Cache-Aided Cooperation with No CSIT

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***Abstract*— This work explores cache-aided interference management in the absence of channel state information at the transmitters (CSIT), focusing on the setting with K transmitter/receiver pairs endowed with caches, where each receiver k is connected to transmitter k via a direct link with normalized capacity 1, and to any other transmitter via a cross link with normalized capacity $\tau \leq 1$. In this setting, we explore how a combination of pre-caching at transmitters and receivers, together with interference enhancement techniques, can a) partially counter the lack of CSIT, and b) render the network self-sufficient, in the sense that the transmitters need not receive additional data after pre-caching. Toward this we present new schemes that blindly harness topology and transmitter-and-receiver caching, to create separate streams, each serving many receivers at a time. Key to the approach here is a combination of rate-splitting, interference enhancement and coded caching.**

***Index Terms*—Coded Caching, CSIT, User Cooperation, Rate-Splitting, Interference Enhancement, Generalized DoF.**

I. INTRODUCTION

Caching has received much attention over the last few years, especially after the emergence of the coded caching technique [1] which allowed — for the single-stream 'wired-like' broadcast setting — for scalable delivery of content to many users. The idea behind coded caching was that during off-peak hours, data is carefully pre-cached at the receiving users, while during peak hours — upon notification of the users' requests — the transmitter would transmit such that each signal would simultaneously deliver different content to different users, by cleverly accounting for each user's cache content. Recently, this attention has shifted to wireless settings which can be considerably different — in terms of how they benefit from caching — from their wired counterparts [2]. Works that seek to exploit such particularities of the wireless medium, can be found for example in [3]–[9]. One interesting such work, is that of Naderializadeh, Maddah-Ali and Avestimehr in [7], which explores a variant of the K-user symmetric interference channel, and which combines transmitter-and-receiver side caches with (perfect) CSIT-assisted precoding and transmitter-cooperation, to achieve a throughput gain that increases linearly with the size of the collective memory of both transmitters and receivers. Moreover, the works in [8], [9] build upon the above effort, the first by combining Interference Alignment with Zero Forcing precoding to further increase the aforementioned gains, while the second implements a 3×3 network with multiple antennas at both sides and caching at the receivers and transmitters. The aforementioned works, study wireless networks that operate under perfect CSIT.

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Our work here belongs to the new line of research that seeks to explore wireless coded caching in the presence of realistic constraints, such as limited feedback and uneven link qualities. In the aforementioned cache-aided setting with K interfering transmitter/receiver pairs, this combination of having transmitter and receiver side caches, of having no CSIT and of having statistically dissimilar link strengths, jointly reflect the reality of a multi-cell network where typically users are strongly connected only to their own base-station, and where CSIT (especially concerning channels that traverse cells) is very limited. These considerations, in addition to being realistic, are also impactful; after all, feedback-quality and topology are known to substantially affect the effectiveness of coded caching techniques (cf. [4], [5]). In our setting of interest, the use of caches is further motivated by the idea that pre-caching valuable content at base-stations during off-peak hours, can be used to alleviate the backhaul load. Here, in the spirit of [7], caching is used to preload all possible data files across the different base-stations, thus conceivably allowing us to 'shut-down' the backhaul after caching.

II. MODEL & NOTATION

We consider a scenario with K transmitter/receiver pairs, where each receiver $k \in [K] \triangleq \{1, 2, \cdots, K\}$ is connected to transmitter k via a strong direct link with unit-normalized capacity, while the cross links from all other transmitters are weak, with capacity $\tau \leq 1$. In this setting, each transmitter and receiver are equipped with a cache of size $M_T f$ and $M_R f$ bits, respectively.

Communication consists of two phases: placement and delivery. During the placement phase, each cache, both at the transmitters and the receivers, is pre-filled with content from a library of $N \geq K$ distinct files $W_1, W_2, \cdots, W_N$ (each of size $|W_n| = f$ bits), without knowing future file requests. We will assume that $KM_T \geq N$ so that the whole library can be found on the transmitters' side. For notational simplicity we will use

$$\gamma_T \triangleq \frac{M_T}{N}, \quad \gamma_R \triangleq \frac{M_R}{N}$$

to respectively denote the normalized cache size at each transmitter and receiver.

In the beginning of the delivery phase, each receiver k requests a single file $W_{d_k}, d_k \in [N]$. Transmitters do not have knowledge of channel state information, i.e., there is no CSIT. At any time t, the received signal at user $k \in [K]$, takes the form

$$y_k(t) = \sqrt{P} h_{k,k}(t) x_k(t) + \sum_{k' \in [K] \setminus \{k\}} \sqrt{P^\tau} h_{k',k} x_{k'}(t) + z_k(t)$$

where an input signal $x_i(t)$ from transmitter i satisfies the power constraint $\mathbf{E}\{|x_i(t)|^2\} \leq 1$, where the channel fading coefficient from transmitter i to receiver k ($i, k \in [K]$) is denoted by $h_{i,k}(t)$, and where the noise follows the normal distribution $z_k(t) \sim \mathbb{CN}(0,1)$. The average received signal-to-noise ratio (SNR) from any link to user k is given as

$$\mathbf{E}\left\{|\sqrt{P} h_{i,k}(t) x_i(t)|^2\right\} = \begin{cases} P, & i = k \\ P^\tau, & i \neq k. \end{cases}$$

A. Coding challenge

This setting brings about interesting challenges but also opportunities. The lack of CSIT is naturally non-beneficial, while the topological factor can be used to counter the lack of CSIT by (occasionally) reducing the interference power. At the same time though, this topological factor, together with the requirement that the transmitters do not receive any additional data after the caching phase, will inevitably force some of the receivers' requested data to reside only at far-away (weak) transmitters. Such 'cross' interference settings have notoriously bad performance, which will be here boosted by using receiver-side (coded) caching and partial cooperation among the transmitters. Hence our main challenge will be to place content in a way that best allows for topology, caching and transmitter cooperation to jointly remove unwanted interference. In the end, the schemes will combine coded caching with rate-splitting (Han-Kobayashi) approaches [10], and with interference enhancement techniques [11].

a) Some Intuition: By carefully combining the above three techniques we want to ameliorate the performance loss due to the lack of CSIT. Specifically, the rate-splitting approach will allow for a partition of a message into "private" (low powered) and "common" (high powered) parts, the first heard only by one receiver (strongly connected) while the second will be heard by all. On the other hand, interference enhancement takes advantage of data redundancy across the transmitters by sending interference in the private part of the message, thus allowing for interference removal at the receiver side. Finally, coded caching exploits the cached content in the receiver side to create multicasting opportunities, i.e. XORed packets desired and decodable by many.

B. Measures of performance

Our aim is to design schemes that reduce the duration $T(\gamma_T, \gamma_R)$ — in time slots, per file served per user — needed to complete the delivery process, *for any set of K requests*. Specifically we will focus on the worst case scenario, where each receiver requests a different file. Moreover, our measure of performance will be a normalized version of the required delay, specifically

$$\mathcal{T}(\gamma_T, \gamma_R) = T(\gamma_T, \gamma_R) \frac{\log P}{f}. \tag{1}$$

Equivalently, when meaningful, we will also consider the per user *Cache-Aided Generalized Degrees of Freedom* (cache-aided GDoF) which is simply[1]

$$d(\gamma_T, \gamma_R, \tau) = \frac{1 - \gamma_R}{\mathcal{T}} \in [0, 1]. \tag{2}$$

C. Notation and definitions

The set of integers is denoted by $\mathbb{Z}$, while the sets of naturals and reals are, respectively, $\mathbb{N}$ and $\mathbb{R}$. In terms of operators, $\binom{n}{k}$ is the n-choose-k operator, $\oplus$ the bitwise XOR and $(x)^+$ is $\max\{x, 0\}$. If Ψ is a set, then $|\Psi|$ will denote its cardinality. For sets A and B we will denote the difference set with $A \backslash B$. Moreover, we will use $\mathcal{Z}_k$ to denote the memory content of receiver $k \in [K]$ and $\Psi_k, k \in [K]$ for the memory content of transmitter k. Finally, $a|b$ and $a \nmid b$ denote that a divides / doesn't divide b, respectively. Also, $S = A \otimes B$ with A, B sets, is the set with elements, $(i, j) : i \in A, j \in B$.

When the requests have been made, each of the requested subfiles resides either in the dominant transmitter or to (some) further away ones. This makes us consider the delivery as having two separate phases, in the first the requested subfiles that are cached by the "direct" transmitter are sent, while the second phase delivers the rest of the request.

The schemes here are presented for integer values of $K\gamma_T$ and $K\gamma_R$. When that is not the case we can perform memory sharing (cf. [1]).

III. Results

We first proceed with a lemma on the achievable GDoF of the K-user topological MISO BC with receiver-side caches, corresponding to a K-antenna transmitter serving K independent users. We then present a second lemma that provides the achievable performance for the K-user topological interference scenario corresponding to K transmitter-receiver pairs, where caching exists only at the transmitters. Finally, using these two lemmas, we cover the general case of the K-user topological interference scenario with caching at both receivers and transmitters. We will use the following notation

$$x_s = \frac{K(1-\tau)}{K(1-\tau) + \left((K\gamma_R + 1) \min\left\{\frac{2\tau - 1}{1 - \gamma_T}, \tau\right\}\right)^+}$$

$$K_g = \frac{K(1 - \gamma_T)\binom{K-1}{K\gamma_R}}{\binom{K}{K\gamma_R + 1} - \gamma_T \binom{K-1}{K\gamma_R} - \binom{K\gamma_T}{K\gamma_R + 1}\frac{K - K\gamma_R - 1}{K}}.$$

We proceed with the results.

Lemma 1. *In the K-user topological MISO BC with receiver-side caches, the achievable cache-aided GDoF here takes the form*

$$d_\Sigma = K(1 - \tau) + (K\gamma_R + 1)\tau \tag{3}$$

[1]We note that $Kd(\gamma, \tau)$ is simply the coding gain $K(1 - \gamma_R)/\mathcal{T}$ that is often used as a measure of performance in coded-caching.

corresponding to

$$\mathcal{T}(\gamma_R) = \frac{K(1-\gamma_R)}{K(1-\tau)+(K\gamma_R+1)\tau}. \tag{4}$$

Lemma 2. *In the K-pair topological interference scenario with caches only at the transmitters, the achievable delay here takes the form*

$$\mathcal{T}(\gamma_T) = \frac{K\gamma_T}{K(1-\tau)+\left(\min\{\frac{2\tau-1}{1-\gamma_T},\tau\}\right)^+} + \frac{x_s K(1-\gamma_T)}{\min\left\{\frac{\tau}{1-\gamma_T},1\right\}}.$$

Theorem 1. *In the K-pair topological interference scenario with caches at the transmitters and the receivers, the achievable delay here takes the form*

$$\mathcal{T}(\gamma_T,\gamma_R) = \begin{cases} \frac{K(1-\gamma_R)}{(K\gamma_R+1)\min\left\{\frac{\tau}{1-\gamma_T},1\right\}} & \gamma_T \le \gamma_R \\ \frac{K\gamma_T(1-\gamma_R)}{K(1-\tau)+(K\gamma_R+1)\left(\min\left\{\frac{2\tau-1}{1-\gamma_T},\tau\right\}\right)^+}+ & \\ +\frac{K(1-\gamma_T)(1-\gamma_R)}{K_g\min\left\{\frac{\tau}{1-\gamma_T+},1\right\}}\cdot x_s & \gamma_T > \gamma_R. \end{cases}$$

IV. Placement and Delivery Schemes

A. Cache Placement

Each of the N files is split into $s=\binom{K}{K\gamma_T}\binom{K}{K\gamma_R}$ subfiles, i.e. W_{n,σ_T,σ_R}, where $n\in[N]$, $\sigma_T\subset[K]$, $|\sigma_T|=K\gamma_T$ and $\sigma_R\subset[K], |\sigma_R|=K\gamma_R$ and cached according to

$$\begin{aligned}\Psi_k =&\{W_{n,\sigma_T,\sigma_R},\quad k\in\sigma_T,\ \forall n\in[N]\}\\ \mathcal{Z}_k =&\{W_{n,\sigma_T,\sigma_R},\quad k\in\sigma_R,\ \forall n\in[N]\}.\end{aligned}$$

We note here that placement is independent of the topological factor τ or the pairing of transmitters with receivers.

B. Delivery Phase

The delivery phase begins when receivers request a single file from the library of N files. We assume that each receiver $k\in[K]$ requests the file indexed by $d_k\in[N]$.

The delivery scheme is dependent on τ and γ_T and consists of four distinct cases: i) the MISO BC case where each transmitter cache has access to the entire library, ii) the interference scenario where $\gamma_T<1$, $\gamma_R<\gamma_T$ and $\tau\le 1/2$, iii) the interference scenario where $\gamma_T<1$, $\gamma_R<\gamma_T$ and $\tau\ge 1/2$ and iv) the interference scenario when $\gamma_T\le\gamma_R$.

a) MISO BC with receiver side caching: We begin by dividing all subfiles into two parts, $W'_{n,1,j}$ and $W''_{n,1,j}$, $\forall n\in[N]$ with respective relative sizes x_s and $1-x_s$

$$x_s = \frac{|W'_{n,1,j}|}{|W_{n,1,j}|} = \frac{(K\gamma_R+1)\tau}{K(1-\tau)+(K\gamma_R+1)\tau}.$$

Then, each transmitted message has the form

$$x_k = \underbrace{c_k}_{\text{common}} + \underbrace{b_k}_{\text{private}}$$

where c_k has power $1-P^{-\tau}$ and rate τ, while b_k, has power $P^{-\tau}$ and rate $1-\tau$. Each transmitter $k\in[K]$ encodes in b_k, W'' subfiles that are requested by its respective receiver, i.e.

$$b_k \leftarrow W''_{d_k,1,j}, k\notin j,\ \forall k\in[K].$$

Moreover, the upper powered, common, part is filled with XORs of $K\gamma_R+1$ elements from the set of W' subfiles, i.e.

$$c_k \leftarrow \bigoplus_{i\in R} W'_{d_i,1,R\setminus\{i\}},\ R\subset[K], |R|=K\gamma_R+1,\ \forall k\in[K].$$

In each slot, all transmitted messages have the exact same common message, while the private is chosen independently. The received signal takes the form

$$y_k = \underbrace{h_{k,k}c_k}_{P-P^{1-\tau},\tau} + \sum_{i\neq k}\underbrace{h_{i,k}c_i}_{P^{1-\tau},\tau} + \underbrace{h_{k,k}b_k}_{P^{\tau},1-\tau} + \sum_{i\neq k}\underbrace{h_{i,k}b_i}_{P^0,1-\tau}. \tag{5}$$

User $k\in[K]$ first decodes message c_k by treating interference as noise (TIN), with rate τ and proceeds to remove all c_i's from Eq. (5). Then, by TIN the receiver can decode the respective private (lower powered) message with rate $1-\tau$.

For the above, the delivery time achieved is

$$\begin{aligned}\mathcal{T} &= \max\left\{\frac{(1-x_s)\binom{K}{K\gamma_T}\binom{K-1}{K\gamma_R}}{\binom{K}{K\gamma_T}\binom{K}{K\gamma_R}(1-\tau)}, \frac{x_s\binom{K}{K\gamma_T}\binom{K}{K\gamma_R+1}}{\binom{K}{K\gamma_T}\binom{K}{K\gamma_R}\tau}\right\}\\ &= \frac{K(1-\gamma_R)}{K(1-\tau)+(K\gamma_R+1)\tau},\end{aligned}$$

which implies a $d_\Sigma = K(1-\tau)+(K\gamma_R+1)\tau$.

b) Interference scenario with $\tau\le\frac{1}{2}$ and $\gamma_T>\gamma_R$: In this setting delivery consists of two subphases. In the first, transmitters deliver all their cached subfiles that are requested by their direct receivers. Each transmitter sends their message with power 1 and rate $1-\tau$, while receivers decode the high powered (direct) signal by TIN with rate of $1-\tau$. Specifically, the transmitted signals take the form

$$\begin{aligned}x_k \leftarrow \big\{W_{d_k,T,R}:\ & T\subset[K], k\in T, |T|=K\gamma_T\\ & R\subset[K], k\notin R, |R|=K\gamma_R\big\}.\end{aligned}$$

This delivery subphase has duration

$$\mathcal{T} = \frac{\binom{K-1}{K\gamma_T-1}\binom{K-1}{K\gamma_R}}{(1-\tau)\binom{K}{K\gamma_T}\binom{K}{K\gamma_R}} = \frac{\gamma_T(1-\gamma_R)}{1-\tau}.$$

The rest of the subfiles are to be transmitted via delivery subphase 2. We note that these messages reside only on far away transmitters, hence need to go through a slower channel. Each transmitter forms all XORs of size $K\gamma_R+1$ apart from those whose *all* elements have already been transmitted. The transmitted signals have the following form

$$\begin{aligned}x_k \leftarrow \Big\{\bigoplus_{i\in R} W_{d_i,T,R\setminus\{i\}}, & \forall T\subset[K], |T|=K\gamma_T, k\in T\\ & \forall R'\subset[K],\ |R|=K\gamma_R+1\\ & R''\subset T,\ |R'|=K\gamma_R+1\\ & R=R'\setminus R''\big\}\end{aligned}$$

i.e. the transmitted signals contain all possible XORs of size $K\gamma_R+1$, apart from those whose all $(K\gamma_R+1)$ elements have been transmitted in delivery subphase 1. Moreover, we can see that some XORs have been partly transmitted in subphase 1. That allows for interference removal opportunities. For example, if $K=4$, $\gamma_T=1/2$ and $\gamma_R=1/4$, receiver 1 would receive subfile $W_{d_1,12,4}$ in subphase 1 (using the high rate of subphase 1), while the same subfile would appear in subphase 2 as part of XOR $W_{d_1,12,4}\oplus W_{d_4,12,1}$. While this XOR is useful for receiver 4, nevertheless receiver 1 can fully "cache-it-out" and thus reduce the experienced interference.

In total, each XOR will be transmitted from $K\gamma_T$ transmitters. Each message is transmitted with power 1 and encoded with *all* the above XORs, while the rate of each XOR is

$$r_{\text{XOR}}=\frac{\min\left\{\frac{\tau}{1-\gamma_T},1\right\}}{\binom{K}{K\gamma_T}\left[\binom{K}{K\gamma_R+1}-\gamma_T\binom{K-1}{K\gamma_R}-\binom{K\gamma_T}{K\gamma_R+1}\frac{K-K\gamma_R-1}{K}\right]}$$

which accounts for all the possible XORs minus those that have been partly transmitted, thus no longer causing interference, and, also, minus those that have been fully transmitted. We can see that the total rate of each transmitted message is

$$\begin{aligned}r_{x_k}&=\frac{\binom{K-1}{K\gamma_T-1}\left[\binom{K}{K\gamma_R+1}-\binom{K\gamma_T}{K\gamma_R+1}\right]\min\left\{\frac{\tau}{1-\gamma_T},1\right\}}{\binom{K}{K\gamma_T}\left[\binom{K}{K\gamma_R+1}-\gamma_T\binom{K-1}{K\gamma_R}-\binom{K\gamma_T}{K\gamma_R+1}\frac{K-K\gamma_R-1}{K}\right]}\\&=\frac{\gamma_T\left[\binom{K}{K\gamma_R+1}-\binom{K\gamma_T}{K\gamma_R+1}\right]\min\left\{\frac{\tau}{1-\gamma_T},1\right\}}{\left[\binom{K}{K\gamma_R+1}-\gamma_T\binom{K-1}{K\gamma_R}-\binom{K\gamma_T}{K\gamma_R+1}\frac{K-K\gamma_R-1}{K}\right]}.\end{aligned}$$

By removing the partially transmitted XORs of subphase 1, from each x_i message, the remaining rate of any x_i is $r_{x_i}=\gamma_T\min\{\frac{\tau}{1-\gamma_T},1\}$.

As evident, there appear two cases, depending on parameters γ_T and τ. If $\tau\leq 1-\gamma_T$, Rx$_k$ decodes x_k by TIN, with rate $\gamma_T\frac{\tau}{1-\gamma_T}\leq 1-\tau$ and proceeds to remove its contents from all the received signals. Since the XORs appearing in message x_k will also be transmitted from some other $K\gamma_T-1$ transmitters the remaining signal has sum-rate

$$\sum_{i\in[K]}r_{x_i}-r_{x_k}=\frac{\tau}{1-\gamma_T}-\gamma_T\frac{\tau}{1-\gamma_T}=\tau$$

which makes them decodable by joint decoding.

On the other hand, if $\tau\geq 1-\gamma_T$, then a receiver proceeds to jointly decode all $x_i, i\in[K]$ messages.

The time required to finish this (sub)phase is

$$\mathcal{T}=\frac{\binom{K-1}{K\gamma_T}\left[\binom{K}{K\gamma_R+1}-\binom{K\gamma_T}{K\gamma_R+1}\right]}{r_{x_k}\binom{K}{K\gamma_T}\binom{K}{K\gamma_R}}=\frac{K(1-\gamma_T)(1-\gamma_R)}{K_g\min\left\{\frac{\tau}{1-\gamma_T+},1\right\}}.$$

c) Interference scenario with $\tau\geq\frac{1}{2}$ and $\gamma_T>\gamma_R$: In this case, similarly to the previous one, delivery consists of two subphases. First, each subfile is split into two parts $W'_{n,i,j}$ and $W''_{n,i,j}$ with relative sizes x_s and $1-x_s$ where

$$x_s=\frac{|W'_{n,i,j}|}{W_{n,i,j}}=\frac{(K\gamma_R+1)\min\left\{\frac{2\tau-1}{1-\gamma_T},\tau\right\}}{(K\gamma_R+1)\min\left\{\frac{2\tau-1}{1-\gamma_T},\tau\right\}+K(1-\tau)}.$$

Then, in delivery subphase 1 the transmitted messages are

$$x_k=\underbrace{c_k}_{1-P^{-\tau},\tau}+\underbrace{b_k}_{P^{-\tau},1-\tau},\ k\in[K]$$

where

$$\begin{aligned}c_k&\leftarrow\Big\{\bigoplus_{i\in R}\quad W'_{d_i,T,R\setminus\{i\}},T\subset[K],\ |T|=K\gamma_T,\ k\in T\\&\qquad\qquad R\subset[K],\ |R|=K\gamma_R+1\Big\}\\b_k&\leftarrow\big\{W''_{d_k,T,R},T\subset[K],\ |T|=K\gamma_T,\ k\in T\\&\qquad\qquad R\subset[K],\ |R|=K\gamma_R,\ k\notin R\big\}\end{aligned}$$

with c_k carrying all XORs composed from W' subfiles cached at transmitter k, while b_k carries all files from the set W'' which are cached at transmitter k and wanted by receiver k.

The required time for this subphase is

$$\begin{aligned}\mathcal{T}&=\frac{1}{f}\cdot\max\left\{\frac{(1-x_s)\binom{K-1}{K\gamma_T-1}\binom{K-1}{K\gamma_R}}{(1-\tau)},\frac{x_s\binom{K}{K\gamma_T}\binom{K}{K\gamma_R+1}}{\min\{\frac{\tau}{1-\gamma_T},\tau\}}\right\}\\&=\frac{K\gamma_T(1-\gamma_R)}{K(1-\tau)+(K\gamma_R+1)\min\left\{\frac{2\tau-1}{1-\gamma_T},\tau\right\}}.\end{aligned}$$

At the end of the above subphase, all W' requested subfiles have been delivered, while the rest of the requested W'' subfiles are to be send via subphase 2. Similarly to the previous paragraph's approach (subphase 2), we will send the rest of the subfiles via the slow channel, achieving a delivery time for this subphase

$$\mathcal{T}=\frac{K(1-\gamma_T)(1-\gamma_R)}{K_g\min\left\{\frac{\tau}{1-\gamma_T+},1\right\}}(1-x_s).$$

d) Interference scenario when $\gamma_T<\gamma_R$: In this case, the benefits of Coded Caching outperform the gain of sending some subfiles using the fast, direct channel. As a result, we will only focus on transmitting messages (XORs) using uniquely the slow channel. Each user sends a message with power 1 and total rate $\gamma_T\min\{\frac{\tau}{1-\gamma_T},1\}$. This message carries the XORs composed of $K\gamma_R+1$ requested subfiles found at the transmitter, with each XOR having a rate of

$$r_{\text{XOR}}=\frac{\min\left\{\frac{\tau}{1-\gamma_T},1\right\}}{\binom{K}{K\gamma_T}\binom{K}{K\gamma_R+1}}.$$

In the two regions, i.e. $\tau\lesseqgtr 1-\gamma_T$ receivers either first decode x_k by TIN, remove the XORs it contains from the received signal and proceed to jointly decode the remaining XORs. On the other hand, if $\tau\geq 1-\gamma_T$ receivers proceed to jointly decode all XORs. The achieved delivery time is

$$\begin{aligned}\mathcal{T}(\gamma_T,\gamma_R)&=\frac{\binom{K}{K\gamma_T}\binom{K}{K\gamma_R+1}}{\binom{K}{K\gamma_T}\binom{K}{K\gamma_R}\min\left\{\frac{\tau}{1-\gamma_T},1\right\}}\\&=\frac{K(1-\gamma_R)}{(K\gamma_R+1)\min\left\{\frac{\tau}{1-\gamma_T},1\right\}}.\end{aligned}$$

V. EXAMPLE

To provide a better understanding of the above, let us consider the $K=4$ pair network with $\tau=6/10$ and normalized caches $\gamma_T=2/4$ and $\gamma_R=1/4$.

For this case, the initial subpacketization is $s = \binom{4}{2}\binom{4}{1} = 24$, while the respective cached content per user is

$$\begin{aligned}
\mathcal{Z}_k &= \{12, 13, 14, 23, 24, 34\} \otimes \{k\},\ \forall k \in [4] \\
\Psi_1 &= \{12, 13, 14\} \otimes [4], \quad \Psi_2 = \{12, 23, 24\} \otimes [4] \\
\Psi_3 &= \{13, 23, 34\} \otimes [4], \quad \Psi_4 = \{14, 24, 34\} \otimes [4]
\end{aligned}$$

a) Delivery Subphase 1: In the first delivery subphase, transmitters split each subfile $W_{n,i,j}$ into $W'_{n,i,j}$ and $W''_{n,i,j}$, according to Eq. (6), i.e. $|W'_{n,i,j}| = \frac{1}{3}, |W''_{n,i,j}| = \frac{2}{3}$. Assuming the standard request, the transmitted signals take the form

$$x_k = \underbrace{c_k}_{1-P^{-\tau}, 2\tau-1} + \underbrace{b_k}_{P^{-\tau}, 1-\tau}. \tag{6}$$

Common messages are filled as follows[2]

$$\begin{aligned}
c_1 \leftarrow \{ & A'_{j,2}B'_{j,1}, A'_{j,3}C'_{j,1}, A'_{j,4}D'_{j,1}, C'_{j,2}B'_{j,3}, \\
& D'_{j,2}B'_{j,4}, C'_{j,4}D'_{j,3} \}, \forall j \in \{12, 13, 14\} \\
c_2 \leftarrow \{ & A'_{j,2}B'_{j,1}, A'_{j,3}C'_{j,1}, A'_{j,4}D'_{j,1}, C'_{j,2}B'_{j,3}, \\
& D'_{j,2}B'_{j,4}, C'_{j,4}D'_{j,3} \}, \forall j \in \{12, 23, 24\} \\
c_3 \leftarrow \{ & A'_{j,2}B'_{j,1}, A'_{j,3}C'_{j,1}, A'_{j,4}D'_{j,1}, C'_{j,2}B'_{j,3}, \\
& D'_{j,2}B'_{j,4}, C'_{j,4}D'_{j,3} \}, \forall j \in \{13, 23, 34\} \\
c_4 \leftarrow \{ & A'_{j,2}B'_{j,1}, A'_{j,3}C'_{j,1}, A'_{j,4}D'_{j,1}, C'_{j,2}B'_{j,3}, \\
& D'_{j,2}B'_{j,4}, C'_{j,4}D'_{j,3} \}, \forall j \in \{14, 24, 34\}
\end{aligned}$$

while the b_i part of the message is filled with

$$\begin{aligned}
b_1 &\leftarrow A''_{i,j}, i \in \{12, 13, 14\}, j \in \{2, 3, 4\} \\
b_2 &\leftarrow B''_{i,j}, i \in \{12, 23, 24\}, j \in \{1, 3, 4\} \\
b_3 &\leftarrow C''_{i,j}, i \in \{13, 23, 34\}, j \in \{1, 2, 4\} \\
b_4 &\leftarrow D''_{i,j}, i \in \{14, 24, 34\}, j \in \{1, 2, 3\}
\end{aligned}$$

In the receiver side, we can, for example, see user 1:

$$y_1 = \underbrace{h_{11}c_1}_{P-P^{1-\tau}} + \sum_{i=2}^{4} \underbrace{h_{i1}c_i}_{P^{\tau}} + \underbrace{h_{11}b_1}_{P^{1-\tau}} + \sum_{i=2}^{4} \underbrace{h_{i1}b_i}_{P^0}. \tag{7}$$

First, user 1 proceeds to decode message c_1 by TIN. Since its rate is $2\tau - 1 < 1 - \tau$ user 1 can decode the message successfully. Then, proceeds to remove c_1 and all subfiles it contains from Eq. (7). From the collective $c_2 + c_3 + c_4$ what is left is half of the XORs, having a sum rate of $2\tau - 1$. Then, user 1 can jointly decode the rest of the common messages by TIN and proceed to remove them from the received signal. Finally, user 1 can decode b_1 by TIN with rate $1 - \tau$.

b) Delivery Subphase 2: This phase aims to deliver the rest of the messages, i.e. subfiles that belong to W'' category and are cached only at far away transmitters. Transmitters send their messages $x_k, k \in [K]$ each with power 1 and rate $1/4$. For example transmitter 1's message content is

$$\begin{aligned}
x_1 \leftarrow \{ & A''_{12,3}C''_{12,1}, A''_{12,4}D''_{12,1}, C''_{12,2}B''_{12,3}, D''_{12,2}B''_{12,4}, \\
& C''_{12,4}D''_{12,3}, A''_{13,2}B''_{13,1}, A''_{13,4}D''_{13,1}, C''_{13,2}B''_{13,3}, \\
& D''_{13,2}B''_{13,4}, C''_{13,4}D''_{13,3}, A''_{14,2}B''_{14,1}, A''_{14,3}C''_{14,1}, \\
& C''_{14,2}B''_{14,3}, D''_{14,2}B''_{14,4}, C''_{14,4}D''_{14,3} \}.
\end{aligned}$$

[2]For simplicity we skip the $\oplus$ symbol.

From the above XORs, receiver 1 removes all those that contain subfiles of file A (since all these subfiles A have been previously delivered while their XORed counterpart is cached at $\mathcal{Z}_1$) and then jointly decodes $x_i, i \in [K]$. The two subphases combined have a delivery time $\mathcal{T} = \frac{31}{24} \approx 1.29$.

VI. Discussion

The work developed cache-aided transmission schemes that ameliorate the lack of CSIT by a careful combination of topology with pre-cached content at both receivers and transmitters. Transmitter side caches allowed partial transmitter cooperation as well as allowed content to move closer to their intended users. On the other hand, receiver side caching enabled local caching gains and multicasting gains which are especially useful for delivering data that resides in far away nodes.

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