

# INFORMATION CAPACITY AND POWER CONTROL IN SINGLE-CELL MULTIUSER COMMUNICATIONS

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**Abstract**—We consider a power control scheme for maximizing the information capacity of the uplink in single-cell multiuser communications with frequency-flat fading, under the assumption that the users' attenuations are measured perfectly. Its main characteristics are that only one user transmits over the entire bandwidth at any particular time instant and that the users are allocated more power when their channels are good, and less when they are bad. Moreover, these features are independent of the statistics of the fading.

Numerical results are presented for the case of single-path Rayleigh fading. We show that an increase in capacity over a perfectly-power controlled (Gaussian) channel can be achieved, especially if the number of users is large. By examining the bit error-rate with antipodal signalling, we show the inherent diversity in multiuser communications over fading channels.

## I INTRODUCTION

In the multiuser cellular systems in use today, there are two primary communication links between the users located in a cell and the cell's central base station, the uplink and downlink. The uplink refers to the information flow from the users to the base station and is an example of a classic multiuser channel, or a many-to-one communication problem (see [1]). The downlink refers to the opposite situation, namely the flow of information from the base station to the users. This is an example of a broadcast channel or a one-to-many communication problem (again see [1]). There has been much recent interest in determining the capacity of such systems. In [2], the information capacity for the uplink channel in single and multicell systems is addressed for non-fading Gaussian channels. The capacity of the uplink channel without fading for the multicell case, modelled both as linear and hexagonal arrays, is treated in [3]. Recent work on the capacity over fading channels includes [4],[5]. In [6] the optimal power control scheme to maximize capacity over single-user Rayleigh fading channels under an average power constraint is considered. Here we extend similar ideas to the multiuser channel by considering the uplink in a single-cell communication system. By single-cell, we mean either that we really only have a single-cell system or that we consider interference from neighbouring cells to be negligible.

In a cellular environment, the signals from different

users are transmitted over channels having different characteristics, and result in different received powers upon reception at the base station. The average received power of a given user is related to his distance from the base station, and correspondingly there is a certain loss in signal strength or *path loss*. The instantaneous power, on the other hand, is usually time-varying due to *multipath fading* [7]. In order to alleviate these situations, *power control* is used to equalize the received powers at the base station. A system employing power control uses estimates of the received power at the base station to control the users' transmit powers. This is usually done in two ways, namely *open-loop* or *closed-loop* power control. The former refers to the case where the uplink and downlink channels are assumed to be strongly correlated and estimation of the received power at the base station is based on the signal received by the user. In the latter, estimation is performed in the base station which then instructs the users, via the downlink channel, to transmit at a certain power. Provided that the received powers do not vary too quickly, the power controllers simply try to keep all the received powers at some nominal level by inverting both the path loss and fading effects of the channel. If we make the assumption that the received powers are perfectly estimated, which will be the case in this paper, we have so-called "perfect" power control. In addition, if the signal at the base station is corrupted by additive white Gaussian noise the channel is transformed into a purely Gaussian multiuser channel, for which the information capacity is known.

The question remains whether or not another power control scheme, tailored to the fading statistics of the channel, can achieve an information capacity higher than that of "perfect" power control. The answer is yes, and the focus of this paper will be to describe a power control scheme which achieves capacity for a channel with arbitrary fading statistics. The capacity of this scheme for the particular case when the users experience independent single-path Rayleigh fading will also be compared with that of a perfectly power controlled system (Gaussian Channel) and a system employing no power control.

## II UPLINK CHANNEL CAPACITY

We can express the signal received by the base station

mathematically as follows

$$r = \sum_{i=0}^{K-1} \alpha_i x_i + n, \quad (1)$$

where  $K$  is the number of users in the cell,  $\alpha_i$  and  $x_i$  are gain and information for the  $i^{\text{th}}$  user. We assume that the information sources  $x_i$  are zero-mean, have unit energy and are mutually uncorrelated. The noise  $n$  is a zero-mean gaussian random variable with variance  $N$ . If the  $\alpha_i$  are deterministic constants, this is simply a gaussian *multiuser channel* whose capacity region is defined by the following set of equations [1]

$$\forall S \subset \{0, 1, \dots, K-1\}, \quad (2)$$

$$\sum_{i \in S} R_i < \frac{1}{2} \log \left( 1 + \sum_{i \in S} \gamma_i \right)$$

where  $R_i$  and  $\gamma_i = \alpha_i^2/N$  are the information rate and received signal-to-noise ratio (SNR) for the  $i^{\text{th}}$  user respectively. In a fading environment the  $\alpha_i$  and consequently the  $\gamma_i$  are usually modelled as random quantities. If we consider *frequency-flat Rayleigh* fading,  $\alpha_i$  is assumed to have a Rayleigh distribution, and in turn, the  $\gamma_i$  assume the following exponential distribution

$$p(\gamma_i) = \begin{cases} \frac{1}{\gamma_{si}} \exp\left(-\frac{\gamma_i}{\gamma_{si}}\right), & \gamma_i > 0 \\ 0, & \text{otherwise,} \end{cases} \quad (3)$$

where  $\gamma_{si}$  is the average received SNR for the  $i^{\text{th}}$  user. In general, these will be different for each user, due to the different path losses between users and the base station.

To compensate for the effects of fading and/or path loss, we introduce some feedback between the base station and the users in the cell allowing the users to adjust their transmit power. This is done by requesting the users to transmit at a power according to the power control law  $\mu_i(\gamma)$ , with  $\gamma = [\gamma_0 \ \gamma_1 \ \dots \ \gamma_{K-1}]$ , which takes the instantaneous received power of all the users in the cell into account. Assuming that we desire a received SNR of  $\gamma_{si}$  for the  $i^{\text{th}}$  at the base station, the perfect power control law would be  $\mu_i(\gamma) = \gamma_{si}/\gamma_i$ . The average value of the power control over a Rayleigh fading channel,

$$\bar{\mu}_i = \int_0^\infty (\gamma_{si}/\gamma_i) p(\gamma_i) d\gamma_i, \quad (4)$$

is infinite. From a practical standpoint, this means that below a certain cutoff SNR, the output power

must be fixed. Ignoring this practical limitation, the sum-of-rates capacity for this case is given by

$$C_G = \frac{1}{2} \log \left( 1 + \sum_{i=0}^{K-1} \gamma_{si} \right) \quad (5)$$

which is simply the equation from the capacity region of a Gaussian multiuser channel (2) which contains the rates of all the users. For a system with no power control, the sum-of-rates capacity is given by

$$C_{npc} = \frac{1}{2} \mathbb{E} \left[ \log \left( 1 + \sum_{i=0}^{K-1} \gamma_i \right) \right]_\gamma \quad (6)$$

This expression is simply an average of the sum-of-rates capacity for each possible realization of  $\gamma$ . It should be clear that in order to attain capacity, the channel code must be changed for each such realization.

We remark that  $C_{npc} \leq C_G$  by Jensen's inequality [8, pg. 85]. Furthermore, if we assume that  $\gamma_{si} = \gamma_s, \forall i$  (i.e. that we equalize the average received powers) and that the fading between users is independent, it is easily shown that  $C_{npc} \rightarrow C_G$  when  $K$  becomes large.

This is somewhat surprising, since it means that controlling the average power is sufficient from a sum-of-rates capacity standpoint, and that "perfect" power control will not yield a significant improvement.

### III OPTIMAL POWER CONTROL

In this section, we concern ourselves with finding the power control laws for maximizing the sum-of-rates capacity of a multiuser channel corrupted by arbitrary fading. As we saw earlier, a disadvantage of perfect power control over a Rayleigh fading channel is that the average power is infinite. To alleviate this problem we constrain the average value of the power control to unity, expressed mathematically as

$$\iint \dots \int \mu_i(\gamma) p(\gamma) d\gamma = 1. \quad (7)$$

The problem, therefore, is to maximize

$$C_{pc} = \frac{1}{2} \iint \dots \int \log \left( 1 + \sum_{i=0}^{K-1} \mu_i(\gamma) \gamma_i \right) p(\gamma) d\gamma \quad (8)$$

subject to (7) and  $\mu_i(\gamma) \geq 0$ . Introducing the Lagrange multipliers,  $\lambda_i$ , corresponding to each constraint in (7), and using the convexity of the logarithm, we obtain the following system of inequalities governing the  $\mu_i(\gamma)$

$$1 + \sum_{j=0}^{K-1} \mu_j(\gamma) \gamma_j \geq \frac{\gamma_i}{\lambda_i}, \text{ with equality iff } \mu_i(\gamma) > 0. \quad (9)$$

Assuming that the  $\gamma_i$  are all different, we have that

$$\mu_i(\gamma) \neq 0 \Rightarrow \mu_j(\gamma) = 0, \forall j \neq i \text{ and consequently}$$

$$\gamma_i \geq \left( \frac{\lambda_i}{\lambda_j} \right) \gamma_j. \quad (10)$$

For the sake of interpreting this relation, let us assume that all the users have the same average received power. By symmetry, all the  $\lambda_i$  must be equal. This means that the only user who is allowed to transmit at any given time is the one with the largest instantaneous power, and the other must remain quiet until one of them becomes the strongest. In general, the  $\lambda_i$  are directly related to the average received power of the users.

An important point is that this scheme is independent of the probability distribution used to model the users' fading processes, and thus it follows that the power control law for user  $i$  is always of the form

$$\mu_i(\underline{\gamma}) = \begin{cases} \frac{1}{\lambda_i} - \frac{1}{\gamma_i}, & \gamma_i > \lambda_i, \gamma_i > \frac{\lambda_i}{\lambda_j} \gamma_j, j \neq i \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

We see that a user is allocated more power when his channel is good and less when his channel is poor. This works in a completely opposite sense to conventional power control and can be interpreted as a *water-filling* phenomenon in time [6,8], since the instantaneous sum of the transmitted power and  $1/\gamma_i$  is constant. The  $\lambda_i$  act both as a cutoff SNR, below which a user may not transmit, and as scaling factors used to determine the "strongest" user. Moreover, the peak power of the  $i^{\text{th}}$  user is  $1/\lambda_i$ .

The interesting thing is that it defines a new type of multiple access scheme. It says that time-sharing should be performed based on measurements of the instantaneous received power. In this sense it is closer to TDMA than to CDMA, since only one user accesses the channel over the entire bandwidth at any given time. It may turn out that this is very difficult to achieve practically, especially if the channel changes too rapidly for accurate estimation to be performed<sup>1</sup>, or if the number of users is large. The latter would cause problems since, as the number of users increases, the average amount of time a particular user's channel is best must decrease correspondingly. In addition, it should also be evident that the fading must not be too slow, so as to insure that the average time any user accesses the channel is not too long. Clearly, in order to assess the viability of such a scheme for a practical application, it would be necessary to determine the average channel access time based on the rate of the fading and  $K$ . Preliminary results in this direction seem quite promising. This

scheme is most likely not suitable for voice transmission because of the uncertainty in the channel access time. For bursty data, however, this may pose less of a problem.

In the single user ( $K = 1$ ) case, these results correspond to those presented in Fig. 6. For the two-user case, the regions in the  $(\gamma_0, \gamma_1)$  space where each user is permitted to transmit are shown in Fig. 1.

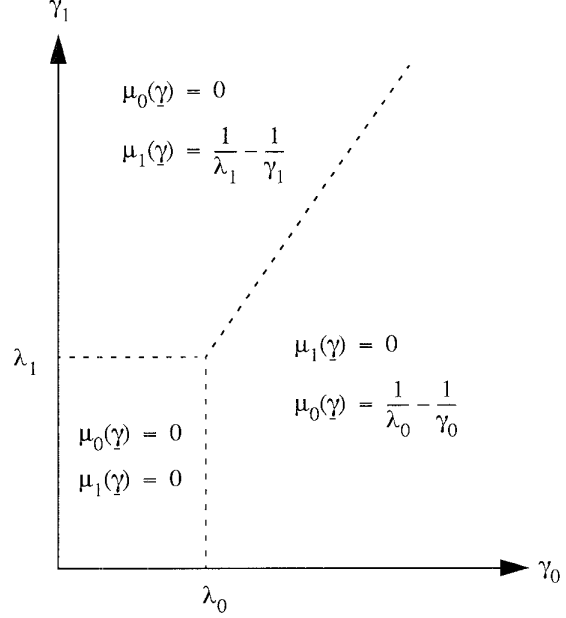


Figure 1: Transmission Regions for a two-user system

#### IV NUMERICAL RESULTS FOR SINGLE-PATH RAYLEIGH FADING

In this section we present some numerical results for the single-path or frequency-flat Rayleigh fading case. First we compare the capacity of the optimal power control scheme with the capacities in (5) and (6), and second we present the probability of error per user with antipodal signalling.

The  $\lambda_i$  are found by solving the constraint equation in (7), resulting in following set of non-linear equations

$$\frac{1}{\gamma_{si}} \int_{\lambda_i}^{\infty} \left( \frac{1}{\lambda_i} - \frac{1}{\gamma_i} \right) e^{-\gamma_i/\gamma_{si}} \prod_{\substack{j=0 \\ j \neq i}}^{K-1} \left( 1 - e^{-\left( \frac{\lambda_j}{\lambda_i} \right) \frac{\gamma_i}{\gamma_{sj}}} \right) d\gamma_i = 1 \quad (12)$$

which may be evaluated numerically. Using (11), the expression for the sum-of-rates capacity in (8) can be simplified to

1. In the IS-95 standard, estimation is performed 800 times/sec in order to update the power control. This appears to be adequate for most environments.

$$C_{pc} = \sum_{i=0}^{K-1} \frac{1}{2\gamma_{si}} \int_{\lambda_i}^{\infty} \log_2 \left( \frac{\gamma_i}{\lambda_i} \right) e^{-\frac{\gamma_i}{\gamma_{si}}} \prod_{j=0, j \neq i}^{K-1} \left( 1 - e^{-\left( \frac{\lambda_j}{\lambda_i} \right) \frac{\gamma_i}{\gamma_{sj}}} \right) d\gamma_i, \quad (13)$$

which again may be evaluated using numerical methods. For the symmetric case (i.e.  $\gamma_{si} = \gamma_s, \forall i$ ), (12) can be simplified to the single non-linear equation

$$\sum_{i=1}^K (-1)^{i-1} \binom{K}{i} \left[ \frac{e^{-i\lambda/\gamma_s}}{\lambda} - \left( \frac{i}{\gamma_s} \right) \text{Ei} \left( \frac{i\lambda}{\gamma_s} \right) \right] = K, \quad (14)$$

where  $\text{Ei}(\cdot)$  is the first order exponential integral. The corresponding sum-of-rates capacity is given by

$$C_{pc} = \frac{1}{2 \ln 2} \sum_{i=1}^K (-1)^{i-1} \binom{K}{i} \text{Ei} \left( \frac{i\lambda}{\gamma_s} \right). \quad (15)$$

It is easily shown from (14) that  $\lambda \rightarrow 1/K$  as  $\gamma_s \rightarrow \infty$ , so that the peak power and cutoff SNR both increase linearly with  $K$ .

In Fig. 2, we compare the average sum-of-rates capacities in (5), (6) and (15) for the case of equal average received powers. We see that even in the two-user case the optimally power-controlled Rayleigh channel has a higher capacity than the Gaussian channel. As  $K$  increases we see that there is a significant increase in capacity for the optimal scheme. This is not surprising, since with many users, the probability that one of the channels is good is high which means that the corresponding user can transmit at a high rate. Moreover, as was mentioned in Section II, the capacity of the Rayleigh channel with no power control tends quickly with the number of users to that of the Gaussian channel.

In Fig. 3 we consider the effect of unequal average received power in a two-user system. We show the two single-user capacities and the sum-of-rates capacity as a function of  $\gamma_{s1}$  when  $\gamma_{s0}$  is fixed. We see that the when the average powers are heavily mismatched the stronger user is favoured considerably, since it dominates the sum-of-rates capacity. This means that when the average powers are not well controlled, the sum-of-rates capacity is not a good figure of merit.

We now examine the average probability of error per user with antipodal signalling. The probability of error as a function of the SNR is given by

$$P_e(\gamma) = Q(\sqrt{2\gamma}). \quad (16)$$

When a given user is transmitting, his received SNR is the largest in a set of  $K$  independent exponential random variables times the optimal power control adjustment. We ignore this adjustment by assuming it

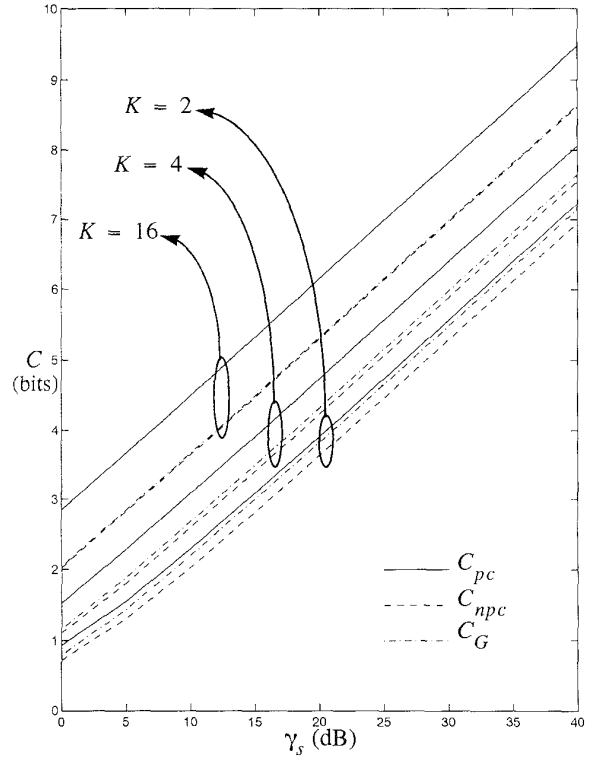


Figure 2:  $C_{pc}$ ,  $C_G$ ,  $C_{npc}$  for  $K = 2, 4, 16$ .

to be unity, since we will see that this yields a closed-form expression for the average error probability. This is, however, a sub-optimal approximation. Defining  $\gamma = \max(\gamma_0, \gamma_1, \dots, \gamma_{K-1})$ , it can be shown that the probability density for  $\gamma$  is given by

$$f_\gamma(\alpha) = \frac{K}{\gamma_s} e^{-\gamma/\gamma_s} (1 - e^{-\gamma/\gamma_s})^{K-1}, \quad \gamma > 0. \quad (17)$$

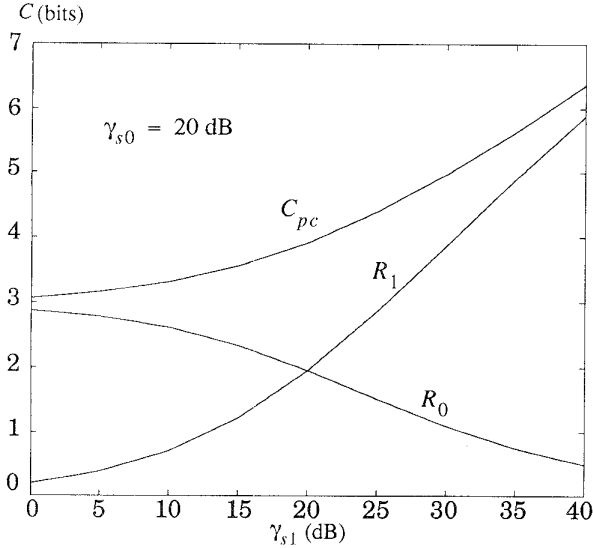
By averaging (16) over this density, we obtain the following expression for the average probability of error

$$\bar{P}_e = \frac{1}{2} \sum_{i=1}^K (-1)^{i-1} \binom{K}{i} \left[ 1 - \frac{1}{\sqrt{1 + i/\gamma_s}} \right], \quad (18)$$

which is completely equivalent to a selection diversity system of order  $K$  [7]. We show  $\bar{P}_e$  in Fig. 4 for several choices of  $K$ . As the number of users increases, there is a significant reduction in  $\bar{P}_e$ , which could be seen as a diversity effect. The major difference is that this form of diversity is inherent in all multiuser channels with fading.

## V CONCLUSIONS

This work addressed power control for the uplink of single-cell multiuser systems corrupted by fading. Using sum-of-rates capacity as a figure of merit and a constraint on the average transmit power, we have found an optimal power control scheme whose main

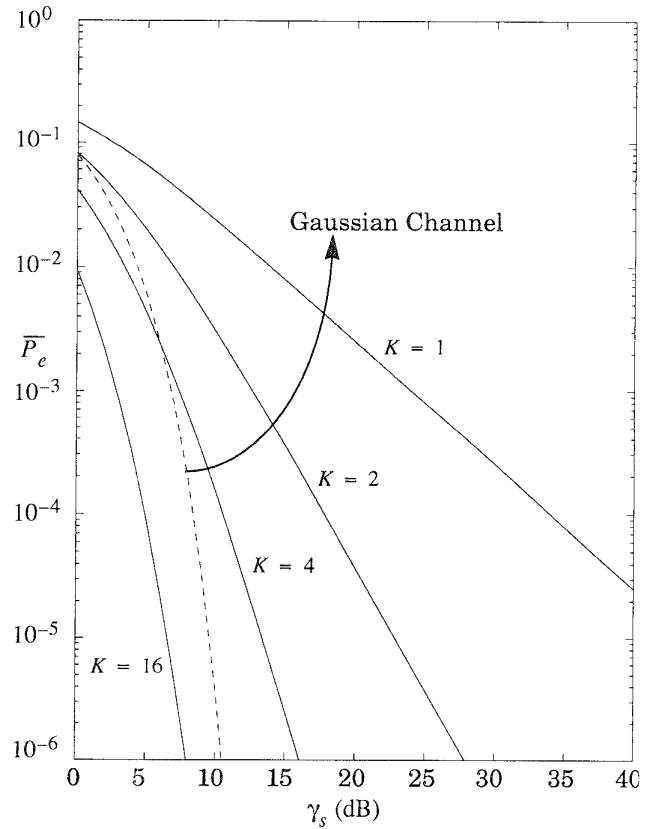


**Figure 3: Capacity vs. Power Level Difference**

characteristics are independent of the fading statistics. The most interesting result is that in order to attain capacity, only one user should transmit at any given time over the entire bandwidth. This user has the strongest signal at that particular time instant, relative to the average received powers of all the users. It is also possible for no user to be using the channel, which will occur if all of the users have received powers below a certain threshold. It is also interesting to see that the optimal scheme works in an opposite sense to conventional power control since it allocates more power to a user when his received power is high, and less when it is low.

We have presented numerical results for the capacity in Rayleigh fading. They show that significant improvements, for large  $K$ , can be made over conventional power control, especially when the number of users is high. In order for this scheme to be effective, the average received powers should be as close to equal as possible. We have determined the bit error rate with antipodal signals using a sub-optimal scheme in which only the user on the strongest instantaneous channel transmits. Exploiting the inherent diversity in this fashion can yield significant performance rewards.

We are currently working on obtaining similar results for the multi-receiver (multi-cell) case and frequency-selective channels. As well, it would be interesting to find numerical results for other fading environments, log-normal and Ricean for instance. In order to assess the viability of such a scheme, the dependence of the fading dynamics and the number of users on the average transmit time should be determined.



**Figure 4: Probability of Error with BPSK**

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