IDENTIFIABILITY CONDITIONS FOR BLIND AND SEMI-BLIND MULTIUSER MULTICHANNEL IDENTIFICATION

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ABSTRACT

We explore the identifiability conditions for blind and semi-blind multiuser multichannel identification. Starting with the deterministic approach, we compare the identifiability conditions on the channel and the sources of training sequence based channel identification, blind and semi-blind channel identification. Further on, we use the stochastic approach with Gaussian priors for the source symbols and do the same derivations. Comparison between the two approaches lead to the conclusion that the latter yields less restricting conditions and, moreover, that the conditions are sufficient for any stochastic approach.

1. INTRODUCTION

Blind multichannel identification has received considerable interest over the last decade. In particular, second-order methods have raised a lot of attention, due to their ability to perform channel identification with relatively short data bursts. A major drawback of these methods is their inability to identify any channel. This motivates the development of various other methods to alleviate this problem. Among these methods, the so-called semi-blind (where some input symbols are known) approaches are very promising, as well for their performance as for their ability to perform identification for any channel, under certain conditions on the sources.

Two different data models have been proposed in the literature, namely the deterministic model, where the input symbols are considered deterministic and the stochastic model, where the input symbols are considered stochastic. In this latter model [2] has considered Gaussian priors for the symbols, which leads to very good performance. In this paper, we study the identifiability conditions for these models and show how semi-blind on one hand, and the stochastic model on the other hand lead to less restrictive identifiability conditions (if any) on the channel and the sources.

2. DATA MODEL AND NOTATIONS

Consider linear digital modulation over a linear channel with additive Gaussian noise. Assume that we have p transmitters at a certain carrier frequency and m antennas receiving mixtures of the signals. We shall assume that $m > p$. The received signals can be written in the baseband as

$$y_i(t) = \sum_{j=1}^{p} \sum_k a_j(k) h_{ij}(t-kT) + v_i(t),\ i = 1, \cdots, m \quad (1)$$

where the $a_j(k)$ are the transmitted symbols from source j, T is the common symbol period, $h_{ij}(t)$ is the (overall) channel impulse response from transmitter j to receiver antenna i. Assuming the $\{a_j(k)\}$ and $\{v_i(t)\}$ to be jointly (wide-sense) stationary, the processes $\{y_i(t)\}$ are (wide-sense) cyclostationary with period T. If $\{y_i(t)\}$ is sampled with period T, the sampled process is (wide-sense) stationary.

We assume the channels to be FIR. In particular, after sampling we assume the (vector) impulse response from source j to be of length N_j. Without loss of generality, we assume the first non-zero vector impulse response sample to occur at discrete-time zero. Let $N = \sum_{j=1}^{p} N_j$ and, w.l.o.g., $N_1 \geq N_2 \geq \cdots \geq N_p$. The discrete-time received signal can be represented in vector form as

$$\boldsymbol{y}(k) = \boldsymbol{H} \boldsymbol{A}_N(k) + \boldsymbol{v}(k); \boldsymbol{H} = [\boldsymbol{H}_1 \cdots \boldsymbol{H}_p] \quad (2)$$

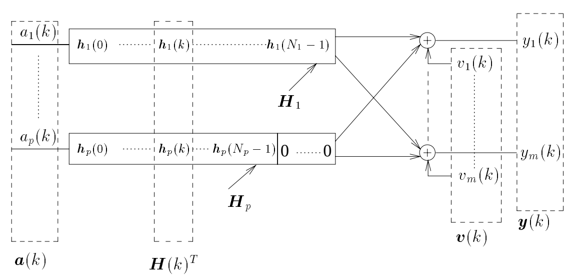

Figure 1: Notations.

We consider additive temporally and spatially white Gaussian circular noise $\boldsymbol{v}(k)$ with $R_{vv}(k-i) = \mathrm{E}\left\{\boldsymbol{v}(k)\boldsymbol{v}^H(i)\right\} = \sigma_v^2 I_m \delta_{ki}$. Assume we receive M samples :

$$\boldsymbol{Y}_M(k) = \mathcal{T}_M(\boldsymbol{H})\ \boldsymbol{A}_{N+p(M-1)}(k) + \boldsymbol{V}_M(k) \quad (3)$$

where $\boldsymbol{Y}_M(k) = \left[\boldsymbol{Y}^H(k) \cdots \boldsymbol{Y}^H(k-M+1)\right]^H$ and $\boldsymbol{V}_M(k)$ is defined similarly whereas $\mathcal{T}_M(\boldsymbol{H})$ is the multichannel multiuser convolution matrix of $\boldsymbol{H}$, with M block lines ($\mathcal{T}_M(\boldsymbol{H})$

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$= [\mathcal{T}_M(\boldsymbol{H}_1)\ldots\mathcal{T}_M(\boldsymbol{H}_p)]$, where $\mathcal{T}_M(\boldsymbol{H}_j)$ is block Toeplitz).

Some notations

We will note K_i (U_i) the number of non trivial known (unknown) symbols for the i^{th} source, $K = \sum_{i=1}^p K_i (U = \sum_{i=1}^p U_i)$, o_i the number of independent excitation modes for the i^{th} source and $o = \sum_{i=1}^p o_i$ the number of independent excitation modes for all the sources (different sources are assumed to have different excitation modes). For details about definition of excitation modes, see [4] and references therein.

3. IDENTIFIABILITY DEFINITION

Parameters θ are considered as identifiable when they are determined uniquely by the probability distribution of the data (i.e. $\forall \boldsymbol{Y}$, $f(\boldsymbol{Y}|\theta) = f(\boldsymbol{Y}|\theta') \Rightarrow \theta = \theta'$). In the models we will consider, data have a Gaussian distribution, so identifiability in this case means identifiability from the mean and the covariance of $\boldsymbol{Y}$. Another indicator of identifiability is regularity of the Fisher Information Matrix (FIM). This point of view is not equivalent however [3]. In particular, discrete valued ambiguities cause unidentifiability but don't lead to singularity of the FIM.

4. DETERMINISTIC MODEL

In this approach, the transmitted symbols $\boldsymbol{A}$ are considered deterministic, the stochastic part is considered to come only from the additive Gaussian white noise whose variance can be identified from the covariance matrix ($= \sigma_v^2$ I). The channel and the unknown symbols can be identified from the mean :

$$m_{\boldsymbol{Y}} = \overline{\boldsymbol{Y}} = \mathcal{T}(\boldsymbol{H})\ \boldsymbol{A} = \mathcal{T}_U(\boldsymbol{H})\ \boldsymbol{A}_U + \mathcal{T}_K(\boldsymbol{H})\ \boldsymbol{A}_K \quad (4)$$

where $\boldsymbol{Y} = \boldsymbol{Y}_M(M-1)$, $\boldsymbol{A}_U$ and $\boldsymbol{A}_K$ are the unknown and known symbols resp. (4) is equivalent to identification from the noiseless data (so identifiability in this approach is like in [4]). We will denote $\overline{\boldsymbol{Y}}$ as $\boldsymbol{Y}$ in the rest of this section.

4.1. Training Sequence Based Identification

To draw complete comparisons of identifiability conditions, we will first express the Training Sequence (TS) case. Here, we only consider $\boldsymbol{y}(k)$ containing known symbols only ($\boldsymbol{A} = \boldsymbol{A}_K$). Identification is based on : $\boldsymbol{Y}_M = \mathcal{T}_M(\boldsymbol{H})\boldsymbol{A} = \mathsf{A}_{N+p(M-1)}\mathbf{h}$
where $\mathsf{A}_{N+p(M-1)} = \left[\mathsf{A}_{N_1+M-1,1}\cdots\mathsf{A}_{N_p+M-1,p}\right]$
$\mathbf{h} = \left[\boldsymbol{h}_1(0)^H\cdots\boldsymbol{h}_1(N_1-1)^H\cdots\boldsymbol{h}_p(0)^H\cdots\boldsymbol{h}_p(N_p-1)^H\right]^H$,
and $\mathsf{A}_{K,i} = \begin{bmatrix} a_i(K-N_i) & \cdots & a_i(K-2N_i+1) \\ \vdots & \ddots & \vdots \\ a_i(0) & \cdots & a_i(1-N_i)\end{bmatrix} \otimes I_m$.

Necessary and sufficient condition TS *The m-channel $\boldsymbol{H}$ is identifiable by Training Sequence iff*

(i) $M \geq N, \qquad (K_i \geq N + N_i - 1, i = 1\cdots p)$;

(ii) $o_i \geq N_i, i = 1\cdots p$.

These conditions are deduced by imposing that A be full column rank. Obviously, there are no conditions on the channel itself.

4.2. Blind Channel Identification

In this case $\boldsymbol{A} = \boldsymbol{A}_U$. Here, we simultaneously identify the channel and the source, so identifiability means that $(\boldsymbol{H}, \boldsymbol{A})$ are identifiable from $\overline{\boldsymbol{Y}}$, up to a factor which will be described here under.

Effective Number of Channels

In the subsequent developments, we will often consider irreducible and column reduced channels, (i.e. such that $\mathbf{H}(z) = [\mathbf{h}_1(z)\cdots\mathbf{h}_p(z)]$ is full-rank $\forall z$ and $[\boldsymbol{h}_1(N_1-1)\ldots\boldsymbol{h}_p(N_p-1)]$ is full-rank), which is equivalent to forcing the tall (i.e. more lines than columns) matrix $\mathcal{T}(\boldsymbol{H})$ to be full column rank. Obviously, if $\boldsymbol{H}_N$ is not full rank, $\mathcal{T}(\boldsymbol{H})$ can not be full column rank under the same matrix size conditions and one must consider a reduced number of channels (equal to the rank of $\boldsymbol{H}_N$), which we will call effective number of channels. From here on, irreducible channels will mean irreducible channels with m being the effective number of channels.

Basic indeterminacy

As in the single user case, where the channel can only be determined up to a scalar gain, there is a basic indeterminacy for the multi-user case. Indeed, let $\mathbf{Y}(z) = \mathbf{H}(z)\mathbf{A}(z) =$
$\mathbf{H}'(z)\mathbf{T}(z)\mathbf{T}^{-1}(z)\mathbf{A}'(z)$. Then $\mathbf{T}(z)$ must be a unimodular matrix (see [5]), i.e. a nonsingular polynomial matrix whose determinant is not a function of z. When the channel is irreducible and column reduced, one can easily show that $\mathbf{T}(z) = \mathbf{R}(z)$ is of the form

$$\mathbf{R}(z) = \begin{bmatrix} R_{11} & 0 & \cdots & 0 \\ R_{12}(z) & R_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ R_{1l}(z) & R_{2l}(z) & \cdots & R_{ll}\end{bmatrix} \quad (5)$$

with $\nu_r \times \nu_s$ polynomial matrices $R_{rs}(z)$ of degree $L_r - L_s, r < s$ and constant non-singular matrices R_{rr}. The $L_r, r = 1, \ldots, l$ are the different values in the sequence $N_1, \ldots, N_p$ and the ν_r are their multiplicities. Indeed, if $\mathbf{H}'(z)$ is irreducible and column reduced, then $\mathbf{H}(z) = \mathbf{H}'(z)\mathbf{T}(z)$ has the same properties and column ranks as $\mathbf{H}'(z)$ if $\mathbf{T}(z)$ is unimodular [5], which leads to the structure shown here above.

Necessary and sufficient condition *In the Deterministic model, $\boldsymbol{H}$ and $\boldsymbol{A}$ are identifiable blindly up to unimodular triangular matrix $\mathbf{R}(z)$ iff*

(i) The channel is irreducible and column reduced;

(ii) $M \geq \underline{L}(p+1) + N$

(iii) $o_i \geq N_i + \underline{L}, \qquad i = 1, \ldots, p,$

where $\underline{L} = \left\lceil \frac{N-p}{m-p} \right\rceil$ ($= 0$ *for* $\frac{0}{0}$)

Proof: **Sufficiency** It has been shown in [6] that, if a channel is irreducible and column reduced, a minimum parameterization of the noise subspace of the data is given by $\overline{\boldsymbol{P}}_{\underline{L}}$ of size $(m-p) \times m(\underline{L}+1)$. The notation $\overline{\boldsymbol{P}}_{\underline{L}}$ indicates that it can be obtained by linear prediction. Using this parameterization, we may write

$$\overline{\boldsymbol{P}}_{\underline{L}}\mathcal{Y}_M = \overline{\boldsymbol{P}}_{\underline{L}}\mathcal{T}_{\underline{L}+1}(\boldsymbol{H})\mathcal{A} = 0 \quad (6)$$

where $\mathcal{Y}_M$ is of size $m(\underline{L}+1) \times M - \underline{L}$ and

$$\mathcal{Y}_M = \begin{bmatrix} \boldsymbol{y}(\underline{L}) & \cdots & \boldsymbol{y}(M-1) \\ \vdots & \ddots & \vdots \\ \boldsymbol{y}(0) & \cdots & \boldsymbol{y}(M-\underline{L}-1) \end{bmatrix}, \mathcal{A} = \begin{bmatrix} \mathcal{A}_1 \\ \vdots \\ \mathcal{A}_p \end{bmatrix},$$

$$\mathcal{A}_i = \begin{bmatrix} a_i(\underline{L}) & \cdots & a_i(M-1) \\ \vdots & \ddots & \vdots \\ a_i(1-N_i) & \cdots & a_i(M-N_i-\underline{L}) \end{bmatrix}.$$

Under condition *(i)*, and if $\mathcal{A}$ is full row rank, (6) implies $\overline{\boldsymbol{P}}_{\underline{L}}\mathcal{T}_{\underline{L}+1}(\boldsymbol{H}) = 0$ which implies that the correct $\overline{\boldsymbol{P}}_{\underline{L}}$ (and hence $\widehat{\boldsymbol{H}}$) can be determined from $\overline{\boldsymbol{P}}_{\underline{L}}\mathcal{Y}_M = 0$. As $\mathcal{A}$ is of size $(\underline{L}p + N) \times (M - \underline{L})$, this leads to a minimum data burst length : $M \geq \underline{L}(p+1) + N$, which is condition *(ii)*. Furthermore, the $\mathcal{A}_i$ must also be full row rank, which leads to condition *(iii)*. Once $\boldsymbol{H}$ is known, $\boldsymbol{A}$ can be determined from (4).

Necessity (i) If the channel is reducible and/or column reduced, than $\mathcal{T}(\boldsymbol{H})$ is never full column rank, hence, $\mathcal{A}'$ fulfills (6), where $\mathcal{A}' = \mathcal{A} + \mathcal{A}''$, and $\mathcal{A}''$ is in the null space of $\mathcal{T}(\boldsymbol{H})$ and independent of $\mathcal{A}$ and $\mathbf{R}(z)$, which shows that *(i)* is a necessary condition. *(ii–iii)* If $\mathcal{A}$ is not full rank, then, as $\overline{\boldsymbol{P}}$ is calculated from $\overline{\boldsymbol{P}}\mathcal{T}(\boldsymbol{H})\mathcal{A}\mathcal{A}^H\mathcal{T}^H(\boldsymbol{H}) = 0$, we can find $\overline{\boldsymbol{P}}' \neq \overline{\boldsymbol{P}}$ such that $\overline{\boldsymbol{P}}'\mathcal{Y} = 0$ but $\overline{\boldsymbol{P}}'\mathcal{T}(\boldsymbol{H}) \neq 0$ and hence another $\boldsymbol{H}' \neq \boldsymbol{H}$ such that $\overline{\boldsymbol{P}}'\mathcal{T}(\boldsymbol{H}') = 0$ exists for which $\boldsymbol{Y} = \mathcal{T}(\boldsymbol{H}')\boldsymbol{A}$, which shows that *(ii-iii)* are necessary conditions.

Futhermore, if $\mathbf{H}(z)$ is irreducible, but not necessarily column reduced, one can always find $\mathbf{H}'(z)$ column reduced by multiplying it by a unimodular matrix. This leads to :

Necessary and sufficient condition *In the Deterministic model, $\boldsymbol{H}$ and $\boldsymbol{A}$ are identifiable blindly up to unimodular matrix $\mathbf{T}(z)$ iff*

(i) *The channel is irreducible;*

(ii) $M \geq \underline{L}(p+1) + N$

(iii) $o_i \geq N_i + \underline{L}, \qquad i = 1, \ldots, p,$

where $\underline{L} = \left\lceil \frac{N-p}{m-p} \right\rceil$ $(= 0$ *for* $\frac{0}{0})$ *and* N *is the order of the equivalent column reduced channel* $\mathbf{H}'(z)$

4.3. Semi-Blind Channel Identification

In this section, we suppose that the channel is blindly identified up to a dynamic triangular matrix by a blind method, and that this matrix is further identified by contiguous known symbols.

Sufficient conditions to identify $\mathbf{R}(z)$

In this paragraph, we will investigate the number of known symbols needed to identify $\mathbf{R}(z)$, considering that the blind identification has already been performed. Consider $\boldsymbol{Y}_{TS} = \mathcal{T}(\boldsymbol{H})\boldsymbol{A}_K = \mathcal{T}(\tilde{\boldsymbol{H}})\mathcal{T}(\boldsymbol{R})\boldsymbol{A}_K$, we can always build $\tilde{\boldsymbol{Y}}_{TS} = \mathcal{T}(\boldsymbol{R})\boldsymbol{A}_K$ to which the TS conditions can be applied. Namely, parameter N in TS is replaced by $N' = \nu_1 + \sum_{j=2}^{l} \nu_j(L_j - L_{j-1} + 1)$, hence $M \geq N'$ (which is equivalent to $K_i \geq N' + L_{j(i)} - L_{j(i)-1}$). When the different users have equal channel lengths ($N_i \equiv N_1$ and $\nu_1 = p$, $l = 1$, $N' = p$), the mixing matrix is static and the number of known symbols per user must be $K_i = p$ (i.e. determining a static mixing matrix of size $p \times p$).

Sufficient conditions to identify any channel

Consider a reducible channel $\mathbf{H}(z)$ and its irreducible part $\underline{\mathbf{H}}(z)$, then, resorting for example to Lemma 6.1 of [1], one can show that $\mathbf{H}(z) = \underline{\mathbf{H}}(z)[r_1(z) \ldots r_p(z)]$, where $r_i(z)$ are $p \times 1$ polynomials of order $N_i - \underline{N}_i$, where $\underline{N}_i$ is the length of $\underline{\mathbf{H}}_i(z)$.

Hence, to identify any channel, sufficient conditions are given by the necessary and sufficient conditions for the blind part, and the TS conditions for $\mathbf{R}(z)R(z)$ with $R(z) = [r_1(z) \ldots r_p(z)]$.

5. GAUSSIAN MODEL

In this model, the input symbols are modeled as Gaussian. Hence, the parameters to estimate are the channel and the noise variance. Identifiability means identifiability from the mean and covariance:

$$\overline{\boldsymbol{Y}} = \mathcal{T}_K(\boldsymbol{H})\boldsymbol{A}_K, \qquad C_{YY}(\theta) = \mathcal{T}_U(\boldsymbol{H})\mathcal{T}_U^H(\boldsymbol{H})\sigma_a^2 + \sigma_v^2 I$$

Identifiability from the Gaussian model implies identifiability from any stochastic model, since such a model can be described in terms of the mean and the covariance plus higher-order moments.

5.1. Blind Channel Identification

One must identify $\theta = [\boldsymbol{H}\ \sigma_v^2]$ from

$$C_{YY}(\theta) = \mathcal{T}_U(\boldsymbol{H})\mathcal{T}_U^H(\boldsymbol{H})\sigma_a^2 + \sigma_v^2 I \tag{7}$$

One can show that the channel is identifiable up to a unitary $p \times p$ mixing matrix.

Considering the complex case, the number of real scalars to identify is $2Nm$ for $\boldsymbol{H}$, +1 for σ_v^2 from which one must substract the $p(p+1)/2$ for the unitary static mixture factor. The number of equations involved (corresponding to the first block of (7)) is $(2M-1)m^2$, which leads to the necessary condition :

Necessary condition $\qquad M \geq \frac{N}{m} + \frac{1}{2} + \frac{2-p(p+1)}{4m^2}$

Sufficient condition *In the Gaussian model, the m-channel $\boldsymbol{H}$ is identifiable blindly up to a unitary static mixture factor if*

(i) *The channel is irreducible and column reduced.*

(ii) $M \geq \underline{L} + 1$

If $\mathcal{T}_U(\boldsymbol{H})$ is tall, (which leads to (ii)), we can identify $\sigma_v^2 = \lambda_{min}(C_{YY})$. $\boldsymbol{H}$ can then be identified from the denoised $C_{YY}(\theta) - \sigma_v^2 I$ by linear prediction [6], provided that condition (i) is fulfilled. The unitary static mixture is in fact block diagonal of the form of $\mathbf{R}(z)$ in (5). Indeed an arbitrary unitary mixture can be undone by forcing the proper channel lengths for the different users.

Sufficient condition *Any minimum-phase channel (i.e.* $\mathbf{H}(z) = \underline{\mathbf{H}}(z)R(z)$ *with* $\underline{\mathbf{H}}(z)$ *irreducible and column reduced, and* $R(z)$ *minimum-phase) can be identified up to a static mixture for a larger* M. Indeed, linear prediction derived methods [6] allow to identify $\underline{\mathbf{H}}(z)$. The correlation sequence of $R(z)$ [6, 7] from which $R(z)$ can be identified up to a unitary mixture by spectral factorization.

5.2. Semi-Blind Channel Identification – Blind+TS method

In this section, we suppose that the channel is blindly identified up to a unitary mixing matrix by a blind method, and that this matrix is further identified by known symbols.

Blind part

Consider an irreducible and column reduced channel. Then, as long as $\boldsymbol{Y}$ contains a block of at least $\underline{L}+1$ samples $\boldsymbol{y}(k)$ that contain only unknown symbols, the channel can be identified up to a unitary instantaneous mixture.

Training Sequence part : identifying the mixing matrix

Assume that at a time instant k_1 user one contains a known symbol. Then from $\overline{\boldsymbol{Y}}_{\underline{L}+1}(k_1) = \mathcal{T}_{\underline{L}+1}(\boldsymbol{H})\boldsymbol{A}_K$ we can obtain $\left(\widetilde{\boldsymbol{h}}^H(0)\widetilde{\boldsymbol{h}}(0)\right)^{-1}\widetilde{\boldsymbol{h}}^H(0)\boldsymbol{P}_{\underline{L}}\overline{\boldsymbol{Y}}_{\underline{L}+1}(k_1) = Q\boldsymbol{a}(k_1)$ where $\boldsymbol{P}_{\underline{L}}$ is the linear prediction filter and Q is the unitary mixing matrix $(\boldsymbol{h}(0) = \widetilde{\boldsymbol{h}}(0)Q)$. Assume now that similarly user i has one known symbol at time k_i and that all the k_i are different. Then we can determine Q from :

$$Q = \left(\widetilde{\boldsymbol{h}}^H(0)\widetilde{\boldsymbol{h}}(0)\right)^{-1}\widetilde{\boldsymbol{h}}^H(0)\boldsymbol{P}_{\underline{L}}\left[\overline{\boldsymbol{Y}}_{\underline{L}+1}(k_1)\ldots\overline{\boldsymbol{Y}}_{\underline{L}+1}(k_p)\right] \left[\boldsymbol{a}(k_1)\cdots\boldsymbol{a}(k_p)\right]^{-1}.$$

Hence :

Sufficient condition *In the Gaussian model; the m-channel $\boldsymbol{H}$ is identifiable if*

- *(i) The channel is irreducible and column reduced ;*
- *(ii) each user has one known symbol, appearing at different times for the different users.*

5.3. Semi-Blind Channel Identification for any channel

In what follows, parameters can be identified by $\overline{\boldsymbol{Y}}$ only.

Sufficient condition *In the Gaussian Model, the m-channel $\boldsymbol{H}$ is identifiable if*

- *(i)* $M \geq N$
- *(ii) each user has one known symbol and these symbols are spaced at least N_i samples apart for the different users and the edges of the burst.*

Note that the $\mathsf{A}_{N_i+M-1,i}$ automatically have full column rank. Indeed, in this case $\overline{\boldsymbol{Y}}$ contains all the samples of the channel impulse responses, multiplied by known symbols.

Another approach consists of assuming that all users have a contiguous block of known symbols that are synchronized, as in the TS case. The difference with the TS case is that in the identification of $\boldsymbol{H}$ from $\overline{\boldsymbol{Y}} = \mathcal{T}(\boldsymbol{H})\boldsymbol{A}_K$, the $N_i - 1$ zeros before and after the block of known symbols also serve as training sequence symbols. Hence we get the following sufficient condition as a modification of the TS sufficient condition.

Sufficient condition *In the Gaussian Model, the m-channel $\boldsymbol{H}$ is identifiable if*

- *(i)* $M \geq N$
- *(ii) each user has $K_i \geq N - N_i + 1$ known symbols, where the bursts are synchronized. The burst of known symbols is at least at $N_{1,p}$ symbols from the edges of the burst.*

6. CONCLUSIONS

We have derived necessary and sufficient conditions for blind multiuser multichannel identification for the deterministic and the Gaussian model. Moreover, we have derived sufficient conditions for semi-blind channel identification for both models, on the one hand for removing the indeterminacies due to the blind approaches, on the other hand for identifying any channel. The results lead to two major conclusions

- Semi-blind approaches are able to remove indeterminacies or identify all channels with little additional information (known symbols).
- Stochastic approaches lead to far less restrictive conditions on the sources.

Moreover, as the Gaussian approach can be viewed as a particular case of any stochastic model resorting to first and second-order statistics only, the conditions derived here for the Gaussian model are sufficient for all stochastic approaches.

7. REFERENCES

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