Braking Strategy for an Autonomous Vehicle in a Mixed Traffic Scenario

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Abstract: During the early deployment phase of autonomous vehicles, autonomous vehicles will share roads with conventional manually driven vehicles. They will be required to adjust their driving dynamically taking into account not only preceding but also following conventional manually driven vehicles. This paper addresses the challenges of adaptive braking to avoid front-end and rear-end collisions, where an autonomous vehicle is followed by a conventional manually driven vehicle. We illustrate via simulations the consequences of independent braking in terms of collisions, on both autonomous and conventional vehicles, and propose an adaptive braking strategy for autonomous vehicles to coordinate with conventional manually driven vehicles to avoid front and rear-end collisions.

1 INTRODUCTION

Today autonomous vehicles are equipped with sensing technologies involving cameras, radars, lidars, etc. and/or communication technologies like Vehicle to Vehicle (V2V) or Vehicle to Infrastructure (V2X). Most of the work on autonomous vehicles is based on *coordinated control* decision making for intersection clearance, lane merging, etc. as found in a survey by Torres (Rios-Torres and Malikopoulos, 2016) considering ideal circumstances. Now assume less than ideal circumstances where an autonomous vehicle is alerted to a potential collision with some delay and/or coordination and negotiations with other vehicles fail (leading to potential collisions). Such a scenario creates an emergency situation (Campos et al., 2014), making it imperative to brake and to come to a halt to avoid collisions. Thus, the objective changes from *coordinated control* to *safety critical braking*.

Collision free braking becomes much more complicated when a mix of autonomous and manually driven vehicles need to come to a halt. It is more likely that an autonomous vehicle will have a manually driven vehicle as its neighbour, either in front or behind, because of the higher number of manually driven vehicles compared to autonomous vehicles. Thus the above described scenario of collision free braking is an important concern today, as depicted in Figure 1 where vehicles *A* and *B* are autonomous and manually driven respectively, and are trying to avoid collision with a potential obstacle *L* in front by braking.

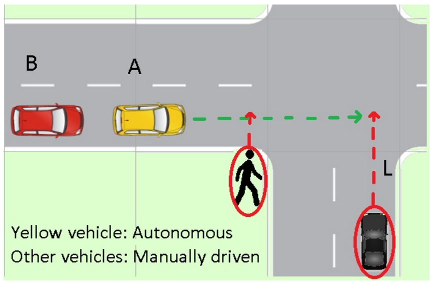

Figure 1: Mixed vehicular traffic scenario involving autonomous and manually driven vehicles.

More than one-fifth of accidents happen with a vehicle immediately behind or ahead in longitudinal direction (Kaempchen et al., 2009), primarily because *human drivers* tend to react based on the vehicle in front and prevent accidents with the vehicle in front (front-end accident avoidance). Generally speaking, the effect of the braking of a vehicle onto the following vehicle is not considered by humans leading to rear-end collisions. On the other hand, an autonomous vehicle can consider potential collisions at both ends. If the following vehicle and ego vehicle are both autonomous, a coordinated braking strategy can be devised. Consider the scenario where a conventional manually driven vehicle without any form of automation is following an autonomous vehicle (ego vehicle). The objective of this paper is to answer the following question: How can an autonomous vehicle anticipate the braking of a following conventional vehicle, modify its controls considering the (anticipated) braking of the following vehicle and guarantee both rear-end and front-end collision avoidance (with the following vehicle and with the obstacle in front respectively) in the above described scenario.

Our contributions are threefold: first we formulate a collision free adaptive (cooperative) braking strategy as a multi-parameter objective based on braking distances and dynamics of following conventional vehicles; second, we propose an adaptive 'smooth' braking strategy for autonomous vehicles and demonstrate its capability to avoid rear and front-end collisions. Finally, we vary the input parameters to illustrate their impact on our proposed coordinated braking strategy.

The rest of this paper is organized as follows: in Section 2, we formulate the coordinated braking problematic with more details and provide related work. In Section 3 we provide a detailed modelling of it, whereas in Section 4 we evaluate our proposed strategy. Finally, Section 5 concludes our work and sheds light on future work.

2 RELATED WORK

In this paper we use various braking strategies and vehicular mobility models to simulate different scenarios for front-end and rear-end collision avoidance in longitudinal motion. First subsection looks at the work related to vehicular mobility models and braking strategies whereas the latter part of this section looks at work related to collision avoidance.

2.1 Vehicular mobility models

In literature, there are lots of vehicular mobility models. Psycho-physical model by Wiedemann (Wiedemann, 1974), is one such mobility model implemented in VISSIM simulator (Fellendorf and Vortisch, 2010). It states that a manually driven vehicle is in one of the following driving modes: free driving, approaching, following or braking. An approaching vehicle would continue at the same velocity until it enters a deceleration perceptual threshold which stimulates the driver to brake. Whereas Trebier proposes Intelligent Driver Model (IDM) (Treiber et al., 2000) in which he suggests that the ego vehicle adjusts its driving dynamics according to that of the vehicle immediately in front to avoid front-end collisions. Subsequent extensions of IDM like Enhanced IDM (Kesting et al., 2010a) and IDM+ (Schakel et al., 2010) optimizes traffic capacity and flow. In such *Follow-the-Leader* models the presence of following vehicles is not considered leaving a big risk of rear-end collisions. [1]

Kesting assumes IDM or a modified version of IDM to be a good basis for implementation of Adaptive Cruise Control (ACC)/ Cooperative ACC (CACC) (Kesting et al., 2010b), thus we assume in this paper, *autonomous vehicles implement IDM*. IDM can be modelled as in equation 1.

$$\begin{aligned} a_\alpha &= a_{max}\,(1-(\frac{v_\alpha}{v_0})^\delta-(\frac{s^*(v_\alpha,\Delta v_\alpha)}{s_\alpha})^2) \\ s^*(v_\alpha,\Delta v_\alpha) &= s_0+v_\alpha\,\tau+\frac{v_\alpha\,\Delta v_\alpha}{2\sqrt{a_{max}b_{max}}} \\ s_\alpha &= x_{\alpha-1}-x_\alpha-l_{\alpha-1} \\ \Delta v_\alpha &= v_\alpha-v_{\alpha-1} \end{aligned} \tag{1}$$

Where α is the vehicle being considered, $\alpha-1$ is the vehicle in front and so on. $a_\alpha, v_\alpha, x_\alpha$ represents acceleration, velocity and the location of vehicle α. a_{max}, b_{max} are maximum acceleration and braking values of the vehicle. s_α, Δv_α, τ represent distance, velocity difference and desired time gap with the vehicle in front. δ is the free acceleration exponent. s_0 is the desired safety distance between two vehicles and v_0 is the desired velocity of vehicle in free traffic. $l_{\alpha-1}$ is the length of the vehicle $\alpha-1$.

On the other hand, *human drivers in manually driven vehicles* are assumed to show realistic characteristics like having a reaction to a situation after some perception response time t_{prt}. In other words, t_{prt} is the measure of attentiveness and responsiveness of a driver. When travelling at high speeds, and noticing the vehicle in front close and braking, humans would tend to immediately hit the brakes. We assume, as this situation is a sudden surprise, the magnitude of applied brakes is maximum. In this paper, we set the value of t_{prt} to 1.3 s, which is the mean value of human perception response time (National Highway Traffic Safety Administration, 2009). To summarize, manually driven vehicles are assumed to brake at maximum braking strength, 1.3 s after the vehicle in front starts braking until they come to a halt. In future, autonomous vehicles with sensors could *learn* about the t_{prt} of the driver in vehicle behind based on the observed driving behaviour.

[1] Through simulations we show a rear-end collision of an autonomous vehicle with a manually following vehicle. Refer to Figure 11, explained in Subsection 4.2.

2.2 Collision avoidance strategies

Most of the work till date has been on collision avoidance between ego vehicle and vehicle in front and comparatively little on the influence of actions of ego vehicle onto following vehicle.

To avoid *front-end* collisions, for manually driven vehicles, traditionally proposed solution is to have larger inter vehicular distances, (Ashley, 2013) states the recommended headway in Germany is 1.8 s. For vehicles with V2V communication capacities, (Liu and Ozguner, 2003) suggests increasing communication range to warn about a potential collision over a larger range. Where as for autonomous vehicles, (Llorca et al., 2011; Durali et al., 2006) propose front-end collision avoidance based on steering rather than braking. Brandt proposes an innovative elastic band theory based approach involving non linear algebraic equations for collision avoidance systems (Brandt et al., 2005). Intent prediction based front-end collision avoidance is proposed by Hamlet in (Hamlet et al., 2015). An approach for collision avoidance during automated lane changing is presented by (Jula et al., 2000). Lu proposes a centralized coordinated braking strategy for ACC vehicles using Model Predictive Control (Lu et al., 2014).

On the other end, to avoid *rear-end collisions*, either the following vehicle should be informed as to when by latest it should start braking as suggested by Zhang (Zhang et al., 2006) or leading vehicle should be informed the latest moment by when it must accelerate as suggested by Cabrera (Cabrera et al., 2012).

All the cited work assumes homogeneous traffic with vehicles having the same level of automation. Little attention has been given to rear-end collision avoidance as evident from above. Most of the accomplished work, requires V2V communication to inform neighbouring vehicles about control strategies. What happens when the following vehicle doesn't have neither any V2V communication nor sensing technology? Are collisions inevitable?

3 MODELLING ADAPTIVE BRAKING STRATEGY

Without loss of generality, we simplify the scenario described in Figure 1, consisting of a potential obstacle L, an autonomous vehicle A and a manually driven vehicle B following A, to a 1D representation in Figure 2. d_e, d_{la} represents the distance covered during an emergency brake at maximum braking strength by vehicle A and the distance at which vehicle A becomes aware of the potential danger by object L over V2X

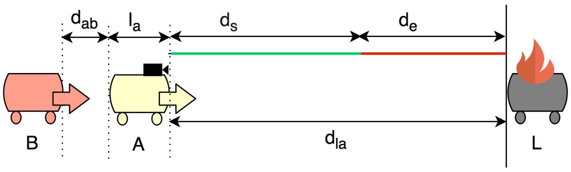

Figure 2: Simplified 1D scenario where autonomous vehicle A detects an obstacle L via V2X communication.

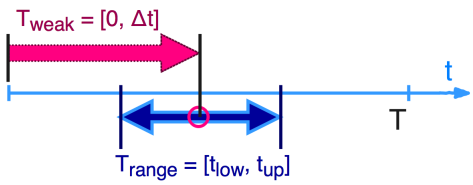

Figure 3: Relation between Δt and T_{range}.

communication respectively. We assume d_{la} is strictly bigger than the sensing range of vehicle A's sensors. WiFi-based ITS-G5/DSRC technology communicating over a 5.9GHz frequency band (V2X/V2V), usually has a communication range(d_{la}) of a few hundred meters, but harsh communication conditions (i.e. Non-Line-of-Sight, channel congestion...) restricts this range d_{la}. We consider in this paper d_{la} to be strictly bigger than d_e ($d_{la} > d_e$), so that vehicle A may use the distance $d_s = d_{la} - d_e$ to adjust its braking strategy. d_{ab} is distance between A and B.
To ensure collision avoidance at both ends, we propose a braking strategy for leading vehicle A (ego vehicle) consisting of two phases: *weak* and *hard*. We illustrate this concept with Figure 3. T is the time any vehicle takes to come to a full halt once it starts braking (T_a is the time vehicle A takes to come to a full halt), and covers a distance shorter than d_{la}. The *weak* braking time interval is represented as T_{weak}, which lasts for Δt s, during which the vehicle will gradually increase its braking magnitude from zero and eventually reach maximum braking strength. Beyond T_{weak}, for a duration of $T - T_{weak}$ s, the vehicle maintains maximum braking strength until it comes to a halt. This time duration is the *hard* braking phase. The challenge is to determine the braking duration Δt which signals the shift from *weak* to *hard* braking manoeuvre. Δt is not unique and can take multiple values within a time interval $T_{range}[t_{low}, t_{up}]$ as shown in Figure 3. t_{up} corresponds to an upper bound to avoid collision with obstacle L and t_{low} corresponds to a lower bound to avoid collision with vehicle B. The same is derived next.

To determine the values of t_{up} and t_{low}, we analyse the deceleration behaviour of vehicles A and B in Figure 4. This image can be understood by decomposing the braking manoeuvres of both vehicles into four phases (or intervals): Phase A, corresponds to vehicle

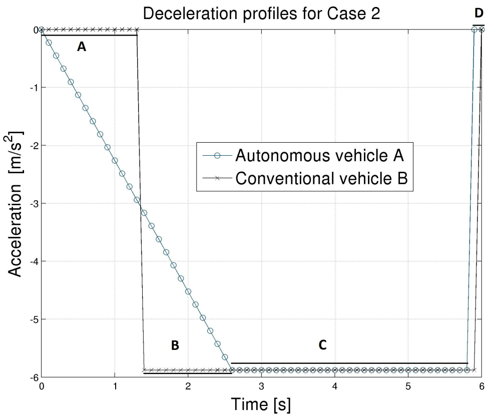

Figure 4: Deceleration profile of vehicles.

B's reaction time during which it doesn't brake where as vehicle A is in *weak* braking phase. Phase B, is the time after t_{prt}, when vehicle A is still in *weak* braking phase and vehicle B has started to brake at maximum strength. Phase C corresponds to both vehicles braking hard, while phase D is when both vehicles come to a halt (collision or not).[2] Now, to ensure collision-free ride, the following conditions need to be ensured:

#1 – $d_{la} > 0$ to avoid front-end collision: corresponds to upper bound of T_{range} (t_{up})

#2 – $d_{ab} > 0$ to avoid rear-end collision: corresponds to lower bound of T_{range} (t_{low})

Ensuring **#1**: Total distance covered by vehicle A before halting must be smaller than initial distance $d_{la(t=0)}$. (L is stationary in longitudinal direction). Simplifying kinematic equations for #1 we get:

$$\Delta t^2 \left(\frac{b_{max,a}}{24}\right) + \Delta t \left(\frac{v_a}{2}\right) + \left(\frac{v_a^2}{2\, b_{max,a}} - d_{la(t=0)}\right) < 0 \tag{2}$$

where v_a is the initial velocity and $b_{max,a}$ is the maximum braking strength of vehicle A.

Ensuring **#2**: As vehicles A and B behave differently in different time phases, but within a phase their braking behaviour remains constant, we split our analysis for **#2** into four intervals previously described:

- Interval A: $t \in [0, t_{prt})$

$$d_{ab}(t) = d_{ab(t=0)} + \frac{b_{max,a}\, t^3}{6\, \Delta t} > 0 \tag{3}$$

where $d_{ab(t=0)}$ is the initial distance between vehicles A and B.

[2]Not always would both vehicles come to a halt at the same time

Table 1: IDM constants and their values

Parameter description	value
Desired speed (v_0)	96 km/h
Free acceleration exponent (δ)	4
Desired time gap (τ)	0.1 s
Maximum acceleration (a_{max})	1.4 m/s^2
Maximum braking strength (b_{max})	-0.6g m/s^2
Length of vehicle (l_a)	4 m
Desired minimum distance (s_0)	5 m

- Interval B: $t \in [t_{prt}, \Delta t)$

$$d_{ab}(t) = d_{ab(t=0)} + \frac{b_{max,a}\, t^3}{6\, \Delta t} - \frac{b_{max,b}\, (t - t_{prt})^2}{2} > 0 \tag{4}$$

where $b_{max,b}$ is the maximum braking strength of vehicle B.

- Interval C: $t \in [\Delta t, T = min(T_a, T_b)$ or $T = T_a = T_b]$

$$d_{ab}(t) = d_{ab(t=0)} + \frac{b_{max,a}\, (\Delta t^2 - 3\, t\, \Delta t - 3\, t^2)}{6} - \frac{b_{max,b}\, (t - t_{prt})^2}{2} > 0 \tag{5}$$

- Interval D: $t \in [T_a, T_b]$ ... for $T_b > T_a$:

$$d_{ab}(t) = d_{ab(t=T)} + \left(\frac{(v_b + b_{max,b}\, (T_a - t_{prt}))^2}{2\, b_{max,b}}\right) \tag{6}$$

or interval $t \in [T_b, T_a]$... for $T_b < T_a$:

$$d_{ab}(t) = d_{ab(t=T)} - \left(\frac{(v_a + b_{max,a}\, (T_b - 0.5\, \Delta t))^2}{2\, b_{max,a}}\right) \tag{7}$$

where $d_{ab(t=T)}$ is the distance between vehicles A and B at time T.

Solving equations mentioned under **#1** and **#2** return a set of possible values which define the time interval T_{range}. The value Δt (between t_{low} and t_{up}) that vehicle A takes depends on its driving strategy but this is out of scope of this paper. The mean value is being taken by default in our calculations: $\Delta t = (t_{up} - t_{low})/2$.

4 PERFORMANCE EVALUATION

We split our experiment into two cases implementing three braking strategies:

Case 1. Vehicle A implements IDM whereas vehicle B implements human behaviour (manual braking)

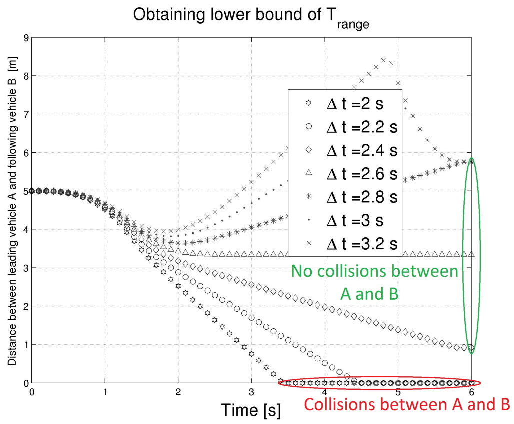

Figure 5: Calculating lower bound for T_{range}.

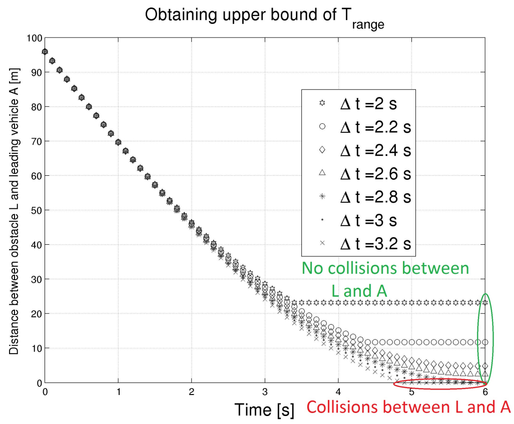

Figure 6: Calculating upper bound for T_{range}.

Case 2. Vehicle A implements the proposed braking strategy whereas vehicle B implements human behaviour (manual braking)
IDM and manual braking were introduced in Section 2 and the new proposed strategy was introduced in Section 3. Values of different parameters used in IDM are summarized in Table 1.[3] We evaluate using Matlab proposed braking strategy against IDM by analysing these two cases.

4.1 Adaptive Braking Strategy

In order to illustrate the role of different parameters influencing T_{range} in the proposed approach, we consider three sets of evaluations: (**I**) we fix all parameters and focus on choosing the right value of Δt; (**II**) we change initial speed of the vehicles and keep rest of the parameters as before; (**III**) we consider the influence of environmental and road conditions (ice, rain, etc..).

Using An's work (An et al., 2011) we calculate that by the time distance reduces to 95.9 m from transmitter of the emergency notification message, receiver can be assumed to have received *at least* one emergency message with 99.5% probability. We thus set $d_{la} = 95.9\ m$. For (**I**) set of evaluation, we fix: $d_{ab} = 5m$, length of vehicle A (l_a) is 4 m, Initial velocities of both vehicles A (v_a) and B (v_b) are assumed to be equal $v_0 = v_a = v_b$. For a highway scenario we assume $v_0 = 96\ km/h$. Moreover we assume that both vehicles A and B can reach a maximum braking capacity (b_{max}) of $-0.6g$, which is the mean of maximum deceleration strengths of vehicles (National Highway Traffic Safety Administration, 2002), g is gravitational constant $9.88 m/s^2$.

[3]For emergency braking situations like the one considered here, limitations on jerks or comfort are not considered for analytical calculations

Set of equations 2- 7 can be used to derive the value T_{range} = [2.4,2.8] s, refer Table 2. The same range can also be obtained graphically. For different Δt values, Figure 5 illustrates the variation of d_{ab} vs time where as Figure 6 illustrates the variation of d_{la} vs time. Intersection of a plot with x-axis indicates zero distance between vehicles. Thus Δt should be chosen such that the plot doesn't intersect x-axis in Figure 5, 6. The Upper bound t_{up} can be determined visually from Figure 6 (thus **#1** resolved), while the Lower bound t_{low} can be determined visually from Figure 5 (thus **#2** is resolved). Now, a value Δt can be chosen: $\Delta t \in T_{range}$; $T_{range} = [t_{up}, t_{down}]$.

Note: in Figure 5, plots for $\Delta t > 2.8\ s$ converge to the same point as seen, because A has collided with L and B comes to halt at the same position; thus d_{ab} at the end of simulations for $\Delta t > 2.8\ s$ is the same.

To further illustrate the consequences of an inaccurate Δt, we consider three different possibilities in Figure 7. The first possibility corresponds to conditions on Δt are not respected, and Δt is chosen smaller than the acceptable T_{range}. In this case, it can be seen that A collides with B (i.e. rear-end collision for ($\Delta t < t_{low}$)).The second possibility corresponds to the desired scenario where Δt is chosen from the calculated T_{range}, and collisions are avoided (i.e. $\Delta t \in T_{range}$). The third possibility corresponds to the case, where Δt is too big and A fails to brake and collides into L (i.e. front-end collision for $\Delta t > t_{up}$).

For (**II**) set of evaluation, we change the initial velocities of vehicle v_0 ($v_a = v_b$), keeping the same $d_{ab} = 5\ m$. The objective is to find the minimum d_{la} and the corresponding T_{range} for Δt to be used by vehicle A to avoid any collisions. Results are summarized in Table 3.[4]

Approaches used in evaluations (**I**) and (**II**) as-

[4]x denotes either a rear-end or a front-end collision

Table 2: Distance between autonomous and conventional vehicle and corresponding time to reach maximum deceleration for v = 96km/h; $d_{la}(t=0) = 95.9m$; highway scenario.

d_{ab} [m]	5	8	10	15
T_{range} [s]	2.4 to 2.8	2.1 to 2.8	2.0 to 2.8	1.6 to 2.8

Table 3: T_{range} corresponding to braking strategies for different vehicular speed.

Velocity $[km/h]$	$d_{la}[m]$	T_{range} $[s]$
v_0 = 30; low speed limit scenario	10	x
	15	1.6 to 2.5
	20	1.6 to 4.6
v_0 = 50; urban city with stricter speed limits	20	x
	30	2.1 only
	35	2.1 to 2.9
	40	2.1 to 3.9
v_0 = 70; urban city scenario	50	x
	55	2.3 to 2.5
	60	2.3 to 3.1
	70	2.3 to 4.3
v_0 = 96; highway scenario	90	x
	95.9	2.4 to 2.8
	100	2.4 to 3.1
	110	2.4 to 4
	120	2.4 to 4.9

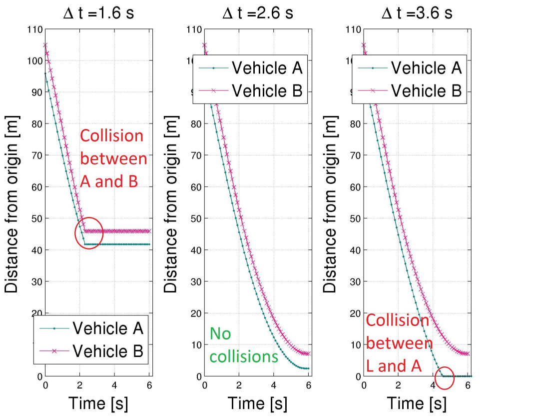

Figure 7: Three cases - rear-end collision, no collision, front-end collision.

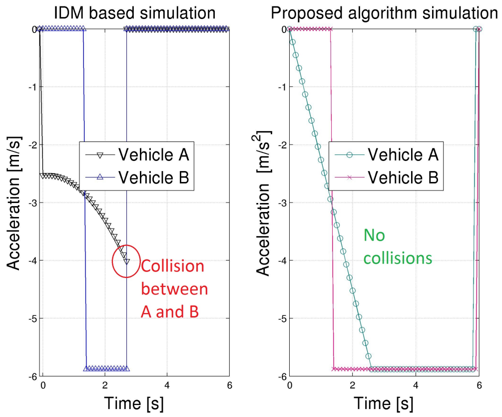

Figure 8: Acceleration profile of vehicles; vehicle A following: IDM (left) proposed algorithm (right).

sume decent road conditions. If a road surface with some oil or sand spill (dirty) is considered, maximum deceleration is physically restricted to $-4\ m/s^2$ (Barbier, 2013). In (**III**) set of evaluation, we limit braking capacity to $-4\ m/s^2$. Simulations show if the vehicles are travelling at 96 km/h with $d_{ab} = 5\ m$, and $d_{la} = 95.9\ m$ collisions can not be averted (i.e. A will collide with either B or L). For these reasons, the maximum speed limit should be capped, say to 80 km/h (50 mph) which in turn returns T_{range} of [2.3,3.2] s. Alternately, under optimal road conditions which could support braking up to $-8\ m/s^2$, maximum velocity permitted can be increased up to 110 km/h (68 mph) such that with Δt values of [2.3,2.4] s, collisions could be avoided.

4.2 Adaptive vs. IDM-ACC braking

We complete our evaluation with a comparison of adaptive braking strategy (proposed algorithm, implemented in Case 2) against the IDM-ACC mechanism (implemented in Case 1) for the same set of parameters. Sub-plots on the right of Figures [8, 9, 10] demonstrates the performance of our algorithm com-

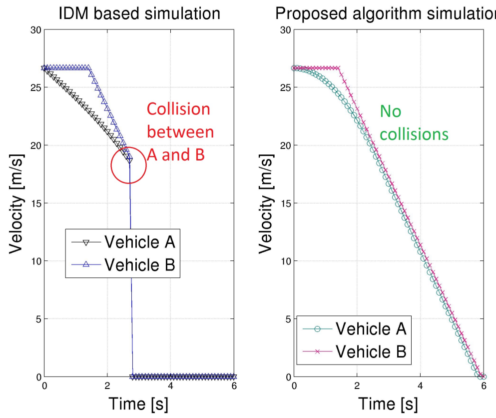

Figure 9: Velocity profile of vehicles; leading vehicle following: IDM (left) proposed algorithm (right).

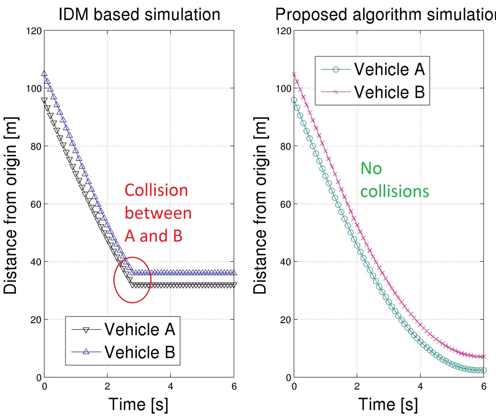

Figure 10: Locations of vehicles; leading vehicle following: IDM (left) proposed algorithm (right).

pared to IDM's on the left; these plots highlight acceleration, velocity and location comparison between the two. IDM would demand an instantaneous increase in braking strength from zero (to -2.5 m/s^2 approximately in this case) as shown in Figure 8 where as the proposed approach is more comfortable as braking strength is increased gradually. At 2.8 seconds, the acceleration jumps from around -4 to 0 m/s^2 in the plot on the left in Figure 8, which is due to rear-end collision of vehicle A with B. Same is the interpretation of Figure 9 which shows sudden fall of velocity to zero after the accident between A and B. Figure 10 shows the vehicles maintaining their position after collision at 2.8 s. Whereas the plot on the right of Figures [8, 9, 10] show smooth collision free braking.

Finally, from Figure 11 it is clear that front-end collisions could be avoided but rear-end accident could not be avoided as d_{ab} reaches zero for Case 1, where as the proposed braking strategy implemented in Case 2 avoids accidents at both ends. These figures

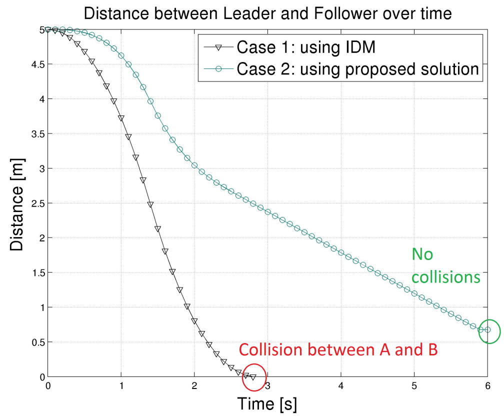

Figure 11: Inter-distance between vehicle A and B

clearly supports our claim that IDM indeed couldn't assure collision avoidance of the following vehicle onto itself where as our proposed algorithm does.

5 CONCLUSIONS

When an autonomous vehicle needs to suddenly brake, it should consider not only the possible front-end collision, but also rear-end collision with the following vehicle. In this paper, we address this aspect and propose a braking strategy for a scenario involving a manually driven vehicle following an autonomous vehicle. First phase of the braking avoids hard brake where as a second phase performs a conventional hard brake.

The proposed approach also suggests that even at high velocities (96km/h) and low inter vehicular distance (5m), safety is not compromised provided the autonomous vehicle gradually increases its braking strength to maximum. Most importantly, via simulation, we show the superiority of the proposed algorithm over ACC/CACC algorithms like IDM in braking circumstances, which usually only manages to avoid front-end collision at the cost of a rear-end collision.

Future work will focus on developing a control theory based approach which would provide control inputs for coordinated braking of multiple vehicles with different levels of automation in a heterogeneous traffic scenario, whilst optimizing a particular cost function.

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