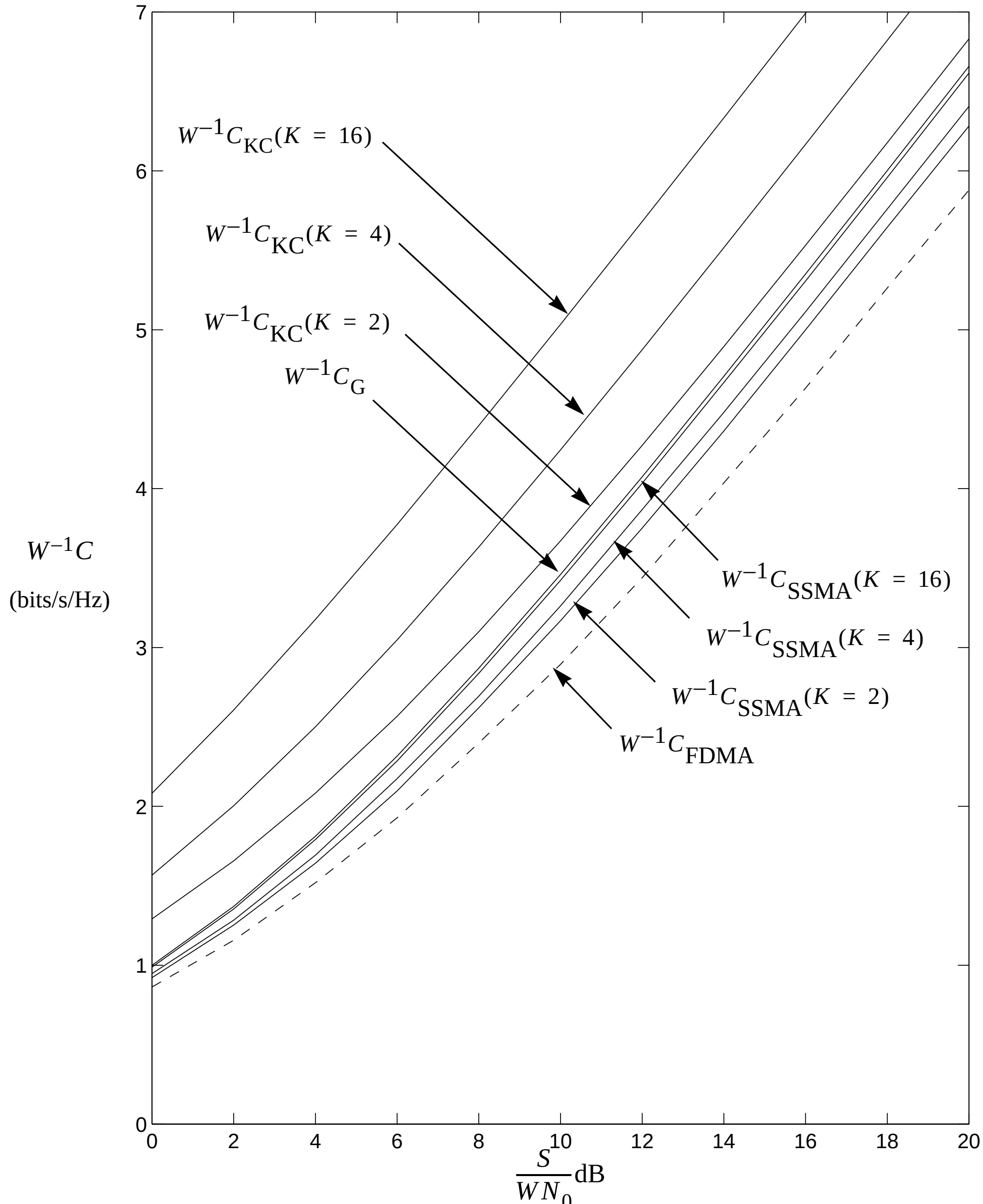

Figure 2: Multiuser Spectral Efficiency in Rayleigh Fading

shown in Fig. 2, along with the spectral efficiency of a non-fading Gaussian channel with the same transmit power S,

$$C_G = W \ln\left(1 + \frac{S}{WN_0}\right). \tag{16}$$

As the number of users increases, the spectral efficiency of SSMA approaches the Gaussian channel. The gap between the curves for SSMA and FDMA is on the order of $0.5772/\ln 2$ (*bits/s/Hz*) for $K = 16$ and high SNR as in (12). In terms of spectral efficiency (with an infinite observation interval), SSMA is not much better than FDMA, however we believe that the capacity vs. outage characteristics of SSMA as defined in [2] are better than those of FDMA because of added diversity. We see that feedback of the channel responses can yield a significant improvement in capacity, especially with a large number of users. Moreover, we believe the same is true for the capacity vs. outage, since a user transmits only where and when his channel is good.

V PRACTICAL CONSIDERATIONS

Partitioning of the available bandwidth in the optimal fashion may be difficult to achieve practically. A more practical alternative would be to divide the entire bandwidth into N equal size sub-bands and allocate a single user to each these sub-bands based on their instantaneous frequency response over the entire bandwidth. In general, a user may occupy more than one sub-band at any given time, or may not occupy any sub-band at all. We can look at this as an OFDM system with one user per carrier and dynamic allocation of the users on the carriers based on the instantaneous frequency responses of the users in each subband. We illustrate this in Fig. 3 for $K = 4$ and $N = 8$, where we see that at a particular time it is possible that a particular user occupies more of the available bandwidth than the others. This would be to take full advantage of the strength of the channels at a particular time. Such a scheme may be very appropriate for high speed wireless data networks.

The bandwidth of each of the smaller bands is $W_s = W/N$. If N is large enough, the sub-bands $[-W/2 + mW_s, -W/2 + (m+1)W_s]$, $n = 0, 1, \ldots, N-1$, can be considered as being frequency-flat (i.e. $W_s \ll 1/T_m$). This reduces the problem to one with N statistically identical, but not independent, parallel chan-

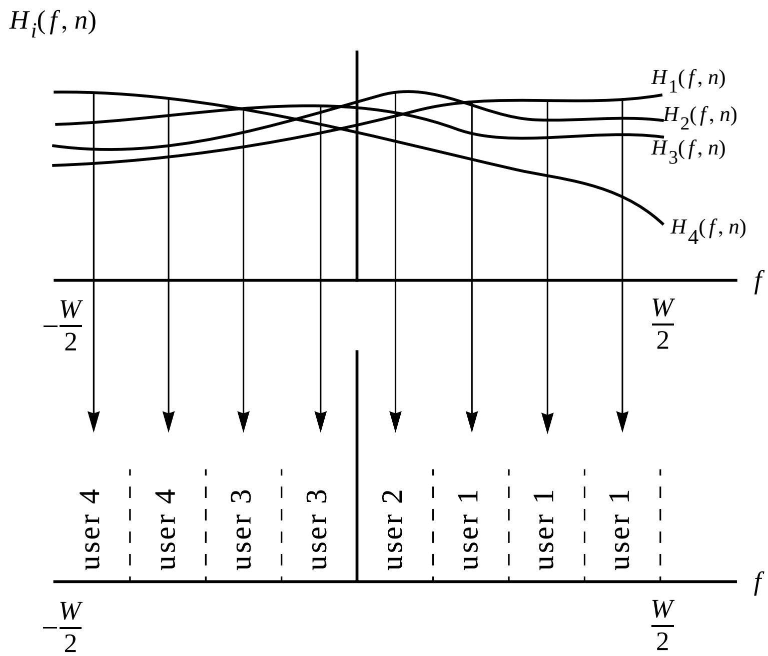

Figure 3: Subband Allocation Example $K = 4, N = 8$.

nels. The sum of rates for this simplified model must also be given by (14). This will be a good approximation to an optimal system provided that the frequency-flat subband assumption holds.

VI CONCLUSIONS

In this work we considered multiple-access methods for frequency-selective fading channels from an information theoretic point of view. Using the achievable sum of rates as a figure of merit for systems with perfect slow power control and unknown channels, we have shown that the spectral efficiency of SSMA exceeds that of FDMA or slow frequency-hopping by .5772 *nats/s/Hz* for a large number of users and high SNR in Rayleigh fading. We then derived the optimal time-varying input power distributions when all the channel impulse responses are known to the transmitters. The form of the input power distributions are multiuser generalizations of the familiar water-filling formula for AWGN channels with a non-flat frequency response, and permit only one user to transmit at any given frequency, at any given time. Expressions for the spectral efficiency in Rayleigh fading were given and compared with those of SSMA and FDMA. We show that for a large number of users there is a significant improvement when perfect channel state information is employed at the transmission end. It would also be interesting to determine the capacity vs. outage characteristics of the various schemes, since this may be a more practical measure of the performance of a multiple-access method.

VII REFERENCES

[1] Gallager R.G., “An Inequality on the Capacity Region of Multiaccess Multipath Channels,” in Communications and Cryptography - *Two Sides of One Tapestry* -, Kluwer 1994, pp. 129-139.

[2] Ozarow L.H., Shamai S., Wyner A.D., “Information Theoretic Considerations for Cellular Mobile Radio,” *IEEE Trans. on Vehic. Tech*., vol. 43, no. 2, May 1994, pp. 359-378.

[3] Steele R., *Mobile Radio Communications*, Pentech Press 1992, chap 8.

[4] Goldsmith A., Varaiya P., “Increasing Spectral Efficiency Through Power Control,” *International Conference on Communications*, ICC ’93, Geneva, Switzerland.

[5] Knopp R., Humblet P.A., “Information Capacity and Power Control in Single-Cell Multiuser Communications,” *International Conference on Communications* (ICC 95), Seattle, Wash, June 1995.

[6] Lee W.C.Y., *Mobile Communications Engineering,* McGraw-Hill, 1982

[7] Ahlswede, R., “Multi-Way Communication Channels,” Proc. 2nd Int. Symp. Information Theory, Tsahkadsor, Armenien SSR, 1971.

[8] Liao, H., “A Coding Theorem for Multiple Access Communications,” Proc. Int. Symp. IT, Asilomar, CA., 1972.

[9] Proakis, John G., *Digital Communications,* McGraw-Hill, 1989

[10]Knopp R., Humblet P.A., “Multiuser Diversity in Fading Communication Channels,” in preparation

[11]Gallager, R.G., *Information Theory and Reliable Communication,* Wiley & Sons, 1968.