Generalized Degrees-of-Freedom of the 2-User Case MISO Broadcast Channel with Distributed CSIT

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***Abstract*—This work[1] analyzes the Generalized Degrees-of-Freedom (GDoF) of the 2-User Multiple-Input Single-Output (MISO) Broadcast Channel (BC) in the so-called Distributed CSIT regime, with application to decentralized wireless networks. This regime differs from the classical limited CSIT one in that the CSIT is not just noisy but also imperfectly shared across the transmitters (TXs). Hence, each TX precodes data on the basis of local CSIT and statistical quality information at other TXs. We derive the GDoF result and obtain the surprising outcome that by specific accounting of the pathloss information, it becomes possible for the decentralized precoded network to reach the same performance as a genie-aided centralized network where the central node has obtained the estimates of both TXs. The key idea allowing this surprising robustness is to let the TXs have asymmetrical roles such that the most informed TX is able to balance the lower CSIT quality at the other TX.**

I. Introduction

Simultaneous transmission between multiple-antennas TXs towards different receivers (RXs) has been widely studied, typically assuming a *Centralized CSIT* setting, where only one channel estimate –possibly noisy– is used for calculating the precoding coefficients [1], [2]. This can also model a joint transmission from different non-colocated TXs in the case where the CSIT is *perfectly shared* among the TXs over a so-called ideal Cloud Radio Access Network (C-RAN) [3].

However, future wireless network topologies will also include heterogeneous scenarios, with a variety of devices, such as user terminals, drone-enabled relays, pico base stations, etc., seeking to cooperate for transmission despite the lack of an ideal backhaul linking them. Other scenarios featuring existing backhaul links may favor local processing over centralized one in order to meet the tight latency constraints derived from 5G and tactile internet applications [4]. In these cases, a full CSI sharing across TXs is not always desired, and there is a need for robust processing on the basis of locally available CSI.

Recently, caching has become increasingly popular due to the low cost of large memory and the shortage of backhaul links [5]. Consequently, it is more and more possible to have a large amount of user's data pre-cached before the transmission at the different transmitter. This provides a strong motivation for distributed processing and distributed transmission. In particular, it is one of the key motivations for the settings considered in this work where the user's data symbols are available at all the TXs, but the CSI is only imperfectly shared due to the imperfections and the delay in the CSI sharing between the TXs.

In this paper, we formalize this scenario under the *Distributed CSIT* label, which refers to each TX being endowed with its own version of the multi-user channel state matrix, with possibly different qualities.

It has been shown in [6] that for the 2-user MISO BC the Distributed CSIT setting achieves the Degrees-of-Freedom (DoF) of the Centralized CSIT setting. The optimal DoF is reached due to a new asymmetrical precoding scheme, so-called Active-Passive Zero-Forcing (AP-ZF), where the most informed TX is able to resolve the error created by the less informed one.

Nevertheless, the DoF is a limited figure of merit, since it does not take into account the differences between channel strengths. In order to study the impact of the network topology, the Generalized DoF (GDoF) concept was introduced in [7]. GDoF approach offers an intermediate step towards finite and constant gap analysis [8], modeling the pathlosses through a dependence in P [9]. In [10] the GDoF for K-user Symmetric MISO BC with Centralized CSIT has been characterized, and it has been shown that for the 2-user case the GDoF only depends on the worst CSIT accuracy towards each RX.

In this work, our key contribution is to provide the GDoF performance of the 2-user MISO BC under Distributed CSIT, for the case where one TX has better CSI quality for all the links. We show that, accounting for pathloss difference in the multi-user channels, the cooperative decentralized network can reach the same performance as a genie-aided centralized network where the best CSI estimate is shared. We propose a scheme achieving the GDoF, which is based on the idea that each TX should precode data according to the CSI that it sees.

Due to space constraint, some of the proofs are sketched, while full proofs can be found in the extended version [11].

Notations: $\doteq$ denotes the *exponential equality*, i.e., $f(P) \doteq P^{\beta}$ denotes $\lim_{P\to\infty} \frac{\log(f(P))}{\log(P)} = \beta$. The *exponential inequalities* $\dot{\leq}$ and $\dot{\geq}$ are defined in the same manner. Being x a number, it holds that $(x)^+ \triangleq \max(x, 0)$. For $i, \bar{i} \in \{1, 2\}$, we define $\bar{i} \triangleq i \pmod 2 + 1$.

II. System Model

A. 2-User MISO BC Transmission Model

This work considers a communication system where 2 single-antenna TXs jointly serve 2 single-antenna RXs over

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a MISO BC. We assume that the RXs have perfect, instantaneous CSI. The signal received at RX i is written as

$$y_i = \boldsymbol{h}_i^{\mathrm{H}}\mathbf{x} + z_i, \tag{1}$$

where $\boldsymbol{h}_i^{\mathrm{H}} \in \mathbb{C}^{1\times 2}$ is the channel to user i and $z_i \in \mathbb{C}$ is the additive Gaussian noise at RX i, distributed in an i.i.d. manner as $\mathcal{N}_{\mathbb{C}}(0,1)$. $\mathbf{x} \in \mathbb{C}^{2\times 1}$ is the transmitted multi-user signal, which is generated from the information symbols s_i that are distributed in an i.i.d. manner as $\mathcal{N}_{\mathbb{C}}(0,1)$. $\mathbf{x}$ fulfills the constraint $\mathrm{E}[\|\mathbf{x}\|^2] = P$, where P is the nominal SNR. The channel is assumed to be drawn from a continuous ergodic distribution such that the channel matrix $\mathbf{H} \triangleq [\boldsymbol{h}_1, \boldsymbol{h}_2]^{\mathrm{H}}$ and all their sub-matrices are almost surely full rank [12].

In the GDoF framework, the channel strength is modeled as a function of the nominal SNR P, where $\gamma_{i,k} \in [0,1]$ is the *strength exponent* that represents the channel power between TX k and RX i, and therefore it holds that

$$\mathrm{E}[|\mathbf{H}_{i,k}|^2] \doteq P^{\gamma_{i,k}-1}, \qquad \forall i,k \in \{1,2\}. \tag{2}$$

Remark 1. *For $\gamma_{i,k} = 1, \forall i,k \in \{1,2\}$, we recover the conventional DoF setting, while choosing $\gamma_{i,i} = 1$, $\gamma_{i,k} = 0$, $\forall i,k \in \{1,2\}$, $k \neq i$ we recover the results of the non-interfering IC.* □

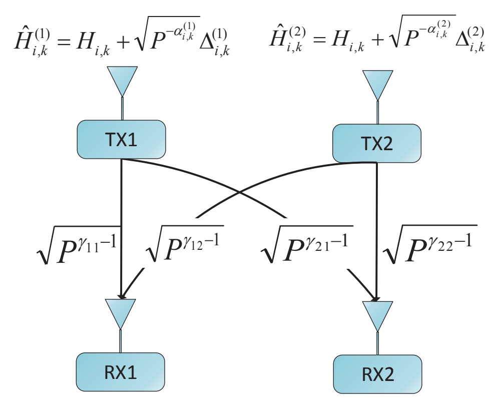

Fig. 1: Illustration of the 2-user MISO Broadcast Channel scenario with Distributed CSIT configuration.

The GDoF approach allows to model more accurately practical transmissions by taking into account the pathloss differences, while the DoF model neglects all pathloss differences. Furthermore, it is also an intermediate step to obtain so-called "finite gap results" [8], [9], bounding the losses with respect to the optimal performance *at any SNR*.

B. Distributed CSIT Model

In that Distributed CSIT setting [13], each TX receives a different estimate of the channel, with possibly different accuracies. The CSI uncertainty at the TX j is modeled as

$$\hat{\mathbf{H}}_{i,k}^{(j)} \triangleq \mathbf{H}_{i,k} + \sqrt{P^{-\alpha_{i,k}^{(j)}}}\boldsymbol{\Delta}_{i,k}^{(j)}, \quad \forall i,j,k \in \{1,2\}, \tag{3}$$

where $\hat{\mathbf{H}}_{i,k}^{(j)}$ is the estimation of the channel from TX k to RX i available at TX j, and the noise terms $\boldsymbol{\Delta}_{i,k}^{(j)}$ are independent random variables with zero mean and bounded covariance matrix satisfying $\left|\boldsymbol{\Delta}_{i,k}^{(j)}\right| \doteq \sqrt{P^{\gamma_{i,k}-1}}$, $\forall i,k \in \{1,2\}$.

The *CSIT quality exponent* at TX j is denoted as $\alpha_{i,k}^{(j)}$ and it is used to parametrize the accuracy of the local CSIT. Note that from a GDoF perspective $\alpha_{i,k}^{(j)} \in [0, \gamma_{i,k}]$ [8]. We assume that TX 1 is the most informed TX throughout the work, i.e.,

$$1 \geq \alpha_{i,k}^{(1)} \geq \alpha_{i,k}^{(2)} \geq 0, \qquad \forall i,k \in \{1,2\}. \tag{4}$$

The more-informed TX assumption is key to the optimality of AP-ZF. Extending the results to the arbitrary CSIT regime is an interesting research topic currently under investigation.

In addition, we extend the bounded density assumption from [12], and therefore we assume that the conditional probability density functions verify that

$$p_{\mathbf{H}_{i,k}|\hat{\mathbf{H}}_{i,k}^{(1)},\hat{\mathbf{H}}_{i,k}^{(2)}} = O\left(\sqrt{P^{\max_{j\in\{1,2\}}\alpha_{i,k}^{(j)}}}\right). \tag{5}$$

This is a technical condition usually satisfied and it is discussed more in detail in the extended version [11].

C. Generalized Degrees-of-Freedom Analysis

The optimal sum GDoF in the MISO BC scenario with imperfect current CSIT is defined as [7]

$$\mathrm{GDoF}^{\star}(\{\alpha_{i,k}^{(j)}\}) \triangleq \lim_{P\to\infty} \frac{\mathcal{C}(P, \{\alpha_{i,k}^{(j)}\}, \{\gamma_{i,k}\})}{\log_2(P)}, \tag{6}$$

where $\mathcal{C}(P, \{\alpha_{i,k}^{(j)}\}, \{\gamma_{i,k}\})$ denotes the sum capacity [14] of the MISO BC studied, and we have omitted the $\mathrm{GDoF}^{\star}(\{\alpha_{i,k}^{(j)}\})$ dependence on $\{\gamma_{i,k}\}$ for ease of notation.

III. Preliminary: Results of the Centralized CSIT Case

We now focus on the Centralized CSIT configuration, which serves as the reference case to discuss the impact of the CSIT inconsistencies between TXs. In this centralized setting, all the transmitting antennas share the same, potentially imperfect, channel estimate. Hence, there is a single channel estimate $\hat{\mathbf{H}}$, formed as in (3), but without the need for a TX index j.

The GDoF of the 2-user MISO BC with Centralized CSIT setting has been derived in [10] and we state it in the following.

Theorem 1. *[10] In the 2-user MISO BC with Centralized CSIT, the optimal sum GDoF satisfies*

$$\mathrm{GDoF}^{CCSIT}(\{\alpha_{i,k}\}) = \min(D_1, D_2), \tag{7}$$

where

$$D_1 \triangleq \max(\gamma_{1,2},\gamma_{1,1}) + \left(\max(\gamma_{2,1}-\gamma_{1,1}+\alpha_1,\, \gamma_{2,2}-\gamma_{1,2}+\alpha_1)\right)^+$$
$$D_2 \triangleq \max(\gamma_{2,2},\gamma_{2,1}) + \left(\max(\gamma_{1,1}-\gamma_{2,1}+\alpha_2,\, \gamma_{1,2}-\gamma_{2,2}+\alpha_2)\right)^+$$

and where we have introduced the short-hand notations

$$\alpha_1 \triangleq \min(\alpha_{1,1}, \alpha_{1,2}), \tag{8}$$
$$\alpha_2 \triangleq \min(\alpha_{2,1}, \alpha_{2,2}). \tag{9}$$

This optimal sum GDoF is achieved by superposition coding and ZF precoding [1] [15]. Interestingly, the GDoF performance is only dependent on the weakest CSIT parameter for each RX. Moreover, the pathlosses can be either advantageous or detrimental depending on the network geometry.

IV. Main Results

This work is focused on how cooperation over imperfect and uneven CSIT settings reduces the performance of the transmission. Our main result is stated in the following theorem.

Theorem 2. *In the 2-user MISO BC with Distributed CSIT, the optimal sum* GDoF *is given by*

$$\mathrm{GDoF}^{DCSIT}(\{\alpha_{i,k}^{(j)}\})=\mathrm{GDoF}^{CCSIT}(\{\max_{j\in\{1,2\}}\alpha_{i,k}^{(j)}\}_{i,k\in\{1,2\}}).$$

In the Distributed CSIT configuration, letting the TXs share perfectly their own estimates leads to a Centralized CSIT configuration with the CSI $\mathcal{H} \triangleq \{\hat{\mathbf{H}}^{(1)},\hat{\mathbf{H}}^{(2)}\}$, such that the optimal GDoF of that genie-aided setting is given by Theorem 1. This provides the upperbound while the lower bound is derived in Section V for a particular case and the general proof is given in the extended version [11].

This theorems leads to several interesting insights. For example, assuming that we start from the centralized, perfectly-shared CSIT case, if we *completely* remove the CSI information from one of the TXs we still achieve the same GDoF for *any* channel strength topology. From the opposite point of view, it is necessary to send CSI information to only one of the two TXs, therefore reducing the transmission overhead.

Assuming $\gamma_{i,k}=1$, $\alpha_{i,k}^{(j)}=\alpha^{(j)}$, $\forall i,k\in\{1,2\}$, we recover the DoF results of [12] and hence

$$\mathrm{DoF}^{DCSIT}(\{\alpha^{(j)}\})=1+\max_{j\in\{1,2\}}\alpha^{(j)}.$$

The main interpretation of this theorem is that we need a single free variable to cancel the interference as we only need to cancel it at one non-intended RX. That means that we can fix the precoder of the TX with less-accurate CSIT and use the other TX to remove the interference.

We present in Fig. 2 some simulation results illustrating our main results, considering a simplified topology where

$$\gamma_{i,1}=0.8,\quad \forall i\in\{1,2\}, \tag{10}$$
$$\gamma_{i,2}=1,\quad \forall i\in\{1,2\}. \tag{11}$$

We further consider that TX 1 has the CSIT quality $\alpha_{i,k}^{(1)}=0.6$, $\forall i,k\in\{1,2\}$ while TX 2 has $\alpha_{i,k}^{(2)}=0$, $\forall i,k\in\{1,2\}$, i.e., no CSIT in terms of GDoF. The AP-ZF scheme has been simulated and compared with two different schemes. The first one is the Centralized CSIT setting of Section III, where both TXs share perfectly their estimations. The second one is the naive Zero-Forcing, where each TX implicitly assumes that the other TX has the same channel estimate [13]. The GDoF is equal to the slope at high SNR of the sum-rate function over the SNR. It can be seen that AP-ZF achieves the same GDoF as the Centralized CSIT case. The gap between the theoretic GDoF upper bound and the simulations comes from the fact that the GDoF metric does not take into account the finite gaps, since they do not increase as function of P (see (6)). The naive ZF is limited by the less accurate estimate, $\alpha^{(2)}=0$, and thus matches the performance of the setting with no CSIT [13].

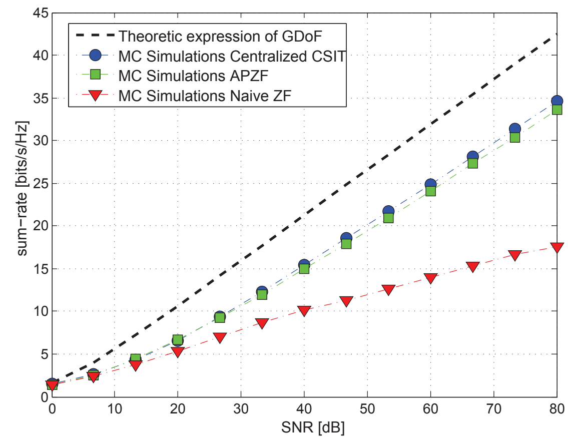

Fig. 2: Sum rate in terms of the SNR for the *Parallel Configuration* with $\alpha^{(1)}=0.5$, $\alpha^{(2)}=0$ and $\gamma=0.8$.

V. Proof of Theorem 2

In the following, we describe the AP-ZF precoder first introduced in [6], as well as the achievable scheme and the power consumption for a particular configuration, so-called *Parallel Configuration*. It is important to note that all the lemmas, corollaries and results hold in the general case and are available in the extended version [11], while we focus on the *Parallel Configuration* so as to easily convey the main intuition without heavy notations, and as the extension to the general case follows easily.

A. AP-ZF Precoder for the 2-user Setting

This precoder has an asymmetrical structure and it is built on the Distributed CSIT basis and the assumption of having an uneven topology where one TX is more accurately informed than the other. In a nutshell, the less informed TX does not use his own CSIT for precoding, which allows the most informed TX to know the interference generated and use that knowledge to cancel the interference.

Let RX i be the intended RX and RX $\bar{i}$ be the interfered RX. As TX 1 is the most informed TX, the AP-ZF beamformer $\boldsymbol{t}_i^{\mathrm{APZF}}=[t_i^{(1)}\ t_i^{(2)}]^{\mathrm{T}}$ is given by

$$t_i^{(2)}\triangleq c_P, \tag{12}$$
$$t_i^{(1)}\triangleq -\hat{h}_{\bar{i},1}^{(1)}\left(\left|\hat{h}_{\bar{i},1}^{(1)}\right|^2+\frac{1}{P}\right)^{-1}\hat{h}_{\bar{i},2}^{(1)\mathrm{H}}c_P, \tag{13}$$

where c_P is a constant that can be made dependent on P. By construction, this precoder satisfies that

$$\begin{aligned}\hat{\boldsymbol{h}}_{\bar{i}}^{(1)}\boldsymbol{t}_i^{\mathrm{APZF}} &= \hat{h}_{\bar{i},1}^{(1)\mathrm{H}}t_i^{(1)}+\hat{h}_{\bar{i},2}^{(1)\mathrm{H}}t_i^{(2)}\\ &= -\left|\hat{h}_{\bar{i},1}^{(1)}\right|^2\left(\left|\hat{h}_{\bar{i},1}^{(1)}\right|^2+\frac{1}{P}\right)^{-1}\hat{h}_{\bar{i},2}^{(1)\mathrm{H}}c_P+\hat{h}_{\bar{i},2}^{(1)\mathrm{H}}c_P\\ &\doteq 0.\end{aligned}$$

As it can be seen in (3), the estimation $\hat{\boldsymbol{h}}^{(1)}_{\bar{i},k}$ has an SNR that scales in $\sqrt{P^{\alpha^{(1)}_{\bar{i},k}}}$ with respect to the estimation error, such that the interference at RX $\bar{i}$ is attenuated by a factor $P^{-\alpha^{(1)}_{\bar{i}}}$ [6], where $\alpha^{(1)}_{\bar{i}} \triangleq \min_{k\in\{1,2\}} \alpha^{(1)}_{\bar{i},k}$, $\forall i \in \{1,2\}$, is the minimum accuracy at TX 1 of the channels towards RX $\bar{i}$.

B. Sketch of the Proof for Parallel Configuration

Since Theorem 1 shows that the GDoF only depends on the weakest CSIT parameter for each RX's channel, let us define the distributed counterparts of α_1, α_2 (see (8)-(9)) as

$$\alpha^{(j)}_i \triangleq \min_{k\in\{1,2\}} \alpha^{(j)}_{i,k}, \qquad \forall i,j \in \{1,2\}. \tag{14}$$

Therefore, $\alpha^{(j)}_1$ (resp. $\alpha^{(j)}_2$) represents the minimum CSIT accuracy of the estimate at the TX j for the channels towards RX 1 (resp. RX 2). The proposed *Parallel Configuration* is represented in Fig. 3 and denotes a particular topology where

$$\gamma_{i,i} = 1, \quad \forall i \in \{1,2\}, \tag{15}$$

$$\gamma_{i,k} = \gamma, \quad \forall i,k \in \{1,2\},\ k \neq i. \tag{16}$$

Thus, the CSIT *quality exponents* are limited by $\alpha^{(j)}_i \leq \gamma$, $\forall i,j \in \{1,2\}$, and we assume that the CSIT quality for each RX is the same, i.e., $\alpha^{(j)} = \alpha^{(j)}_i$ $\forall i \in \{1,2\}$. Hence, Theorem 2 gives

$$\mathrm{GDoF}^{DCSIT}(\{\alpha^{(j)}_{i,k}\}_{i,j,k\in\{1,2\}}) = 2 - \gamma + \alpha^{(1)}. \tag{17}$$

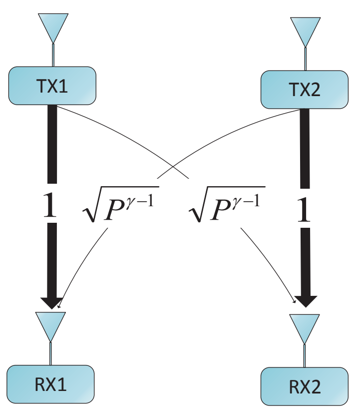

Fig. 3: Network topology for the *Parallel Configuration*.

1) Power consumption: One of the main technical contributions of this work consists in deriving the need for a proper power control at the TXs, as shown below.

Lemma 1. *In the* Parallel Configuration *of the* 2*-user MISO BC, the AP-ZF precoder* $\boldsymbol{t}^{\mathrm{APZF}}_1$ *for the symbols intended by RX* 1*, transmitted with unitary power* $||\boldsymbol{t}^{\mathrm{APZF}}_1||^2_2 \doteq 1$, *satisfies*

$$\left|t^{(1)}_1\right|^2 \doteq 1, \tag{18}$$

$$\left|t^{(2)}_1\right|^2 \doteq P^{\gamma-1}. \tag{19}$$

By symmetry, this also holds for the AP-ZF precoder $\boldsymbol{t}^{\mathrm{APZF}}_2$ *aimed to RX* 2 *by switching the TX-index.*

Proof. Let us choose the constant c_P in (12) equal to

$$\left|t^{(2)}_1\right| \doteq \sqrt{P^x}, \qquad x \in [-1, 0]. \tag{20}$$

It then holds from (13) that the coefficient designed at TX 1 satisfies

$$\left|t^{(1)}_1\right| = \left|\hat{h}^{(1)}_{2,1}\right| \left| -\left(\left|\hat{h}^{(1)}_{2,1}\right|^2 + \frac{1}{P}\right)^{-1}\right| \left|\hat{h}^{(1)\mathrm{H}}_{2,2}\right| \sqrt{P^x}. \tag{21}$$

By definition (see equation (2)), it also holds

$$|\hat{h}^{(1)}_{2,1}| \doteq \sqrt{P^{\gamma-1}}, \tag{22}$$

$$|\hat{h}^{(1)}_{2,2}| \doteq 1. \tag{23}$$

Inserting these values in (21) yields that

$$\begin{aligned}\left|t^{(1)}_1\right| &\doteq \sqrt{P^{\gamma-1}}\left|P^{\gamma-1}(1+P^{-\gamma})\right|^{-1}\sqrt{P^x} \\ &\doteq \sqrt{P^{x+(1-\gamma)}}.\end{aligned} \tag{24}$$

From (20) and (24), given that the final precoder should have a power of $\|\boldsymbol{t}^{\mathrm{APZF}}_1\|^2_2 \doteq 1$, the optimal choice for x is

$$x = \gamma - 1, \tag{25}$$

which concludes the proof. □

Remark 2. *The key factor of the achievability is the harmonization at each RX of the interference power received from each TX, since that allows us to cancel the interference through Zero-Forcing. The precoding scheme always compensates the pathloss differences transmitting with different power from each of the TXs.*

Building upon Lemma 1, the following results on the scaling of the received signals are easily obtained, such that the detailed proof is relegated to the extended version [11].

Corollary 1. *In the* Parallel Configuration *of the* 2*-user MISO BC with unit transmit power, the intended signal at RX* i, $i \in \{1,2\}$, *satisfies*

$$\left|\boldsymbol{h}^{\mathrm{H}}_i \boldsymbol{t}^{\mathrm{APZF}}_i\right|^2 \doteq 1 \tag{26}$$

and the interference power from the signal intended to the other RX $\bar{i}$, $\bar{i} \neq i$ *satisfies*

$$\left|\boldsymbol{h}^{\mathrm{H}}_i \boldsymbol{t}^{\mathrm{APZF}}_{\bar{i}}\right|^2 \dot{\leq} P^{\gamma-\alpha^{(1)}-1}. \tag{27}$$

2) Encoding: In the proposed achievable scheme, the transmitted signal is

$$\mathbf{x} = \sqrt{P - 2P^{\mathrm{APZF}}}\,\boldsymbol{t}_0 s_0 + \sqrt{P^{\mathrm{APZF}}}\left(\boldsymbol{t}^{\mathrm{APZF}}_1 s_1 + \boldsymbol{t}^{\mathrm{APZF}}_2 s_2\right) \tag{28}$$

where

- $s_0 \in \mathbb{C}$ is a common symbol of rate $(\gamma - \alpha^{(1)})\log_2(P)$ bits that is decoded at both users and $\boldsymbol{t}_0 = \frac{1}{\sqrt{2}}[1 \ \ 1]^{\mathrm{T}}$.
- $s_i \in \mathbb{C}$, with $i \in \{1,2\}$ is a symbol of rate $(1 + \alpha^{(1)} - \gamma)\log_2(P)$ bits intended to user i. $\boldsymbol{t}^{\mathrm{APZF}}_i \in \mathbb{C}^2$ is the unitary AP-ZF precoder and $P^{\mathrm{APZF}} \doteq \frac{1}{2}P^{1+\alpha^{(1)}-\gamma}$.

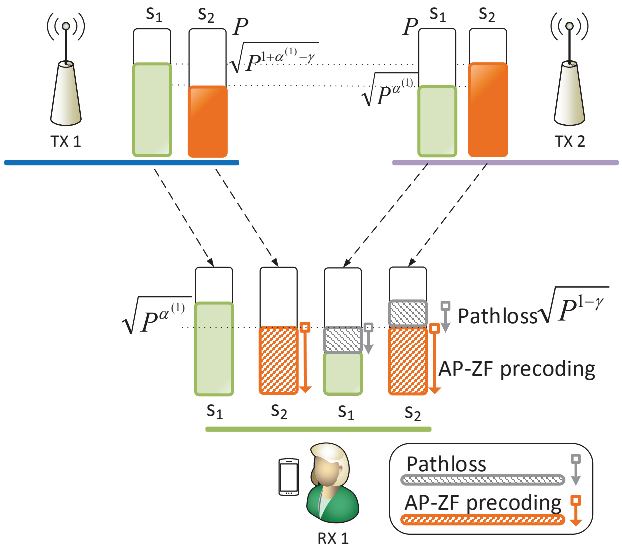

Fig. 4: Illustration of the different power scalings for the *Parallel Configuration*. Attenuation of the signal power due to pathloss and AP-ZF precoding are emphasized using arrows.

3) Received signal: The received signal at RX 1 is

$$y_1 = \underbrace{\sqrt{P-2P^{\mathrm{APZF}}}\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{t}_0 s_0}_{\doteq\sqrt{P}} + \underbrace{\sqrt{P^{\mathrm{APZF}}}\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{t}_1^{\mathrm{APZF}} s_1}_{\doteq\sqrt{P^{1+\alpha^{(1)}-\gamma}}} + \underbrace{\sqrt{P^{\mathrm{APZF}}}\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{t}_2^{\mathrm{APZF}} s_2}_{\doteq\sqrt{P^0}}. \quad (29)$$

To help the reader, we have written the power scaling of the received signals explicitly below equation (29). In Fig. 4, we have illustrated the precoding and the different power levels for the transmission towards RX 1.

The power scaling of s_1 comes from (26) in Corollary 1, since it holds that $\left|\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{t}_1^{\mathrm{APZF}}\right|^2 \doteq 1$ and $\sqrt{P^{\mathrm{APZF}}} \doteq \sqrt{P^{1+\alpha^{(1)}-\gamma}}$. Turning to interference power scaling, the contribution of the interfering symbol s_2 lies on the noise floor thanks to the precoding and power control: TX 1 reduces his transmitted power for s_2 to compensate that the channel from TX 2 towards RX 1 is weaker, as it is shown in Fig. 4, so that the interference power received at RX 1 from both TXs has the same scaling $P^{\alpha^{(1)}}$. Then, it holds from (27) in Corollary 1 that $\left|\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{t}_2^{\mathrm{APZF}}\right|^2 \doteq P^{\gamma-\alpha^{(1)}-1}$, and since $\sqrt{P^{\mathrm{APZF}}} \doteq \sqrt{P^{1+\alpha^{(1)}-\gamma}}$ the interference power scales as

$$\left|\sqrt{P^{\mathrm{APZF}}}\boldsymbol{h}_1^{\mathrm{H}}\boldsymbol{t}_2^{\mathrm{APZF}}\right| \doteq \sqrt{P^{1+\alpha^{(1)}-\gamma}}\sqrt{P^{\gamma-\alpha^{(1)}-1}} \quad (30)$$

$$= \sqrt{P^0}. \quad (31)$$

4) Decoding and GDoF analysis: From (29), the common symbol s_0 has a SNR that scales in $P^{\gamma-\alpha^{(1)}}$ when treating s_1 as noise. After decoding the common symbol and removing its contribution to the received signal, s_1 can be decoded with a SNR scaling in $P^{1+\alpha^{(1)}-\gamma}$. By symmetry, the received signal and decoding at RX 2 is studied in the same way.

Therefore, decoding the messages as above, it is possible to transmit with a GDoF of $\gamma-\alpha^{(1)}$ through the common symbol s_0, and a GDoF of $1+\alpha^{(1)}-\gamma$ through each private symbol s_1, s_2, i.e., a sum-GDoF of $2+\alpha^{(1)}-\gamma$. This corresponds to the Centralized CSIT result, which concludes the proof.

VI. Conclusion

For the 2-user MISO BC scenario with Distributed CSIT setting, with one TX being more informed than the other, we have developed an achievable scheme extended from the Active-Passive ZF scheme presented in [6], whose GDoF performance matches the genie-aided Centralized CSIT setting. These results enlighten that cooperation under distributed settings, where the CSIT is unevenly shared, may not lead to a performance reduction if we develop schemes that are responsive to these disparities, in contrast to the current schemes which lead to important losses. Extending the GDoF characterization to a K-user setting is a challenging problem, since the number of different topologies –and therefore cooperative precoding cases– increases exponentially with the number of users K and the optimal transmission strategy may be different for any of these cases.

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