SPATIO-TEMPORAL ARRAY PROCESSING FOR MATCHED FILTER BOUND OPTIMIZATION IN SDMA DOWNLINK TRANSMISSION

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ABSTRACT

We address the problem of performing optimum spatio-temporal processing when using adaptive antenna arrays at base stations for multiuser downlink transmission, assuming the knowledge of the channel related to each user. This assumption typically holds in the context of time division duplex (TDD), time division multiple access (TDMA) based mobile communication systems. For frequency division duplex (FDD) based systems that assumption is still valid if the base station is provided with feedback from each mobile about the downlink channel. We consider the Spatial Division Multiple Access (SDMA) strategy for using antenna arrays to gain system capacity. In that case the interfering users are located in the same cell and communicate with the same base station. The base station performs transmission through m channels resulting from an array of antennas and/or oversampling of the transmitted signals, towards d co-channel users. The goal is designing the $m \times d$ transmission FIR filters at the base station in order to maximize the minimum Matched Filter Bound (MFB) among the d users.

We address Zero-Forcing (ZF), Minimum Mean Squared Error (MMSE) and other approaches to solve that problem and we provide the related solutions, under specified assumptions and constraints concerning transmitter and receiver complexity.

1. INTRODUCTION

The use of adaptive antenna arrays at base stations can increase the capacity of mobile radio networks by an improved spectrum efficiency, in the uplink as well as in the downlink.

The problem is performing optimum spatio-temporal processing when using adaptive antenna arrays at base stations for multiuser downlink transmission, in the context of time division duplex (TDD), time division multiple access (TDMA) based mobile communication systems.

Note that in TDD based systems the uplink and the downlink channels can be considered to be practically the same, assuming the mobile velocity low enough and the receiver and transmitter appropriately calibrated. In such circumstances since the channel is known (or estimated) from the uplink, efficient spatio-temporal processing can be performed at the base station during transmission as well as during reception.

On the contrary the lack of channel knowledge represents a strong limit inherent to frequency division duplex (FDD) based systems. Indeed the base station has no direct knowledge of the downlink channel, since it can not be directly observed and therefore estimated. A solution to that problem consists in providing the base station with a feedback from the mobile station about the downlink channel. Obviously such solution involves a reduction in spectral efficiency. On the other hand, if such feedback is not provided, the downlink channel characterization can only be based on the estimates of those parameters related to the uplink channel, which are relatively frequency independent and whose changing rate is slow with respect to the frame duration. Actually in FDD based mobile communication systems, in the absence of feedback, only the downlink channel covariance matrix can be estimated, and not the channel itself. In addition even a robust and reliable estimation of the channel covariance matrix represents a non trivial issue. In spite of that complication, solutions to perform optimum transmit array processing have been previously proposed only for FDD based systems [2]–[9]. Moreover, in those solutions only purely spatial filtering (i.e., beamforming) has been considered. At present we are not aware of any publication considering spatio-temporal array processing for FDD and TDD based systems for downlink transmission.

Here we deal with the problem of the Matched Filter Bound (MFB) optimization assuming a TDD mobile communication system operating with SDMA frequency reuse technique to gain system capacity. Then the interfering users are located in the same cell while the interference coming from other cells is neglected. The maximization of the MFB leads to the minimum probability of error for an optimal receiver.

We assume that reciprocity between up-link and downlink channels holds, i.e., the uplink and the downlink channels are the same. The base station performs transmission through m channels resulting from an array of antennas and/or oversampling of the transmitted signals, towards d co-channel users. Each one of all the d mobile receivers is assumed to have one antenna and to sample at the symbol rate (i.e., no oversampling is provided at the receivers). The goal is designing the $m \times d$ transmission FIR filters in order to maximize the minimum MFB among the d users.

2. MFB OPTIMIZATION PROBLEM FORMULATION

In order to provide a consistent problem formulation we start considering a base station that performs oversampled (OS) prefiltering before transmitting with only one antenna towards a generic mobile user where a single antenna signal gets sampled at the symbol rate. Then referring to figure 1 we denote $a(k)$ the transmitted symbols, $h(t) = w(t) \star c(t)$ the continuous time impulse response of the convolution between the transmit pulse shape filter $w(t)$ and the physical channel $c(t)$, and $H(z_r)$ the discrete-time transfer function corresponding to $h(t)$ sampled at rate r/T where T is the symbol period (when no OS is performed we denote $z_1 = z$). The receive filter is included in $c(t)$. We assume $H(z_r)$ to be identified by the base station on the uplink, the base station uses the same pulse shape filter $w(t)$ as the mobile receiver and the clocks between transmitter and receiver are synchronized both at the base station and the mobile. On this basis, we can formulate the problem for all the d users regardless of the OS factor r, also considering an array of antennas at the base station as fol-

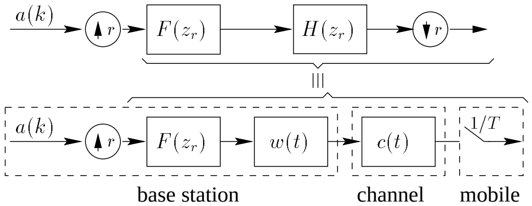

Figure 1: Oversampled signal transmission chain to one mobile with one antenna at the base station

lows. Actually the ith user discrete-time received signal, for $i = 1\ \ldots,\ d$, is

$$y_i(k) = \mathbf{H}_i^T(q) \sum_{j=1}^{d} \mathbf{F}_j(q) a_j(k) + v_i(k) \tag{1}$$

where the $a_j(k)$ are the transmitted symbols intended for the jth user, q^{-1} is the unit sample delay operator (i.e., $q^{-1} y_i(k) = y_i(k-1)$), $\mathbf{H}_i^T(z)$ is the channel transfer function between the base station and the ith user, $\mathbf{F}_j(z)$ is the spatio-temporal filter for the transmitted symbols $a_j(k)$, and $v_i(k)$ is the additive noise at the ith receiver. The superscript ^{T} denotes transpose. Note that $\mathbf{F}_j(z)$ is a $m \times 1$ column vector and $\mathbf{H}_i^T(z)$ is a $1 \times m$ row vector. Note that $\mathbf{H}_i(z)$ is the $m \times 1$ channel in the uplink from the ith user to the m base station channels.

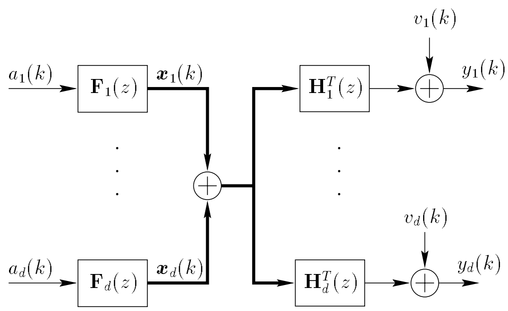

Figure 2: Transmission filters and channels for d users

2.1. Frequency domain problem formulation

The frequency domain MFB definition for the ith user, considering interferers as Gaussian noise, is

$$\mathrm{MFB}_i = \frac{1}{2\pi \mathrm{j}} \oint \frac{\sigma_a^2 \mathbf{G}_{ii}^\dagger(z) \mathbf{G}_{ii}(z)}{\sigma_a^2 \sum_{j \neq i} \mathbf{G}_{ji}(z) \mathbf{G}_{ji}^\dagger(z) + \sigma_{v_i}^2} \frac{\mathrm{d}z}{z} \tag{2}$$

where $\mathbf{G}_{ji}(z) = \mathbf{H}_i^T(z)\mathbf{F}_j(z)$, $\sigma_a^2 = \mathrm{E}\{|a_i(k)|^2\}$, for $i = 1,\ \ldots,\ d$, $\sigma_{v_i}^2$ is the variance of the additive noise $v_i(k)$, assumed temporally and spatially white hereafter, and, in general, $\mathbf{H}^\dagger(z) = \mathbf{H}^H(1/z^*)$. The superscripts ^{H} and * denote Hermitian transpose and conjugate respectively. The symbols are assumed i.i.d. and the symbol constellation is assumed circular (for a real constellation, the complex signals should be split into in phase and in quadrature components).
The cost function is given by

$$\max_{\{\mathbf{F}_j(z)\}} \min_i \{\mathrm{MFB}_i\} \tag{3}$$

2.2. Burst processing time domain problem formulation

Consider the ith user I/O transmission chain (see Fig. 2) regardless of the contributions intended for the other users. The channel $\boldsymbol{h}_i^T(t)$ and the filter $\boldsymbol{f}_i(t)$ are assumed to be FIR filters with duration $N_i T$ and LT respectively (approximately), where T is the symbol period.

In the discrete-time representation we have

$$\begin{aligned}
\boldsymbol{x}_i(k) &= \textstyle\sum_{l=0}^{L-1} \boldsymbol{f}_i(l) a_i(k-l) = \boldsymbol{F}_i A_{i,\,L}(k) \\
y_i(k) &= \textstyle\sum_{n=0}^{N_i-1} \boldsymbol{h}_i^T(n) \boldsymbol{x}_i(k-n) + v_i(k) \\
&= \boldsymbol{H}_i^t \boldsymbol{X}_{i,\,N_i}(k) + v_i(k) \\
\boldsymbol{H}_i^t &= [\boldsymbol{h}_i^T(N_i-1)\ \ldots\ \boldsymbol{h}_i^T(0)] \\
\boldsymbol{F}_i &= [\boldsymbol{f}_i(L-1)\ \ldots\ \boldsymbol{f}_i(0)] \\
\boldsymbol{X}_{i,\,N_i}(k) &= [\boldsymbol{x}_i^H(k-N_i+1)\ \ldots\ \boldsymbol{x}_i^H(k)]^H \\
A_{i,\,L}(k) &= [a_i^H(k-L+1)\ \ldots\ a_i^H(k)]^H
\end{aligned} \tag{4}$$

where superscript ^{t} denotes transposition of the blocks in a block matrix. If we consider M consecutive samples

$$\begin{aligned}
Y_{i,\,M}(k) = {}& \mathcal{T}_M(\boldsymbol{H}_i^t) \mathcal{T}_{M+N_i-1}(\boldsymbol{F}_i) A_{i,\,M+N_i+L-2}(k) \\
&+ V_{i,\,M}(k)
\end{aligned}$$

where $Y_{i,\,M}(k) = [y_i^H(k-M+1)\ \ldots\ y_i^H(k)]^H$ and similarly for $V_{i,\,M}(k)$. $\mathcal{T}_M(\boldsymbol{G})$ is in general a block Toeplitz matrix with M block rows and $[\boldsymbol{G}\ \mathbf{0}_{p\times q(M-1)}]$ as first block row, where $\boldsymbol{G}$ is a matrix with $p \times q$ block entries.
Then, introducing also the contributions of all the other cochannel users, for the ith user we have

$$\begin{aligned}
Y_{i,\,M}(k) = {}& \sum_{j=1}^{d} \mathcal{T}_M(\boldsymbol{H}_i^t) \mathcal{T}_{M+N_i-1}(\boldsymbol{F}_j) A_{j,\,M+N_i+L-2}(k) \\
&+ V_{i,\,M}(k)
\end{aligned} \tag{5}$$

and in the corresponding burst covariance matrix

$$\boldsymbol{R}_i^{(M)} = \sum_{j=1}^{d} \boldsymbol{R}_{ji}^{(M)} + \sigma_{v_i}^2 \mathbf{I}_M$$

we can distinguish the following contributions

$$\begin{aligned}
\boldsymbol{R}_{ii}^{(M)} &= \sigma_a^2 \mathcal{T}_M(\boldsymbol{H}_i^t) \mathcal{T}_{M+N_i-1}(\boldsymbol{F}_i) \mathcal{T}_{M+N_i-1}^H(\boldsymbol{F}_i) \mathcal{T}_M^H(\boldsymbol{H}_i^t) \\
\boldsymbol{R}_{ji}^{(M)} &= \sigma_a^2 \mathcal{T}_M(\boldsymbol{H}_i^t) \mathcal{T}_{M+N_i-1}(\boldsymbol{F}_j) \mathcal{T}_{M+N_i-1}^H(\boldsymbol{F}_j) \mathcal{T}_M^H(\boldsymbol{H}_i^t)
\end{aligned} \tag{6}$$

where $\boldsymbol{R}_{ii}^{(M)}$ and $\boldsymbol{R}_{ji}^{(M)}$ are the contributions of the ith and jth transmitted signals respectively at the ith receiver, for $j \neq i$. Note that $\sum_{j\neq i} \boldsymbol{R}_{ji}^{(M)}$ represents the burst covariance matrix of the whole Inter-User-Interference (IUI) at the ith receiver.
Then the burst processing MFB is defined as

$$\mathrm{MFB}_i^{(M)} = \frac{1}{M} \mathrm{tr}\{\boldsymbol{R}_{ii}^{(M)} [\sum_{j\neq i} \boldsymbol{R}_{ji}^{(M)} + \sigma_{v_i}^2 \mathbf{I}_M]^{-1}\} \tag{7}$$

where $\mathrm{tr}\{\cdot\}$ denotes the trace operator. Remark that as $M \to \infty$, $\mathrm{MFB}_i^{(M)} \to \mathrm{MFB}_i$ in (2).
Similarly to the frequency domain formulation (3) the optimization criterion results in

$$\max_{\{\boldsymbol{F}_j\}} \min_i \{\mathrm{MFB}_i^{(M)}\} \tag{8}$$

2.3. Further assumptions

Both problem formulations (3), (8) are too complicated to allow any analytical approach to fined the optimum solution. Nevertheless analytical solutions can be found under the following assumption that the optimal solution corresponds to a low Interference-to-Noise Ratio (INR) for all the users, i.e., for all the i's we have

$$\mathrm{INR}_i = \frac{\sigma_a^2}{2\pi \mathrm{j} \sigma_{v_i}^2} \sum_{j\neq i} \oint \mathbf{G}_{ji}^\dagger(z) \mathbf{G}_{ji}(z) \frac{\mathrm{d}z}{z} \ll 1 \tag{9}$$

In that case, it is easy to see that maximizing the MFB is approximately equivalent to maximizing the Signal-to-Interference-plus-Noise Ratio (SINR) and vice versa. Hence, referring to the burst processing problem formulation, the SINR definition for the ith user is

$$\mathrm{SINR}_i = \frac{\mathrm{tr}\{\boldsymbol{R}_{ii}^{(M)}\}}{\mathrm{tr}\{\sum_{j\neq i}\boldsymbol{R}_{ji}^{(M)} + \sigma_{v_i}^2\mathbf{I}_M\}} \tag{10}$$

By introducing $\boldsymbol{F}_i^t = [\boldsymbol{f}_i^T(L-1)\ \ldots\ \boldsymbol{f}_i^T(0)]$, it can be written as

$$\mathrm{SINR}_i = \frac{\sigma_a^2\boldsymbol{F}_i^t\boldsymbol{R}_i\boldsymbol{F}_i^{tH}}{\sigma_a^2\sum_{j\neq i}\boldsymbol{F}_j^t\boldsymbol{R}_i\boldsymbol{F}_j^{tH} + \sigma_{v_i}^2} \tag{11}$$

where $\boldsymbol{R}_i$ is a properly defined covariance matrix related to the channel $\boldsymbol{H}_i^t$, whose derivation is straightforward. In the continuous-processing case, we have $\boldsymbol{R}_i = \mathcal{T}_L(\boldsymbol{H}_i)\mathcal{T}_L^H(\boldsymbol{H}_i)$, where $\boldsymbol{H}_i = [\boldsymbol{h}(N_i-1)\ \ldots\ \boldsymbol{h}(0)]$.

According to the definition (11) we denote $\mathrm{SINR}_i = \gamma_i$ in the sequel. Then let $\boldsymbol{F}_i^t = \sqrt{p_i}\boldsymbol{U}_i^t$, where $\boldsymbol{U}_i^t$ is a vector with unit norm (e.g., $\|\boldsymbol{U}_i^t\|_2 = 1$ or $\boldsymbol{U}_i^t\boldsymbol{R}_i\boldsymbol{U}_i^{tH} = 1$), the vector of the inverse SINR's $\boldsymbol{\gamma}^{-1} = [\gamma_1^{-1}\ \ldots\ \gamma_d^{-1}]^T$ and the vector of the transmit powers $\boldsymbol{p} = [p_1, \ldots, p_d]^T$. In addition we need to constrain the overall power transmitted by the base station to be less that or equal to $p_{\max}$. Given that, the optimization criterion is

$$\min_{\boldsymbol{p},\{\boldsymbol{U}_i\}} \|\boldsymbol{\gamma}^{-1}\|_\infty \quad \text{s.t.} \quad \boldsymbol{g}^T\boldsymbol{p} \leq p_{\max} \tag{12}$$

where[1] $\boldsymbol{g} = [\|\boldsymbol{U}_1^t\|_2^2\ \ldots\ \|\boldsymbol{U}_d^t\|_2^2]^T$. In the rest of this paper we shall consider the SINR optimization criterion (12), regardless of its relationship to the MFB criterion in (3). In that case $\sigma_{v_i}^2$ can account for the variance of the inter-cell interference also. Then we define the normalized power delivered by the jth transmission filter $\boldsymbol{F}_j$ to the ith user as

$$c_{ji} = \boldsymbol{U}_j^t\boldsymbol{R}_i\boldsymbol{U}_j^{tH}$$

For any i it results

$$\gamma_i^{-1}p_i c_{ii} = \sum_{j\neq i} p_j c_{ji} + \nu_i \tag{13}$$

where we introduced $\nu_i = \sigma_{v_i}^2/\sigma_a^2$ for all the i's. In order to account for all the users we introduce the matrix $\boldsymbol{D}_c = \mathrm{diag}(c_{11}, \ldots, c_{dd})$, the matrix $\boldsymbol{C}^T$ defined as

$$[\boldsymbol{C}^T]_{ij} = \begin{cases} c_{ji} & \text{for } j \neq i \\ 0 & \text{for } j = i \end{cases}$$

the vector $\boldsymbol{\nu} = [\nu_1\ \ldots\ \nu_d]^T$ and the matrix $\boldsymbol{P} = \mathrm{diag}(\boldsymbol{p})$. Then we have the following equation

$$\boldsymbol{\gamma}^{-1} = \boldsymbol{D}_c^{-1}\boldsymbol{P}^{-1}(\boldsymbol{C}^T\boldsymbol{p} + \boldsymbol{\nu}) \ . \tag{14}$$

So the criterion (12) generally leads to a set of coupled problems which cannot be solved analytically. It can be shown however that the optimum (12) leads to the same γ for all the users. Indeed if some γ_i's are not the same, then we can scale the $\{p_i\}$ to improve $\gamma_{\min}$ (refer to [1] for a detailed proof).

3. MFB OPTIMIZATION PROBLEM SOLUTIONS

Generally the optimization problem cannot be solved analytically for both $\boldsymbol{p}$ and $\{\boldsymbol{U}_i^t\}$ at the same time. Nevertheless under certain assumptions the optimization can be carried out in a decoupled way for $\boldsymbol{p}$ and $\{\boldsymbol{U}_i^t\}$ allowing analytical approaches to find the optimum.

[1]Actually, the proper norm for the $\boldsymbol{U}_i^t$'s in $\boldsymbol{g}$ is $\boldsymbol{U}_i^t\boldsymbol{W}\boldsymbol{U}_i^{tH}$, where $\boldsymbol{W}$ depends on the pulse shape filter, but we shall ignore this issue in this paper.

3.1. Normalized transmit filters optimization for $m > d$

3.1.1. Zero-Forcing (ZF) solution

In the noiseless case or assuming the assumption (9) holds, the MFB optimization becomes

$$\max_{\|\boldsymbol{U}_i^t\|_2=1}\{\boldsymbol{U}_i^t\boldsymbol{R}_i\boldsymbol{U}_i^{tH}\} \quad \text{s.t.} \quad \sum_{j\neq i} p_j\boldsymbol{U}_j^t\boldsymbol{R}_i\boldsymbol{U}_j^{tH} = 0 \tag{15}$$

Note that the condition $\sum_{j\neq i} p_j\boldsymbol{U}_j^{tH}\boldsymbol{R}_i\boldsymbol{U}_j^t = 0$ is equivalent to a set of Zero-Forcing (ZF) conditions in the form $\boldsymbol{U}_i^t\boldsymbol{R}_j\boldsymbol{U}_i^{tH} = 0$, for $j \neq i$. Then the optimization problem reduces to

$$\max_{\|\boldsymbol{U}_i^t\|_2=1} \|\boldsymbol{U}_i^t\mathcal{T}_L(\boldsymbol{H}_i)\|_2^2 \text{ s.t. } \boldsymbol{U}_i^t\mathcal{T}_L(\boldsymbol{H}_j) = \boldsymbol{0} \text{ for } j \neq i \tag{16}$$

Defining $\boldsymbol{B}_i = [\mathcal{T}_L(\boldsymbol{H}_j)]_{j\neq i}$, which is a block Toeplitz matrix accounting for all the channels but the channel $\boldsymbol{H}_i$, the solution of the problem (16) is $\boldsymbol{U}_i^{tH} = V_{\max}(\mathbf{P}_{\mathbf{B}_i}^\perp\boldsymbol{R}_i\mathbf{P}_{\mathbf{B}_i}^\perp)$.

In order for a non trivial solution to this problem to exist, the constraints should not fix all the available degrees of freedom and we require

$$L > \frac{\sum_{j\neq i} N_j - (d-1)}{m - (d-1)} \tag{17}$$

The constraints present in the optimization problem (16) lead to perfect IUI cancelation. This is obtained at the expense of increased ISI at the receiver. In order to consider the ISI as well as the IUI rejection in the optimization problem we rely on the ZF pre-equalization conditions.

3.1.2. ZF conditions for IUI and ISI rejection

In order to ensure ZF conditions for IUI and ISI for the ith user the set of constraints to be considered is

$$\begin{aligned}&\boldsymbol{U}_i^t[\mathcal{T}_L(\boldsymbol{H}_1)\ldots\mathcal{T}_L(\boldsymbol{H}_d)]\\ &= [0\ldots0\ldots\overbrace{|\,0\ldots0\,\alpha\,0\ldots0\,|}^{i\text{th user}}\ldots0\ldots0]\end{aligned} \tag{18}$$

where $\alpha \neq 0$ is an arbitrary constant to be fixed in order to satisfy the constraint on the norm of $\boldsymbol{U}_i^t$. To be able to satisfy all the constraints (18) we need to choose the length of each filter $\boldsymbol{U}_i$, L, such that the previous system is exactly or underdetermined. Hence

$$L \geq \underline{L} = \left\lceil \frac{N-d-1}{m-d} \right\rceil \tag{19}$$

where $N = \sum_{j=1}^d N_j$.

Then assuming $L \geq \underline{L}$ we can consider two limiting set of constraints:

- IUI rejection, no ISI rejection, as in section 3.1.1.
- IUI and ISI rejection: in this case the set of constraints is (18), i.e., we have $N_i + L - 1$ more constraints.

The goal is to maximize the MFB which, in absence of IUI (equal to zero due to ZF), is proportional to the energy in the prefilter-channel cascade. This MFB decreases if all the energy is constrained in one tap.

Hence if no ISI rejection is provided the best performance will be achieved, for a specified L, due to the larger number of degrees of freedom. However, in that case the ith receiver needs to equalize a delay spread of up to $N_i + L - 1$ symbol periods, corresponding to the whole delay spread due to the convolution between the channel and the transmission filter. We may prefer that the introduction of the prefilter is done without the consequential increase in delay spread. Or we may want to limit the delay spread seen by the mobile to limit the complexity for the equalization task in the mobile. In those cases additional constraints in order to obtain at least partial ISI rejection, i.e., limited delay spread, can be added, leading to intermediate solutions between the previous two limiting cases. In general to have complete IUI and partial ISI rejection we add $(N_i + L - 1) - L_{\text{ISI}}$ constraints (coefficients of the prefilter-channel cascade being zero), with $1 \leq L_{\text{ISI}} \leq (N_i + L - 1)$, where L_{ISI} corresponds to the residual delay spread, i.e., residual ISI. Actually that means to trade between performance and receiver complexity. This optimization problem has to be carried out for all possible positions of the nonzero part of length L_{ISI} of the prefilter-channel cascade, and the best position should be chosen. Finally, note that as L increases the MFB increases as well. So, we shall choose the actual length of the transmission filters L according to a trade-off between performance and transmitter complexity.

3.2. Normalized transmit filters optimization for $m \leq d$

When $m \leq d$, IUI and ISI cancelation with FIR filters cannot be obtained. Also when $m \leq d$ we cannot specify a priori any constraint on the length of the transmission filters for the IUI rejection. However, solutions are possible to get at least a partial IUI rejection in those circumstances also. Unfortunately, even if the assumption (9) holds, analytical approaches are not possible in the presence of noise when $m \leq d$. Direct MFB optimization will again be analytically intractable in this case. Then we propose to consider the Signal-to-Interference Ratio (SIR) instead. The SIR for the ith user is defined as

$$\text{SIR}_i = \frac{\boldsymbol{F}_i^t \boldsymbol{R}_i \boldsymbol{F}_i^{tH}}{\sum_{j=1,\, j\neq i}^{d} \boldsymbol{F}_j^t \boldsymbol{R}_i \boldsymbol{F}_j^{tH}} \tag{20}$$

The equation (14) in the absence of noise reduces to

$$\boldsymbol{\gamma}^{-1} = \boldsymbol{D}_c^{-1} \boldsymbol{P}^{-1} \boldsymbol{C}^T \boldsymbol{p} \tag{21}$$

where now $\gamma_i = \text{SIR}_i$ for any i. Considering the criterion (12) and the definition (20) it is straightforward to see that the optimum is achieved when all the inter-user-interference (IUI) is zero so that $\gamma_i^{-1} = 0$ for all i's. Then, if $m > d$ the optimum approach in the absence of noise would lead to the ZF solution (16). Since $m \leq d$ we shall consider other non-ZF approaches. Note that since the optimum still involves $\gamma_i = \gamma$ for any i, the equation (21) reduces to

$$\gamma^{-1} \boldsymbol{p} = \boldsymbol{A}^T \boldsymbol{p} \tag{22}$$

where $\boldsymbol{A}^T = \boldsymbol{D}_c^{-1} \boldsymbol{C}^T$ is a non-negative matrix. Moreover $\boldsymbol{p}$ has to be a non-negative vector and γ^{-1} has to be non-negative as well. On the basis of the following theorems ([10],[1])

Theorem 1
For a non-negative matrix, the eigenvalue of the largest norm is positive, and its corresponding eigenvector can be chosen to be non-negative.

Theorem 2
For a non-negative matrix $\boldsymbol{A}^T$, the non-negative eigenvector corresponding to the eigenvalue of the largest norm is positive.

Theorem 3
Given the matrix $\boldsymbol{A}^T$ there exists only one solution to equation (22).

we can say that for a given set of unit norm vectors $\{\boldsymbol{U}_i^t\}$ then the optimum yields $\gamma^{-1} = \lambda_{\max}(\boldsymbol{A}^T)$ and $\boldsymbol{p} = V_{\max}(\boldsymbol{A}^T)$.

Having an estimate of $\boldsymbol{p}$, we can optimize $\{\boldsymbol{U}_i^t\}$. Indeed the optimization criterion is given by

$$\min_{\{\boldsymbol{U}_i^t\}} \lambda_{\max}(\boldsymbol{A}^T) \tag{23}$$

In order to simplify the problem formulation without loss of generality, we consider $\boldsymbol{U}_i^t$'s normalized such that $\boldsymbol{U}_i^t \boldsymbol{R}_i \boldsymbol{U}_i^{tH} = 1$, so that $\boldsymbol{D}_c = \mathbf{I}_d$ and $\boldsymbol{A}^T = \boldsymbol{C}^T$. Then the criterion (23) becomes

$$\min_{\{\boldsymbol{U}_i^t\}} \boldsymbol{q}^T \boldsymbol{A}^T \boldsymbol{p} \quad \text{s.t.} \quad \boldsymbol{U}_i^t \boldsymbol{R}_i \boldsymbol{U}_i^{tH} = 1 \tag{24}$$

where $\boldsymbol{q} = V_{\max}(\boldsymbol{A})$. The criterion (24) leads to a set of d decoupled problems whose solution is given by $\boldsymbol{U}_i^{tH} = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H \boldsymbol{R}_i \boldsymbol{e}_i}}$, where $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_i, \sum_{j\neq i}^{d} q_j \boldsymbol{R}_j)$ for any i. The new set of vectors $\{\boldsymbol{U}_i^t\}$ can be used to re-optimize the powers $\boldsymbol{p}$ according to (22).

3.2.1. $\|\boldsymbol{A}^T\|_1$ *minimization based solution*

As sub-optimal approach or initialization we can use the following criterion

$$\min_{\{\boldsymbol{U}_i^t\}} \|\boldsymbol{A}^T\|_1 \quad \text{s.t.} \quad \boldsymbol{U}_i^t \boldsymbol{R}_i \boldsymbol{U}_i^{tH} = 1 \tag{25}$$

This approach has the advantage of optimizing the direction vectors $\{\boldsymbol{U}_i^t\}$ independently from the powers $\boldsymbol{p}$. In that sense it is suitable to initialize an iterative procedure to find the global optimum. Indeed it leads to a set of d decoupled minimization problems whose solution is given by $\boldsymbol{U}_i^{tH} = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H \boldsymbol{R}_i \boldsymbol{e}_i}}$, where, in this case, $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_i, \sum_{j=1}^{d} \boldsymbol{R}_j)$ for any i.
Note that the criterion (25) corresponds to minimizing the power delivered to the undesired users while maximizing the power delivered to the desired user, by each each filter $\boldsymbol{F}_i^t$.
A similar criterion was already proposed in [4, 5] to optimize the weight vectors for transmit beamforming in a non-SDMA context.

3.2.2. $\lambda_{\max}(\boldsymbol{A}^T)$ *minimization based algorithm*

According to the previous arguments, we propose the iterative procedure summarized in table 3.2.2 to find the global optimum in the absence of noise.

Table 1: $\lambda_{\max}(\boldsymbol{A}^T)$ minimization based algorithm

(i)	Initialize $\boldsymbol{U}_i^t$ using (25) for $i = 1, \ldots, d$;
(ii)	Compute $\boldsymbol{q} = V_{\max}(\boldsymbol{A})$;
(iii)	Compute $\boldsymbol{e}_i = V_{\max}(\boldsymbol{R}_i, \sum_{j\neq i} q_j \boldsymbol{R}_j)$;
(iv)	Compute $\boldsymbol{U}_i^{tH} = \frac{\boldsymbol{e}_i}{\sqrt{\boldsymbol{e}_i^H \boldsymbol{R}_i \boldsymbol{e}_i}}$;
(v)	Go back to (ii) until convergence;
(vi)	Compute $\boldsymbol{p} = V_{\max}(\boldsymbol{A}^T)$;
(vi)	Compute $\boldsymbol{F}_i^t = \sqrt{p_i} \boldsymbol{U}_i^t$.

3.3. ZF conditions for ISI rejection

The problem of the ISI rejection remains the same as in the case of the ZF solution for $m > d$, i.e., we can add constraints in order to limit the whole delay spread as it was previously explained. In general, to reject ISI at the ith mobile receiver we need to design $\boldsymbol{U}_i^t$ such that

$$L_i \geq \underline{L}_i = \left\lceil \frac{N_i - 2}{m - 1} \right\rceil$$

so that perfect ISI rejection (pre-equalization) for the ith user is achieved when

$$\boldsymbol{U}_i^t \mathcal{T}_{L_i}(\boldsymbol{H}_i) = [0 \ldots 0 \, \alpha \, 0 \ldots 0] \tag{26}$$

where α has to be fixed in order to satisfy the constraint on the norm of $\boldsymbol{U}_i^t$. That set of constraints can be added to (24), (25). For a partial ISI rejection the same arguments in section 3.1.2 hold.

However we shall chose L_i large enough to allow also a partial IUI rejection at least. In case we want to design all the transmission filters with the same length L for all the users, to obtain perfect ISI rejection we need

$$L \geq \left\lceil \frac{N_{\max} - 2}{m - 1} \right\rceil$$

where $N_{\max} = \max_i\{N_i\}$. Once again, the actual length of the transmission filter L has to be chosen according to a trade-off between performance and transmitter complexity.

3.4. Minimum-mean-square-error (MMSE) solution

The MMSE criterion for the ith user is given by

$$\min_{\{\boldsymbol{F}_j\}} \max_i \mathrm{E}\|y_i(k) - a_i(k-n)\|_2^2 \tag{27}$$

where n is a properly chosen delay to minimize the MMSE and

$$y_i(k) = \sum_{j=1}^{d} \boldsymbol{F}_j^t \mathcal{T}_L(\boldsymbol{H}_i) A_{j,\,N_i+L-1}(k) + v_i(k)$$

Then the criterion (27) can be written as

$$\begin{aligned} \min_{p_j,\, \|\boldsymbol{U}_j\|_2 = 1} \max_i \{ & \mathrm{E}\|p_i \boldsymbol{U}_i^t \mathcal{T}_L(\boldsymbol{H}_i) A_{i,\,N_i+L-1}(k) - a_i(k-n)\|_2^2 \\ & + \sigma_a^2 \textstyle\sum_{j \neq i} p_j \boldsymbol{U}_j^t \mathcal{T}_L(\boldsymbol{H}_i) \mathcal{T}_L^H(\boldsymbol{H}_i) \boldsymbol{U}_j^{tH} + \sigma_{v_i}^2 \} \end{aligned} \tag{28}$$

where the first term corresponds to the ISI and the second one to the IUI. Hence it is straightforward to see that the MMSE corresponds to ZF on ISI and IUI when ZF conditions (18) can be applied. Otherwise, MMSE leads to a set of coupled problems which in general cannot be solved analytically.

Finally we point out that MMSE problem has been formulated for purely spatial processing in [2] but no solution has been provided in that paper.

3.5. Power assignment optimization

Assuming a given set $\{\boldsymbol{U}_i\}$, since the optimum involves all the γ_i's to be the same, the expression (14) can be arranged in order to include the constraint on the transmitted power as follows

$$\boldsymbol{Q}\tilde{\boldsymbol{p}} = \gamma^{-1} \boldsymbol{S} \tilde{\boldsymbol{p}} \tag{29}$$

where $\tilde{\boldsymbol{p}} = [\boldsymbol{p}^T \; 1]^T$,

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{A}^T & \boldsymbol{\mu} \\ \boldsymbol{0}_{1 \times d} & 0 \end{bmatrix} \qquad \boldsymbol{S} = \begin{bmatrix} \mathbf{I}_d & \boldsymbol{0}_{d \times 1} \\ \boldsymbol{g}^T & -p_{\max} \end{bmatrix}$$

where $\boldsymbol{\mu} = \boldsymbol{D}_c^{-1} \boldsymbol{\nu}$ and $\boldsymbol{g}^T \boldsymbol{p} = p_{\max}$. Then similarly to [1] since $\boldsymbol{S}$ is invertible we have

$$\boldsymbol{E}\tilde{\boldsymbol{p}} = \gamma^{-1} \tilde{\boldsymbol{p}}, \qquad \boldsymbol{E} = \boldsymbol{S}^{-1}\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{A}^T & \boldsymbol{\mu} \\ \frac{\boldsymbol{g}^T \boldsymbol{A}^T}{p_{\max}} & \frac{\boldsymbol{g}^T \boldsymbol{\mu}}{p_{\max}} \end{bmatrix} \tag{30}$$

which is a non-negative matrix. Relying on theorems 1–3 we can say that $\gamma^{-1} = \lambda_{\max}(\boldsymbol{E}^T)$ and $\tilde{\boldsymbol{p}} = V_{\max}(\boldsymbol{E})$. Further, note that we can always re-scale $\tilde{\boldsymbol{p}}$ in order to make its last element equal to one.

3.6. Implementation issues

The presence of the noise makes the optimization of the filters $\{\boldsymbol{U}_i^t\}$ involve a set of coupled problems that does not allow any analytical approach to find a solution. Therefore, we suggest to compute the vectors $\{\boldsymbol{U}_i^t\}$ applying ZF conditions (16) or MMSE criterion (27) when $m > d$, or using the algorithm described in table 3.2.2 when $m \leq d$. Then, given $\{\boldsymbol{U}_i^t\}$ optimize the power assignment according to the criterion (30).

When the noise is present, since the base station cannot estimate the noise variance $\sigma_{v_i}^2$ at each receiver, unless such an estimate is provided by the mobile, the vector $\boldsymbol{\nu}$ cannot be estimated. To remedy this drawback we shall properly define the SNR at the receiver. A possible definition is given by

$$\mathrm{SNR}_i = \frac{p_i}{\nu_i} \lambda_{\max}(\boldsymbol{R}_i)$$

for any i. In practice we need

$$\min_i \{\mathrm{SNR}_i\} \geq \mathrm{SNR}_{\min} \tag{31}$$

where $\mathrm{SNR}_{\min}$ is a value necessary for the mobile receiver to work with an outage probability below a specified maximum. Assuming all the users using the same receiver the worst case for the ith user occurs when $p_i = p_{\max}$ while $\nu_i = \nu_{\max} = \|\boldsymbol{\nu}\|_\infty$. Therefore a sufficient condition to satisfy the requirement (31) is given by setting

$$\mathrm{SNR}_{\min} = \frac{p_{\max}}{\nu_{\max}} \min_i \{\lambda_{\max}(\boldsymbol{R}_i)\} \tag{32}$$

Given $\mathrm{SNR}_{\min}$ and $p_{\max}$, $\nu_{\max}$ can be derived. Then setting $\nu_i = \nu_{\max}$ for all the i's the condition (31) is satisfied. Finally, note that for $p_{\max} \to \infty$ the optimum solution is the one in the absence of noise, for any $\nu_{\max} > 0$.

4. SIMULATIONS

The following simulation is provided to illustrate a practical implementation of the proposed solutions. Here we consider an SDMA scenario in the presence of three co-channel users ($d = 3$) which receive signals transmitted from a base station. The channels $\boldsymbol{H}_i$ (for $i = 1, \ldots d$) are assumed known (or estimated from the uplink) and they are characterized by four paths for the first and the second user and six paths for the third user respectively, resulting in $N_1 = N_2 = 9$ and $N_3 = 11$ symbol periods.

For the first set of simulations the base station is equipped with a four elements antenna array and it is assumed performing oversampling by a factor two, so that $m = 8$. In that case, since $m > d$ ZF conditions (18) can be applied. By setting the length of all the transmit filters equal to $L = \underline{L} = 6$ symbol periods we obtain the performances plotted in figure 3, in terms of SINR at each receiver versus the minimum SNR.

For the second set of simulations the base station is equipped with a two elements antenna array and it does not perform oversampling, so that $m = 2$. Then, since $m < d$, ZF conditions (18) cannot be applied and the algorithm described in Table 3.2.2 is used instead. Figure 4 shows the performances in terms of SINR at each receiver versus the minimum SNR, in the presence of the same user scenario as above, having chosen the length of the transmitted filters equal to $L = 14$ symbol periods for all the users.

Note that increasing L better performances can be obtained in both cases of ZF and non-ZF solutions, as it is shown in figures 5 and 6, where we set $L = 10$ and $L = 20$ symbol periods respectively.

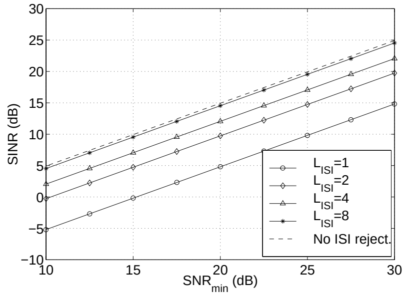

Figure 3: Optimum SINR vs. SNR$_{\min}$, ZF solution for different values of $L_{\rm ISI}$ and $L = 6$

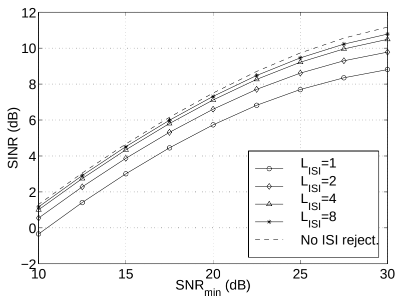

Figure 4: Optimum SINR vs. SNR$_{\min}$, non-ZF solution for different values of $L_{\rm ISI}$ and $L = 14$

5. CONCLUSIONS

We addressed the problem of the optimization of the MFB with respect to the transmit filters at a base station performing spatio-temporal processing. A general problem formulation yielded the proper cost function to be minimized. We showed that the ZF solution allows independent optimization of the transmit filters $\boldsymbol{F}_i$, for $i = 1, \ldots, d$, and under certain assumptions it is optimal for the MFB maximization. We showed that MMSE leads to the same solution as ZF conditions applied to both ISI and IUI, in those cases where ZF (conditions (18) can be achieved. We proposed also a non-ZF solution for those cases where ZF conditions (18)) cannot be applied. We showed that when the number of users is greater than or equal to the number of the channels available at the base station only a partial IUI rejection can be achieved in general. An algorithm was derived for such cases to optimize the MFB. Simulations have shown the performances of both ZF and non-ZF solutions for different values of the transmit filter length and different introduced delay spreads due to the prefilter-channel cascade.

The criteria and the algorithms proposed here can also be applied to perform only purely spatial processing. Although we shall observe that purely spatial processing, i.e., beamforming, is intrinsically sub-optimal since it requires a larger number of array antennas to perform like spatio-temporal processing. For instance, to achieve the ZF solution in the purely spatial case we need $m > d$ and $m \geq N$. Such conditions are hard to satisfy in most applications (e.g., in the previous simulated scenario with purely spatial processing, the ZF solution requires $m \geq 29$).

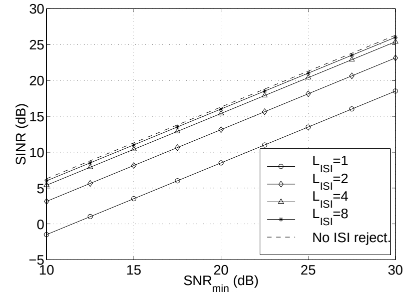

Figure 5: Optimum SINR vs. SNR$_{\min}$, ZF solution for different values of $L_{\rm ISI}$ and $L = 10$

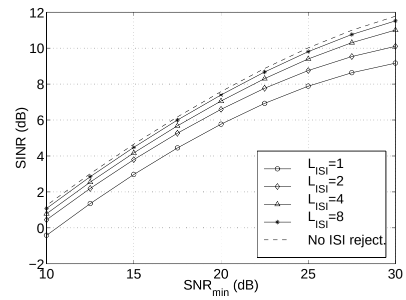

Figure 6: Optimum SINR vs. SNR$_{\min}$, non-ZF solution for different values of $L_{\rm ISI}$ and $L = 20$

6. REFERENCES

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