Fractionally Spaced Equalization of Linear Polyphase Channels and Related Blind Techniques Based on Multichannel Linear Prediction

Constantinos B. Papadias, *Member, IEEE*, and Dirk T. M. Slock, *Member, IEEE*

***Abstract*— In this paper, we consider the problem of linear equalization of polyphase channels and its blind implementation. These channels may result from oversampling the single output of a transmission channel or/and by receiving multiple outputs of an antenna array. A number of recent contributions in the field of blind channel identification have shown that polyphase channels can be blindly identified using only second-order statistics (SOS) of the output. In this work, we are mostly interested in the blind linear *equalization* of these channels: After some elaboration on the specifics of the equalization problem for polyphase channels, we show how optimal settings of various well-known types of linear equalization structures can be obtained blindly using only the output's SOS by using multichannel linear prediction or related techniques.**

I. Introduction

IN DIGITAL communications, a sequence of symbols $a(k)$ gets modulated and transmitted over a channel. We assume the modulation to be linear and the channel to be a linear time-invariant system with additive white circular noise. In practice, small degrees of nonlinearity and slow variations in time can always be accommodated. Let $h(t)$ be the overall impulse response of modulation and channel. Then, the continuous-time received signal can be written as

$$x(t) = \sum_{i=-\infty}^{\infty} a(i)h(t - iT) + v(t) \quad (1)$$

where $v(t)$ is the additive noise. The signal part of this single-input single-output (SISO) system is cyclostationary with period T, which is the symbol period. Its cycle spectrum is the discrete set $\{\alpha_m = \frac{m}{T}, m = 1, 2, \ldots\}$. In the presence of stationary noise, the noisy received signal has the same cyclostationarity properties.

If we sample the received continuous-time signal $x(t)$ at a rate $\frac{m}{T}$ greater than the symbol rate $\frac{1}{T}$ $(m > 1)$, the discrete-time received signal is also cyclostationary with period T and contains m distinct cycle frequencies. On the other hand, if m were chosen to be equal to 1, the sampled signal would be purely stationary. This manifestation of cyclostationarity as the sampling rate exceeds the baud rate is of critical importance for channel identification since it results in the presence of phase information in the cyclic second-order statistics (SOS) of the output. It is exactly this property that allows for blind identification of polyphase channels from SOS, as shown in [4]–[6].

Instead of being interested in the identification of the channel itself, in this paper, we will rather focus on the blind acquisition of simple equalizer settings whose output estimates the transmitted symbols in compliance with some optimality criterion. Linear and decision-feedback equalizers implemented through tap-delay lines have been extensively used in order to equalize received signals in many communication systems; however, their calibration had to be based on the use of a training sequence (a sequence of fixed symbols). Blind equalizers that were based on implicit (e.g., decision-directed or Bussgang-type) or explicit (cumulant-based) higher order statistics (HOS's) have been proposed since the 1970's in order to avoid the use of training signals. In this work, we will show how second-order blind equalization can be performed in the light of the recent results on SOS identifiability of polyphase channels. We will also examine some further implications of polyphase channels on linear equalization, irrespective of the blind aspect.

The rest of the paper is organized as follows. In Section II, we introduce the channel-equalizer model and some notation. Section III focuses on nonblind aspects of linear zero-forcing (ZF) equalization of polyphase channels, whereas in Section IV, we present multichannel linear prediction techniques for blind equalization. In Section V, we analyze minimum-mean-square-error (MMSE) polyphase equalization and show its connections with ZFE. Section VI shows some simulation results, whereas Section VII presents some conclusions. In this paper, we shall focus on the interpretations of polyphase channels arising from oversampling. However, apart from some discussions in Section III, most of the results apply to any polyphase channel.

II. Linear Fractionally-Spaced Equalization

At this point, we introduce the notation and the basic assumptions that will be used throughout the rest of the paper. The continuous-time channel $h(t)$ is assumed to be FIR with duration of approximatively NT. The oversampling factor (OF) is assumed to be m, and the sampling instants for the received signal $x(t)$ in (1) are $t_0 + T(k + \frac{j-1}{m})$ for integer k and $j = 1, 2, \ldots, m$. j represents the m symbol rate sampling

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C. B. Papadias is with Bell Laboratories, Lucent Technologies, Holmdel, NJ 07733-0400 USA (e-mail: papadias@bell-labs.com).

D. T. M. Slock is with the Eurécom Institute, Sophia Antipolis, France (e-mail: slock@eurecom.fr).

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phases of the oversampling pattern. t_0 represents the initial sampling time instant. In principle, it suffices to introduce a restricted $t_0 \in [0, T)$ to be fully general. However, we shall take $t_0 = t'_0 + dT$, where $t'_0 \in [0, T)$ in order to incorporate also an inherent delay due to transmission. d is chosen as the smallest integer such that

$$\left[h(t'_0 + dT) \cdots h\left(t'_0 + \left(d + \frac{m-1}{m}\right)T\right)\right] \neq 0. \quad (2)$$

The channel being causal implies that d will be non-negative.

We now introduce the *polyphase description* of the oversampled received signal, channel impulse response, and additive noise, respectively.

$$\begin{aligned} x_j(k) &= x\left(t_0 + T\left(k + \frac{j-1}{m}\right)\right) \\ h_j(k) &= h\left(t_0 + T\left(k + \frac{j-1}{m}\right)\right) \quad j = 1, 2, \ldots, m. \\ v_j(k) &= v\left(t_0 + T\left(k + \frac{j-1}{m}\right)\right) \end{aligned} \quad (3)$$

In the sequel, we will refer to the polyphase components of such quantities as *phases*. The oversampled received signal can now be represented in vector form at the symbol rate as

$$\mathbf{x}(k) = \sum_{i=0}^{N-1} \mathbf{h}(i)a(k-i) + \mathbf{v}(k) = \mathbf{H}_N A_N(k) + \mathbf{v}(k) \quad (4)$$

where $\mathbf{x}(k)$, $\mathbf{h}(k)$, $\mathbf{v}(k)$ are defined as

$$\mathbf{x}(k) = \begin{bmatrix} x_1(k) \\ \vdots \\ x_m(k) \end{bmatrix}, \quad \mathbf{v}(k) = \begin{bmatrix} v_1(k) \\ \vdots \\ v_m(k) \end{bmatrix}, \quad \mathbf{h}(k) = \begin{bmatrix} h_1(k) \\ \vdots \\ h_m(k) \end{bmatrix}. \quad (5)$$

The subchannels are defined as

$$H_i = [h_i(0) \quad \cdots \quad h_i(N-1)] \quad (6)$$

and the channel matrix $\mathbf{H}_N$ is a $m \times N$ matrix defined as

$$\begin{aligned} \mathbf{H}_N &= \begin{bmatrix} h_1(0) & \cdots & h_1(N-1) \\ \vdots & \cdots & \vdots \\ h_m(0) & \cdots & h_m(N-1) \end{bmatrix} \\ &= [\mathbf{h}(0) \quad \cdots \quad \mathbf{h}(N-1)] = \begin{bmatrix} H_1 \\ \vdots \\ H_m \end{bmatrix}. \end{aligned} \quad (7)$$

Finally, we denote by $A_N(k)$ the $N \times 1$ symbol vector

$$A_N(k) = [a(k) \cdots a(k-N+1)]^T. \quad (8)$$

We formalize the finite duration NT assumption of the channel as follows.

C1): FIR Assumption: $\mathbf{h}(0) \neq 0$, $\mathbf{h}(N-1) \neq 0$ and $\mathbf{h}(i) = 0$ for $i < 0$ or $i \geq N$.

In order to equalize the fractionally spaced channel, we will use a fractionally spaced linear equalizer, whose output is the sum of the outputs of symbol rate linear filters F_j in each subchannel. The channel-equalizer cascade in the case of an oversampling factor $m = 2$ will then look as in Fig. 1. The equalizer output produces an estimate $\hat{a}(k–d–n)$ of the delayed symbols with part of the delay (d) due to the inherent delay in the channel and part (n) intentional for improved performance (see further). In what follows, we shall ignore the inherent delay d.

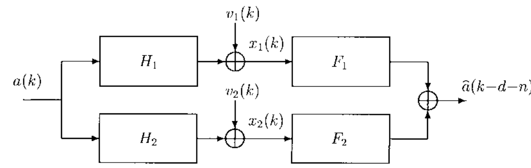

Fig. 1. Polyphase representation of the T/m fractionally spaced channeland equalizer for $m = 2$.

In the frequency domain, the z-transform of the channel response at the sampling rate $\frac{m}{T}$ is given as

$$H(z) = \sum_{i=1}^{m} z^{-(i-1)} H_i(z^m). \quad (9)$$

Similarly, the z-transform of the fractionally spaced $(\frac{T}{m})$ equalizer can also be decomposed into its polyphase components as

$$F(z) = \sum_{i=1}^{m} z^{(i-1)} F_i(z^m). \quad (10)$$

Although the equalizer defined by (10) is slightly noncausal, this does not cause a problem because the discrete-time filter is not a sampled version of an underlying continuous-time function. In fact, a particular equalizer phase $z^{(j-1)}F_j(z^m)$ follows in cascade the corresponding channel phase $z^{-(j-1)}H_j(z^m)$ so that the cascade $F_j(z^m)H_j(z^m)$ is causal. We assume the equalizer phases to be causal and FIR of length L:

$$F_j(z) = \sum_{k=0}^{L-1} f_j(k) z^{-k}, \quad j = 1, \ldots, m. \quad (11)$$

We also denote by $\mathbf{f}(k)$ the $1 \times m$ vector that contains the kth sample of each one of the m equalizer phases and by $\mathbf{F}_L$ a $1 \times Lm$ vector that contains the L consecutive vectors $\mathbf{f}(k)$, $k = 0, \ldots, L-1$

$$\begin{aligned} \mathbf{f}(k) &= [f_1(k) \cdots f_m(k)] \\ \mathbf{F}_L &= [\mathbf{f}(0) \cdots \mathbf{f}(L-1)]. \end{aligned} \quad (12)$$

Finally, we introduce the following multichannel z-transforms of the channel and the equalizer:

$$\begin{aligned} \mathbf{H}(z) &= \sum_{i=0}^{N-1} \mathbf{h}(i) z^{-i} = [H_1(z) \cdots H_m(z)]^T \\ \mathbf{F}(z) &= \sum_{i=0}^{L-1} \mathbf{f}(i) z^{-i} = [F_1(z) \cdots F_m(z)]. \end{aligned} \quad (13)$$

With the delay operator q^{-1} (such that $q^{-1}a(k) = a(k-1)$), we can represent the vectorized received signal as

$$\mathbf{x}(k) = \mathbf{H}(q)a(k) + \mathbf{v}(k) \quad (14)$$

in which the signal part $\mathbf{H}(q)a(k)$ corresponds to a single-input multiple-output (SIMO) system. Assuming the transmitted symbols and independent noise to be stationary, the vectorization has turned the cyclostationary scalar signal $x(t_0 + T(k + \frac{j-1}{m}))$ into a stationary vector signal $\mathbf{x}(k)$. Thus far, we have obtained multiple received signals by unraveling the multiple phases of the oversampled continuous-time received signal. An alternative way to arrive at the same picture is to have several antennas. Each of the antenna signals can then be oversampled or not [if not, then the representations at oversampled rate as in (9) or (10) are not applicable]. Hence, the total number of received signals is the product of the number of antennas times the oversampling factor. The SIMO deconvolution problem now boils down to the calculation of the optimal equalizer coefficients $f_j(i),\ j = 1, \ldots, m,\ i = 0, \ldots, L-1$.

III. FIR Zero-Forcing (ZF) Equalization

A. FIR Equalizability

We consider first the noise-free case. In the absence of noise, the optimal equalizer is a zero-forcing equalizer, i.e., one whose cascade with the channel gives a (possibly delayed) Dirac impulse response. The z transform of the equalizer output $\hat{a}(k-n)$ can be written as

$$\hat{A}(z) = \mathbf{F}(z)\mathbf{H}(z)A(z). \tag{15}$$

In order to achieve zero-forcing equalization in the absence of noise, we should have $\hat{A}(z) = A(z)z^{-n}$, where we allowed for a certain delay n. This gives the following ZF condition for the equalizer parameters:

$$\mathbf{F}(z)\mathbf{H}(z) = z^{-n}, \quad n \in \{0, 1, \ldots, N+L-2\}. \tag{16}$$

In the polyphase representation depicted in Fig. 1, we can recognize the channel and the equalizer to correspond to a cascade of an analysis filterbank followed by a synthesis filterbank. The ZF equalization condition corresponds to the perfect reconstruction property for the filterbank. In the filterbank literature [7], it is well known that perfect reconstruction is possible with a FIR synthesis bank for a FIR analysis bank. Equation (16) is the ZF condition in the z-domain. The counterpart of (16) in the time domain is

$$\mathbf{f}(k) * \mathbf{h}(k) = \delta(k-n) \tag{17}$$

where $*$ denotes convolution. By expressing this convolution as a matrix-vector product, (17) takes the form

$$[\mathbf{f}(0) \quad \cdots \quad \mathbf{f}(L-1)] \times \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(N-1) & 0_{m\times 1} & \cdots & 0_{m\times 1} \\ 0_{m\times 1} & \mathbf{h}(0) & \cdots & \mathbf{h}(N-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0_{m\times 1} \\ 0_{m\times 1} & \cdots & 0_{m\times 1} & \mathbf{h}(0) & \cdots & \mathbf{h}(N-1) \end{bmatrix} = \begin{bmatrix} 0_{n\times 1} \\ 1 \\ 0_{(N+L-n-2)\times 1} \end{bmatrix}^T \tag{18}$$

or equivalently

$$\mathbf{F}_L \mathcal{T}_L(\mathbf{H}_N) = [0_{1\times n} \quad 1 \quad 0_{1\times(N+L+n-2)}] \tag{19}$$

where we define $\mathcal{T}_M(\mathbf{x})$ as a (block) Toeplitz matrix with M (block) rows and $[\mathbf{x}\ 0_{p\times(M-1)}]$ as first (block) row (p is the number of rows in $\mathbf{x}$).

Equation (19) is a linear system of $L+N-1$ equations in the Lm unknowns $\mathbf{f}(0), \ldots, \mathbf{f}(L-1)$. For the existence of a solution, the vector on the right-hand side of (19) needs to be in the row space of $\mathcal{T}_L(\mathbf{H}_N)$. This can possibly happen for very short values of L. Indeed, if, e.g., after removal of the coefficient $\mathbf{h}(n)$ the rows in $\mathbf{H}_N$ are linearly dependent, then $L = 1$ suffices. In general, however, the matrix $\mathcal{T}_L(\mathbf{H}_N)$ needs to have full column rank. This imposes

$$L \geq \underline{L} = \left\lceil \frac{N-1}{m-1} \right\rceil \tag{20}$$

on the equalizer length L [2]. The matrix $\mathcal{T}_L(\mathbf{H}_N)$ is a generalized Sylvester matrix. It can be shown that for $L \geq \underline{L}$, it has full column rank if the following condition holds:

C2): No-Common-Zeros Condition: $\mathbf{H}(z) \neq 0, \ \forall z$, that is, if the subchannels $H_j(z)$ have no zeros in common. The same condition was given (in a different form) by Tong *et al.* in [4] and by Tugnait in [8]. Indeed, it is easy to see that if the subchannels have a zero in common, then this zero can be factored out, and the equalization for this factor becomes the equalization of a SISO system for which no FIR solution exists. It may occur that subsets of the subchannels have a shorter length (than N). In that case, $\underline{L}$ needs to be replaced by the minimum of (20) over all sets of subchannels. On the other hand, if the rows of $\mathbf{H}_N$ are not linearly independent, then m in (20) needs to be replaced by $m_{\text{eff}} = \text{rank}(\mathbf{H}_N) \leq \min\{m, N\}$, which is the effective number of (linearly independent) channels (as remarked in [9]). Therefore, we have the following result.

Theorem 1: Under the FIR channel assumption, a ZF FIR equalizer can be found from (19), provided that the equalizer length L satisfies

$$L \geq \underline{L} = \left\lceil \frac{N-1}{\text{rank}\{\mathbf{H}_N\} - 1} \right\rceil \tag{21}$$

and that the channel $\mathbf{H}(z)$ has no zeros [condition C2)].

In the formula for $\underline{L}$ in (21), we get, for the case of a frequency flat channel ($N = 1$), $\frac{0}{0} = 1$. A sufficient condition for any channel without zeros is $L \geq N-1$. However, if we consider the channel coefficients as random with continuous distributions, then the equalizer length condition in (20) is necessary and sufficient with probability one. We will henceforth assume $\underline{L}$ as in (20). For $m = 2$ channels, the minimal equalizer length is $\underline{L} = N-1$, which is about the same as the channel length N. However, the minimal equalizer length decreases with the number of channels. In particular, $\underline{L} = 1$ for $m = N$ channels. Assuming that the multiple channels arise due to the use of multiple antennas only, then such an equalizer corresponds to a purely spatial filter or beamformer. Hence, for a given delay spread N, a pure beamformer can perform equalization if enough antennas are used, as remarked in [10]. The advantage of the spatio-temporal approach is that ZF equalization can be done with fewer antennas.

Discussion: The multichannel FIR equalizability is in remarkable contrast to the single-channel problem, in which case

$$F_1(z)H_1(z) = z^{-n} \Rightarrow F_1(z) = \frac{1}{H_1^{\min}(z)} \frac{z^{-n}}{H_1^{\max}(z)} \quad (22)$$

where $H_1(z)$ has been factored into its minimum- and maximum-phase factors $H_1^{\min}(z)$ and $H_1^{\max}(z)$, respectively (assuming $H_1(z)$ has no zeros on the unit circle). Since $1/H_1^{\min}(z)$ is IIR and causal while $1/H_1^{\max}(z)$ is IIR and anticausal, $F_1(z)$ is noncausal and doubly infinitely long. For a given approximation error, $F_1(z)$ can be truncated to be of finite length and made causal for a judiciously chosen delay n. The length required for $F_1(z)$ depends on the proximity of the zeros of $H_1(z)$ to the unit circle.

The polyphase representation of (16) is

$$\sum_{i=1}^{m} F_i(z)H_i(z) = z^{-n} \quad (23)$$

which for $n = 0$ is known as the Bezout identity [11]. This identity states the existence of FIR equalizers for FIR subchannels that are coprime. Therefore, the Bezout identity is well known in the control literature and in the filter-bank/transmultiplexing literature. It appeared in the communications literature for the first time in [12], where it was applied over finite fields in convolutional coding. Indeed, a rate $1/m$ convolutional coder allows a m-channel representation. An FIR decoder then is, in fact, a multichannel equalizer that ZF equalizes the filtering introduced by the encoder. The Bezout identity was also used in image processing [13], although the formulation there was in continuous time (or rather, space). Fractionally spaced equalizers were introduced in the mid 1970's [14], [15]. However, it was not until much later [16] that it was realized that such equalizers are, in general, FIR when the channel is FIR. The next step, after the establishment of the existence of FIR equalizers as done by the Bezout identity, is then the issue of the minimal FIR equalizer length required: the subject of Theorem 1. It appears that this issue was first addressed in [2], [9], and [16].

FIR equalizability was addressed in a different fashion in [4], [6]. There, a packet oriented transmission mode was considered. In the absence of noise, the packet of received data can be written as

$$\mathbf{X}_M(k) = \mathcal{T}_M(\mathbf{H}_N)A_{M+N-1}(k) \quad (24)$$

where $\mathbf{X}_M(k) = [\mathbf{x}^H(k) \cdots \mathbf{x}^H(k-M+1)]^H$. Hence, a packet mode ZF equalizer is $\hat{A}_{M+N-1}(k) = \mathcal{T}_M^{\#}(\mathbf{H}_N)\mathbf{X}_M(k)$, where $\mathcal{T}_M^{\#}$ $(=(\mathcal{T}_M^H\mathcal{T}_M)^{-1}\mathcal{T}_M^H)$ denotes the pseudo-inverse of $\mathcal{T}_M$. Since $\mathcal{T}_M^{\#}\mathcal{T}_M = I_{mM}$, we have indeed ZF equalization, and since M is finite, the equalization is FIR. Now, every row in $\mathcal{T}_M^{\#}$ can be interpreted as an FIR (MMSE) ZF equalizer corresponding to a certain delay (that is different for every row). However, $\mathcal{T}_M^{\#}$ corresponds to a time-varying equalizer ($\mathcal{T}_M^{\#}$ is not block Toeplitz). The (z-domain) FIR equalizers discussed here are time-invariant filters corresponding to a fixed delay.

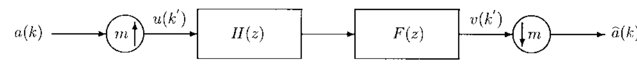

Fig. 2. Multirate representation of fractionally spaced channel and equalizer.

One important issue raised by Theorem 1 is the practical significance of the "no-common-zeros" condition C2): How likely is it for a practical channel to satisfy this condition? Sporadic answers to this question can be found in the literature. The occurrence of exact zeros in common is a zero probability event as discussed in [17]. In [18], a number of measurements of real wireless channel impulse responses has been performed and analyzed: Often enough, the subchannels of these impulse responses have several zeros close to each other when oversampled in time. However, these close-to-common zeros do not always lead to significant performance degradation of the corresponding equalizers. On the other hand, Ding has shown in [19] that there exist some specific classes of realistic multipath channels that always suffer from the problem of common zeros when oversampled in time, thus concluding their unidentifiability from second-order statistics (and unequalizability with FIR equalizers). However, it was later shown in [20] that the same channels do not suffer from this problem when oversampled in space (with the use of uniform linear antenna arrays). Temporal and spatial oversampling may thus lead to different conditionings with respect to this problem.

B. Multirate Representation

The upsampled by a factor L version of a discrete signal $x(kT)$ (with $T = 1$) is defined as

$$x^u(kT_L^u) = x^u\left(k\left(\frac{T}{L}\right)\right) = \begin{cases} x\left(\frac{k}{L}T\right), & \text{if } k \bmod L = 0 \\ 0, & \text{else} \end{cases} \quad (25)$$

whereas the downsampled by a factor M version of $x(kT)$ is defined as

$$x^d(kT_M^d) = x^d(k(TM)) = x(kMT). \quad (26)$$

The corresponding relations in the z-domain are

$$X^u(z) = \mathcal{Z}\left(x^u(kT_L^u)\right) = X(z^L) \quad (27)$$

$$X^d(z) = \mathcal{Z}\left(x^d(kT_M^d)\right) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(w^i z^{\frac{1}{M}}\right) \quad (28)$$

where $w = e^{-j\frac{2\pi}{M}}$ (see [7]). We may now formulate the following theorem.

Theorem 2: The fractionally spaced channel and equalizer corresponding to an integer oversampling factor of $OF = m$ can be represented as in Fig. 2, where $H(z)$ and $F(z)$ are defined in (9), (10), respectively.

Outline of Proof: With $G(z) = F(z)H(z)$ denoting the channel-equalizer cascade and with $w = e^{-j\frac{2\pi}{m}}$, the i/o relation in the z-domain for the setup of Fig. 2 is

$$\hat{A}(z) = \frac{1}{m}\sum_{i=0}^{m-1} V(w^i z^{1/m}) = \left(\frac{1}{m}\sum_{i=0}^{m-1} G(w^i z^{1/m})\right) A(z) \tag{29}$$

since $V(z) = G(z)A(z^m)$. Combining (29) with (9) and (10), we get, after some computation, $\hat{A}(z) = \frac{1}{m}\sum_{i=0}^{m-1} G(w^i z^{1/m}) A(z) = \sum_{i=1}^{m} F_i(z)H_i(z)A(z)$, which concludes the proof. □

Therefore, the SISO setup of Fig. 2 is an alternative to the polyphase representation of Fig. 1. The polyphase aspect is now contained in the upsampling and downsampling elements, as well as in the construction of the oversampled (fractionally spaced) channel and equalizer. The oversampling setup of Fig. 2 has been applied advantageously to CDMA in [21].

An interesting interpretation of the ZF condition in the light of the setup of Fig. 2 is the following. Focusing on (29), it is clear that the transfer function $\sum_{i=0}^{m-1} G(w^i z^{1/m})$ represents just a downsampled by a factor m version of $G(z)$ [compare with (28)]. Now let us consider the m phases of $g(k) = f(k)*h(k)$ in the time domain: $g_1(k) = g(mk+i-1)$, $i = 1, \ldots, m$. The ZF requirement then takes the form

$$g_1(k) = \delta(k-n). \tag{30}$$

Therefore, in order to be ZF, *one only* among the m different symbol-rate phases of the channel-equalizer cascade needs to be a delta function, whereas the other phases can be arbitrary. This increase in degrees of freedom, keeping the number of constraints fixed, is another way to explain the FIR ZF equalizability of a polyphase channel. The ZF requirement (30) is the oversampling equivalent of the continuous-time Nyquist condition [22], which states that the symbol-rate sampled version of the continuous-time equalizer (RX filter) and channel (TX filter) cascade should be a delta function.

C. Equalizability in the Frequency Domain

Therefore, from (29), the ZF condition in the frequency domain is [with $G(f) = G(e^{j2\pi fT/m})$ and delay $n = 0$]

$$\frac{1}{m}\sum_{i=0}^{m-1} G\left(f - \frac{i}{T}\right) = 1, \quad -\frac{1}{2T} < f < \frac{1}{2T}. \tag{31}$$

$G(f)$ is a periodic function with period m/T, but the left-hand side in (31) is periodic with period $1/T$. Equation (31) is the Nyquist condition for oversampling and is very similar to the corresponding condition in continuous time. The following interpretation can now be drawn: In order to have (31) satisfied, there needs to be some aliasing between adjacent frequency characteristics $G(f - \frac{i}{T})$ (otherwise, if there are some frequency regions with no aliasing, it will be impossible to have a nonzero sum within these regions). Let us suppose now that $G(f)$ is bandlimited with a bandwidth f_d. Since the distance between adjacent frequency pulses in (31) is $1/T$ and each pulse occupies a frequency range of width $2f_d$, it turns out that the condition for aliasing is

$$f_d > \frac{1}{2T}. \tag{32}$$

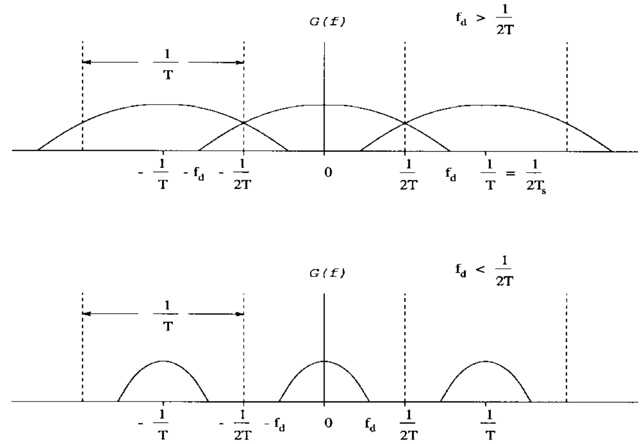

Fig. 3. Nyquist condition for the oversampled channel.

A graphical representation of the condition (32) can be found in Fig. 3, which shows the two different situations that may arise when (32) is satisfied or not, respectively. Now, as $G(f) = F(f)H(f)$, in order to have $G(f)$ satisfy (32), it is necessary that $H(f)$ satisfies it as well. This leads to the following theorem.

Theorem 3: Let f_d^h denote the bandwidth of the channel transfer function $H(f)$. Then, a necessary condition in order to achieve zero ISI in the multichannel setup is that

$$f_d^h > \frac{1}{2T}. \tag{33}$$

Theorem 3 gives us some insight on whether bandwidth limitations influence (or not) the channel estimation problem. If the channel is bandlimited with bandwidth $f_d^h \in (\frac{1}{2T}, \frac{m}{2T})$, this poses no particular problem for the determination of a ZF equalizer (assuming infinite length) ($f_d^h < \frac{m}{2T}$ is desirable in order to exploit all excess bandwidth). If $f_d^h < \frac{1}{2T}$, however, no aliasing occurs even at symbol rate sampling. Since $H_k(z)$ is the m-downsampled version of $z^{k-1}H(z)$, we get from (28)

$$H_k(f) = \frac{1}{m}\sum_{i=0}^{m-1} e^{j2\pi \frac{T}{m}(f-\frac{i}{T})(k-1)} H\left(f - \frac{i}{T}\right) \qquad k = 1, \ldots, m. \tag{34}$$

By replacing $G(f)$ with $H(f)$ in Fig. 3, it is now clear that if $f_d^h < \frac{1}{2T}$, the polyphase components $H_k(f)$ of the channel will be zero simultaneously in the frequency regions that correspond to nonoverlapping, rendering ZF equalization impossible. Therefore, Theorem 3 is the infinite length equivalent of the condition C2) of no common zeros in the FIR case. Note that Theorem 3 is only a necessary condition, however, whereas C2) is necessary and sufficient.

D. ZF Equalization and Noise Enhancement

In this section, we will work in the frequency domain in order to study the problem of noise enhancement of ZF equalizers in polyphase systems. We begin with a remark: We consider the case $m = 2$ channels and, w.l.o.g., $n = 0$ delay [for infinite length (and noncausal) equalizers, the delay is irrelevant; for causal equalizers however, the delay is crucial].

We assume that the setting $\mathbf{F}^o = [F_1^o \;\; F_2^o]$ corresponds to a ZF equalizer and, therefore, satisfies $\mathbf{F}^o(z)\mathbf{H}(z) = 1$. Now, consider another setting, namely, $\mathbf{F}'(z) = \mathbf{F}^o(z) + G(z)[-H_2(z) \;\; H_1(z)]$, where G is any stable filter of finite or infinite length. It can be easily verified that $\mathbf{F}'(z)\mathbf{H}(z) = 1$, which means that *any* equalizer of this form is also ZF. The variety of filters G that can be used represents a lot of degrees of freedom to determine different ZF equalizers for a given equalizer length. These will be all equivalent in the absence of noise; however, one will be optimal in the presence of noise in terms of noise enhancement. A linear equalizer with delay $n = 0$ is a linear estimator of the symbol a_k in terms of the received signal. The noise at the ZF equalizer output is the error in estimating a_k, and its variance is the MSE. Now, the optimal equalizer for a given length is only a special case of an equalizer of greater length, which can still be optimized (due to the degrees of freedom introduced by increasing the length), resulting in a better performing ZFE (lower MSE). We can sum up this discussion as follows.

- For an FIR polyphase channel, an FIR ZF equalizer exists if the FIR ZFE filter is long enough. If the ZFE filter length is longer than the minimum required value, then an infinity of FIR ZFE's exist. Among these, an optimal one exists in terms of noise enhancement (the MMSE ZFE).
- By increasing the length of the ZFE further, the noise enhancement can be further reduced.

We now focus on the derivation of the optimal infinite-length ZF equalizer.

1) Optimal Infinite-Length ZF Equalizer: Considering white noise $(E\mathbf{v}(k)\mathbf{v}^H(i) = \sigma_v^2\delta_{ki})$, the variance at the ZF equalizer output is

$$\sigma^2_{\mathrm{ZFE}} = \sigma_v^2 \sum_{i=1}^{m}\sum_{k=0}^{L-1} |f_i(k)|^2 = \sigma_v^2 \sum_{i=1}^{m} \int_{-\frac{1}{2}}^{\frac{1}{2}} |F_i(f)|^2\, df \tag{35}$$

using Parseval's identity. In the case of an infinite-length ZF equalizer, we get the following optimization criterion:

$$\begin{cases} \min\limits_{F_i(f)} \displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \sum_{i=1}^{m} |F_i(f)|^2\, df \\ \text{subject to } \displaystyle\sum_{i=1}^{m} F_i(f)H_i(f) = 1 \end{cases} \tag{36}$$

which reduces to the following frequency-wise criterion (since the constraint is frequency-wise and the cost function sums up non-negative contributions at different frequencies)

$$\begin{cases} \min\limits_{F_i(f)} \displaystyle\sum_{i=1}^{m} |F_i(f)|^2 \\ \text{subject to } \displaystyle\sum_{i=1}^{m} H_i(f)F_i(f) = 1 \end{cases} \tag{37}$$

at any frequency f. Using vector notation, we get

$$\begin{cases} \min\limits_{\mathbf{F}(f)} \|\mathbf{F}^T(f)\|^2 \\ \text{subject to } \langle \mathbf{H}^*(f), \mathbf{F}^T(f)\rangle = 1. \end{cases} \tag{38}$$

The solution is

$$\mathbf{F}_{\mathrm{MMSE\,ZFE}}(f) = \frac{\mathbf{H}^H(f)}{\sum_{i=1}^{m} |H_i(f)|^2} = \frac{\mathbf{H}^H(f)}{\|\mathbf{H}(f)\|^2} \tag{39}$$

where ^{H} denotes Hermitian transpose. This is the optimum (MMSE) infinite-length ZF equalizer. We remark that the MMSE ZFE consists of a cascade of the MISO filter $\mathbf{H}^H(f)$ (the matched filter) followed by a SISO filter $\|\mathbf{H}(f)\|^{-2}$. The matched filter combines the signal components in the m channels into a single signal in an optimal fashion. The SISO filter that follows then performs the zero forcing. The minimal noise variance (MMSE) at the ZF equalizer output is

$$\sigma^2_{\mathrm{MMSE\,ZFE}} = \sigma_v^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{df}{\|\mathbf{H}(f)\|^2}. \tag{40}$$

To gain insight in the dependence of the MMSE on the channels, we can rewrite (40) as

$$\sigma^2_{\mathrm{MMSE\,ZFE}} = \sigma_v^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{|H_1(f)|^2} \prod_{i=2}^{m} \left(1 - \frac{|H_i(f)|^2}{\sum_{k=1}^{i} |H_k(f)|^2}\right) df. \tag{41}$$

The first factor in the integral represents the contribution to $\sigma^2_{\mathrm{ZFE,min}}$ of the case of symbol-rate sampling $(m = 1)$. The other factors (which are smaller than 1) represent the reduction in MSE obtained by adding more channels. It is useful to compare the SNR of the MMSE ZFE with the matched filter bound (MFB), which is an upper bound on the SNR at the output of any (unbiased) equalizer. The MFB SNR is the SNR at the output of the matched filter. Therefore, we have

$$\begin{aligned} \mathrm{SNR}_{\mathrm{MMSE\,ZFE}} &= \frac{\sigma_a^2}{\sigma^2_{\mathrm{MMSE\,ZFE}}} = \frac{\sigma_a^2}{\sigma_v^2}\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \|\mathbf{H}(f)\|^{-2}\, df\right)^{-1} \\ &\le \frac{\sigma_a^2}{\sigma_v^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \|\mathbf{H}(f)\|^2\, df = \mathrm{SNR}_{\mathrm{MFB}}. \end{aligned} \tag{42}$$

For a single channel, the performance of a ZFE can be quite suboptimal when $|H_1(f)|$ shows significant dips. For multiple channels, and with sufficient diversity, the chance that the $|H_i(f)|$ have a dip at the same frequency and, hence, that $\|\mathbf{H}(f)\|$ shows a dip becomes smaller as the number of channels increases. In fact, $\|\mathbf{H}(f)\|$ tends to show less variation with frequency as m increases. Ideally, if $\|\mathbf{H}(f)\|$ becomes constant (allpass channel), then equality is obtained in (42); the MMSE ZF equalizer then performs maximum likelihood detection (equals the Viterbi equalizer). For multiple channels obtained by oversampling, it is interesting to investigate performance in terms of oversampling factor in the case of limited excess bandwidth. It can be shown that if the oversampling exceeds the Nyquist sampling frequency, then

$$\mathrm{SNR}_{\mathrm{MFB}} = \frac{\sigma_a^2}{N_o} \int_{-\infty}^{+\infty} |H(f)|^2\, df \tag{43}$$

where we used the fact that $\sigma_v^2 = mN_o/T$ with $N_o/2$ is the power spectral density of the white noise per component, and $H(f)$ is the Fourier transform of $h(t)$. Since the expression in (43) does not depend on m, we see that once the Nyquist sampling frequency has been exceeded, further oversampling does not lead to a further increase in MFB.

Another interesting comparison is with the work in [23] and [24]. In that work, an optimal receiver front end is used, consisting of a continuous-time matched filter (matched to the continuous-time channel impulse response) followed by symbol-rate sampling. This approach is impractical since it is hardly possible to know the continuous-time channel. Therefore, the approach taken here is to oversample and use simple antialiasing filters (which leave the noise white) before sampling. Therefore, $h(t)$ includes the anti-aliasing filter (which becomes transparent if the oversampling satisfies Nyquist). The MFB for the optimal approach in [23] and [24] is given by (43) as well [with $H(f)$ not including the anti-aliasing filter]. Therefore, the oversampling approach equals the optimal approach once the oversampling exceeds the Nyquist sampling frequency. The MMSE ZFE SNR for the optimal approach in [23] and [24] can be found to be

$$\mathrm{SNR}_{\mathrm{MMSE\,ZFE}} = \frac{\sigma_a^2}{N_o T^2}\left[\int_{-\frac{1}{2T}}^{\frac{1}{2T}}\left(\sum_{k=-\infty}^{\infty}\left|H\left(f-\frac{k}{T}\right)\right|^2\right)^{-1} df\right]^{-1} \quad (44)$$

where $H(f)$ again does not include the anti-aliasing filter. The MMSE ZFE SNR in (44) is again inferior to the MFB in (43) unless the sum in (44) is constant. On the other hand, the optimal MMSE ZFE SNR in (44) is an upper bound to the one in (42) and is again reached as the oversampling exceeds the Nyquist frequency for the channel.

IV. ZFE and Channel ID by Multichannel Linear Prediction

It is well known that in the case of symbol-rate sampling, the channel can be identified by spectral factorization if it is minimum-phase (MP) [i.e., if its scalar channel response $H(z)$ has no zeros out of the unit circle]. The counterpart of spectral factorization in the time domain is linear prediction: In the absence of noise, the input sequence equals the innovations process (prediction errors) if the channel is MP. This provides an elementary SOS technique for SISO blind equalization. However, this approach is highly restrictive as it only applies to MP SISO channels, which are rare in practice.

On the other hand, the MP property is much less restrictive in the SIMO case. A SIMO channel is again minimum phase if its (vector) channel response $\mathbf{H}(z)$ has no zeros outside the unit circle (no z_0 with $|z_0| > 1$ exists for which $\mathbf{H}(z_0) = 0$). Hence, all channels that satisfy the no-common-zero condition C2) are by definition MP (as they have no zeros at all, much less outside the unit circle). Therefore, $\mathbf{H}(z)$ is typically MP, even though none of its components $H_i(z)$ is MP. This fact has lead to frequency-domain blind SOS techniques for SIMO channel equalization that are based on spectral factorization [5], [25]. In this section, we will investigate the time-domain counterpart of this approach; namely, we will study the possibility of blind SIMO equalization and/or channel identification (ID) based on linear prediction (LP) of the polyphase channel output.

A. Multichannel Linear Prediction

In a first step, we are interested in identifying the channel coefficients of the SIMO setup based on linear prediction. Then, we will use this channel estimate in order to derive optimal MISO equalizers. We consider the following (forward) linear prediction problem:

Predict $\mathbf{x}(k)$ as a linear combination of the components of $\mathbf{X}_L(k-1)$.

The predicted vector sample can be written as

$$\hat{\mathbf{x}}(k) = \mathbf{p}_1\mathbf{x}(k-1) + \cdots + \mathbf{p}_L\mathbf{x}(k-L) = \mathbf{P}_L\mathbf{X}_L(k-1) \quad (45)$$

where $\{\mathbf{p}_i\}$ are $m \times m$ matrices and represent the LP coefficients, and $\mathbf{P}_L = [\mathbf{p}_1 \ \cdots \ \mathbf{p}_L]$. The prediction error can then be written as

$$\begin{aligned}\tilde{\mathbf{x}}(k)|_{\mathbf{X}_L(k-1)} &= \mathbf{x}(k) - \hat{\mathbf{x}}(k)|_{\mathbf{X}_L(k-1)} \\ &= [I_m \quad -\mathbf{P}_L]\mathbf{X}_{L+1}(k).\end{aligned} \quad (46)$$

The $m \times m$ prediction error variance is by definition

$$E\tilde{\mathbf{x}}(k)\tilde{\mathbf{x}}^H(k) = [I_m \quad -\mathbf{P}_L]R^{\mathbf{x}}_{L+1}[I_m \quad -\mathbf{P}_L]^H \quad (47)$$

where $R^{\mathbf{x}}_L = E(\mathbf{X}_L(k)\mathbf{X}_L^H(k))$. The minimization of the prediction error variance leads, therefore, to the following optimization problem:

$$\min_{\mathbf{P}_L}[I_m \quad -\mathbf{P}_L]R^{\mathbf{x}}_{L+1}[I_m \quad -\mathbf{P}_L]^H = \sigma^2_{\tilde{\mathbf{x}},L} \quad (48)$$

which gives

$$[I_m \quad -\mathbf{P}_L]R^{\mathbf{x}}_{L+1} = \left[\sigma^2_{\tilde{\mathbf{x}},L} \quad 0\cdots 0\right]. \quad (49)$$

Equations (49) are the normal equations. By partitioning $R^{\mathbf{x}}_{L+1}$, (49) can be written as

$$[I_m \quad -\mathbf{P}_L]\begin{bmatrix}\mathbf{r}_0 & \mathbf{r}\\ \mathbf{r}^H & R^{\mathbf{x}}_L\end{bmatrix} = \left[\sigma^2_{\tilde{\mathbf{x}},L} \quad 0\cdots 0\right] \quad (50)$$

which gives

$$\begin{cases}\sigma^2_{\tilde{\mathbf{x}},L} = \mathbf{r}_0 - \mathbf{r}\left(R^{\mathbf{x}}_L\right)\#\,\mathbf{r}^H\\ \mathbf{P}_L = \mathbf{r}\left(R^{\mathbf{x}}_L\right)\#.\end{cases} \quad (51)$$

Equation (51) shows how both the prediction error variance and the prediction coefficients can be computed from the SOS of the cyclostationary received signal. We now proceed to obtain channel estimates and optimal equalizers from these quantities.

B. LP-Based Multichannel Identification/ZF Equalization

We will perform LP on the noise-free signal ($v(t) \equiv 0$). The input–output relation of the SIMO channel can be written in the absence of noise as in (24). Hence, the covariance matrix $R^{\mathbf{x}}_L$ of the received signal $\mathbf{x}(k)$ has the following structure:

$$R^{\mathbf{x}}_L = \mathcal{T}_L(\mathbf{H}_N)R^a_{L+N-1}\mathcal{T}^H_L(\mathbf{H}_N) \quad (52)$$

where $R_L^a = EA_L(k)A_L^H(k) > 0$. The $R_L^{\mathbf{x}}$ is of dimension $Lm \times Lm$, and its rank is $L + N - 1$ [assuming C2) and $L \geq \underline{L}$]. Therefore, we have

$$R_L^{\mathbf{x}} = \begin{cases} \text{full-rank}, & L < \underline{L} \\ \text{singular}, & L > \underline{L}. \end{cases} \tag{53}$$

When $R_L^{\mathbf{x}}$ is singular, each further increase of L by 1 results in an increase of rank$(R_L^{\mathbf{x}})$ by 1 and an increase of the dimension of its nullspace by $m - 1$ (in fact, $\frac{\text{rank}}{\text{dimension}} = \frac{L+N-1}{mL} \overset{L\to\infty}{\to} \frac{1}{m}$). Note that in the presence of white noise, we have $\sigma_v^2 = \lambda_{\min}(R_L^{\mathbf{x}})$ for $L > \underline{L}$, and hence, the noise-free covariance matrix can always be found as $R_L^{\mathbf{x}} - \sigma_v^2 I_{mL}$.

When $L \geq \underline{L}$, $\mathcal{T}_L(\mathbf{H}_N)$ has full column rank. Hence, estimation in terms of $\mathbf{X}_L(k-1) = \mathcal{T}_L(\mathbf{H}_N)A_{L+N-1}(k-1)$ boils down to estimation in terms of $A_{L+N-1}(k-1)$. Therefore, we get

$$\begin{aligned} &\tilde{\mathbf{x}}(k)|_{\mathbf{X}_L(k-1)} \\ &\quad= \tilde{\mathbf{x}}(k)|_{A_{L+N-1}(k-1)} = \mathbf{x}(k) - \hat{\mathbf{x}}(k)|_{A_{L+N-1}(k-1)} \\ &\quad= \sum_{i=0}^{N-1} \mathbf{h}(i)a(k-i) - \sum_{i=0}^{N-1} \mathbf{h}(i)\hat{a}(k-i)|_{A_{L+N-1}(k-1)} \\ &\quad= \sum_{i=0}^{N-1} \mathbf{h}(i)a(k-i) - \sum_{i=1}^{N-1} \mathbf{h}(i)a(k-i) \\ &\qquad - \mathbf{h}(0)\hat{a}(k)|_{A_{L+N-1}(k-1)} \\ &\quad= \mathbf{h}(0)\tilde{a}(k)|_{A_{L+N-1}(k-1)}. \end{aligned} \tag{54}$$

Now, let us consider the prediction problem for the transmitted symbols. We get, similarly

$$\hat{a}(k)|_{A_M(k-1)} = \mathbf{Q}_M A_M(k-1) \tag{55}$$

$$[1 \quad -\mathbf{Q}_M]R_{M+1}^a = \sigma_{\tilde{a},M}^2[1 \quad 0\cdots 0] \tag{56}$$

where now, the elements of $\mathbf{Q}_M$ are scalars. From (54) and (56) for $M = L + N - 1$, we find

$$\begin{aligned} &\tilde{\mathbf{x}}(k)|_{\mathbf{X}_L(k-1)} \\ &\quad= [I_m \quad -\mathbf{P}_L]\mathbf{X}_{L+1}(k) = [I_m \, -\mathbf{P}_L]\mathcal{T}_{L+1}(\mathbf{H}_N)A_{L+N}(k) \\ &\quad= \mathbf{h}(0)\tilde{a}(k)|_{A_{L+N-1}(k-1)} = \mathbf{h}(0)[1 \, -\mathbf{Q}_{L+N-1}]A_{L+N}(k). \end{aligned} \tag{57}$$

Hence, the minimum prediction error variance is

$$\sigma_{\tilde{\mathbf{x}},L}^2 = \sigma_{\tilde{a},L+N-1}^2 \mathbf{h}(0)\mathbf{h}^H(0). \tag{58}$$

Therefore, for $L \geq \underline{L}$, the prediction error variance $\sigma_{\tilde{\mathbf{x}},L}^2$ is rank 1. Moreover, (58) allows us to find $\mathbf{h}(0)$ up to a scalar multiple from $\sigma_{\tilde{\mathbf{x}},L}^2$. At this point, we consider two cases separately.

1) Uncorrelated Input Sequence: In this case, $R_{L+N}^a = \sigma_a^2 I_{L+N}$. Combining with (56), this gives

$$\mathbf{Q}_{L+N-1} = 0_{1\times(L+N-1)}, \quad \sigma_{\tilde{a},L+N-1}^2 = \sigma_a^2. \tag{59}$$

Since (57) is valid for all $A_{L+N}(k)$, we have

$$\begin{aligned} [I_m \quad -\mathbf{P}_L]\mathcal{T}_{L+1}(\mathbf{H}_N) &= \mathbf{h}(0)\,[1 \quad -\mathbf{Q}_{L+N-1}] \\ &= \mathbf{h}(0)[1 \quad 0_{L+N-1}] \end{aligned} \tag{60}$$

and therefore

$$(\mathbf{h}^{\#}(0)[I_m \quad -\mathbf{P}_L])\mathcal{T}_{L+1}(\mathbf{H}_N) = [1 \quad 0_{L+N-1}] \tag{61}$$

where $\mathbf{h}^{\#}(0) = (\mathbf{h}^H(0)\mathbf{h}(0))^{-1}\mathbf{h}^H(0)$. Therefore, we have the following result.

Theorem 4: When the transmitted data are uncorrelated, the channel satisfies the no-common-zeros condition C2) and $L \geq \underline{L}$; then, a (0 delay) ZF equalizer can be found from linear prediction as

$$\mathbf{F}_{L+1,0}^{\text{ZF}} = \mathbf{h}^{\#}(0)[I_m \quad -\mathbf{P}_L]. \tag{62}$$

Note that to implement the closed-form solution (62), all we need are the SOS $R_{L+1}^{\mathbf{x}}$. Alternatively, least-squares linear prediction, applied to the noise-free signal (SOS), can be modified into a total least-squares approach for the noisy signal. Appropriate adaptive algorithms can be extrapolated from [26].

Using (60), we could also determine the channel $\mathbf{H}_N$ up to a scalar multiple. We may note that (60), written in the z-domain, is nothing but $\mathbf{P}(z)\mathbf{H}(z) = \mathbf{h}(0)$; hence, $\mathbf{H}(z) = \mathbf{P}^{-1}(z)\mathbf{h}(0)$. This is a relationship between two MP filters. Although $\mathbf{P}^{-1}(z)$ is IIR, $\mathbf{P}^{-1}(z)\mathbf{h}(0)$ is FIR. Note that the singular (because $\sigma_{\tilde{\mathbf{x}},\infty}^2$ is singular) vector process $\mathbf{x}(k) = \sum_{i=1}^{N-1}\mathbf{h}(i)a(k-i) = -\sum_{i=1}^{L}\mathbf{p}_i\mathbf{x}(k-i) + \mathbf{h}(0)a(k)$ is, at the same time, MA and AR. Therefore, in addition, the linear prediction approach is robust to channel length overdetermination. If the normal equations (50) are solved in an order-recursive fashion (using, e.g., the multichannel Levinson algorithm), the recursions will terminate at the correct order $\underline{L}$, as is typical when predicting an AR process.

The channel can alternatively be found from

$$\begin{aligned} &\mathbf{F}_{L+1,0}^{\text{ZF}} E\big(\mathbf{X}_{L+1}(k)\mathbf{X}_N^H(k+N-1)\big) \\ &\quad= \sigma_a^2[\mathbf{h}^H(N-1)\cdots\mathbf{h}^H(0)]. \end{aligned} \tag{63}$$

In the uncorrelated case, the prediction problem allows us (in theory) to also check whether the $H_j(z)$ have zeros in common. Indeed, the common factor colors the transmitted symbols (MA process), and hence, once $\sigma_{\tilde{\mathbf{x}},L'}^2$ becomes of rank 1, its one nonzero eigenvalue $\sigma_{\tilde{a},L'+N'-1}^2\mathbf{h}^H(0)\mathbf{h}(0)$ continues to decrease as a function of L' since for an MA process, $\sigma_{\tilde{a},L}^2$ is a decreasing function of L.

2) Correlated Input Sequence: If the transmitted symbols are correlated, we proceed as follows (Pisarenko-style [27, p. 500]). Linear prediction corresponds to the LDU factorization $LR^{\mathbf{x}}L^H = D$. The prediction filters are rows of L, whereas the prediction variances are the diagonal elements of D. Let us take l prediction filters corresponding to singularities in D and assume the longest one has block length L. Therefore, we obtain $\mathbf{F}_L^b$ of size $l \times Lm$. We introduce a block-componentwise transposition operator ^{t}, viz.

$$\begin{aligned} \mathbf{H}_N^t &= [\mathbf{h}(0)\cdots\mathbf{h}(N-1)]^t = [\mathbf{h}^T(0)\cdots\mathbf{h}^T(N-1)] \\ \mathbf{F}_N^t &= [\mathbf{f}(0)\cdots\mathbf{f}(N-1)]^t = [\mathbf{f}^T(0)\cdots\mathbf{f}^T(N-1)] \end{aligned} \tag{64}$$

where ^{T} is the usual transposition operator. Due to the singularities, we have

$$\mathbf{F}_L^b\mathcal{T}_L(\mathbf{H}_N) = 0 \Leftrightarrow \mathbf{H}_N^t\mathcal{T}_N\big(\mathbf{F}_L^{bt}\big) = 0. \tag{65}$$

Since $\mathbf{F}_L^b \mathbf{X}_L(k) = 0$ for the noise-free signal, we call $\mathbf{F}_L^b$ a set of *blocking* equalizers. We find that if $l(L+N-1) \geq mN-1$, then

$$\dim\big(\text{Range}^\perp\big\{\mathcal{T}_N\big(\mathbf{F}_L^{bt}\big)\big\}\big) = 1. \tag{66}$$

In that case, we can identify the channel $\mathbf{H}_N^{tH}$ (up to a scalar multiple) as the last right singular vector of $\mathcal{T}_N(\mathbf{F}_L^{bt})$. In particular, let $\mathbf{h}(0)^\perp$ be $m \times (m-1)$ of rank $m-1$ such that $\mathbf{h}(0)^{\perp H}\mathbf{h}(0) = 0$; then, with $L = \underline{L}+1$ and $l = m-1$, we can take

$$\mathbf{F}_{\underline{L}+1}^b = \mathbf{h}(0)^{\perp H}[I_m \quad -\mathbf{P}_{\underline{L}}]. \tag{67}$$

From (57), we can furthermore identify $\mathbf{Q}_{L+N-1}$, and via (56), this leads to the identification of the (Toeplitz) symbol covariance matrix R_{L+N}^a (assuming σ_a^2 is known).

3) Arbitrary-Delay ZFE We have previously been able to blindly obtain zero-delay ZF equalizers. As it is well known that for most practical channels, a better performance is achieved by equalizers that introduce some greater delay, we are interested in blindly obtaining such solutions as well. This is possible if instead of one-step-ahead prediction, as above, we use n-step-ahead prediction. This will allow us to avoid the dependence on $\mathbf{h}(0)$. The $(n+1)$-step-ahead (forward) linear prediction of the noise-free $\mathbf{x}(k)$ of order $L \geq \underline{L}$ can be written as

$$\tilde{\mathbf{x}}_{L,n}^f(k) = \tilde{\mathbf{x}}(k)|_{\mathbf{X}_L(k-n-1)} = \mathbf{P}_{\tilde{\mathbf{x}}_{L,n}^f}(q)\mathbf{x}(k) = \sum_{i=0}^{n} \mathbf{h}(i) a_{k-i} \tag{68}$$

where $\mathbf{P}_{\tilde{\mathbf{x}}_{L,n}^f}(z)$ is the prediction error filter. The use of the optimal predictor results, indeed, in the last expression in (68) for the prediction error, which is the part of $\mathbf{x}(k)$ that cannot be predicted from $\mathbf{X}_L(k-n-1)$ or, hence, $A_{L+N-1}(k-n-1)$. For $n = 0$, we retrieve the results of Section IV-B. Note that we can regard $\tilde{\mathbf{x}}_{L,n}^f(k)$ as the received signal from a truncated channel. If we now apply backward linear prediction of sufficient order M [replace N by $n+1$ in the expression in (20) for $\underline{L}$] to the signal $\tilde{\mathbf{x}}_{L,n}^f(k)$, we then obtain, as optimal prediction error

$$\tilde{\tilde{\mathbf{x}}}_M^b(k) = \mathbf{P}_{\tilde{\mathbf{x}}_M^b}(q)\tilde{\mathbf{x}}_{L,n}^f(k) = \mathbf{h}(n) a_{k-n-M}. \tag{69}$$

From (69) and (68), we deduce that we can obtain a ZF equalizer with delay $n+M$ (with M depending on n) as

$$\mathbf{F}_{n+L+M-1,n+M}^{\text{ZF}}(z) = \mathbf{h}^{\#}(n)\mathbf{P}_{\tilde{\mathbf{x}}_M^b}(z)\mathbf{P}_{\tilde{\mathbf{x}}_{L,n}^f}(z). \tag{70}$$

Notice that in (70), the dependence on $\mathbf{h}(0)$—see (62)—has been replaced by the dependence on $\mathbf{h}(n)$. This should allow for better conditioning of the solution in the presence of noise in most cases. A better way to use these results should be, perhaps, through the combination of several ZFE's corresponding to different delays $(=0, 1, \ldots, n)$ such that the chance for $\sum_{i=0}^{n} ||\mathbf{h}(i)||^2$ being small becomes small. The outputs of these ZFE's should be properly delayed to align them at the same a_{k-n} (see [28] and [29] for approaches along these lines).

V. MMSE Polyphase Equalization

Minimum-mean-square-error equalizers (MMSEE's) are known to perform better in general than ZFE's in the presence of noise. When the channel ($||\mathbf{H}(f)||$) has very deep spectral nulls, then the noise enhancement introduced by a ZF equalizer is very high (in the extreme case of channel zeros *on* the unit circle, the noise enhancement introduced by the ZFE is infinite). On the other hand, MMSEE's avoid this problem by compromising the noise amplification and the ISI reduction. In this section, we are interested in (especially blind) MMSE equalization in the context of the multichannel setup. In order to justify the superiority of MMSE equalizers, however, we first provide a comparison of ZF and MMSE equalizers in terms of noise enhancement. In the following, we assume that both the input $\{a(k)\}$ and the noise $\{\mathbf{v}(k)\}$ are white of variance σ_a^2, $\sigma_v^2 I_m$, respectively.

A. Comparison of ZF and MMSE Equalization Performance

In the frequency domain, the MMSEE minimizes the following quantity (assuming infinite length equalizers)

$$\sigma^2 = \min_{F_i(f)} \int_{\frac{1}{2}}^{\frac{1}{2}} \left\{ \sigma_a^2 \left| \sum_{i=1}^{m} H_i(f)F_i(f) - 1 \right|^2 + \sigma_v^2 \left(\sum_{i=1}^{m} |F_i(f)|^2 \right) \right\} df \tag{71}$$

which represents the SNR at the equalizer output. In the right-hand side of (71), the first term represents the ISI and the second term the noise contribution. It is actually due to this second term that the MMSEE differs from the ZFE [compare with (36)]. Now, the solution to the problem (71) is

$$\mathbf{F}_{\text{MMSE}}(f) = \frac{\mathbf{H}^H(f)}{||\mathbf{H}(f)||^2 + \frac{\sigma_v^2}{\sigma_a^2}} \tag{72}$$

which gives the optimal infinite-length MMSE equalizer. Equation (72) should be compared with (39). The additive term in the denominator of the expression appearing in (72) protects against the infinite noise enhancement that can be produced by a ZFE since the denominator in (72) is always strictly positive.

It is also worth noting that in contrast with the symbol-rate case in which the problem of infinite noise amplification of the ZFE appears when the channel has zeros on the unit circle, according to (39) and (72), in the multichannel case, this will only happen when the subchannels have zeros in common on the unit circle.

In the noiseless case, according to (72) and (39), the optimal (infinite-length) MMSE and ZF equalizers coincide. However, in the noisy case, the MMSE equalizer has a superior performance since

$$\sigma_{\text{MMSE}}^2 = \sigma_v^2 \int_{\frac{1}{2}}^{\frac{1}{2}} \frac{df}{||\mathbf{H}(f)||^2 + \frac{\sigma_v^2}{\sigma_a^2}} < \sigma_{\text{MMSE ZFE}}^2 \tag{73}$$

[see (40)]. Therefore, as in the symbol-rate case, the optimal MMSE equalizer will always be superior to the corresponding

ZF equalizer. We now proceed to the derivation of the MMSE equalizer.

B. Blind MMSE Polyphase Equalization Based on Channel Estimation

The MMSE criterion is of the form

$$\min_{\mathbf{F}_L} E(|a(k-n) - \mathbf{F}_L \mathbf{X}_L(k)|^2) \tag{74}$$

where $\mathbf{F}_L$ is the sought-after equalizer setting, n is the delay (lag) that allows the equalizer to be noncausal by n samples (and whose choice, as mentioned, may influence the equalizer performance considerably), and $\mathbf{F}_L \mathbf{X}_L(k) = \hat{a}(k-n)$ is an estimate of $a(k-n)$. The solution to the criterion (74) is the MMSE equalizer corresponding to delay n, which is given by the closed-form solution

$$\begin{aligned} \mathbf{F}_{L,n}^{\text{MMSE}} &= E\big(a(k-n)\mathbf{X}_L^H(k)\big)\big[E\big(\mathbf{X}_L(k)\mathbf{X}_L^H(k)\big)\big]^{-1} \\ &= \mathbf{d}_n \big(R_L^{\mathbf{x}}\big)^{-1}. \end{aligned} \tag{75}$$

Notice that the form of the equalizer given in (75) is the same as the one of the classical (symbol-rate) MMSE equalizer, the only difference being the different composition of the regression and equalizer vectors. In the presence of noise, the channel *i/o* relationship takes the form

$$\mathbf{X}_L(k) = \mathcal{T}_L(\mathbf{H}_N) A_{L+N-1}(k) + V_L(k) \tag{76}$$

and therefore, the matrix $R_L^{\mathbf{x}}$ is equal to

$$R_L^{\mathbf{x}} = \mathcal{T}_L(\mathbf{H}_N) R_{L+N-1}^a \mathcal{T}_L^H(\mathbf{H}_N) + R_L^{\mathbf{v}} \tag{77}$$

whereas the cross-correlation vector $\mathbf{d}_n$ has the general form (keeping in mind that the input is white of variance σ_a^2)

$$\mathbf{d}_n = \sigma_a^2 [\mathbf{0} \cdots \mathbf{0} \quad \mathbf{h}^H(N-1) \cdots \mathbf{h}^H(0) \quad \mathbf{0} \cdots \mathbf{0}] \tag{78}$$

where the number of zeros preceding and succeeding the channel coefficients depends on n (for low or high values of n, some channel coefficients may not appear at all in $\mathbf{d}_n$, e.g., for $n = 0$ $\mathbf{d}_n$ takes the form $\mathbf{d}_0 = \sigma_a^2[\mathbf{h}^H(0) \ \ 0 \cdots 0]$). Concerning the delay n, the only way to find its best value is to evaluate $\sigma_{\text{MMSE},n}^2 = \sigma_a^2 - \mathbf{d}_n (R_L^{\mathbf{x}})^{-1} \mathbf{d}_n^H$ for all n. A practical guideline is that the delay should be such that all the channel coefficients appear in $\mathbf{d}_n$ and preferably near the middle. When not all channel coefficients are contained in $\mathbf{d}_n$, performance may degrade significantly.

According to (75), (77), and (78), the MMSE equalizer can be determined blindly if the channel has already been identified. The following algorithm can, hence, be used for blind MMSE equalization:

Algorithm 1—LP-Based Blind MMSEE Using Channel Estimation:

1) Choose the delay parameter n in (74).
2) Estimate $R_{L+1}^{\mathbf{x}}$ from the received data $\{\mathbf{x}(k)\}$, estimate σ_v^2 (e.g. from the minimal eigenvalues of $R_{L+1}^{\mathbf{x}}$), and subtract the noise contribution from $R_{L+1}^{\mathbf{x}}$ using (77).
3) Compute the prediction coefficients $\mathbf{P}_L$ and error variance $\sigma_{\tilde{\mathbf{x}},L}^2$ from (50) and (51).
4) Compute an estimate of the channel impulse response $\mathbf{H}$ using (58) to find $\mathbf{h}(0)$ (up to a scalar multiple) and (62) combined with (19) or (63) to estimate the channel.
5) Compute $\mathbf{F}_{L,n}^{\text{MMSE}}$ from (78) and (75).

The above algorithm allows for the blind computation of the MMSE equalizer corresponding to a given delay n, based on the channel estimate given by the LP method of Section IV. In the sequel, we are interested in alternative approaches to compute blindly the MMSE equalizer by side stepping the channel estimation stage (in order to improve estimation accuracy and computational complexity). We call these *direct* methods.

C. Direct Methods for Blind MMSE Polyphase Equalization

1) Zero-Delay MMSEE: A straightforward blind approach to obtain the MMSE solution in the zero-delay case [$n = 0$ in (74)] is the following. We have from Section IV (using the noisy signal now)

$$[I_m \quad -\mathbf{P}_L] = \sigma_{\tilde{\mathbf{x}},L}^2 [I_m \quad 0 \cdots 0] \big(R_{L+1}^{\mathbf{x}}\big)^{-1}. \tag{79}$$

In the case of zero delay, according to (75) and (78), the MMSE equalizer takes the form

$$\mathbf{F}_{L,0}^{\text{MMSE}} = \sigma_a^2 \mathbf{h}^H(0) [I_m \quad 0 \cdots 0] \big(R_L^{\mathbf{x}}\big)^{-1}. \tag{80}$$

From (79) and (80), we deduce that

$$\mathbf{F}_{L,0}^{\text{MMSE}} = \sigma_a^2 \mathbf{h}^H(0) \sigma_{\tilde{\mathbf{x}},L-1}^{-2} [I_m \quad -\mathbf{P}_{L-1}]. \tag{81}$$

Equation (81) offers an alternative for the blind computation of the zero-delay MMSE equalizer. Now, the MMSEE is obtained by performing first linear prediction (using the denoised SOS) and requires only the inversion of the $m \times m$ prediction-error variance matrix. In [30], a similar approach has been taken, but only $x_1(k)$ is predicted from $\mathbf{X}_L(k-1)$. The resulting prediction error is $h_1(0)a_k$. In that case, the zero-delay MMSE ZF equalizer is also an unbiased MMSE equalizer and, hence, is just proportional to the corresponding MMSE equalizer [filtering $x_1(k)$ and $\mathbf{X}_L(k-1)$ to obtain $\hat{a}(k)$].

2) Maximal Delay MMSE: The maximal delay for the channel impulse response still to be contained in $\mathbf{d}_n$ is $n = L-1$. For such a delay, the following two-step approach can be used:

Step 1) Do blind zero delay ZF equalization. The equalizer output will be ($M + 1 \geq \underline{L}$)

$$\hat{a}(k) = a(k) + \mathbf{F}_{M+1,0}^{\text{ZF}} V_{M+1}(k). \tag{82}$$

Step 2) Obtain $\mathbf{F}_{L,L-1}^{\text{MMSE}}$ as a linear combination of a Wiener filter with $\mathbf{X}_L(k)$ as input vector, $\hat{a}(k-L+1)$ as desired response, and the backward linear prediction filter on the vector $\mathbf{X}_L(k)$. Preferably, $L \geq N$.

For Step 1, the computation of $\mathbf{F}_{M+1,0}^{\text{ZF}}$ has been discussed before. For Step 2, consider the FIR Wiener filtering problem

$$\min_{\mathbf{F}_L^W} E\big|\hat{a}(k-L+1) - \mathbf{F}_L^W \mathbf{X}_L(k)\big|^2 \tag{83}$$

which leads to the normal equations

$$\begin{aligned}\mathbf{F}_L^W R_L^{\mathbf{x}} &= E\hat{a}(k-L+1)\mathbf{X}_M^H(k)\\ &= Ea(k-L+1)\mathbf{X}_L^H(k)\\ &\quad + \mathbf{F}_{M+1,0}^{\mathrm{ZF}} EV_{M+1}(k-L+1)V_L^H(k)\\ &= \mathbf{F}_{L,L-1}^{\mathrm{MMSE}} R_L^{\mathbf{x}} + [0\cdots 0 \quad \mathbf{f}(0)]\sigma_v^2 \end{aligned} \tag{84}$$

where $\mathbf{f}(0)$ represents the first vector coefficient of $\mathbf{F}_{M+1,0}^{\mathrm{ZF}}$. Now, consider the multichannel backward prediction problem (on the noisy signal)

$$\tilde{\mathbf{x}}_{L-1}^b(k) = \tilde{\mathbf{x}}(k-L+1)|_{\mathbf{X}_{L-1}(k)} = \mathbf{P}_{\tilde{\mathbf{x}}_{L-1}^b}\mathbf{X}_L(k) \tag{85}$$

where $\mathbf{P}_{\tilde{\mathbf{x}}_{L-1}^b} = [-\mathbf{P}_{\hat{\mathbf{x}}_{L-1}^b} \ I_m]$ with normal equations

$$\mathbf{P}_{\tilde{\mathbf{x}}_{L-1}^b} R_L^{\mathbf{x}} = \left[0\cdots 0 \quad \sigma_{\tilde{\mathbf{x}}_{L-1}^b}^2\right]. \tag{86}$$

From (84) and (86), we conclude that

$$\mathbf{F}_{L,L-1}^{\mathrm{MMSE}} = \mathbf{F}_L^W - \sigma_v^2 \mathbf{f}(0)\sigma_{\tilde{\mathbf{x}}_{L-1}^b}^{-2}\mathbf{P}_{\tilde{\mathbf{x}}_{L-1}^b}. \tag{87}$$

In this expression for $\mathbf{F}_{L,L-1}^{\mathrm{MMSE}}$, all quantities can easily be found by adaptive filtering (an estimate for σ_v^2 also results as a byproduct of Step 1).

3) Arbitrary-Delay Blind MMSEE: A simple approach to blindly acquire the n-delay MMSE equalizer can be obtained by exploiting the relationship between ZF and MMSE equalizers. The n-delay MMSEE can be written as

$$\mathbf{F}_{\mathrm{MMSE},n} = \mathbf{d}_n\left(R_L^{\mathbf{x},s} + \sigma_v^2 I_{Lm}\right)^{-1} \tag{88}$$

where $R_L^{\mathbf{x},s}$ represents the noiseless correlation matrix (note that $\mathbf{d}_n$ is unaffected by the additive noise). The corresponding (MMSE) ZFE is given by

$$\mathbf{F}_{\mathrm{ZFE},n} = \mathbf{d}_n\left(R_L^{\mathbf{x},s}\right)^{\#}. \tag{89}$$

From (88) and (89), we obtain

$$\begin{aligned}\mathbf{F}_{L,n}^{\mathrm{MMSE}} &= \mathbf{d}_n\left(R_L^{\mathbf{x}}\right)^{-1} = \mathbf{d}_n\left(R_L^{\mathbf{x},s}\right)^{\#}R_L^{\mathbf{x},s}\left(R_L^{\mathbf{x}}\right)^{-1}\\ &= \mathbf{F}_{L,n}^{\mathrm{ZF}}\left(R_L^{\mathbf{x}} - \sigma_v^2 I_{Lm}\right)\left(R_L^{\mathbf{x}}\right)^{-1}\\ &= \mathbf{F}_{L,n}^{\mathrm{ZF}}\left(I_{Lm} - \sigma_v^2\left(R_L^{\mathbf{x}}\right)^{-1}\right).\end{aligned} \tag{90}$$

The second equality in (90) holds because $(R_L^{\mathbf{x},s})^{\#}R_L^{\mathbf{x},s}$ is the projection matrix on the signal subspace (the column space of $R_L^{\mathbf{x},s}$), and $\mathbf{d}_n^H$ belongs to the signal subspace. Equation (90) shows that there exists a simple linear relation that allows us to obtain an MMSEE from the corresponding ZFE for *any given delay* n. The merit of (90) is in that it allows the MMSEE to be obtained directly and blindly for an arbitrary delay n since the corresponding ZFE can be obtained by multichannel ZFE, as per Section IV. Hence, the following algorithm can be used for direct blind polyphase MMSE equalization.

Algorithm 2—LP-Based Direct Blind MMSE:

1) Choose the delay parameter n in (74).
2) Estimate $R^{\mathbf{x}}$ from the received data $\{\mathbf{x}(k)\}$, estimate σ_v^2 (e.g., from the minimal eigenvalues of $R^{\mathbf{x}}$), and compute $R^{\mathbf{x},s} = R^{\mathbf{x}} - \sigma_v^2 I$.
3) Compute $\mathbf{P}_{\tilde{\mathbf{x}}_{M'}^b}$, $\mathbf{P}_{\tilde{\mathbf{x}}_{L',n'}^f}$, $\mathbf{h}(n')$ from normal equations of the type (50) and (58).
4) Use (70) to compute the ZF equalizer corresponding to delay n.
5) Use (90) to obtain the n-delay MMSE equalizer.

D. Discussion

In the above, we have shown that MMSE equalization can be blindly achieved with the help of SOS and linear prediction. This approach has a number of advantages over other blind equalization methods, including the following.

- *Asymptotic optimality*: The proposed blind equalizers achieve asymptotically optimal Wiener (MMSE) performance. This improves on CMA equalizers that converge at best (i.e., even if they attain their global optima) to the *vicinity* of a Wiener solution—their output MSE contains nonvanishing bias terms w.r.t. the Wiener MSE (see [31], [32]).
- *Convergence*: As a result of the above property, these LP-based techniques do not suffer from the problem of ill-convergence, which is typically encountered by blind equalization methods.
- *Flexibility*: The proposed approach allows us to choose the delay parameter n to optimize the performance. This is typically not the case in most blind equalization algorithms that are not able to preselect the value of n. For example, the CMA may converge to solutions corresponding to different n's, depending on its initialization.
- *Robustness*: The proposed LP-based blind methods are robust in that
  a) they can cope with colored input signals;
  b) they are insensitive to the distance of the input signal to Gaussianity;
  c) they alleviate asymptotically the effects of additive white Gaussian noise;
  d) channel order overestimation does not degrade the performance (provided that a good delay has been chosen).
- *Data efficiency*: As these LP-based methods are based on the SOS of the channel output, they will also be more data efficient than their HOS-based (batch) counterparts, which need to collect more data to estimate higher order cumulants. This may, however, not necessarily always be the case for adaptive HOS algorithms such as the CMA, which are gradient-type and often converge to a local minimum at a relatively high speed (especially when they are normalized—see [33]). It is quite conceivable, however, that other SOS-based techniques such as subspace fitting [1], [6] or maximum-likelihood [1], [2], [34] may yield better statistical efficiency at the cost

of higher computational complexity and possibly channel order overestimation problems.

VI. Computer Simulation Examples

We first derive the expression of the output SNR for the multichannel setup that will be used in the sequel as a measure of equalizer performance. If we denote by $\{g(i)\}$ the (symbol rate) impulse response of the channel-equalizer cascade, then the equalizer output can be written as

$$\begin{aligned}\hat{a}(k-n) &= \mathbf{F}_{L,n}\mathbf{X}_L(k)\\ &= g(n)a(k) + \sum_{i\neq n} g(i)a(k-i) + b(k)\end{aligned} \quad (91)$$

where $b(k)$ represents the noise at the equalizer *output*. The output noise $\{b(k)\}$ is the input noise filtered by the equalizer bank

$$b(k) = \sum_{i=1}^{m}\sum_{j=0}^{L-1} v_i(k-j)f_i(j). \quad (92)$$

Therefore, the variance of the equalizer output is

$$\begin{aligned}E|\hat{a}(k)|^2 &= E\left|g(n)a(k) + \sum_{i\neq n} g(i)a(k-i) + b(k)\right|^2\\ &= \sigma_a^2\left(|g(n)|^2 + \sum_{i\neq n}|g(i)|^2\right) + \sigma_v^2\sum_{i=1}^{m}\sum_{j=0}^{L-1}|f_i(j)|^2\end{aligned} \quad (93)$$

assuming the symbol sequence to be white. Therefore, the SNR at the equalizer output is

$$\text{SNR}_o = \frac{\sigma_a^2|g(n)|^2}{\sigma_a^2\sum_{i\neq n}|g(i)|^2 + \sigma_v^2\sum_{i=1}^{m}\sum_{j=0}^{L-1}|f_i(j)|^2}. \quad (94)$$

In order to demonstrate the performance of LP-based blind equalizers, we consider the multipath radio channel given in [35], which has been oversampled twice ($m = 2$). The real and imaginary parts of the channel impulse response are shown in Fig. 4. Fig. 5 shows an example that demonstrates the output SNR's (in decibels) achieved by the linear prediction-based equalizer for different sizes of data samples. The channel of Fig. 5 has been truncated to retain the $N = 5$ most important coefficients of its two phases, and we assume an SNR of 30 dB. As mentioned in Section IV, the truncation allows us to achieve good performance with a zero-delay equalizer ($\mathbf{h}(0)$ not too small). The fractionally spaced equalizer used also has length $L = 5$ and was computed based on the estimated sample data covariance matrix of the received signal according to (81). Notice the good performance of the equalizer (dashed line), as well as how fast it approaches the ideal MMSE solution [the solid line is (80) using the true quantities].

In order to demonstrate the dependence of the equalizer performance on the delay parameter n [see (75)], we show in Fig. 6 the output SNR of the ideal MMSE equalizer (75) as

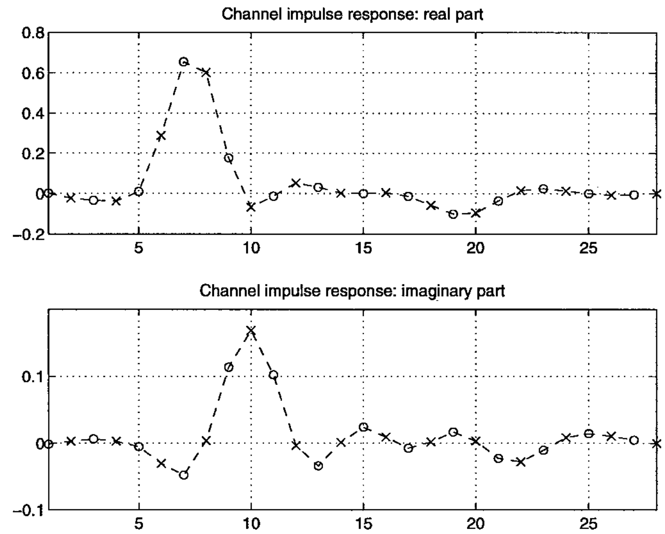

Fig. 4. Wireless channel impulse response.

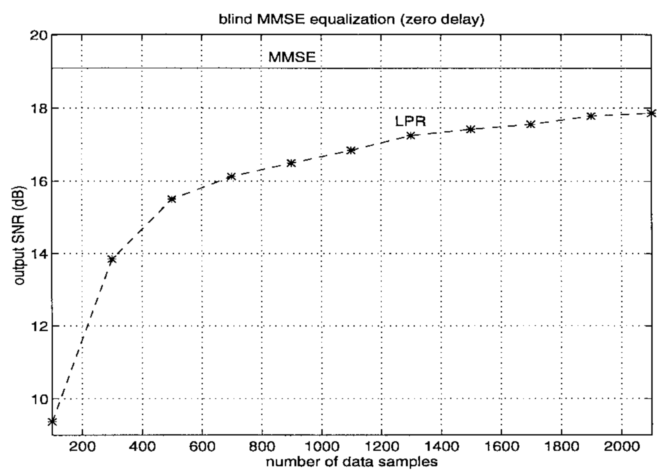

Fig. 5. Blind LP-based equalizer performance as a function of data sample size.

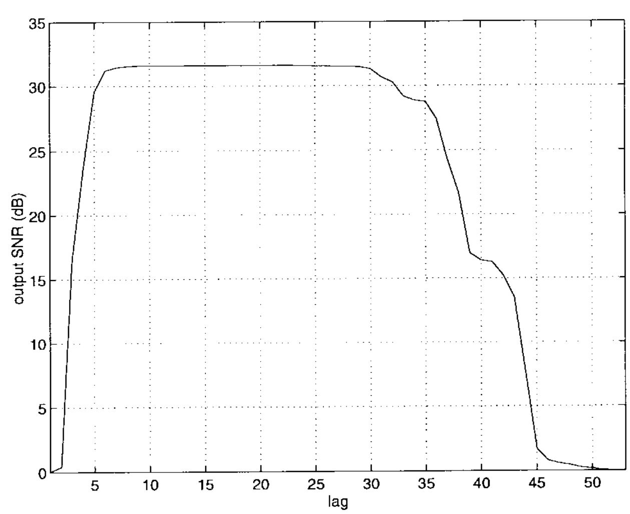

Fig. 6. MMSEE performance: Influence of the lag.

a function of the delay n for the channel of Fig. 4. We now use the full length of the channel (no truncation, $N = 14$) and plot the output SNR as a function of all the achievable delays for a $L = 40$ tap/phase equalizer (the maximum delay equals $L + N - 1 = 53$). Notice how very small and very large values of n lead to degraded performance, whereas an important number of intermediate delays provide practically the best achievable MMSE-type performance for the given equalizer length.

It is also worth commenting on the comparison between Figs. 5 and 6: in Fig. 5, the zero-delay equalizer has a satisfactory performance because $\mathbf{h}(0)$ of the truncated channel is not small. On the other hand, in Fig. 6, the low-delay performance is poor (due to a negligible $\mathbf{h}(0)$); however, the optimal performance is better that the one of Fig. 5 because $\mathbf{d}_n$ captures all the channel energy and is filled up with zeros on both ends (equalizer long enough).

VII. Conclusion

In this work, we have focused on the linear equalization of polyphase linear SIMO channels and have been interested in both nonblind (equalizability) and blind (SOS-based techniques) aspects. The advantages of polyphase as compared with single-phase equalization have been pointed out, and in particular, the use of oversampling when excess bandwidth is available and second-order blind techniques based on multichannel linear prediction for obtaining optimal equalizer settings (of the (MMSE) ZF or the MMSE type) have been proposed. These techniques present a number of advantages over their HOS-based counterparts. We believe that such simple blind equalization methods may be, in a number of cases, good alternatives to more complex techniques, combining relatively low complexity, ease of implementation, and flexibility with good performance.

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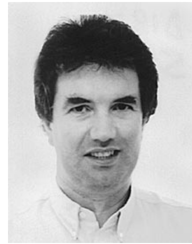
Constantinos B. Papadias (S'88–M'96) was born in Athens, Greece, in 1969. He received the diploma of electrical engineering from the National Technical University of Athens (NTUA) in 1991. He then enrolled as a graduate student in the doctoral program of the Ecole Nationale Supérieure des Télécommunications (ENST), Paris, France, from which he received the Ph.D. degree in signal processing (with highest honors) in 1995.

From 1992 to 1995, he was a teaching and research assistant at the Mobile Communications Department, Eurécom Institute, Sophia Antipolis, France, where he taught undergraduate courses in digital communications and signal processing and performed most of the research work of his thesis. After his graduation from the ENST, he joined the Information Systems Laboratory, Stanford University, Stanford, CA, as a Post-Doctoral Researcher, working in the Smart Antennas Research Group. In November 1997, he joined the Wireless Communications Research Department of Bell Laboratories, Lucent Technologies, Holmdel, NJ, where he is currently a Member of Technical Staff. His current research interests lie in the areas of signal processing and digital communications theory, including adaptive filtering, blind channel equalization and identification, source separation, smart antennas, multiuser detection, and wireless network optimization.

Dr. Papadias is a member of the Technical Chamber of Greece.



Dirk T. M. Slock (M'89) was born in Gent, Belgium, in 1959. He received the degree of engineer in electrical engineering from the University of Gent in 1982, and the M.S. degrees in electrical engineering and statistics and the Ph.D. degree in electrical engineering from Stanford University, Stanford, CA, in 1986, 1989, and 1989 respectively.

From 1982 to 1984, he was a Research and Teaching Assistant at the State University of Gent on a fellowship from the National Science Foundation of Belgium. In 1984, he received a Fulbright grant and went to Stanford University, where he held appointments as a Research and Teaching Assistant. From 1989 until 1991, he was a Member of the Scientific Staff of the Philips Research Laboratory, Leuven, Belgium, where he worked on the application of fast and efficient adaptive filtering algorithms to telecommunications problems. In 1991, he joined the Mobile Communications Department of the Eurécom Institute, Sophia Antipolis, France, where he has been an Associate Professor since 1994. His present areas of interest are geared toward signal processing for wireless communications and include multichannel equalization, spatio-temporal filtering, multiuser detection, interference cancellation, downlink, and speech processing.

Dr. Slock received the IEEE Signal Processing Society Paper Award and the EURASIP Best Paper Award in 1994. He was an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 1994 to 1996.