SPATIO-TEMPORAL ARRAY PROCESSING FOR DS-CDMA DOWNLINK TRANSMISSION

Giuseppe Montalbano[†], *Irfan Ghauri*[*], *and Dirk T. M. Slock*[†]

[†]Institut Eurécom, B.P. 193, 06904 Sophia–Antipolis CEDEX, France
[*]Ericsson Mobile Communications AB, Nya Vattentornet SE-22183 Lund Sweden
{Giuseppe.Montalbano,Dirk.Slock}@eurecom.fr, Irfan.Ghauri@ecs.ericsson.se

ABSTRACT

We address the problem of optimal downlink transmission in a DS-CDMA system where periodic orthogonal Walsh-Hadamard spread different users' symbols followed by scrambling by a symbol aperiodic cell specific overlay sequence. We assume that base-stations have multiple-antenna transceivers and each user's receiver consists of a RAKE. Multiple transmit antennas can reduce the interference and increase the signal-to-noise ratio at mobile receivers. It is assumed that at least partial knowledge of the intracell users' downlink channels is available at the base-station. This information combined with the knowledge of the type of mobile receiver, suffices to perform optimal spatio-temporal downlink transmission at base-stations. Although spatial and temporal dimensions can no longer be jointly exploited due to the aperiodic overlay scrambling, pure spatial processing or beamforming, can still be performed in order to optimize the whole spatio-temporal processing through the transmitter-channel-receiver cascade. We provide closed-form relations for the signal, interference and noise terms at the output of a mobile station RAKE receiver as functions of the transmit power allocation and beamforming weight vectors. We show that analytical solutions to two optimization problems can be found for the multiuser transmit beamformers, relying on the spatio-temporal structure of the propagation channel.

1. INTRODUCTION

Third generation wireless communication systems envision the use of DS-CDMA employing aperiodic spreading sequences for the downlink, typically consisting of periodic Walsh-Hadamard sequences followed by masking by a symbol aperiodic base-station specific overlay sequence. Alternatively, even the scrambling sequence can be user dependent. The standard receiver is the RAKE and more complex receivers are considered to be too power and space hungry to be candidates for mobile terminals.

The use of adaptive antenna arrays at the base station can increase the capacity of a mobile radio network by increasing transmit diversity and due to the capability of mitigating the interference. In the downlink however, the possibility of spatial diversity reception by multiple antennas (MA) is limited due to complexity and space limitations. The presence of the spatial dimension, eventually combined with oversampling of the transmitted signals and/or to other forms of diversity (e.g., polarization diversity, in-phase and quadrature modulations for mono-dimensional symbol constellations) allows the increase of the system loading fraction which is limited by the multipath multiuser interference, by increasing diversity.

When periodic spreading sequences are adopted, effective spatial-temporal processing can be carried out at the base station transmitter relying on symbol-rate wide sense stationarity. Under these circumstances in [1] it was demonstrated that orthogonality between the spread signals can be restored at each receiver by properly filtering/spreading the symbols intended for different users based upon the information of the channel state associated with each user. Thus, a number of interfering users more than the processing gain may be located in the same cell, in particular accounting for the users in soft hand-off mode.

The application of these techniques is not straightforward when the symbol-rate cyclostationarity no longer exists due to the use of aperiodic overlay spreading sequences which spread/randomize the orthogonal user sequences. It has to be noted that, assuming the fading processes slow enough, in the structure of this downlink problem, the only entity fixed over the processing interval is the propagation channel. The actual channel as seen from the base station to a certain user consists of the cascade of spreading, transmit filters, propagation channel, receive filters and RAKE receiver. Due to the aperiodicity of spreading sequences this cascade results in a time-variant filter from symbol to symbol, which precludes the possibility of performing feasible adaptive temporal pre-filtering at the base station, as in [1], because the pre-filters need to be updated every symbol period. Some related work is also found in [5].

Although spatial and temporal dimensions can no longer be jointly exploited, pure spatial processing can be performed to optimize the whole spatio-temporal transmitter(TX)-channel-receiver(RX) cascade. This problem has also been addressed in [5] with several approximations and simplifying assumptions, that although yielding to analytically tractable optimization problems, do not actually account for the real structure and behavior of the spatio.temporal TX-channel-RX cascade.

Here we consider a scenario with K intracell users, each with a RAKE receiver capturing signals from a base station with M antennas. We provide fairly general closed-form expressions for the receiver as a function of the transmit power allocation and beamforming weight vectors.

We consider at first, a problem where the transmit beamformers are optimized to minimize the total transmit power while the signal-to-interference-plus-noise ratio (SINR) is set at each mobile to a target value. Later, we also address the problem where the minimum SINR among all K users' receivers is to be maximized under the constraint of a limited total transmit power at the base station. We shall see that, contrary to the simpler case (see [1] [3] [4] [10] [7] [6] and references therein) where the receiver consists of a symbol pulse-shaping matched filter, the presence of the RAKE renders the above optimization problems unsolvable in an analytical way. However we show that relying on the spatio-temporal structure of the propagation channel, analytical solutions can be found for both optimization problems by

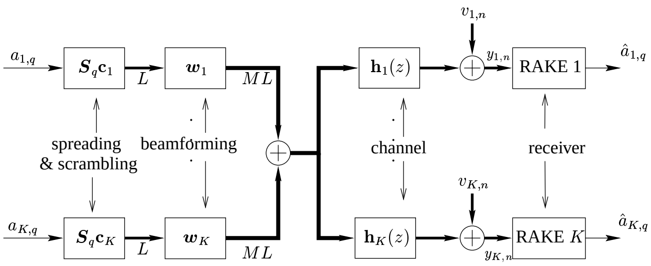

Figure 1: Transmission filters, beamformers, channels and RAKE receivers for K users.

adding *ad-hoc* constraints. By inspecting these constrained optimization problems, we also see that only partial downlink channel state information is needed for the optimization to be carried out. For simplicity, we assume that this information is available at the base station either due to reciprocal time-division duplex (TDD) channels, or due to a feedback mechanism in frequency-division duplex (FDD) and non-reciprocal TDD channels [11].

2. SIGNAL MODEL

We describe a K-user DS-CDMA system employing aperiodic signature waveforms, $b_k(t;\, kT) = c_k(t)s(t;\, nT)$, for $k = 1, \ldots, K$, where T is the symbol period and n is an integer. Note that $c_k(t)$ are the symbol periodic spreading sequences and are very often mutually orthogonal, being columns of the Hadamard matrix. $s(t;\, nT)$ is the scrambler and is aperiodic in nature.
Fig. 1 shows in detail the scenario addressed. The RAKE receivers account for the descrambling and the despreading operations and are shown to be preceded by the channel matched filters. Notice that the actual temporal channel as seen from the base station is given by the autocorrelation sequence of the channel itself. Indeed, this is straightforward to notice if we commute the de-spreading and channel matched filtering operations in the receiver.
The spreading factor is L and $T_c = T/L$ denotes the chip period. Due to the random nature of the scrambler, the cascade of the code filter, the transmit filter, the channel and the receive filter results in a time-variant system. Assuming the channels, $\mathbf{h}_i(z)$, time-invariant for the observation time, the signal at the input to the ith user RAKE receiver can be written as

$$y_{i,n} = \sum_{k=1}^{K} g_{i,k}(z) b_{k,n} a_{k,\lceil \frac{n}{L} \rceil} + v_{i,n} \qquad (1)$$

where, the $a_{k,q}$ are the transmitted symbols intended for the kth user, $g_{i,k}(z) = \boldsymbol{w}_k^H \mathbf{h}_i(z)$ is the cascade of the M-dimensional beamformer weight vector, $\boldsymbol{w}_k$, for the kth user and the channel between the base station and the ith user, and $v_{k,n}$ is the additive, zero mean, white noise at the input of the kth RAKE receiver. We assume that the base station is transmitting through M antennas, and evaluate the signal and interference energies transmitted through the TX-channel-RX cascade averaged over the spreading codes' statistics.
The channel for the kth, $h_k(t)$ is assumed to be a FIR filter of duration approximately equal to N_k chip periods. Let $\boldsymbol{h}_k = [\boldsymbol{h}_k^T(0) \,\ldots\, \boldsymbol{h}_k^T(N_k - 1)]^T$ denote the discrete-time representation of the kth channel. The RAKE receiver for user k is the channel matched filter followed by a descrambler and a despreader for this user.

We can write the RX signal over N_i successive data vectors y_n^i at the ith receiver as

$$\boldsymbol{Y}_n^i = \sum_{k=1}^{K} (I_{L+N-i-1} \otimes \boldsymbol{w}_k^H) \mathcal{T}(\boldsymbol{h}_i) \widetilde{\boldsymbol{S}}_n \widetilde{\boldsymbol{C}}_k \boldsymbol{A}_{k,n} + \boldsymbol{V}_n^i, \quad (2)$$

where, $\boldsymbol{Y}_n^i = [\underline{Y}_{n,l_2}^T \; Y_{n-1}^T \ldots Y_{n-l_3}^T \; \overline{Y}_{n-l_3-1,l_4}^T]^T$, $N_i + L - 1 - l_2 = l_3 L + l_4$, and the sub-vectors are given by

$$Y_n = \begin{bmatrix} y_{n,L-1} \\ \vdots \\ y_{n,0} \end{bmatrix}, \underline{Y}_{n,j} = \begin{bmatrix} y_{n,j-1} \\ \vdots \\ y_{n,0} \end{bmatrix}, \overline{Y}_{n,j} = \begin{bmatrix} y_{n,L-1} \\ \vdots \\ y_{n,L-j} \end{bmatrix},$$

and, $y_{n,j} = y_{nL+j}$. $\widetilde{\boldsymbol{C}}_k = \text{block diag}\{\underline{\boldsymbol{c}}_k, \boldsymbol{c}_k, \ldots, \boldsymbol{c}_k, \overline{\boldsymbol{c}}_k\}$, and,

$$\boldsymbol{c}_k = \begin{bmatrix} c_{k,L-1} \\ \vdots \\ c_{k,1} \\ c_{k,0} \end{bmatrix}, \overline{\boldsymbol{c}}_k = \begin{bmatrix} c_{k,L-1} \\ \vdots \\ \\ c_{k,L-l_6} \end{bmatrix}, \underline{\boldsymbol{c}}_k = \begin{bmatrix} c_{k,l_2-1} \\ \vdots \\ \\ c_{k,0} \end{bmatrix}.$$

$\mathcal{T}(\boldsymbol{h}_i)$ is the $M(L + N_i - 1) \times (L + 2N_i - 2)$ block Toeplitz channel matrix with first block row given by $[\boldsymbol{h}_i^{'}, \mathbf{0}_{M \times L+N_i-2}]$, and $\boldsymbol{h}_i^{'} = [\boldsymbol{h}_i(N_i - 1), \ldots, \boldsymbol{h}_i(0)]$. The code matrix $\widetilde{\boldsymbol{C}}_k$ is the $(l_5 L + l_2 + l_6) \times (l_5 + 2)$ matrix accounting for the contribution of $l_5 + 2$ symbols in the received signal $\boldsymbol{Y}_n$. $\underline{\boldsymbol{c}}_k$ and $\overline{\boldsymbol{c}}_k$ denote the partial contribution of the end symbols of the data block. We shall denote the $l_5 + 2$ columns of $\widetilde{\boldsymbol{C}}_k$ as $\boldsymbol{C}_{k,l}$, for $l \in \{0, \ldots, l_5 + 1\}$. $\boldsymbol{A}_{k,n} = [a_{k,n}, \ldots, a_{k,n-l_5-1}]^T$ is the symbol sequence vector, and $\widetilde{\boldsymbol{S}}_n$ denotes the $L + 2N_i - 2 = l_5 L + l_2 + l_6$ diagonal scrambling code matrix with the diagonal given by $[s_{n,l_2-1}, \ldots, s_{n,0}, s_{n-1,L-1}, \cdots, s_{n-l_1,0}, s_{n-l_1-1,L-1}, \ldots, s_{n-l_5-1,L-l_6}]$. The filter $g_{i,i}^{\dagger}(z) = \boldsymbol{h}_i^{\dagger}(z) \boldsymbol{w}_i$ is matched to the beamformer/channel cascade. We moreover define the correlation sequence, $\alpha_{i,k}(z) = g_{i,i}^{\dagger}(z) g_{i,k}(z) = \sum_{l=0}^{2N_i-2} \alpha_{i,k,l} z^{-l}$. The central tap, corresponding to the desired chip delay, d, for the ith user is denoted as $\alpha_{i,i,d}$. In principle, if the channel is causal, then the channel matched filter is anti-causal. We have considered the filter $g_{i,i}^{\dagger}(z)$ to be causal, so that the receiver output symbol estimates have an inherent delay (precisely $l_1 + 1$ symbols). This symbol estimate delay is due to the delay of $d = l_1 L + l_2$ chips ($l_1 = \lfloor \frac{d}{L} \rfloor$, $l_2 = d \bmod L$). A scaled estimate of the nth symbol is obtained at the output of the RAKE receiver as

$$\hat{a}_{i,n-l_1-1} = \boldsymbol{c}_i^H \boldsymbol{X}_n, \qquad (3)$$

where, $\boldsymbol{X}_n$ is the vector of appropriately descrambled filter outputs.

$$\boldsymbol{X}_n = \boldsymbol{S}_{n-l_1-1}^H \boldsymbol{Z}_n, \qquad (4)$$

with

$$\boldsymbol{Z}_n = \mathcal{T}^H(\boldsymbol{g}_{i,i}) \boldsymbol{Y}_n, \qquad (5)$$

being the filter output, $\boldsymbol{S}_n = \text{diag}\{s_{n,L-1}, \ldots, s_{n,1}, s_{n,0}\}$, and $\boldsymbol{g}_{i,k} = [g_{i,k}(N_i - 1), \ldots, g_{i,k}(0)]$.
In the time domain we can write the cascade of the filter and the propagation channel as the following set of equations.

$$\mathcal{T}^H(\boldsymbol{g}_{i,k}) \mathcal{T}(\boldsymbol{g}_{i,k}) = \mathcal{T}(\boldsymbol{\alpha}_{i,k}) = \mathcal{T}(\underline{\boldsymbol{\alpha}}_{i,k}) + \mathcal{T}(\overline{\boldsymbol{\alpha}}_{i,k}), \qquad (6)$$

where, $\mathcal{T}^H(\boldsymbol{g}_{i,k})$ is a $L \times (L + N_i - 1)$ Toeplitz matrix filled up with the channel/beamformer matched filter coefficients. $\mathcal{T}(\boldsymbol{\alpha}_{i,k})$

denotes a Toeplitz matrix with the first row $[\boldsymbol{\alpha}_{i,k} \quad \mathbf{0}_{|L-N_i|-1}]$. Same holds for $\mathcal{T}(\underline{\boldsymbol{\alpha}}_{i,k})$ and $\mathcal{T}(\overline{\boldsymbol{\alpha}}_{i,k})$, with,

$$\begin{aligned}
\boldsymbol{\alpha}_{i,k} &= [\alpha_{i,k,0}\ \alpha_{i,k,1}\ \ldots\ \alpha_{i,k,2N_i-2}],\\
\underline{\boldsymbol{\alpha}}_{i,k} &= [0\ldots0\ \alpha_{i,k,d}\ 0\ldots0]\\
\overline{\boldsymbol{\alpha}}_{i,k} &= [\alpha_{i,k,0}\ \ldots\ \alpha_{i,k,d-1}\ 0\ \alpha_{i,k,d+1}\ \ldots\ \alpha_{i,k,2N_i-2}].
\end{aligned}$$

The output energy of the RAKE receiver (user i) can be shown to be [2]

$$E|c_i^H \boldsymbol{X}_n|^2 = \boldsymbol{g}_{i,i}^H \boldsymbol{R}_{vv}^i \boldsymbol{g}_{i,i} + p_i \alpha_{i,i,d}^2 + \frac{1}{L}\sum_{k=1}^{K} p_k \|\overline{\boldsymbol{\alpha}}_{i,k}\|_2^2, \quad (7)$$

in the case of complex scrambling sequences. $p_k = E|a_k|^2 \pi_k$, where, π_k is the received power for the kth user. A further term given by $\sum_{k=1}^{K} p_k\, \mathrm{tr}\{\boldsymbol{B}_{i,k}\boldsymbol{D}_k\boldsymbol{D}_i\boldsymbol{B}_{i,k}^*\boldsymbol{D}_k^*\boldsymbol{D}_i^*\}$ adds up to (7) in the case of real scrambling sequences [2], where,

$$\boldsymbol{B}_{k,i} = \begin{bmatrix} 0 & \alpha_{k,i,d+1} & \cdots & \alpha_{k,i,d+L-1} \\ \alpha_{k,i,d-1} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \alpha_{k,i,d+1} \\ \alpha_{k,i,d-L+1} & \cdots & \alpha_{k,i,d-1} & 0 \end{bmatrix},$$

and $\boldsymbol{D}_k = \mathrm{diag}\{\boldsymbol{c}_k\}$. The SINR for the RAKE receiver is thus written as

$$\gamma_i = \frac{p_i \alpha_{i,i,d}^2}{\boldsymbol{g}_{i,i}^H \boldsymbol{R}_{vv}^i \boldsymbol{g}_{i,i} + \frac{1}{L}\sum_{k=1}^{K} p_k \|\overline{\boldsymbol{\alpha}}_{i,k}\|_2^2}, \quad (8)$$

3. OPTIMIZATION PROBLEM FORMULATION

The ith user SINR given by (8) can also be expressed in terms of the beamforming weights as

$$\gamma_i = \frac{p_i \boldsymbol{w}_i^H \boldsymbol{R}_i(0) \boldsymbol{w}_i}{\sigma_{\nu_i}^2 + \frac{1}{L}\frac{1}{\boldsymbol{w}_i^H \boldsymbol{R}_i(0)\boldsymbol{w}_i}\sum_{k=1}^{K} p_k \sum_{j\neq 0} |\boldsymbol{w}_k^H \boldsymbol{R}_i(j)\boldsymbol{w}_i|^2} \quad (9)$$

where, $\sigma_{\nu_i}^2 = \boldsymbol{g}_{i,i}^H \boldsymbol{R}_{vv}^i \boldsymbol{g}_{i,i}$ is the noise variance at the output of the ith RAKE. This will, ingeneral be a colored noise due to the channel matched filter. $\boldsymbol{R}_i(j) = \sum_l \boldsymbol{h}_i(l)\boldsymbol{h}_i^H(l+j)$, $\forall i$, and ν_i is the colored noise term at the output of the ith RAKE receiver. We consider two cost functions for the optimization problem. The first one is given by

$$\min_{\{\boldsymbol{w}_i\},\{p_i\}} \sum_{i=1}^{K} p_i \quad \text{s.t.} \quad \gamma_i \geq \Gamma_i \ \forall i \quad (10)$$

where Γ_i is the target SINR for the user i. The second one

$$\max_{\{\boldsymbol{w}_i\},\{p_i\}} \min_i \{\gamma_i\} \quad \text{s.t.} \quad \sum_{i=1}^{K} p_i \leq p_{\max} \quad (11)$$

where $p_{\max}$ is the maximum allowed power at the base station. Unfortunately, (9) depends on fourth-order expressions of the weight vectors $\boldsymbol{w}_k$'s. Thus the above optimization problems have no simple analytical solution.

However the dependence of the SINR (9) on $\boldsymbol{w}_k$'s can be reduced to second order expressions by adding some *ad hoc* constraints to the above optimization problems.

For this purpose we shall consider the structure of the propagation channels and rely on some heuristic considerations.

4. STRUCTURED CHANNEL MODEL

A classical discrete-multipath channel model is the following [8]

$$\boldsymbol{h}(\tau, t) = \sum_{q=1}^{Q} \alpha_q(t)\mathbf{a}(\theta_q)\psi(\tau - \tau_q) \quad (12)$$

where τ_q, θ_q, and $\alpha_q(t)$ denote the delay, the angle and the fading attenuation associated to the qth path, respectively, $\psi(t)$ denotes the chip pulse shaping filter, and $\mathbf{a}(\theta)$ represents the M-dimensional array response vector. In practice, since the most scattering phenomena occur near the mobile rather than near the base stations the spatial component of channel, as seen from the base station, typically exhibits only a few significant angles of propagation. However, several temporally resolvable paths can be associated with approximately the same angle especially in spread-spectrum systems. In this way temporally resolvable multipath components associated with approximately the same angle can be collected in clusters. Most channel energy is typically located around one main direction with a fairly little angle spread that is associated with the strongest multipath cluster. Other propagation angles which strongly deviate from the nominal angle of the strongest cluster are generally associated with smaller portions of the whole channel energy and often have very reduced angle spread (see [8] and references therein). This situation is illustrated in figure 2.

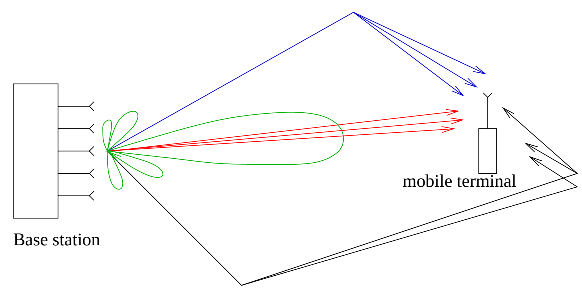

Figure 2: Downlink propagation channel, as seen from base station and the mobile terminal.

5. CONSTRAINED OPTIMIZATION PROBLEM

In the light of the above considerations the ith propagation channel discrete-time spatio-temporal impulse response at time n can be written as

$$\boldsymbol{h}_i(n) = \sum_{m=1}^{M_i} \mathbf{a}_{i,m} f_{i,m}(n) \quad (13)$$

where $\mathbf{a}_{i,m}$ and $f_{i,m}(n)$ denote the array response vector and the temporal channel response respectively for the mth path of the ith user's channel and M_i is the total number of multipath components for the ith channel. Notice that $f_{i,m}(n)$ can include several temporally resolvable paths (especially in a CDMA system) and accounts also for the fading coefficients. In other words there can be more than one temporally resolvable path associated with the same angle, i.e., with the same array response vector.

Then, the matrix $\boldsymbol{R}_i(j)$ takes the form

$$\boldsymbol{R}_i(j) = \sum_{m=1}^{M_i}\sum_{q=1}^{M_i} \mathbf{a}_{i,m}\mathbf{a}_{i,q}^H \sum_l f_{i,m}(l) f_{i,q}^*(l+j) \quad (14)$$

Assume that we re-order the multipath components of each user's channel so as the first angle is the one carrying the most channel energy.
Then we constrain the beamforming weight vectors as follows

$$|\boldsymbol{w}_i^H \mathbf{a}_{i,1}|^2 = 1 \quad \text{and} \quad |\boldsymbol{w}_i^H \mathbf{a}_{i,m}|^2 = 0 \text{ for } m \neq 1 \quad \forall i \tag{15}$$

This set of constraints leads to focusing the whole power intended for a certain user on the angle corresponding to the stronger channel cluster while zero-forcing the angles corresponding to weak clusters (see fig.2).
In this way the SINR expression (9) reduces to

$$\gamma_i = \frac{p_i r_{i,1}(0)}{\sigma_{\nu_i}^2 + \frac{1}{L r_{i,1}(0)} \sum_{k=1}^K p_k |\boldsymbol{w}_k^H \mathbf{a}_{i,1}|^2 \sum_{j\neq 0} |r_{i,1}(j)|^2} \tag{16}$$

where we defined the autocorrelation sequences of the temporal channel response associated with the strongest angle as $r_{i,1}(j) = \sum_l f_{i,1}(l) f_{i,1}^*(l+j)$, $\forall i$. The constrained SINR expression (16) depends on quadratic forms of the weight vectors $\boldsymbol{w}_k$'s and analytical approaches to solve the optimization problems (10) and (11) can be pursued.

5.1. Remarks

- The set of constraints (15) is at least partially consistent with an optimal spatial-power allocation strategy for the transmit power intended for a certain user that would suggest to allocate more power to the stronger clusters.
- The constraints (15) can be satisfied if the number of antennas at the base station $M \geq M_i$ for $i = 1, \ldots, K$.
- In practical systems the beamforming resolution is limited by the limited number of antennas. Therefore the condition $|\boldsymbol{w}_i^H \mathbf{a}_{i,1}|^2 = 1$ is not very critical even in the presence of relatively large angle spread around the nominal angle. On the contrary the zero-forcing conditions $|\boldsymbol{w}_i^H \mathbf{a}_{i,m}|^2 = 0$ for $m \neq 1$ can be critical when deep nulls are placed at nominal angles of weak clusters with large angle spreads. In this case, one may add further constraints (e.g., derivative constraints) to extend zero-forcing to the whole cluster angle spread.
- Zero-forcing weak multipath clusters has the advantage of reducing the channel delay spread, reducing the impact of the inter-symbol interference and the need of further equalization. Notice that for traditional DS-CDMA downlinks, the presence of ISI translates to an increase in multiaccess interference due to loss of orthogonality due to dispersion [2].
- By inspecting the SINR expression (16) a joint optimization of the powers p_i's and the (normalized) weight vectors $\boldsymbol{w}_i$'s requires the knowledge of the downlink array response vectors $\mathbf{a}_{i,m}$ for $m = 1, \ldots, M_i$ and $i = 1, \ldots, K$, of the energy in the strongest cluster temporal channel component (i.e. $r_{i,1}(0)$, the middle tap of the autocorrelation sequence of the temporal channel), of the sum of energies $\sum_{j\neq 0} |r_{i,q}(j)|^2$ of the other taps but the middle one of the autocorrelation sequence of the strongest cluster temporal channel component, and estimates of the RAKE output noise variances $\sigma_{\nu_i}^2$. It is assumed that at least this partial knowledge of the downlink channel is available at the base-station, due to either a time-division duplex (TDD) or some feedback structure in the network. Under fast fading conditions, the above derivation can be adapted by taking the averaged versions of $\boldsymbol{R}_i(j)$ in (14) over fading characterstics [9].

6. CONSTRAINED OPTIMIZATION PROBLEM SOLUTION

In the sequel we shall focus on the first optimization problem (10). We provide a detailed derivation of an algorithm that allows to find the global optimum of the problem (10) under the constraints (15). Define the coefficients

$$\beta_i = \frac{\sum_{j\neq 0} |r_{i,1}(j)|^2}{r_{i,1}(0)}, \quad \tilde{\beta}_i = \frac{\beta_i}{\sigma_{\nu_i}^2}, \quad \tilde{r}_{i,1}(0) = \frac{r_{i,1}(0)}{\sigma_{\nu_i}^2}$$

then the optimization problem (10) with the constraints (15) reduces to the form

$$\begin{aligned} &\min_{\{\boldsymbol{w}_i\},\{p_i\}} \sum_{i=1}^K p_i \qquad \text{s.t.} \\ &\gamma_i = \frac{p_i \tilde{r}_{i,1}(0)}{1 + \frac{1}{L}\tilde{\beta}_i \left(p_i + \sum_{k\neq i} p_k |\boldsymbol{w}_k^H \mathbf{a}_{i,1}|^2 \right)} \geq \Gamma_i \\ &|\boldsymbol{w}_i^H \mathbf{a}_{i,1}|^2 = 1 \\ &|\boldsymbol{w}_i^H \mathbf{a}_{i,m}|^2 = 0 \qquad \text{for } m \neq 1 \ \forall i \end{aligned} \tag{17}$$

The optimal solution to this problem can be shown to be the one of the following equivalent *virtual uplink* problem (see [4] and references therein)

$$\begin{aligned} &\min_{\{\boldsymbol{w}_i\},\{p_i\}} \sum_{i=1}^K p_i \qquad \text{s.t.} \\ &\frac{p_i \tilde{r}_{i,1}(0)}{\|\boldsymbol{w}_i\|^2 \left(1 + \frac{1}{L}\tilde{\beta}_i p_i\right) + \frac{1}{L}\tilde{\beta}_i \sum_{k\neq i} p_k |\boldsymbol{w}_i^H \mathbf{a}_{k,1}|^2} \geq \Gamma_i \\ &|\boldsymbol{w}_i^H \mathbf{a}_{i,1}|^2 = 1 \quad |\boldsymbol{w}_i^H \mathbf{a}_{i,m}|^2 = 0 \quad \text{for } m \neq 1 \ \forall i \end{aligned} \tag{18}$$

It can be shown that for problems (10), (17) and (18) the optimal solution must satisfy the constraints with equality [4].

6.1. Optimization Algorithm

The algorithm to compute the optimum beamforming vectors $\boldsymbol{w}_i$'s can be stated as the follows. At the nth iteration do

1. For $i = 1, \ldots, K$ compute

$$\begin{aligned} \boldsymbol{w}_i^{(n)} &= \arg\min_{\boldsymbol{w}_i} \frac{\tilde{\beta}_i}{L} \sum_{k\neq i} p_k^{(n)} |\boldsymbol{w}_i^H \mathbf{a}_{k,1}|^2 + \|\boldsymbol{w}_i\|^2 \left(1 + \frac{\tilde{\beta}_i}{L} p_i\right) \\ &\text{s.t. } |\boldsymbol{w}_i^H \mathbf{a}_{i,1}|^2 = 1 \quad |\boldsymbol{w}_i^H \mathbf{a}_{i,m}|^2 = 0 \quad \text{for } m \neq 1 \end{aligned}$$

2. For $i = 1, \ldots, K$ compute the updated virtual uplink powers

$$p_i^{(n+1)} = \frac{\Gamma_i \left(\|\boldsymbol{w}_i\|^2 + \frac{\tilde{\beta}_i}{L} \sum_{k\neq i} p_k^{(n)} |\boldsymbol{w}_i^{(n)H} \mathbf{a}_{k,1}|^2 \right)}{\tilde{r}_{i,1}(0) - \Gamma_i \|\boldsymbol{w}_i\|^2 \frac{\tilde{\beta}_i}{L}}$$

3. For $i = 1, \ldots, K$ compute the updated downlink powers $\tilde{p}_i$'s

$$\tilde{p}_i^{(n+1)} = \frac{\Gamma_i \left(1 + \frac{\tilde{\beta}_i}{L} \sum_{k\neq i} \tilde{p}_k^{(n)} |\boldsymbol{w}_k^{(n)H} \mathbf{a}_{i,1}|^2 \right)}{\tilde{r}_{i,1}(0) - \Gamma_i \frac{\tilde{\beta}_i}{L}}$$

Once the algorithm has converged compute the optimum transmit weight vectors as $\sqrt{\tilde{p}_i}\boldsymbol{w}_i$.

It can be shown that the preceding algorithm converges to the global minimum if a feasible solution there exists. A solution to the second optimization problem (11) under constraints (15) can be found by applying the algorithms proposed in [3] and [5] with some slight modification.

An interesting observation ralated to the power update expressions in steps 2 and 3 of the algorithm is that when the denominator term is zero or negative, there is no solution to the power allocation problem. This phenomenon can be interpreted as failure to meet the target SINR due to the inability of spatial filtering to control the ISI.

7. NUMERICAL RESULTS

In this section, we consider a typical propagation scenario where a base station with $M = 4$ antennas transmits to $K = 4$ users in a DS-CDMA system. The spreading factor is $L = 16$ and users are in near-far propagation conditions (up to 40 dB of difference in channel energy attenuation).

Users' channels are characterized by a finite set of Rayleigh fading multipath components. The channel parameters in terms of normalized angles w.r.t. the array broadside, path delays (w.r.t. the first path delay), relative paths variances (normalized w.r.t. the the strongest path of each user's channel), and the path losses of different users are as follows:

- 1st user: $(-10, -10, 33, -30)$, degrees, $(0, 0.3, 1.2, 2.1)$ T_c, $(1, 0.64, 0.16, 0.16)$, 10^{-4}
- 2nd user: $(12, 12, -40, 28)$ degrees $(0.5, 0.9, 1.5, 2.8)$ T_c, $(1, 0.81, 0.25, 0.09)$, 10^{-2}
- 3rd user: $(12, 12, -40, 28)$, degrees, $(0.5, 0.9, 1.5, 2.8)$ T_c, $(1, 0.81, 0.25, 0.09)$, 10^{-6}
- 4th user: $(38, 38, -50, 30)$ degrees, $(0.1, 1.4, 4.16, 4.76)$ T_c, $(1, 0.70, 0.35, 0.17)$, 10^{-4}

In our simulations we choose the target SINRs Γ_i to be all the same. Figure 3 shows the result of the optimization algorithm described in section 6.1, in terms of total transmit power for different target SINRs. Figure 4 shows the radiation pattern obtained with the converged beamforming vectors $\sqrt{\tilde{p}_i}\boldsymbol{w}_i$, for $i = 1, \ldots, 4$ when the target SINR is fixed to 10 dB.

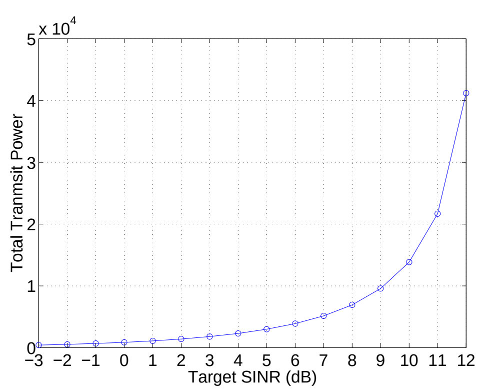

Figure 3: Optimal total TX power vs. target SINR.

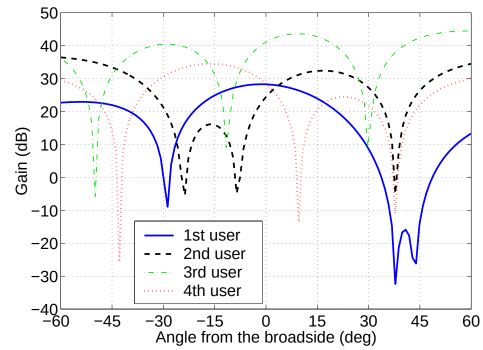

Figure 4: Optimal radiation beam patterns for a fixed target SINR=10dB.

8. CONCLUSIONS

We addressed the problem of SINR maximization at the mobile stations by performing spatial filtering at the base-station and employing a RAKE receiver at the mobile stations. It was shown that in cases where spatio-temporal processing cannot be employed to pre-cancel the interference, significant performance gains can still be achieved by spatial filtering only. An algorithm for determining the optimum beamforming weights was also presented. It was observed that in the instance of clustered channel models, a wide range of target SINRs can be met if the beamforming effort is geared towards placing nulls at nominal angles of weak channel clusters, thus reducing ISI in the received signal.

9. REFERENCES

[1] G. Montalbano, I. Ghauri, and D. T. M. Slock,"Spatio-Temporal Array Processing for CDMA/SDMA Downlink Transmission," *Proc. 32nd Asilomar Conf. on Signals, Systems, and Computers, Pacific Grove, CA,* Nov. 1998.

[2] D. T. M. Slock and I. Ghauri,"Blind maximum SINR receiver for the DS-CDMA downlink," *Proc. ICASSP'2000,*, Mar. 2000

[3] G. Montalbano and D. T. M. Slock,"Matched filter bound optimization for multiuser downlink transmit beamforming," *Proc. ICUPC'98*, Oct. 1998.

[4] E. Visotsky and U. Madhow,"Optimum beamforming using transmit antenna arrays," *Proc. VTC'99*, May 1999.

[5] F. Rashid-Farrokhi and L. Tassiulas, "Power control and space-time diversity for CDMA systems, " *Proc. GLOBECOM'98*, Nov. 1998.

[6] F. Rashid-Farrokhi, L. Tassiulas and K.J.R. Liu, "Transmit beamforming and power control for cellular wireless systems, " *IEEE JSAC.*, vol.26(8), pp. 1437-1449, Oct. 1998.

[7] F. Rashid-Farrokhi, L. Tassiulas, "Joint optimal power control and beamforming in wireless networks with antenna arrays, " *IEEE Trans. on Comm.*, vol.46(10), pp. 1313-1323, Oct. 1998.

[8] R. B. Ertel, P. Cardieri, K. Sowerby, T. Rappaport and J. Reed, "Overview of spatial channel models for antenna array communication systems, " *IEEE Pers. Commun. Mag.,*, pp. 10-22, Feb. 1998.

[9] G. G. Raleigh and V. K. Jones, "Adaptive antenna transmission for frequeny duplex digital wireless communications," *Proc. ICC'97*, June 1997.

[10] P. Zetterberg *Mobile cellular communications with base station antenna arrays: Spectrum efficiency, algorithms and propagation models*, Ph.D. thesis, RTK, Stockholm, Sweden, 1997.

[11] D. Gerlach and A. Paulraj,"Adaptive transmit antenna arrays with feedback," *IEEE Signal Processing Lett.*, vol. 1(10), pp. 150–182, Oct. 1994.