M–ARY PHASE CODING FOR CORRELATED RAYLEIGH FADING CHANNELS

Raymond Knopp *
Institut Eurécom
2229 Route des Crêtes, BP 193
B.P. 193, 06904 Sophia–Antipolis Cedex, FRANCE
Tel: (33) 93 00 26 57 Fax: (33) 93 00 26 27
E-Mail: knopp@eurecom.fr

Harry Leib
Telecommunications and Signal Processing Lab
Dept. of Electrical Eng., McGill University
Montréal, Québec, Canada, H3A 2A7

Abstract— **This work considers coded MPSK systems with non–coherent detection over correlated Rayleigh fading channels. We extend the results of [5], in which a block coded–modulation technique for non–coherent detection on AWGN channels was developed, by considering the performance of this technique in a fading environment. The performance of the maximum–likelihood decoder is studied for various fade rates using union bounding techniques and the exact expression for the pairwise error event probability. It is shown that significant performance improvements can be obtained over differentially–coherent detection with the use of little or no symbol interleaving. Finally, we address the performance of a reduced–complexity/sub–optimal decoding strategy with the aid of computer simulations.**

I. Introduction

The increased demand for effective mobile and multi–user communication systems in recent years has created substantial interest in non–coherent communications with improved performance. The reason being that carrier phase tracking is extremely difficult to achieve in many systems. This may be attributed mainly to the burst nature of the signals, for systems employing TDMA or frequency hopping, or due to the presence of multipath fading and the *Doppler effect* in mobile systems, or very often a combination of both.

In this paper we will focus on coded MPSK (M–ary Phase Shift Keying) schemes for non–coherent detection over the correlated Rayleigh fading channel, which is an appropriate model for the mobile radio environment. The key technique used is the idea of performing non–coherent detection on blocks of MPSK symbols, which was introduced in [1]–[3] for the AWGN channel. In these works it is shown that the performance of this block detection technique approaches that of coherent detection as the block length is increased. Similar ideas were used in [4] to improve the performance of trellis–coded DPSK modulation. The basic technique was taken one step further in [5] by tailoring coded–modulation schemes for this detection strategy through the use of an appropriate distance measure. This work also considered the design of reduced–complexity/sub–optimal decoder strategies partly based on ideas used for the uncoded case in [2].

The performance of non–coherent block detection of MPSK over correlated Rayleigh channels is treated in [6]. Using an exact expression for the pairwise error event probability, it is shown that some improvement over differentially–coherent detection is possible, most notably the significant reduction of the irreducible error–floor. Using the same expression, we will analyse the performance of the coded–modulation schemes outlined in [5] for optimal maximum–likelihood decoding. With the help of computer simulations, we will also investigate the performance of a reduced–complexity/sub–optimal decoder. It will be shown that significant performance gains can be achieved with *little or no* symbol interleaving, depending on the rate of the fading.

II. Channel Model

Consider the transmission of vectors in the ring of integers modulo–M, $\mathcal{Z}_M$. We denote a particular such vector as $\mathbf{c}_m = (\, c_{m0} \;\; c_{m1} \;\; \cdots \;\; c_{m(N-1)}\,)$, where $c_{mi} \in \mathcal{Z}_M$, and the set of all possible transmitted vectors as the code $\mathcal{C}$. Using MPSK modulation the transmitted vectors are mapped into the signal space to the

*This work was performed while with McGill University vector

$$\mathbf{f}_m = \left(e^{j\frac{2\pi}{M}c_{m0}} \quad \cdots \quad e^{j\frac{2\pi}{M}c_{m(N-1)}} \right). \qquad (1)$$

Correlated Rayleigh fading can be modeled as a correlated complex gaussian process, u_l, which multiplies the transmitted symbols. This assumes that the fading process is sufficiently slow, so that it may be considered as being constant over the symbol period, but not over the codeword. The received symbols, y_l, which are assumed to be the output of a matched filter, are given by

$$y_l = Tu_l f_{ml} + n_l \quad l = 0, \cdots, N-1, \qquad (2)$$

where the n_l are independent zero mean Gaussian random variables with variance N_0, and T is the symbol duration. It should be noted that the u_l's are statistically independent of the n_l's. In order to express (2) in vector form, we create the diagonal matrix $\mathbf{F}_m$, whose main diagonal is the vector $\mathbf{f}_m$, so that the received vector may be expressed as

$$\mathbf{y} = T\mathbf{u}\mathbf{F}_m + \mathbf{n} \qquad (3)$$

where $\mathbf{u}$ and $\mathbf{n}$ are the vectors made up of the u_l's and n_l's. The power spectrum of the u_l's may take on various forms to model different situations. Here we will use the *land-mobile* model for the power spectrum which has the following form

$$S_u(f) = \begin{cases} \dfrac{1}{\sqrt{\pi^2(f^2 - f_D^2)}} & |f| \leq f_D \\ 0 & |f| > f_D \end{cases}. \qquad (4)$$

The constant f_D is the maximum *Doppler frequency.* The autocorrelation function corresponding to $S_u(f)$ is given by

$$\phi_{uu}(i) = \frac{1}{2}E(u_l^* u_{l+i}) = J_0(2\pi i f_D T), \qquad (5)$$

where $J_0(\cdot)$ is the zero–order bessel function, and $f_D T$ is the fade rate.

From the form of (3), we see that this is simply a general gaussian detection problem. It is shown in [7, p. 98] that the maximum–likelihood(ML) decoding rule is the following minimization

$$\min_{m=0,1,\cdots,|\mathcal{C}|-1} \mathbf{y}\mathbf{\Phi}_m^{-1}\mathbf{y}^* \qquad (6)$$

where $\mathbf{\Phi}_m$ is the autocorrelation matrix for $\mathbf{y}$ assuming the m^{th} codeword was transmitted, $|\mathcal{C}|$ is the cardinality of the code $\mathcal{C}$, and $(\cdot)^*$ denotes the conjugate transpose operation. The autocorrelation matrix is expressed as

$$\mathbf{\Phi}_m = \frac{1}{2}E(\mathbf{y}^*\mathbf{y}) = \mathbf{F}_m^*(\mathbf{\Phi}_{uu} + \frac{1}{\gamma}\mathbf{I})\mathbf{F}_m, \qquad (7)$$

where γ is the signal–to–noise ratio(SNR) and $\mathbf{\Phi}_{uu}$ is the autocorrelation matrix of $\mathbf{u}$, whose elements are given by

$$\mathbf{\Phi}_{uu}(i,j) = \phi_{uu}(|i-j|). \qquad (8)$$

The decoding rule is therefore as follows,

$$\min_{m=0,1,\cdots,|\mathcal{C}|-1} \mathbf{y}\mathbf{F}_m(\mathbf{\Phi}_{uu} + \frac{1}{\gamma}\mathbf{I})^{-1}\mathbf{F}_m^*\mathbf{y}^*. \qquad (9)$$

It turns out for a flat fading channel (ie. when $f_D T = 0$) that (9) is equivalent to the decision rule for the non–coherent AWGN channel [8]. It should be clear that estimates of both f_D and the SNR are required for the decoding process. For the sake of simplicity, we assume perfect estimation of these parameters.

III. Performance of Module–Phase Codes over Rayleigh Channels

We will now examine the performance of *Module–Phase Codes* which were originally designed for the non–coherent AWGN channel in [5]. The codes are linear (both in the code and signal spaces) (N, K) block codes defined over $\mathcal{Z}_M$. The distance measure used in the design of the codes is the non–coherent distance, d_{NC}^2 (see [5], [9]), which is monotonically related to the pairwise error event probability, $P(\mathbf{c}_i \to \mathbf{c}_j)$, between two arbitrary codewords, $\mathbf{c}_i$ and $\mathbf{c}_j$. Although the non–coherent distance is not a precise indication of the pairwise error event probability over Rayleigh channels, we will nonetheless see that codes designed using it still provide significant performance rewards.

The error performance of non–coherent block detection over correlated Rayleigh fading channels is treated in [6] for uncoded systems. The results will be summarized here, since the expression for the pairwise error probability is the same. Given that the actual transmitted codeword is $\mathbf{c}_i$, we will make a wrong decision (ie. that we choose a codeword $\mathbf{c}_j$), if the random variable

$$D = \mathbf{y}\mathbf{B}\mathbf{y}^* \qquad (10)$$

is negative. The matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{F}_j(\mathbf{\Phi}_{uu} + \frac{1}{\gamma}\mathbf{I})^{-1}\mathbf{F}_j^* - \mathbf{F}_i(\mathbf{\Phi}_{uu} + \frac{1}{\gamma}\mathbf{I})^{-1}\mathbf{F}_i^*. \qquad (11)$$

The pairwise error probability is then given by

$$P(\mathbf{c}_i \to \mathbf{c}_j) = \mathrm{Prob}(D \leq 0) = -\sum_i \mathrm{Residue}\left[\frac{\phi_D(s)}{s}\right]_{s=p_i}, \qquad (12)$$

where p_i are all the right–hand plane poles of $\phi_D(s)/s$. The characteristic function, $\phi_D(s)$, is given by

$$\phi_D(s) = \prod_{k=0}^{N-1} \frac{1}{2\lambda_k s + 1}, \qquad (13)$$

where λ_k is the kth eigenvalue of the matrix

$$\mathbf{\Lambda} = \mathbf{\Phi}_m \mathbf{B}. \tag{14}$$

Using (12) we will generate union bounds on the bit error–rate for two rate 1/2 QPSK codes from [5]. We will examine the simple (8,4) code in $\mathcal{Z}_4$ with the generator matrix

$$\mathbf{G_C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 3 & 3 & 2 \\ 0 & 1 & 0 & 0 & 1 & 3 & 1 & 3 \\ 0 & 0 & 1 & 0 & 3 & 2 & 3 & 3 \\ 0 & 0 & 0 & 1 & 3 & 0 & 2 & 3 \end{pmatrix}, \tag{15}$$

and the more powerful (14,7) code in $\mathcal{Z}_4$ with generator matrix

$$\mathbf{G_C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 2 & 3 & 3 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 1 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 & 3 & 2 & 2 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 3 & 2 & 0 & 3 & 3 & 3 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 3 & 1 & 2 & 2 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & 2 & 1 & 0 & 2 \end{pmatrix}. \tag{16}$$

Codewords are created in the same fashion as traditional block codes, except that the operations are now carried out over $\mathcal{Z}_4$. It should be noted that in this work we also assume that codeword overlapping is performed (see [5]). This just means that each symbol of the transmitted codeword is incremented (modulo–M) by the last symbol of the previously transmitted codeword. It may be looked at as performing differential encoding on a block basis, by using the last symbol of the previous codeword as a phase reference for the entire block. This was shown to be completely equivalent to differential encoding on a per symbol basis, provided that detection is carried out on the vector of length $N+1$ which includes the last symbol of the previous codeword.

It is shown in [6] that the all–zero codeword may be used to assess the error performance without any loss of generality. This remains valid for the coded case, as long as the code is linear in the signal space, and follows directly from the form of the decoding rule in (9). The union bound on the bit error rate is given by the following summation

$$P_b \leq \sum_{m=1}^{|\mathcal{C}|-1} \frac{e_m}{K \log_2 M} P(\mathbf{c}_0 \rightarrow \mathbf{c}_m), \tag{17}$$

where e_m is the number of bit errors located in the K information positions of $\mathbf{c}_m$.

Unlike the AWGN channel, codewords sharing the same d_{NC}^2 cannot be considered as being equivalent in terms of $P(\mathbf{c}_0 \rightarrow \mathbf{c}_m)$, since, not surprisingly, $P(\mathbf{c}_0 \rightarrow \mathbf{c}_m)$ is somewhat dependent on the location of symbols within a codeword. We may not, therefore, group codewords having the same d_{NC}^2 (ie. by using the weight distribution in terms of d_{NC}^2) together in the union bound in order to evaluate it more quickly. As was the case for the AWGN channel, we do not compute the entire sum in (17), but rather include only those terms which are dominant. Unlike the uncoded case in [6], we use more than just the nearest neighbours to evaluate (17), which is necessary for the coded case, especially at higher fade rates.

In Fig.1 we show the performance of uncoded binary DPSK and the union bounds on the performance of the (8,4) and (14,7) QPSK codes for fade rates of $f_D T = .001, .01$ and $.1$. We see that as the fade rate increases there is a diversity effect, since the symbols within a codeword become less correlated. Consequently, the performance of the coded system improves with increasing fade rates while that of the uncoded system degrades. For the slow fading case ($f_D T = .001$) we see that the union bound is worse than the uncoded system for error rates greater than 10^{-4}.

In order to alleviate the situation at very low fade rates, we can improve the performance by artificially increasing the fade rate by using some symbol interleaving. Let us assume that we use an interleaver with depth D_i. This means that the symbols of the codeword will be spaced D_i transmitted symbols apart, and we may therefore look at this as artificially increasing the fade rate by a factor of D_i, as far as the codeword is concerned. More precisely, the matrix $\mathbf{\Phi}_{uu}$ is calculated using a fade rate of $f_D T D_i$, rather than $f_D T$. For both codes at $f_D T = .001$, the performance is quite poor for low SNR. If, however, we use an interleaving depth of only ten symbols, which is rather small, we have an effective fade rate of $f_D T D_i = .01$, which yields a significant improvement. The amount of interleaving required depends, of course, on the fade rates experienced in the environment. We see, however, that even the rather simple (8,4) code performs quite well at moderate fade rates without interleaving and at lower fade rates with only a small amount of interleaving.

IV. Simulation Results for Reduced–Complexity/Sub–Optimal Decoders

We will now present some simulation results for the reduced–complexity/sub–optimal decoding strategy described in [5] over correlated fading channels. This decoding strategy is a combination of ***information set decoding*** [10] and Wilson *et al.*'s reduced–complexity methods for non–coherent block detection [2]. A detailed description of this decoding strategy can also be found in [9]. The (8,4) and (14,7) bandwidth efficient codes over $\mathcal{Z}_4$ have both been simulated over a correlated Rayleigh fading channel with $f_D T = .01$, and the (8,4) code over a channel with $f_D T = .1$ as well. Figs.2–4 show the results of the simulations. In each figure, we compare the simulation result with the union bound for a ML decoder for that code, the result of a simulation for a differentially-coherent system ($M = 2$) over the same channel and the performance of ideal coherent BPSK. We see that the simulation of the differentially-coherent systems match the analytical curves of Fig.1 exactly, verifying the correctness of our channel simulation. In each case, we see that the union bound is quite pessimistic for low SNR, which is consistent with the results over the AWGN channel. It is interesting to note that the two codes with $f_D T = .01$ in Figs.2) and 3 attain the same slope for two orders of magnitude in P_b (between 10^{-3} and 10^{-5}). For both codes at $f_D T = .01$, we also notice that the irreducible error-floor of the differentially-coherent system is completely eliminated (at least down to $P_b = 10^{-5}$). For the (8,4) code with $f_D T = .1$ in Fig. 4, which represents a fairly high fade rate, significant performance improvement is obtained even at fairly high error-rates. We do see, however, that the reduced-complexity decoding strategy breaks down at low error rates. This can be attributed to the first stage of decoding which uses differential detection and results in an irreducible error floor around $P_b = 10^{-6}$. It is very important to point out that, in each case, these performance enhancements are obtained *without* the use of symbol interleaving. Most other coding schemes for fading channels with fade rates as high as $f_D T = .01$, and sometimes higher, use some interleaving (see for instance [11, Chap.9],[12]).

V. Conclusions

In this paper we have studied the performance of two of the Module-Phase Codes presented in [5] over correlated Rayleigh fading channels. Using a union bound approach, the performance of the optimum maximum-likelihood receiver is presented for various fade rates. Through the use of computer simulations, we have also examined the performance of these codes combined with a reduced-complexity/sub-optimal decoding method. It was shown that significant performance gains over uncoded differentially-coherent systems can be attained using little or no symbol interleaving.

References

[1] D. Divsalar and M. K. Simon, "Multiple-symbol differential detection of MPSK", *IEEE Trans. Commun.*, vol. COM-**38**, pp. 300-308, March 1990.

[2] S. G. Wilson , J. Freebersyser, C. Marshall , "Multi-symbol detection of M-DPSK, Global Telecomm. Conf., GLOBECOM'89 Dallas, Texas, Nov.27-30, 1989, Conf.Rec.,1692-1697.

[3] H. Leib and S. Pasupathy, "Optimal Noncoherent Block Demodulation of Differential Phase Shift Keying (DPSK)", *Archiv für Elektronik und Übertragungstechnik*, Vol. 45,pp. 299-305, 1991.

[4] D. Divsalar and M. K. Simon, "The Performance of Trellis-Coded MDPSK with Multiple Symbol Detection", *IEEE Trans. Commun.*, vol. COM-**38**, pp.1391-1403, Sept. 1990.

[5] R. Knopp, H. Leib, "*M*-ary Phase Coding for the Non-Coherent AWGN Channel", accepted for publication in *IEEE Trans. Information Theory*, March 1994.

[6] P. Ho, D. Fung, "Error Performance of Multiple-Symbol Differential Detection of PSK Signals Transmitted Over Correlated Rayleigh Fading Channels", *IEEE Trans. Communications*, vol. COM-**40**, pp. 1566-1569, 1992.

[7] H. Van Trees, *Detection, Estimation, and Modulation Theory: Part I.*, New York: Wiley, 1968.

[8] P. Ho, D. Fung, "Error performance of multiple symbol differential detection of PSK signals transmitted over correlated Rayleigh fading channels," *Proc. IEEE ICC'91*, Denver, CO, June 1991, pp. 19.6.1-19.6.7.

[9] R. Knopp, "Module Phase Codes with Non-Coherent Detection and Reduced-Complexity Decoding", M.Eng. Thesis, McGill University, Montreal, Canada, Sept. 1993.

[10] E. Prange, "The Use of Information Sets in Decoding Cyclic Codes", *IRE Trans. Inform. Theory*, Vol. IT-8, pp.S5-S9, 1962.

[11] E. Biglieri, D.Divsalar, P. J. McLane, M. K. Simon, *Introduction to Trellis-Coded Modulation with Applications*, McMillan, 1991.

[12] K. Yu, P. Ho, "Trellis Coded Modulation with Multiple Symbol Differential Detection", *Proc. IEEE ICC'93*, Geneva, Switzerland, May 1993.

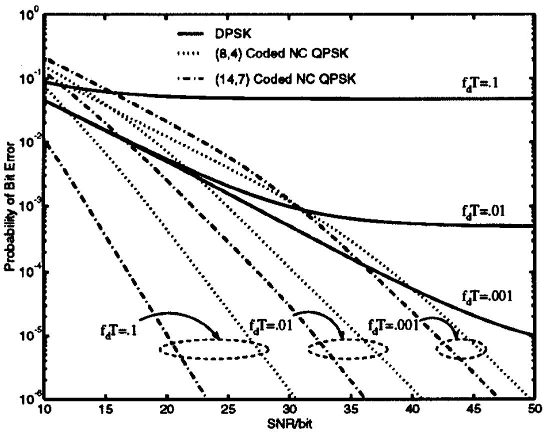

Fig. 1. Union bound on the performance of two rate 1/2 QPSK codes for fade rates of $f_D T = .1, .01$ and $.001$

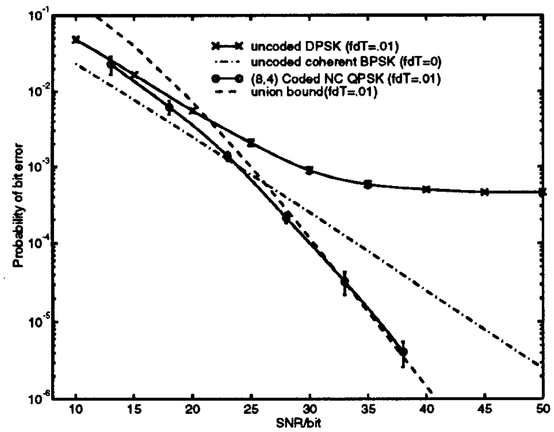

Fig. 2. Simulation results for an (8,4) QPSK code ($f_D T = .01$)

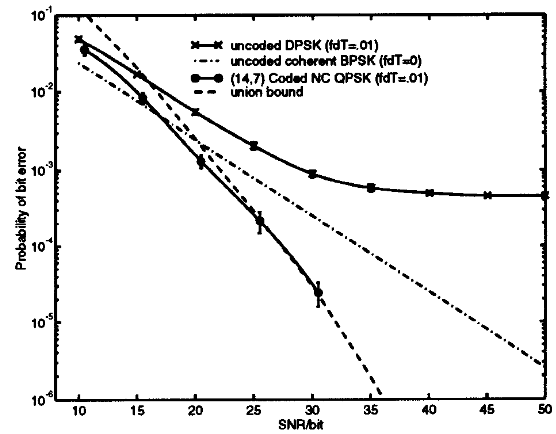

Fig. 3. Simulation results for a (14,7) QPSK code ($f_D T = .01$)

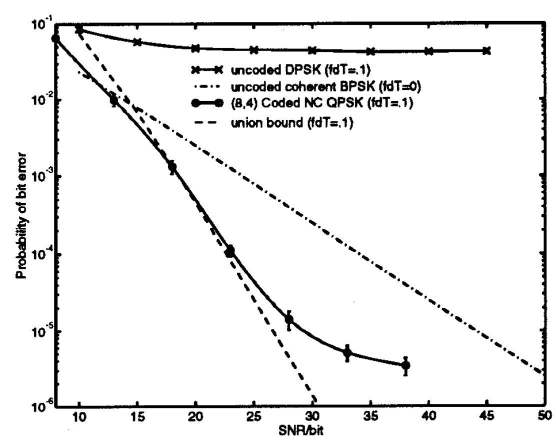

Fig. 4. Simulation results for an (8,4) QPSK code ($f_D T = .1$)