Optimum array signal processing in the presence of imperfect spatial coherence of wavefronts

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Abstract— **Traditionally optimum-adaptive beamforming algorithms have been developed assuming fully coherent plane wavefronts, i.e., assuming a data model of point sources. In most applications this assumption is inappropriate, since the channel model has to account for different kinds of dispersion phenomena due to both the propagation environment and the array itself. Significant examples are SONAR and underwater communication systems. Indeed, in such circumstances, the resulting wavefronts can be randomly distorted, usually suffering a loss of spatial coherence. Here, assuming a more realistic stochastic channel model, we analyze the performance of a traditional optimum-adaptive beamformer for point sources, when the signal or the interference undergo a spatial coherence degradation. It is shown, with analytical details, that the same coherence loss, for the interference results in larger performance degradation than for the signal. Furthermore, we provide a theoretical comparison among different beamforming algorithms, based on the estimate of the channel parameters and on spatial smoothing methods.**

I. Introduction

Mostly, sensor array beamforming algorithms have been developed based on the assumption of a data model of point sources with only a direct fully coherent plane-wave path between the source and the receiving array. Nevertheless, in most real scenarios this assumption is clearly inappropriate because many kinds of dispersion phenomena make received signals exhibit a limited spatial coherence, even when the corresponding sources are fully coherent for any propagation distance. Causes of such a spatial coherence degradation can generally be attributed to the propagation medium and/or to the array itself. Indeed, when non-rigid large aperture acoustic arrays are used, random mechanical deformations can occur because the array is in motion and/or due to random fluctuations of the medium wherein it is located. These effects are well known in SONAR and underwater communication systems operating in shallow water with non-rigid acoustic arrays. Since spatially scattered sources and not point sources exhibit limited spatial coherence, i.e., angular spreading, we shall assume a more realistic channel model, accounting for both the propagation medium and the array dispersive effects, where the signal energy arrives at the array in clusters of rays/modes, distributed around the nominal directions of the signals sources. This model has been already developed for array processing in radio mobile communications (see [1] and references therein). The effects of a signal reduced spatial coherence on the performance of optimum-adaptive beamformers have been analyzed by many authors [1]–[12], although the most attention has been focused on the effects on Direction Of Arrival (DOA) estimation where several algorithms have been derived [1]–[6]. Concerning the analysis of the effects, Cox [11] derived a simple exponential correlation model to describe the coherence degradation of a plane-wave signal, to compare the array output Signal-to-Noise Ratio (SNR) with the output SNR attainable with only one sensor, assuming a linear array in the absence of interference. Other authors provided spatial coherence loss models based on measured data in real testbeds [14], [15]. More recently, Morgan and Smith [9] analyzed the same effects on the detection performance of linear and quadratic array processors using an exponential-power-law model for the signal wave-front coherence in the presence of uncorrelated noise, but always in the absence of interference. A first analysis of the effects due to the interference coherence loss was given in [7], [8]. Here we show that the detection performance of an optimum-adaptive array can be substantially degraded due to an interference with limited spatial coherence even though the signal is fully coherent. In order to give a complete insight, here we study both the effects of the signal and the interference spatial coherence degradation on the performance of an optimum-adaptive array processor, which has been designed to maximize the output Signal-to-Interference-plus-Noise Ratio (SINR), when receiving fully coherent signal and interference (i.e., when the angular spread is zero, i.e., for point sources) [13]. The single signal and the single interference cases are studied with analytical details in order to identify the main causes of performance loss. Analytical expressions for the output SINR are derived in terms of signal and interference coherence coefficients (source coherence length or angular spread) and parameters related to the array processor. Then, on the basis of that analysis, the performances of different beamforming algorithms are compared.

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II. Model and Assumptions

We consider a linear uniform array with m sensors and inter-element spacing of d, receiving narrowband signals.

Each signal wavefront can be modeled as resulting from the superposition of a large number of fully coherent plane-wavefronts, i.e., rays or modes, due to the scattering phenomena described above. Each mode has a random complex gain factor γ and arrives from a random direction $\theta + \phi$, where θ is the nominal DOA of the source and ϕ is a random zero mean angular deviation. In addition, we shall assume the absence of channel delay spread, i.e., the differences in path delay between the different modes very small so that they can be considered as phase shifts in the gain factors.

The complex baseband signals related to q independent sources, received at the antenna array can be represented by the vector

$$\boldsymbol{x}(t) = \sum_{k=1}^{q} \boldsymbol{x}_k(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{n}(t)$ is temporally and spatially white Gaussian noise such that $\mathrm{E}\boldsymbol{n}(t_1)\boldsymbol{n}^H(t_2) = \sigma_n^2 \mathbf{I}\delta(t_1 - t_2)$, and the contribution $\boldsymbol{x}_k(t)$ from the kth source is modeled as

$$\boldsymbol{x}_k(t) = s_k(t) \sum_{j=1}^{M_k} \gamma_{jk}(t)\mathrm{a}(\theta_k + \phi_{jk}(t)) = s_k(t)\boldsymbol{v}_k(t) \tag{2}$$

where $s_k(t)$ is the signal transmitted by the kth source, $\mathrm{a}(\theta)$ is the array response vector of a point source at DOA θ, M_k is the number of modes related to the kth source. We assume that the Probability Density Function (PDF) of ϕ_{jk} is the same for all j's and obeys a known PDF with unknown standard deviation σ_{ϕ_k}. Note that from the central limit theorem, since usually we have $M_k \gg 1, \boldsymbol{v}_k(t) \sim \mathcal{N}(0, \boldsymbol{R}_v(\theta_k, \sigma_{\phi_k}))$. Since we assumed a linear uniform array it can be shown that

$$\boldsymbol{R}_v(\theta, \sigma_\phi) \approx [\mathrm{a}(\theta)\mathrm{a}^H(\theta)] \odot \boldsymbol{B}(\theta, \sigma_\phi) \tag{3}$$

where $\odot$ denotes the Schur Hadamard product (i.e., the elementwise product) and $\boldsymbol{B}(\theta, \sigma_\phi)$ is a Toeplitz matrix with $[\boldsymbol{B}(\theta, \sigma_\phi)]_{ij} = C(i-j, \theta, \sigma_\phi)$, the characteristic function corresponding to the PDF of ϕ/σ_ϕ. In the derivation of (3) we normalized the gain factors such that $\sum_{j=1}^{M_k} \mathrm{E}|\gamma_{jk}|^2 = 1$ for all k's. We denote $\boldsymbol{B}(\theta, \sigma_\phi)$ spatial coherence matrix throughout the paper. In underwater acoustic wave propagation it is usual to refer to the spatial characteristic coherence length L_ϕ related to a specified source. Note that since L_ϕ is a measurable quantity at the receiver, contrary to σ_ϕ, it can be estimated without assuming any specified PDF's for ϕ/σ_ϕ. It is straightforward to show that $L_\phi \sim 1/\sigma_\phi$ anyway. We also introduce the equivalent angle spread $\sigma_\theta = \sigma_\phi \cos\theta$ and the equivalent characteristic coherence length $L_\theta = L_\phi / \cos\theta$.

Several authors have suggested different PDF's for ϕ/σ_ϕ (e.g., see [1], [5]). A general model in terms of coherence length is given by

$$C(i-j, L_\theta) = \mathrm{e}^{-[(d/L_\theta)|i-j|]^r} \tag{4}$$

where r is a parameter that typically varies between 1 (exponential coherence decay) [11], and 2 (Gaussian coherence decay) [1], [2], [15]. In the case of $r = 1$, we can define $\beta = \exp(-d/L_\theta)$ so that expression (4) assumes the form

$$C(i-j, L_\theta) = \beta^{|i-j|} \tag{5}$$

Finally, note that if the sources are uncorrelated, the covariance matrix of $\boldsymbol{x}(t)$ is given by

$$\boldsymbol{R}_x = \sum_{k=1}^{q} p_k \boldsymbol{R}_v(\theta_k, \sigma_{\phi_k}) + \sigma_n^2 \mathbf{I} \tag{6}$$

where $p_k = \mathrm{E}|s_k|^2$.

III. Signal waveform estimation

Assume that among the q different signals received at the array the 1st is the signal of interest. Form the estimate $\hat{s}_1(t) = \boldsymbol{w}^H \boldsymbol{x}(t)$. Then we divide the estimate in three terms, $\hat{s}_1 = y_s + y_i + y_n$ where $y_n = \boldsymbol{w}^H \boldsymbol{n}(t)$, $y_i = \boldsymbol{w}^H \sum_{k=2}^{q} \boldsymbol{v}_k(t)s_k(t)$ and $y_s = \boldsymbol{w}^H \boldsymbol{v}_1(t)s_1(t)$ are the contributions from the noise, the interference and the signal respectively. Then the SINR is given by

$$\mathrm{SINR} = \frac{\mathrm{E}|y_s|^2}{\mathrm{E}|y_i|^2 + \mathrm{E}|y_n|^2} \tag{7}$$

or, using the previous model in terms of coherence length,

$$\mathrm{SINR} = \frac{\boldsymbol{w}^H \boldsymbol{R}_s \boldsymbol{w}}{\boldsymbol{w}^H \boldsymbol{R}_{in} \boldsymbol{w}} \tag{8}$$

where $\boldsymbol{R}_s = p_1 \boldsymbol{R}_v(\theta_1, L_{\phi_1})$, $\boldsymbol{R}_{in} = \sum_{k=2}^{q} p_k \boldsymbol{R}_v(\theta_k, L_{\phi_k}) + \sigma_n^2 \mathbf{I}$. The optimal weight vector in terms of SINR is the generalized principal eigenvector $\boldsymbol{w}_{\mathrm{opt}} = V_{\max}(\boldsymbol{R}_s, \boldsymbol{R}_{in})$ and the optimal SINR is the associated eigenvalue $\lambda_{\max}(\boldsymbol{R}_s, \boldsymbol{R}_{in})$ (e.g., [1]). We refer to this beamforming criterion as the Generalized Optimum Beamformer (GOB). When the source are fully coherent, i.e., the angular spread is zero, the optimal weight vector coincides with the Minimum Variance Beamformer (MVB)(e.g., [13])

$$\boldsymbol{w}_{\mathrm{MVB}} \propto \boldsymbol{R}_{in}^{-1} \mathrm{a}(\theta_1) \tag{9}$$

IV. Effects of coherence loss using the MVB

On the basis of the previous results, we compare the performance degradation due to both the signal and the interference coherence loss when the MVB is assumed as SINR optimization criterion. In order to provide analytical results we shall consider the following scenarios:

- A partially coherent signal in the presence of a fully coherent interference and white noise;
- A fully coherent signal in the presence of a partially coherent interference and white noise.

For the sake of notation all the quantities related to the signal and to the interference in the sequel will be denoted $(\cdot)_s$ and $(\cdot)_i$ respectively.

A. Signal coherence loss

When the interference is a point source, if the angular separation between the signal and the interference is larger than the mainlobe width and the interference-to-noise ratio (INR) is large enough (i.e., INR ≥ 1), it can be shown that the beamformer output SINR is practically the same as when assuming the absence of interference. For the sake of our analysis we can assume that those conditions hold, i.e., $\boldsymbol{R}_{in} \approx \sigma_n^2 \mathbf{I}$. Remark that even though the interference and the signal DOA's are very close, the results obtained in the previous approximation are still indicative of the order of magnitude of the output SINR (note that the error increases as the signal coherence increases).

The maximum SNR in the presence of white noise only is equal to

$$\mathrm{SNR}_o = m p_s / \sigma_n^2 = m \nu_s \tag{10}$$

In the following we consider the ratio $G = \mathrm{SINR_{MVB}}/\mathrm{SNR}_o$ to provide a normalized quantity for the performance loss evaluation. Then, in the specified scenario we have

$$G = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} C(i-j,\, L_s) \tag{11}$$

where we denoted $L_s = L_{\theta_s}$. If $r = 1$, according to (5) equation (11) reduces to

$$G = \frac{1}{m} + \frac{2}{m^2} \sum_{l=1}^{m-1} \beta_s^l (m - l) \tag{12}$$

Note that in the case of fully coherent signal $\beta_s \approx 1$ so that $G \approx 1$. On the other hand, if $\beta_s \approx 0$, then $G \approx 1/m$, i.e., the array does not provide any gain compared to a single sensor.

B. Interference coherence loss

Here we evaluate the effects of interference with limited spatial coherence considering a MVB receiving a fully coherent signal and a single partially coherent interference. Since the signal is fully coherent then $\mathrm{SINR_{MVB}} = \mathrm{SINR_{GOB}}$ and it reduces to [11], [13]

$$\mathrm{SINR_{opt}} = p_s \mathbf{a}^H(\theta_s) \boldsymbol{R}_{in}^{-1} \mathbf{a}(\theta_s) \tag{13}$$

According to (6) we have $\boldsymbol{R}_i = p_i[\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)] \odot \boldsymbol{B}(\theta_i,\, \sigma_{\phi_i})$ and the corresponding inverse

$$\boldsymbol{R}_i^{-1} = (1/p_i)[\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)] \odot \boldsymbol{B}^{-1}(\theta_i,\, \sigma_{\phi_i}) \tag{14}$$

Unfortunately, a closed form expression for $\boldsymbol{R}_i^{-1}$ exists only for $r = 1$. In this case the matrix $\boldsymbol{B}_i^{-1} = \boldsymbol{B}^{-1}(\theta_i,\, \sigma_{\phi_i})$ can be written as

$$\boldsymbol{B}_i^{-1} = \frac{1}{1-\beta_i^2} \begin{bmatrix} 1 & -\beta_i & 0 & \cdots & 0 \\ -\beta_i & 1+\beta_i^2 & -\beta_i & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\beta_i & 1+\beta_i^2 & -\beta_i \\ 0 & \cdots & & -\beta_i & 1 \end{bmatrix} \tag{15}$$

In order to achieve an analytical closed form expression for G in this scenario we need to assume INR $\gg 1$ so that $\boldsymbol{R}_i \approx \boldsymbol{R}_{in}$. In that first order approximation the MVB reduces to the Zero-Forcing Beamforming criterion (ZFB) [16]. Note that the MVB for high INR performs very close to the ZFB and they yield the same optimum SINR in the absence of noise. Then G is given by

$$G = \frac{1}{m\nu_i} \mathbf{a}^H(\theta_s)[\mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i)] \odot \boldsymbol{B}^{-1}(\theta_i,\, \sigma_{\phi_i})\mathbf{a}(\theta_s)$$

where we denoted the INR with $\nu_i = p_i/\sigma_n^2$. If $r = 1$, $\boldsymbol{B}_i^{-1}$ is given by (15) and G reduces to

$$G = \frac{1}{m\nu_i(1-\beta_i^2)}[m + (m-2)\beta_i^2 - 2\beta_i(m-1)\cos u_{si}] \tag{16}$$

where, assuming d equal to half a wavelength $u_{si} = \pi(\sin\theta_s - \sin\theta_i)$. From expression (16) it can be noted that the optimum beamformer gain G is a monotonically decreasing function of β_i and ν_i. In the case of $m \gg 1$, from expression (16) we have

$$G \approx \frac{1}{\nu_i(1-\beta_i^2)}(1 + \beta_i^2 - 2\beta_i \cos u_{si}) \tag{17}$$

If in addition we have $u_{si} = 0$, then $G \approx (1-\beta_i)/[(1+\beta_i)\nu_i]$. Remark that the ZFB yields to $G \to \infty$ for $\beta_i \to 1$. Hence, when the interference is fully coherent we shall replace (16) with the following expression [13]

$$G = 1 - \frac{m\nu_i}{1 + m\nu_i} |\mathbf{a}^H(\theta_i)\mathbf{a}(\theta_s)|^2 \tag{18}$$

Figures 1, 2 show the dependence of G on the ratio L_s/L_a and L_i/L_a respectively, for different values of r, where $L_i = L_{\theta_i}$ and L_a denotes the array aperture. The array has $m = 16$ half a wavelength spaced elements. Figure 1 shows that G is weakly dependent on r, for a specified coherence length L_s. On the contrary, in figure 2 it is shown that G is strongly dependent on r. In particular by increasing the parameter r the output SINR increases as well, if the ratio L_i/L_a is large enough (if $(L_i/L_a) > 0.1$ in the presented example). Otherwise, for very small values of the L_i/L_a ratio, the output SINR is quite independent on r.

C. Effects of the signal–interference angular separation

We observed that the function G is strongly influenced by the parameter u_{si}, especially when the parameter r increases. In general, for a specified r, interference DOA and array aperture L_a, we can identify a critical interference coherence length $L_{i(c)}$ corresponding to a minimum of the function G. If $r = 1$ an analytical expression for $L_{i(c)}$ can be found

$$L_{i(c)}/L_a = -\frac{1}{(m-1)\ln\beta_{i(c)}} \tag{19}$$

where $\beta_{i(c)} = (1 - \sin u_{si})/(\cos u_{si})$. Note that a real positive value for $L_{i(c)}$ requires $\beta_{i(c)} > 0$, that is $0 \leq u_{si} < \pi/2$

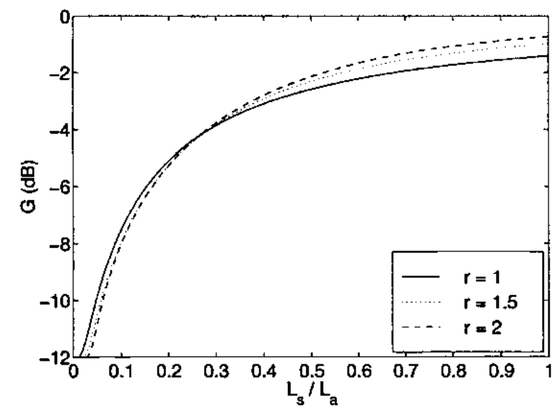

Fig. 1. Signal coherence loss effects on G for $\nu_s = 20\,\mathrm{dB}$ and $r = 1,\ 1.5,\ 2$

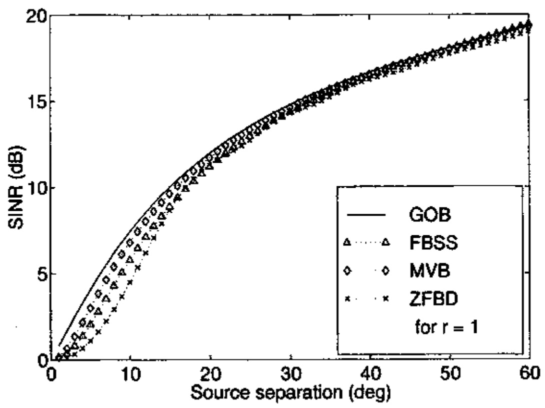

Fig. 3. Output SINR vs. source separation for different algorithms and $r = 1$

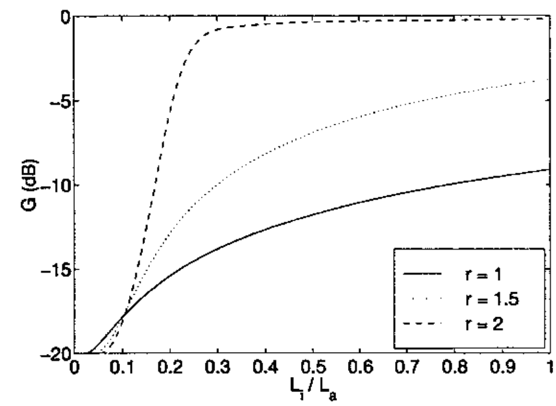

Fig. 2. Interference coherence loss effects on G for $\nu_i = 20\,\mathrm{dB}$ and $r = 1,\ 1.5,\ 2$

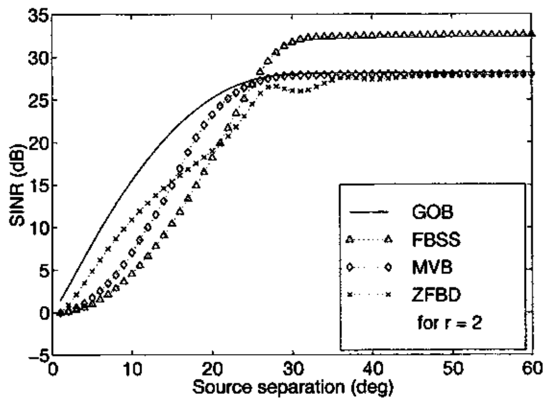

Fig. 4. Output SINR vs. source separation for different algorithms and $r = 2$

or $3\pi/2 < u_{si} \leq 2\pi$ (i.e., $\theta_i < 30\,\mathrm{deg}$ in the considered example). Finally, note that for a specified value of u_{si}, as the number of array elements m decreases, the ratio $L_{i(c)}/L_a$ rapidly increases.

V. Algorithms

Here we provide a preliminary theoretical comparison among some beamforming algorithms. Some of them have been designed to deal with scattered sources as well. We consider a typical SONAR system operating in shallow water, where the channel delay spread is negligible, but we can have significant angle spread. We assume perfect estimates of the channel parameters and the covariance matrices given by (3). Hence we compare the GOB [1], the MVB, the ZFB with Derivative constraints (ZFBD)[16] and the Forward/Backward Spatial Smoothing (FBSS) beamformer [17].

Note that the GOB needs structured covariance matrices estimates, i.e., the estimates of $\boldsymbol{R}_v(\theta_k, \sigma_{\phi_k})$, p_k for all k's and σ_n in order to avoid severe signal cancelation phenomena. In the ZFBD, zero-forcing conditions are applied to both the beampattern up to its nth derivative in correspondence of the interference DOA's. It can provide significant performance improvement with respect to the classical ZFB for large number of array sensors and high INR. Finally, spatial-smoothing techniques are well known to provide good performance in the presence of scattered sources at the cost of a reduced spatial resolution but without the need of structured covariance matrices.

In the following we provide a preliminary theoretical comparison among the algorithms mentioned above. Hence we assume the array having $m = 16$ sensors, receiving one signal of interest and one interference. We consider a scenario where the signal DOA is fixed at $\theta_s = 0^o$ and the interference DOA θ_i varies in the range $[1^o, 60^o]$ with respect to the array broadside. Both the signal and the interference have coherence length $L = 4.2$ half wavelengths and they are 20 dB stronger than the noise.

Figures 3, 4 show the dependence of the SINR on the signal-interference angular separation for $r = 1$ and $r = 2$ respectively. Note that much larger differences arise in the case of $r = 2$ than in the case of $r = 1$. Moreover, for large source separations the FBSS algorithm performs better than the GOB for $r = 2$.

Extensive simulations have shown r (i.e., the coherence loss model) to be the most influential parameter on the differences in the beamforming algorithms performances. As a general trend, we observed that such differences increase as r increases. Hence the choice of the coherence loss model represents the most important issue to compare the performances of different algorithms.

VI. Conclusions

In this paper we considered a receiving array of acoustic sensors operating in shallow water. In such a scenario the resulting transmission channel makes the signals received at the array exhibit a considerable angle spread, i.e., a reduced spatial coherence. We provided a unified analysis of the performance loss inherent to the use of an optimum beamformer for fully coherent sources (i.e., point sources), when it operates in the presence of partially coherent sources. A general coherence loss model has been proposed and analytical results have been provided for both the cases of signal and interference coherence loss, in order to identify the main causes of the beamformer performance degradation. Then we compared the performances of four different beamforming algorithms in the presence of partially coherent signal and interference, providing a numerical example. Moreover, extensive simulations have shown a high sensitivity of the algorithms performances to the coherence loss model. We point out that there are many possible strategies available to improve the beamformer performance and we have only covered a few. Further investigations are needed to find a set of criteria for the choice of the beamforming algorithm, including the problems concerned with parameter estimation and with real-time implementation, when also impulsive sources are present and only short data samples are available.

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