Covariance Shaping for Massive MIMO Systems

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***Abstract*—The low-rank behavior of massive multiple-input multiple-output (MIMO) channel covariance matrices and its exploitation for pilot decontamination and statistical beamforming are well documented. Existing algorithms, however, rely on signal subspace separation among user equipments (UEs) and, as such, they tend to fail when the distance between UEs becomes small. This paper proposes a solution to this problem via *covariance shaping* at the UE-side in the case where the UEs are equipped with (a small number of) multiple antennas. The key resides in: *i)* exploiting general non-Kronecker MIMO channel structures that allow the transmitter to suitably alter the channel statistics perceived by the base station, and *ii)* sacrificing some spatial degrees of freedom at each UE so as to improve the statistical orthogonality between closely spaced UEs. Numerical results illustrate the sum-rate performance gains of the proposed covariance shaping method with respect to existing ones.**

***Index Terms*—Covariance shaping, massive MIMO, pilot contamination, statistical beamforming.**

I. Introduction

Massive multiple-input multiple-output (MIMO) is expected to be a key enabler towards successful 5G deployments [1], [2]. While the promises of massive MIMO are numerous and well investigated, some challenges remain and are mainly related with the high dimensionality of the channel state vectors associated with each user equipment (UE). Such difficulties stem from both the complexity of implementing precoders/decoders that involve large matrix operations and the overhead linked with the pilot-aided training and feedback of channel state information.

In response to these issues, several important works have pointed out the significant role played by statistical information in the massive MIMO regime [3]–[8]. In particular, massive MIMO channel covariance matrices tend to crystallize into low-rank matrices whose rank is dictacted by the angle spread spanned by the multipath's angles of arrival (AoAs) when inpinging on the massive array [4], [6]. This low-rank behavior of the channel covariance matrices can be leveraged, for instance, to reduce feedback overhead as in [6], [8], [9]. Alternatively, it can be exploited to mitigate interference when sufficiently distant UEs exhibit non-overlapping signal subspaces: in fact, two distant UEs with AoAs that do not overlap at the base station (BS) can be discriminated based on statistical information only, both in the channel estimation stage (when using non-orthogonal pilot sequences [4]) and in the precoding stage [8]. On the downside, the performance of schemes such as [4], [6] relies on the structure of the UE's channel covariance matrix, i.e., its rank and the degree of separation from the other UEs' signal subspaces: these characteristics are given by the physical scattering environment and are generally beyond the designer's control.

Building on the fact that most current and future UEs are, and will be, equipped with a small-to-moderate number of antenna elements, this paper proposes to exploit their inherent spatial selectivity properties towards a suitable shaping of the channel covariance matrix performed at the UE-side. Our covariance shaping method is obtained by means of a statistical beamforming that effectively allows the UE to excite a suitable subset of all the available propagation paths between itself and the BS. In particular, we show that this method is effective in restoring partial or full orthogonality between the signal subspaces of UEs that are placed too close to each other. In turn, this scheme can be exploited for tasks such as pilot decontamination in time division duplex (TDD) systems and statistical beamforming at the BS. Specifically, in this paper, we propose a covariance shaping method for massive MIMO systems that is applicable when the UEs are equipped with at least two antenna elements. The antennas at the UEs are combined to enforce *statistical orthogonality* between UEs in the context of pilot-contaminated channel estimation [10]. We show substantial gains in terms of sum rate over a reference scenario in which the antennas at the UEs are directly used for spatial multiplexing without any concern for pilot contamination.

Notation. We use $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^*$ to denote the Hermitian, transpose, and conjugate operators, whereas $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm. The operator $\otimes$ indicates the Kronecker product, $\mathbf{I}_I$ is the I-dimensional identity matrix, and $(a_{ij})_{i,j=1}^{I}$ denotes the I-dimensional square matrix with indexed elements a_{ij}.

II. System Model

Consider a multiuser massive MIMO system where a BS equipped with M antennas communicates with K UEs with N antennas each. Let $\mathbf{H}_k \triangleq [\mathbf{h}_{k,1} \ldots \mathbf{h}_{k,M}] = [\mathbf{g}_{k,1}^T \ldots \mathbf{g}_{k,N}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{N\times M}$ denote the channel matrix between the BS and UE k, where $\mathbf{h}_{k,m} \in \mathbb{C}^{N\times 1}$ and $\mathbf{g}_{k,n} \in \mathbb{C}^{1\times M}$ are the channel vectors between the mth BS antenna and UE k and between the nth antenna of UE k and the BS, respectively. Focusing on the downlink transmission, we use $\mathbf{s}_k \in \mathbb{C}^{L_k\times 1}$ to denote the data symbol vector transmitted to UE k, with $\mathbb{E}[\mathbf{s}_k\mathbf{s}_k^{\mathrm{H}}] = \mathbf{I}_{L_k}$, and $\mathbf{s} \triangleq [\mathbf{s}_1^{\mathrm{T}} \ldots \mathbf{s}_K^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{L\times 1}$, with $L \triangleq \sum_{k=1}^{K} L_k$ being the total number of transmitted symbols. The multiuser precoding

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matrix $\mathbf{W} \triangleq [\mathbf{W}_1 \dots \mathbf{W}_K] \in \mathbb{C}^{M\times L}$, with $\|\mathbf{W}\|_{\mathrm{F}}^2 = 1$, is used at the BS to precode $\mathbf{s}$, where $\mathbf{W}_k \triangleq [\mathbf{w}_{k,1} \dots \mathbf{w}_{k,L_k}] \in \mathbb{C}^{M\times L_k}$ is the precoding matrix corresponding to $\mathbf{s}_k$. The receive signal at UE k is then expressed as

$$\mathbf{y}_k \triangleq \sqrt{\rho}\,\mathbf{H}_k\mathbf{W}\mathbf{s} + \mathbf{z}_k \tag{1}$$

$$= \sqrt{\rho}\,\mathbf{H}_k\mathbf{W}_k\mathbf{s}_k + \sqrt{\rho}\sum_{j\neq k}\mathbf{H}_k\mathbf{W}_j\mathbf{s}_j + \mathbf{z}_k \tag{2}$$

where ρ is the normalized transmit power at the BS and $\mathbf{z}_k \sim \mathcal{CN}(0, \mathbf{I}_N)$ is the normalized noise at UE k. Finally, UE k decodes $\mathbf{s}_k$ as $\hat{\mathbf{s}}_k \triangleq \mathbf{V}_k^{\mathrm{H}}\mathbf{y}_k$, where $\mathbf{V}_k \triangleq [\mathbf{v}_{k,1} \dots \mathbf{v}_{k,L_k}] \in \mathbb{C}^{N\times L_k}$, with $\|\mathbf{V}_k\|_{\mathrm{F}}^2 = 1$, is the corresponding combining matrix. The sum rate of such system is given by

$$R \triangleq \sum_{k=1}^{K}\sum_{\ell=1}^{L_k}\log_2\left(1+\frac{|\mathbf{v}_{k,\ell}^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_{k,\ell}|^2}{\sum_{j\neq k}|\mathbf{v}_{k,\ell}^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_{j,\ell}|^2 + \rho^{-1}\|\mathbf{v}_{k,\ell}\|^2}\right). \tag{3}$$

We assume a general channel model where the entries of $\mathbf{H}_k$ satisfy $\mathrm{vec}(\mathbf{H}_k) \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_k)$ [11, Ch.3]: here, the channel covariance matrix $\mathbf{\Sigma}_k \in \mathbb{C}^{NM\times NM}$ may be written as

$$\mathbf{\Sigma}_k \triangleq \begin{bmatrix} \mathbf{\Sigma}_{k,11} & \mathbf{\Sigma}_{k,12} & \dots & \mathbf{\Sigma}_{k,1M} \\ \mathbf{\Sigma}_{k,12}^{\mathrm{H}} & \mathbf{\Sigma}_{k,22} & & \vdots \\ \vdots & & \ddots & \\ \mathbf{\Sigma}_{k,1M}^{\mathrm{H}} & \dots & & \mathbf{\Sigma}_{k,MM} \end{bmatrix} \tag{4}$$

where $\mathbf{\Sigma}_{k,mn} \triangleq \mathbb{E}[\mathbf{h}_{k,m}\mathbf{h}_{k,n}^{\mathrm{H}}] \in \mathbb{C}^{N\times N}$ represents the cross-covariance matrix between the mth and nth columns of $\mathbf{H}_k$. Lastly, we define the covariance matrix seen by UE k as $\mathbf{R}_k \triangleq \mathbb{E}[\mathbf{H}_k\mathbf{H}_k^{\mathrm{H}}] \in \mathbb{C}^{N\times N}$ and the covariance matrix relative to UE k seen at the BS as $\mathbf{T}_k \triangleq \mathbb{E}[\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k] \in \mathbb{C}^{M\times M}$, respectively.[1]

III. Covariance Shaping at the UE-Side

In a massive MIMO setting, the BS can spatially separate the signals corresponding to different UEs if their covariance matrices lie on orthogonal supports, i.e., if $\mathbf{\Sigma}_k\mathbf{\Sigma}_j = \mathbf{0}$ for a given pair of UEs k and j. This is a property determined by the physical scattering environment that is rarely satisfied in practice [3]. In this context, we propose a *covariance shaping* method at the UE-side that relies uniquely on statistical information of the channels, aiming at enforcing the aforementioned orthogonality of channel statistics. The UEs preemptively apply a transmit/receive[2] beamforming vector (different for each UE) that aims at spatially separating their transmissions, thus drastically reducing interference. Hence, the MIMO channel is transformed into an effective multiple-input single-output (MISO) channel by combining the signal transmitted or received at the UE's antennas.

Let $\mathbf{v}_k \in \mathbb{C}^{N\times 1}$ denote the transmit/receive beamforming vector preemptively applied by UE k, with $\|\mathbf{v}_k\|^2 = 1$: in the rest of the paper, we refer to $\mathbf{v}_k$ as *covariance shaping vector*. The effective MISO channel between the BS and UE k is given by $\bar{\mathbf{g}}_k \triangleq \mathbf{v}_k^{\mathrm{H}}\mathbf{H}_k \in \mathbb{C}^{1\times M}$ and is distributed as $\bar{\mathbf{g}}_k \sim \mathcal{CN}(\mathbf{0}, \bar{\mathbf{\Phi}}_k)$, where $\bar{\mathbf{\Phi}}_k \in \mathbb{C}^{M\times M}$ is the effective covariance matrix defined as

$$\bar{\mathbf{\Phi}}_k \triangleq \mathbb{E}[\bar{\mathbf{g}}_k^{\mathrm{T}}\bar{\mathbf{g}}_k^*] \tag{5}$$

$$= \big((\mathbf{I}_M \otimes \mathbf{v}_k^{\mathrm{H}})\mathbf{\Sigma}_k(\mathbf{I}_M \otimes \mathbf{v}_k)\big)^{\mathrm{T}} \tag{6}$$

with $\mathbf{\Sigma}_k$ introduced in (4), and $\mathbb{E}[\|\bar{\mathbf{g}}_k\|^2] = \mathbf{v}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{v}_k$. Keeping our focus on the downlink transmission, the BS now transmits only one symbol $s_k \in \mathbb{C}$ to each UE k (i.e., $L_k = 1$, $k = 1, \dots, K$): thus, we have $\mathbf{s} = [s_1 \dots s_K]^{\mathrm{T}} \in \mathbb{C}^{K\times 1}$ and the multiuser precoding matrix becomes $\mathbf{W} = [\mathbf{w}_1 \dots \mathbf{w}_K] \in \mathbb{C}^{M\times K}$, where $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$ is the precoding vector corresponding to s_k. The receive signal at UE k reads as

$$\bar{y}_k \triangleq \sqrt{\rho}\,\bar{\mathbf{g}}_k\mathbf{W}\mathbf{s} + \bar{z}_k \tag{7}$$

$$= \sqrt{\rho}\,\bar{\mathbf{g}}_k\mathbf{w}_k s_k + \sqrt{\rho}\sum_{j\neq k}\bar{\mathbf{g}}_k\mathbf{w}_j s_j + \bar{z}_k \tag{8}$$

with $\bar{z}_k \triangleq \mathbf{v}_k^{\mathrm{H}}\mathbf{z}_k \sim \mathcal{CN}(0,1)$, and the sum rate of such system is finally given by

$$\bar{R} \triangleq \sum_{k=1}^{K}\log_2\left(1+\frac{|\bar{\mathbf{g}}_k\mathbf{w}_k|^2}{\sum_{j\neq k}|\bar{\mathbf{g}}_k\mathbf{w}_j|^2 + \rho^{-1}}\right). \tag{9}$$

Hence, with the proposed covariance shaping method, we sacrifice some spatial degrees of freedom at the UE-side (i.e., the possibility of transmitting multiple streams to each UE) in exchange for improved effective channel separation between the UEs.

Consider two UEs k and j with similar channel statistics, i.e., such that $\mathbf{\Sigma}_k \simeq \mathbf{\Sigma}_j$:[3] our objective is to design the covariance shaping vectors $\mathbf{v}_k$ and $\mathbf{v}_j$ in order to reduce the spatial correlation of the two UEs. In this respect, we use

$$\Omega_{kj}(\mathbf{v}_k, \mathbf{v}_j) \triangleq \mathrm{tr}(\bar{\mathbf{\Phi}}_k\bar{\mathbf{\Phi}}_j) \tag{10}$$

$$= \mathrm{tr}\big((\mathbf{I}_M \otimes \mathbf{v}_k^{\mathrm{H}})\mathbf{\Sigma}_k(\mathbf{I}_M \otimes \mathbf{v}_k) \times (\mathbf{I}_M \otimes \mathbf{v}_j^{\mathrm{H}})\mathbf{\Sigma}_j(\mathbf{I}_M \otimes \mathbf{v}_j)\big) \tag{11}$$

$$= \textstyle\sum_{m,n=1}^{M}\mathbf{v}_k^{\mathrm{H}}\mathbf{\Sigma}_{k,mn}\mathbf{v}_k\mathbf{v}_j^{\mathrm{H}}\mathbf{\Sigma}_{j,mn}^{\mathrm{H}}\mathbf{v}_j \tag{12}$$

as a metric to measure such a correlation. Observe that $\Omega_{kj}(\mathbf{v}_k, \mathbf{v}_j) = 0$ implies $\bar{\mathbf{\Phi}}_k\bar{\mathbf{\Phi}}_j = \mathbf{0}$, i.e., that $\bar{\mathbf{\Phi}}_k$ and $\bar{\mathbf{\Phi}}_j$ lie on orthogonal supports [3]: considering the eigenvalue decomposition $\bar{\mathbf{\Phi}}_k = \mathbf{U}_{\bar{\mathbf{\Phi}}_k}\mathbf{\Lambda}_{\bar{\mathbf{\Phi}}_k}\mathbf{U}_{\bar{\mathbf{\Phi}}_k}^{\mathrm{H}}$, this occurs when $\mathbf{U}_{\bar{\mathbf{\Phi}}_k} = \mathbf{U}_{\bar{\mathbf{\Phi}}_j}$ and $\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathbf{\Phi}}_k}\mathbf{\Lambda}_{\bar{\mathbf{\Phi}}_j}) = 0$ (i.e., $\bar{\mathbf{\Phi}}_k$ and $\bar{\mathbf{\Phi}}_j$ need to be rank deficient). Clearly, in the general case, $\bar{\mathbf{\Phi}}_k\bar{\mathbf{\Phi}}_j = \mathbf{0}$ imposes M^2 conditions whereas only $2N$ variables can be adjusted: this means that the resulting system of equations can be solved when $N \geq \frac{M^2}{2}$, which is generally not verified in practice. Some of these conditions might be removed by assuming a

[1] In case of downlink transmission, $\mathbf{R}_k$ and $\mathbf{T}_k$ represent the receive and transmit covariance matrices, respectively.

[2] Although this paper focuses on the downlink transmission, the concept of covariance shaping is equally meaningful in the uplink direction.

[3] In general, two UEs have similar channel statistics when the distance between them is much smaller than their distance to the scatterers and to the BS.

specific channel model (e.g., [12], [13]) that can reveal the low-rank structure of the covariance matrices, which will be considered in future works. Nevertheless, it is of interest to minimize, in the general case, the spatial correlation between UEs k and j as

$$\begin{aligned} \min_{\mathbf{v}_k,\mathbf{v}_j} \quad & \Omega_{kj}(\mathbf{v}_k,\mathbf{v}_j) \\ \text{s.t.} \quad & \|\mathbf{v}_k\|^2 = \|\mathbf{v}_j\|^2 = 1, \\ & \mathbf{v}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{v}_k \geq p_k, \\ & \mathbf{v}_j^{\mathrm{H}}\mathbf{R}_j\mathbf{v}_j \geq p_j \end{aligned} \tag{13}$$

where the minimum power constraint $\mathbf{v}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{v}_k \geq p_k$ ensures that the effective channel separation is not achieved by beamforming towards directions carrying exceedingly low power. In this regard, the values of p_k and p_j can be chosen so as to guarantee a sufficient receive signal-to-noise ratio (SNR) at the UEs during the downlink transmission or at the BS during the uplink channel estimation. Alternatively, one can minimize the spatial correlation between UEs k and j over the average power of their effective channels as

$$\begin{aligned} \min_{\mathbf{v}_k,\mathbf{v}_j} \quad & \frac{\Omega_{kj}(\mathbf{v}_k,\mathbf{v}_j)}{(\mathbf{v}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{v}_k)(\mathbf{v}_j^{\mathrm{H}}\mathbf{R}_j\mathbf{v}_j)} \\ \text{s.t.} \quad & \|\mathbf{v}_k\|^2 = \|\mathbf{v}_j\|^2 = 1. \end{aligned} \tag{14}$$

Unfortunately, a closed-form solution to (13) and (14) is not available in the general case and one needs to resort to exhaustive search algorithms. A suboptimal, low-complexity solution to problem (14) can be achieved, for instance, via alternate optimization. For a fixed $\mathbf{v}_j$, defining $\eta_{j,mn}(\mathbf{v}_j) \triangleq \mathbf{v}_j^{\mathrm{H}}\mathbf{\Sigma}_{j,mn}\mathbf{v}_j/(\mathbf{v}_j^{\mathrm{H}}\mathbf{R}_j\mathbf{v}_j)$, we have the following optimization problem in $\mathbf{v}_k$:

$$\begin{aligned} \min_{\mathbf{v}_k} \quad & \frac{\mathbf{v}_k^{\mathrm{H}}\big(\sum_{m,n=1}^{M}\eta_{j,mn}^{*}(\mathbf{v}_j)\mathbf{\Sigma}_{k,mn}\big)\mathbf{v}_k}{\mathbf{v}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{v}_k} \\ \text{s.t.} \quad & \|\mathbf{v}_k\|^2 = 1. \end{aligned} \tag{15}$$

where the objective has the form of a generalized Rayleigh quotient and thus its solution is given by the minimum eigenvector of $\mathbf{R}_k^{-1}\big(\sum_{m,n=1}^{M}\eta_{j,mn}^{*}(\mathbf{v}_j)\mathbf{\Sigma}_{k,mn}\big)$. A similar optimization problem in $\mathbf{v}_j$ is obtained for a fixed $\mathbf{v}_k$. The values of $\mathbf{v}_k$ and $\mathbf{v}_j$ are obtained by alternating the minimization with respect to one of the two, until the difference between the objective in consecutive iterations is sufficiently small. Note that each UE can obtain its covariance shaping vector without any information exchange with the other UE, provided that the channel statistics of the latter are known.

The result of (14) and (15) heavily depends on the physical scattering environment. Consider the scenario in Figure 1 where there is no direct line-of-sight (LoS) path between the UEs and the BS: the covariance shaping vectors tend to align with the channel directions that are the most orthogonal to each other while carrying sufficient power, which results in a nearly interference-free scenario, as shown in Figure 2. In the case where a LoS path exists, this would generally carry more power than any other path and, therefore, the directions selected by the covariance shaping vector would not sensibly deviate from it: this would be necessary to avoid an excessive loss in useful power, although it might not be effective in restoring the orthogonality between the signal subspaces of the UEs.

A. Kronecker Channel Model

Let us consider the particular case where the channels $\mathbf{H}_k$ are modeled using the Kronecker channel model [14], i.e., $\mathbf{H}_k = \mathbf{R}_k^{\frac{1}{2}}\mathbf{H}_k^{(\mathrm{w})}\mathbf{T}_k^{\frac{1}{2}}$, with $\mathbf{R}_k$ and $\mathbf{T}_k$ defined in Section II, and $\mathrm{vec}(\mathbf{H}_k^{(\mathrm{w})}) \sim \mathcal{CN}(\mathbf{0},\mathbf{I}_{NM})$. Under this model, the channel covariance matrix in (4) is expressed as $\mathbf{\Sigma}_k = \mathbf{T}_k^{\mathrm{T}} \otimes \mathbf{R}_k$ with block elements given by $\mathbf{\Sigma}_{k,mn} = t_{k,mn}^{*}\mathbf{R}_k$, where $t_{k,mn}$ denotes the (m,n)th element of $\mathbf{T}_k$. Hence, we have that

$$\Omega_{kj}(\mathbf{v}_k,\mathbf{v}_j) = \mathrm{tr}\big((\mathbf{v}_k^{\mathrm{H}}\mathbf{\Sigma}_{k,mn}^{\mathrm{H}}\mathbf{v}_k)_{m,n=1}^{M}(\mathbf{v}_j^{\mathrm{H}}\mathbf{\Sigma}_{j,mn}^{\mathrm{H}}\mathbf{v}_j)_{m,n=1}^{M}\big) \tag{16}$$

$$= \mathrm{tr}\big((t_{k,mn}\mathbf{v}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{v}_k)_{m,n=1}^{M} \times (t_{j,mn}\mathbf{v}_j^{\mathrm{H}}\mathbf{R}_j\mathbf{v}_j)_{m,n=1}^{M}\big) \tag{17}$$

$$= \mathrm{tr}(\mathbf{T}_k\mathbf{T}_j)\mathbf{v}_k^{\mathrm{H}}\mathbf{R}_k\mathbf{v}_k\mathbf{v}_j^{\mathrm{H}}\mathbf{R}_j\mathbf{v}_j. \tag{18}$$

It is straightforward to see that, in this case, the solution to problem (13) is attained when the covariance shaping vectors $\mathbf{v}_k$ and $\mathbf{v}_j$ are chosen as the minimum eigenvectors of $\mathbf{R}_k$ and $\mathbf{R}_j$, respectively, that satisfy the minimum power constraints (note that these can be computed independently by UEs k and j without any information exchange between them). However, this strategy corresponds to beamforming towards directions carrying low power. On the other hand, the objective in (14) reduces to $\mathrm{tr}(\mathbf{T}_k\mathbf{T}_j)/(\mathrm{tr}(\mathbf{T}_k)\mathrm{tr}(\mathbf{T}_j))$ and it is thus independent on $\mathbf{v}_k$ and $\mathbf{v}_j$. This is in accordance with the properties of the Kronecker channel model, whereby the transmit and receive covariance matrices are independent. Hence, no meaningful effective channel separation can be performed under the Kronecker channel model. The orthogonality between covariance matrices remains a property of particular scenarios, i.e., when the physical scattering environment is such that $\mathrm{tr}(\mathbf{T}_k\mathbf{T}_j) = 0$ (which is rarely satisfied in practice [12], [14]).

B. The Case of More Than Two UEs

So far, we have considered the case where only $K = 2$ closely spaced UEs are present. The extension to more than two UEs will be analyzed in depth in the longer version of this paper; however, an implementable procedure for this case is briefly illustrated next. Assume that the UEs exchange statistical information (i.e., their channel covariance matrices) between themselves and the BS on a low-rate control channel. Each UE can thus pair with its nearest neighbor, which likely represents its major source of interference, forming a two-UE cluster characterized by similar channel statistics (as in [6]). Assuming that the BS is informed of the outcome of this clustering procedure, it assigns orthogonal pilot sequences to each cluster. The covariance shaping method is thus executed separately within each cluster, whereby each UE solves (14) or (15), with inputs given by its own covariance matrix and that of its nearest neighbor. In this paper we focus on the

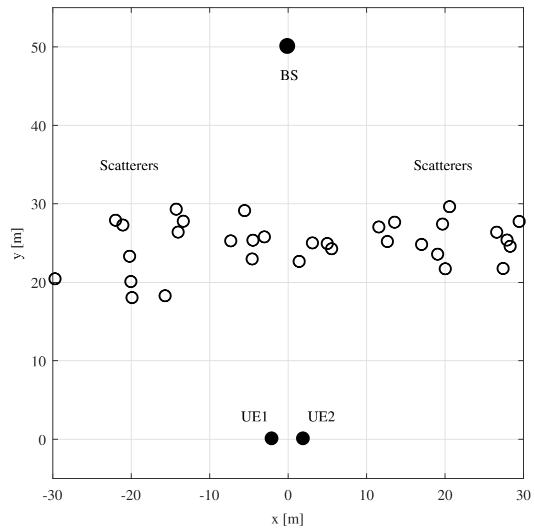

Figure 1. Position of the BS, UEs, and scatterers in the considered scenario.

case of one cluster with two UEs in order to better present the novelty of this approach. The case of more than one cluster is considered in [15], where the performance of the proposed covariance shaping method is evaluated in several realistic scenarios.

IV. Numerical Results and Discussion

In this section, we present numerical results to analyze the benefits of the proposed covariance shaping at the UE-side. We examine the following alternative scenarios, where the BS needs to transmit data to $K = 2$ UEs:

1) **Reference scenario.** The BS estimates the MIMO channels $\mathbf{H}_1$ and $\mathbf{H}_2$ using the same uplink pilot sequence and obtains the corresponding estimates $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$. Then, based on $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$, the BS adopts block diagonalization precoding and the UEs apply minimum mean square error (MMSE) combining.
2) **Covariance shaping scenario.** UEs 1 and 2 apply the covariance shaping vectors $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively, whereas the BS estimates the effective MISO channels $\bar{\mathbf{g}}_1$ and $\bar{\mathbf{g}}_2$ using the same uplink pilot sequence and obtaining the corresponding estimates $\hat{\bar{\mathbf{g}}}_1$ and $\hat{\bar{\mathbf{g}}}_2$. Then, the BS adopts zero-forcing precoding based on $\hat{\bar{\mathbf{g}}}_1$ and $\hat{\bar{\mathbf{g}}}_2$, while the UEs decode their data symbol in a simple, statistical, manner by applying the covariance shaping vectors to combine their receive signal.

Figure 1 shows the physical scattering environment taken into consideration for our simulations. A set of 32 randomly placed scatterers lies between the BS and the UEs, obstructing the LoS path (as often happens in practice). The expression of $\mathbf{H}_k$ follows the discrete physical channel model (see, e.g., [13]): let $\mathbf{a}(\theta) \in \mathbb{C}^{N\times 1}$ and $\mathbf{b}(\phi) \in \mathbb{C}^{M\times 1}$ denote the uniform

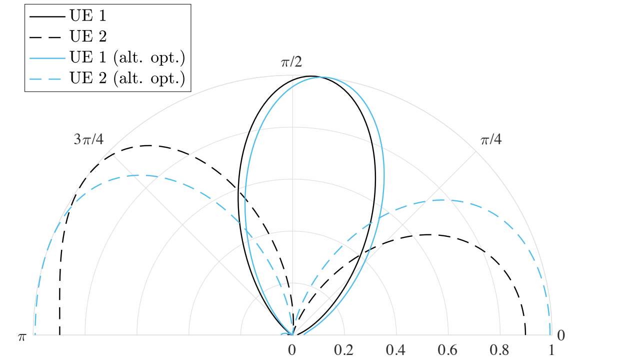

Figure 2. Antenna diagram of the covariance shaping vectors at the UE-side obtained solving (14) and (15) (alternate optimization).

linear array responses of UE k and of the BS defined as

$$\mathbf{a}(\theta) \triangleq \frac{1}{\sqrt{N}}[1 \; e^{-2\pi\delta\sin(\theta)} \dots e^{-2\pi(N-1)\delta\sin(\theta)}]^{\mathrm{T}}, \quad (19)$$

$$\mathbf{b}(\phi) \triangleq \frac{1}{\sqrt{M}}[1 \; e^{-2\pi\delta\sin(\phi)} \dots e^{-2\pi(M-1)\delta\sin(\phi)}]^{\mathrm{T}} \quad (20)$$

respectively, where δ is the ratio between the antenna spacing and the signal wavelength. The channel matrix between the BS and UE k is thus given by

$$\mathbf{H}_k = \sum_{p=1}^{P} \frac{\alpha_{k,p}}{d_{k,p}^{\beta/2}} \mathbf{a}(\theta_{k,p})\mathbf{b}^{\mathrm{H}}(\phi_{k,p}) \quad (21)$$

where P is the total number of paths, $\alpha_{k,p}$ is the random phase delay for path p, $d_{k,p}$ is the distance of path p, $\beta = 2$ is the pathloss exponent, and $\theta_{k,p}$ (resp. $\phi_{k,p}$) is the angle of impingement of path p on the antenna array of UE k (resp. of the BS).

The antenna diagram of the covariance shaping vectors obtained solving (14) via exhaustive search and (15), i.e., alternate optimization, are depicted in Figure 2. The two UEs clearly choose separate channel directions, which renders their covariance matrices nearly orthogonal. The covariance shaping method is indeed particularly effective when the channel statistics of the UEs are similiar and hence strongly interfering, as in the considered scenario. This effect is expected to reduce as the distance between the UEs increases. The near-orthogonality of the channel statistics can be exploited during both channel estimation (to eliminate pilot contamination) and the downlink data transmission (to aid interference cancellation techniques).

A. Uplink Pilot-Aided Channel Estimation

Considering a TDD setting, let us assume that the downlink transmission is preceded by a channel estimation stage via uplink pilots, where the two UEs are assigned the same pilot sequences. On the one hand, for the reference scenario, consider the pilot matrix $\mathbf{P} \in \mathbb{C}^{N\times\tau_{\mathrm{p}}}$, with $\mathbf{P}\mathbf{P}^{\mathrm{H}} = \mathbf{I}_N$. We use $\mathbf{Y}_{\mathrm{p}} \in \mathbb{C}^{M\times\tau_{\mathrm{p}}}$ to denote the receive uplink signal at the BS, which is given by

$$\mathbf{Y}_{\mathrm{p}} \triangleq \sqrt{\rho_{\mathrm{p}}}(\mathbf{H}_1^{\mathrm{H}} + \mathbf{H}_2^{\mathrm{H}})\mathbf{P} + \mathbf{Z}_{\mathrm{p}} \quad (22)$$

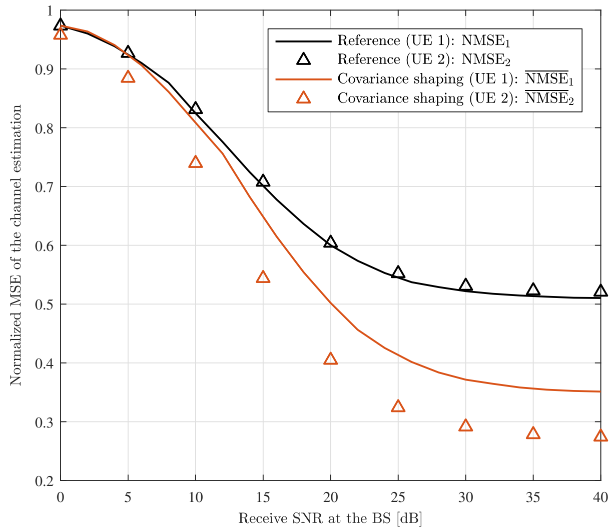

Figure 3. Normalized MSE of the channel estimation versus receive SNR at the BS with the same uplink pilot sequences for the reference and covariance shaping scenarios.

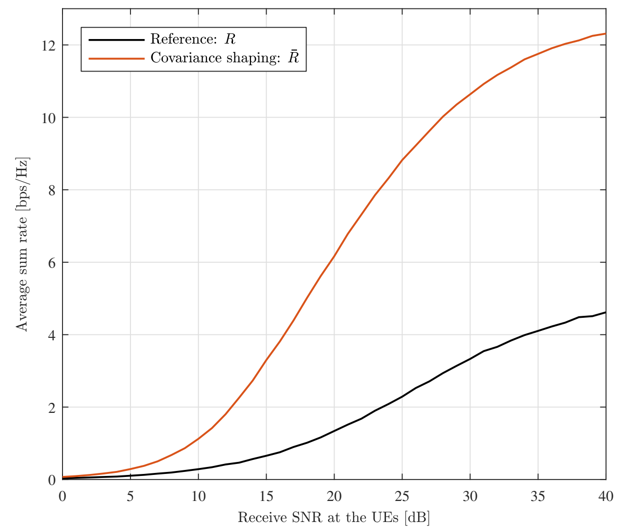

Figure 4. Average sum rate versus receive SNR at the UEs with the same uplink pilot sequence for the reference and covariance shaping scenarios.

where ρ_{p} is the normalized pilot power and $\mathbf{Z}_{\mathrm{p}} \in \mathbb{C}^{M \times \tau_{\mathrm{p}}}$ is the normalized noise at the BS with elements distributed independently as $\mathcal{CN}(0,1)$. Defining $\mathbf{\Phi}_{k,nn} \triangleq \mathbb{E}[\mathbf{g}_{k,n}^{\mathrm{T}} \mathbf{g}_{k,n}^{*}] \in \mathbb{C}^{M \times M}$ as the covariance of $\mathbf{g}_{k,n}$ (i.e., the nth row of $\mathbf{H}_k$) and $\widetilde{\mathbf{Z}} \triangleq \mathbf{Z}_{\mathrm{p}} \mathbf{P}^{\mathrm{H}} = [\tilde{\mathbf{z}}_1 \ldots \tilde{\mathbf{z}}_N] \in \mathbb{C}^{M \times N}$, the MMSE estimate of $\mathbf{g}_{k,n}$ reads as

$$\hat{\mathbf{g}}_{k,n}^{\mathrm{H}} \triangleq \mathbf{\Phi}_{k,nn} \left(\mathbf{\Phi}_{k,nn} + \mathbf{\Phi}_{j,nn} + \frac{1}{\rho_{\mathrm{p}}} \mathbf{I}_M \right)^{-1} \times \left(\mathbf{g}_{k,n}^{\mathrm{H}} + \mathbf{g}_{j,n}^{\mathrm{H}} + \frac{1}{\sqrt{\rho_{\mathrm{p}}}} \tilde{\mathbf{z}}_n \right) \tag{23}$$

with $j \neq k$. The full MIMO channel is thus estimated as $\hat{\mathbf{H}}_k \triangleq [\hat{\mathbf{g}}_{k,1}^T \ldots \hat{\mathbf{g}}_{k,N}^T]^T$. On the other hand, for the covariance shaping scenario, consider the pilot vector $\mathbf{p} \in \mathbb{C}^{1 \times \tau_{\mathrm{p}}}$, with $\|\mathbf{p}\|^2 = 1$. We use $\mathbf{Y}_{\mathrm{p}} \in \mathbb{C}^{M \times \tau_{\mathrm{p}}}$ to denote the receive uplink signal at the BS, which is given by

$$\bar{\mathbf{Y}}_{\mathrm{p}} \triangleq \sqrt{\rho_{\mathrm{p}}} (\bar{\mathbf{g}}_1^{\mathrm{H}} + \bar{\mathbf{g}}_2^{\mathrm{H}}) \mathbf{p} + \mathbf{Z}_{\mathrm{p}}. \tag{24}$$

Here, the MMSE estimate $\hat{\bar{\mathbf{g}}}_k$ reads as

$$\hat{\bar{\mathbf{g}}}_k^{\mathrm{H}} \triangleq \bar{\mathbf{\Phi}}_k \left(\bar{\mathbf{\Phi}}_k + \bar{\mathbf{\Phi}}_j + \frac{1}{\rho_{\mathrm{p}}} \mathbf{I}_M \right)^{-1} \left(\bar{\mathbf{g}}_k^{\mathrm{H}} + \bar{\mathbf{g}}_j^{\mathrm{H}} + \frac{1}{\sqrt{\rho_{\mathrm{p}}}} \mathbf{Z}_{\mathrm{p}} \mathbf{p}^{\mathrm{H}} \right) \tag{25}$$

with $j \neq k$.

Figure 3 shows the normalized mean square error (MSE) of the channel estimation against the receive SNR at the BS. For the reference and covariance shaping scenarios, this is defined as

$$\mathrm{NMSE}_k \triangleq \frac{1}{N} \sum_{n=1}^{N} \mathbb{E} \left[\frac{\|\hat{\mathbf{g}}_{k,n} - \mathbf{g}_{k,n}\|^2}{\|\mathbf{g}_{k,n}\|^2} \right] \tag{26}$$

$$\overline{\mathrm{NMSE}}_k \triangleq \mathbb{E} \left[\frac{\|\hat{\bar{\mathbf{g}}}_k - \bar{\mathbf{g}}_k\|^2}{\|\bar{\mathbf{g}}_k\|^2} \right] \tag{27}$$

respectively. The reference scenario is severely interference-limited and the normalized MSE between the actual and the estimated channels saturates quickly as the SNR increases. On the contrary, the covariance shaping is particularly effective and the channel estimation is only noise-limited up to a higher SNR range. Note that the normalized MSE relative to UE 1 is slightly higher than that of UE 2: indeed the paths excited by $\mathbf{v}_2$ carry more power than the ones excited by $\mathbf{v}_1$, resulting in a slightly lower signal-to-interference-plus-noise ratio (SINR). This happens because the covariance shaping vectors are chosen to maximize the system-level performance and not that of the single UEs.

B. Downlink Data Transmission

For the reference scenario, given the model for the downlink receive signal in (1), the multiuser precoding matrix $\mathbf{W}$ is computed based on the channel estimates $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ using block diagonalization [16]. Each UE k decodes its data streams by employing MMSE combining, i.e.,[4]

$$\mathbf{V}_k = \frac{\left(\hat{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^{\mathrm{H}} \hat{\mathbf{H}}_k^{\mathrm{H}} + \frac{1}{\rho} \mathbf{I}_N\right)^{-1} \hat{\mathbf{H}}_k \mathbf{W}_k}{\left\| \left(\hat{\mathbf{H}}_k \mathbf{W}_k \mathbf{W}_k^{\mathrm{H}} \hat{\mathbf{H}}_k^{\mathrm{H}} + \frac{1}{\rho} \mathbf{I}_N\right)^{-1} \hat{\mathbf{H}}_k \mathbf{W}_k \right\|_{\mathrm{F}}}. \tag{28}$$

Note that this implies that the UEs have the same knowledge of the estimated instantaneous channels as the BS and, moreover, they have perfect knowledge of the particular precoder employed by the latter. Observe that these assumptions require extra feedback resources between the BS and the UEs: this represents an additional point in favor of the proposed covariance shaping approach, which relies only on statistical information at the UE-side.

[4] Note that in (28) and (29) we use matrix normalization to enforce the power constraints $\|\mathbf{V}_k\|_{\mathrm{F}}^2 = 1$ and $\|\mathbf{W}\|_{\mathrm{F}}^2 = 1$, respectively. However, vector normalization can be alternatively applied (see, e.g., [17]).

For the covariance shaping scenario, defining $\hat{\hat{\mathbf{H}}} \triangleq [\hat{\hat{\mathbf{g}}}_1^{\mathrm{T}} \ \hat{\hat{\mathbf{g}}}_2^{\mathrm{T}}]^{\mathrm{T}}$, the BS adopts zero-forcing precoding, with multiuser precoding matrix given by

$$\mathbf{W} = \frac{\hat{\hat{\mathbf{H}}}^{\mathrm{H}} (\hat{\hat{\mathbf{H}}} \hat{\hat{\mathbf{H}}}^{\mathrm{H}})^{-1}}{\big\| \hat{\hat{\mathbf{H}}}^{\mathrm{H}} (\hat{\hat{\mathbf{H}}} \hat{\hat{\mathbf{H}}}^{\mathrm{H}})^{-1} \big\|_{\mathrm{F}}}. \quad (29)$$

Each UE decodes its data stream by simply applying the covariance shaping vector $\mathbf{v}_k$, obtained solving (14) or (15). Note that, in this case, the UEs need not know the channel estimate and the precoder used by the BS instantaneously.

Figure 4 shows the average sum rate for the reference and covariance shaping scenarios against the receive SNR at the UEs; here, it is assumed that the receive SNR at the UEs during downlink transmission is the same as the receive SNR at the BS during uplink channel estimation. The superior performance of the covariance shaping scenario stems from the fact that the channel estimates are considerably less corrupted by pilot contamination and, in addition, the effective channels of the UEs are more orthogonal to each other. On the other hand, applying block-diagonalization precoding in the reference scenario tends to cancel both interfering and useful channels due to insufficient signal subspace separation between the UEs. As a result, the corresponding average sum rate saturates quickly, since the system is heavily interference-limited and increasing the transmit power does not bring any substantial gain.

V. Conclusions

In this paper, we propose a covariance shaping method for massive MIMO TDD systems that allows the UEs to suitably alter the channel covariance structure seen at the BS to maximize some network utility. This is achieved by preemptively applying a statistical beamforming vector at the UE-side, which is obtained as the result of a simple optimization problem and requires to exchange only statistical information between the UEs. In particular, we show how the average sum rate can be substantially improved with respect to systems limited by interference. This is possible even if the spatial degrees of freedom are sacrificed to enforce statistical orthogonality. The gains arise mainly from accurate uplink channel estimation, which suffers considerably less from pilot contamination, and effective downlink data transmission, which exploits the near-orthogonality of the channel statistics between UEs. Additionally, the proposed scheme allows the UEs to decode their data stream without the knowledge of the instantaneous channel estimate and of the precoder employed by the BS. Future works will consider the case of more than two UEs and specific channel models.

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