On Coding for Fading Channels

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Abstract

Wireless mobile communication systems are characterized by time-varying propagation channels, generally referred to as "fading channels". Under this common name, we find a variety of channel models, suited to different applications. The rationale behind the selection of coding schemes for wireless systems depends critically on the underlying fading channel model, which in turns depends on the application. In this paper, we discuss the coding design criteria for some classical fading channel models. If the channel is not stationary, as it happens for example in a mobile-radio communication system where it may fluctuate in time between the extremes of Rayleigh and AWGN, then a code designed to be optimum for a fixed channel model might perform poorly when the channel varies. Therefore, a optimal code for a given channel may be actually suboptimum for a substantial fraction of time. In these conditions, the issue of robustness plays an important role: a good code should provide acceptable performance over a wide range of channel models.

A different approach to efficient communication over fading channels consists of "changing the game's rules" and modifying the propagation channel by introducing diversity and/or exploiting transmitter channel state information. For example, it has been shown that antenna reception diversity with maximal-ratio combining turns asymptotically a fading channel into an AWGN channel. Then, optimal codes for the latter will provide good performance over wireless channels if used jointly with antenna reception diversity. More recently, joint antenna transmission diversity and coding has gained a great attention, because of its capability of providing large spectral efficiency. Finally, in some applications the transmitter can be provided with some channel state information. Then, it can counteract the effect of fading by dynamically selecting the appropriate coding scheme and/or by allocating the transmitted power.

1 Introduction: The fading channel

In the simplest communication channel model (the "additive white Gaussian noise channel", or AWGN) the received signal is assumed to be affected only by a constant attenuation and a constant delay. Digital transmission over radio channels often needs a more elaborate model, since it may be necessary to account for propagation vagaries, referred to as "fading," which affect the signal strength. These are connected with a propagation environment referred to as "multipath" and with the relative movement of transmitter and receiver, which causes time variations of the channel.

Multipath propagation occurs when the electromagnetic energy carrying the modulated signal propagates along more than one "path" connecting the transmitter to the receiver. Examples of such situation occur for example in indoor propagation when the electromagnetic waves are perturbed by structures inside the building, and in terrestrial mobile radio when multipath is caused by large fixed or moving objects (buildings, hills, cars, etc.).

In general, a time-varying linear channel is defined by its (low-pass equivalent) impulse response $c(\tau; t)$. The complex envelope of channel output at time t, for a given input with complex envelope $x(t)$, is given by

$$y(t) = \int c(\tau; t)x(t - \tau)d\tau$$

Equivalently, we can look at a general fading channel as a linear filter whose frequency response

$$C(f; t) = \int c(\tau; t)e^{-j2\pi f\tau}d\tau$$

depends on t.

Since the wireless environment is characterized by several random events, $c(\tau; t)$ is usually modeled as a family of (wide-sense stationary) complex Gaussian random processes with respect to the "time" variable t, indexed by the "delay" variable τ. A usual assumption is that processes for different delay values are uncorrelated, i.e., $\mathrm{cov}(c(\tau; t), c(\tau - \Delta\tau; t - \Delta t)) = \phi_c(\tau; \Delta t)\delta(\Delta\tau)$. This yields to the classical *wide-sense stationary uncorrelated-scattering* model [1]. The time-frequency correlation of the fading channel is described by the function

$$\Phi_c(\Delta f; \Delta t) = E[C(f; t)C(f - \Delta f; t - \Delta t)^*]$$

The channel *coherence bandwidth* B_c is defined as the maximum spacing Δf between frequencies at which two sinusoids are affected by approximately the same complex gain, i.e., for which $\Phi_c(\Delta f; 0) \approx \Phi_c(0; 0)$. The channel *coherence time* T_c is defined as the maximum spacing Δt between instants at which two impulses are affected by approximately the same complex gain, i.e., for which $\Phi_c(0; \Delta t) \approx \Phi_c(0; 0)$. Also, we define the channel *delay spread* T_m as the maximum delay difference between multipath components, and

the *Doppler bandwidth* B_d as the maximum frequency-shift difference due to the relative motion of antennas and scattering objects. The approximate relations between these quantities are $T_c \approx 1/B_d$ and $B_c \approx 1/T_m$.

Let B_x denote the bandwidth of the transmitted signal. If $B_x \ll B_c$, there is no linear distortion. Otherwise, different frequency components of the signal undergo different gains, and we have *frequency selectivity*. Let T_x denote the duration of the transmitted signal. If $T_x \ll T_c$, the channel gain is constant over the signal transmission and we have no multiplicative distortion. Otherwise, different portions of the signal undergo different gains, and we have *time selectivity*.

Fading-channel classification. From the previous discussion we have seen that the two quantities B_c and T_c describe how the channel behaves for the transmitted signal. Specifically,

(i) If $B_x \ll B_c$, the channel is called *flat* (or *frequency non-selective*). Otherwise, it is *frequency-selective*.

(ii) If $T_x \ll T_c$, the channel is called *slow* (or *time non-selective*). Otherwise, it is *fast*.

The channel flat in t and f is not subject to fading, neither in time nor in frequency. It might be characterized by a random gain, which stays constant for a very long time (in principle, for the whole transmission duration). The channel flat in time and selective in frequency is a simple *intersymbol interference (ISI) channel*. The channel flat in frequency is a good model for narrowband terrestrial mobile systems [2] and for satellite mobile systems [3] and most of the following will be devoted to its analysis. The fast frequency-selective fading is not so well understood as the flat-fading and the ISI channels. It is suited to particular applications, such as avionic communications, characterized by long delays due to earth reflections (small B_c) and high speeds (small T_c).

We conclude this section by recalling a well-known discrete-time channel general model and some particular cases. Assume that the channel input $x(t)$ is bandlimited over $[-W/2, W/2]$. Then, from the sampling theorem [1] we can write

$$y(t) = \sum_n c_n(t)x(t - n/W) \tag{1}$$

where $c_n(t) = \int c(\tau; t)\mathrm{sinc}(W(t - \tau) - n)d\tau$. For small Doppler bandwidths, $y(t)$ is still approximately bandlimited over $[-W/2, W/2]$. Then, by sampling at rate W, we obtain the discrete-time representation

$$y[i] = \sum_n c_n[i]x[i - n] \tag{2}$$

where $y[i] = y(i/W)$, $x[i] = x(i/W)$ and $c_n[i] = c_n(i/W)$. The above channel model is a tapped delay-line with time-varying tap weights.

If the channel is flat, then $y[i] = a[i]x[i]$, where $a[i] = c_0[i]$ is the fading gain. Moreover, if the channel varies very slowly, $a[i]$ is a correlated discrete-time process, approximately constant over $N \simeq T_cW$ symbols.

2 Coding for the flat-slow fading channel

We focus here on the flat-fading channel model, and assume that the receiver has (perfect) Channel State Information (CSI), i.e., it knows $a[i]$ for each i. Under these conditions, interleaving is optimal, in the sense that the channel capacity does not depend on the ordering of the transmitted and received sequences [4, 5, 2]. Then, a suitable coding strategy is to concatenate a channel encoder with an interleaver. If the fading is an ergodic process (in particular, if $a[i]$ and $a[i + k]$ tend to be independent as $k \to \infty$), the interleaved fading process can be treated as an i.i.d. sequence, provided that a sufficiently large interleaving depth can be used. In many wireless systems, typical Doppler bandwidths range from 1 to 100 Hz [6], while data rates go from 20 to 200 kbaud (B_x ranges from 20 to 200 kHz). Then, blocks of at least $L \simeq B_x/B_d \geq 200$ symbols undergo approximately the same fading gain. Consider the transmission of a code word $\mathbf{x} = (x[0], \ldots, x[n-1])$ of length n channel symbols. In order to make the fading gains affecting the symbols of $\mathbf{x}$ appear as independent via interleaving, the actual time interval spanned by $\mathbf{x}$ must be at least nL, so that each symbol of $\mathbf{x}$ can be transmitted over a different fading "block". Then, the (interleaving) delay of the system is large and, above all, it is characterized by L, which is not under the control of the coding scheme designer.

In some applications, like wireless data networks or broadcasting, large delays are acceptable. Then, deep interleaving is possible and the fading i.i.d. assumption holds. In other applications, like real-time speech transmission, a strict decoding delay is imposed (e.g., 100 ms, at most [6]). In this case, small interleaving is a constraint and the transmission of a code word may span only a few TDMA channel bursts, over which the channel fading is strongly correlated. These two situations yield rather different coding design criteria, as it will be illustrated in the following.

2.1 Coding without delay constraints

Let us focus on the transmission of a coded modulation (CM) scheme over a fading channel with i.i.d. fading, perfect CSI at the receiver and soft decoding. In general, upper bounds and approximations to the bit, symbol or frame error probability can be derived by using as basic building block the pairwise error probability (PEP) $P(\mathbf{x} \to \mathbf{x}')$ of mistaking the transmitted sequence $\mathbf{x}$ with a different code sequence $\mathbf{x}'$, as if these two were the only possible outcomes of the decoder [7, 8, 9]. Then, simple coding optimization criteria can be found from the analysis of the PEP.

In a Gaussian fading channel, we can write the received vector $\mathbf{y}$ as

$$\mathbf{y} = \mathbf{Ax} + \mathbf{z} \tag{3}$$

where $\mathbf{z}$ is a complex circularly-symmetric Gaussian

noise vector with variance per component N_0 and $A = \text{diag}(a)$ is the diagonal matrix of the (interleaved) fading gains. Then, with maximum-likelihood decoding, the PEP is given by

$$
\begin{aligned}
P(x \to x') &= P(|y - Ax|^2 \geq |y - Ax'|^2 | x) \\
&= P(\Delta(d, a) \leq 0)
\end{aligned}
\tag{4}
$$

where $d = x' - x$ and

$$
\Delta(d, a) = 2\text{Re}\{z^\dagger Ad\} - d^\dagger A^\dagger Ad
\tag{5}
$$

Since $\Delta(d, a)$ is conditionally Gaussian given a, with mean $-d^\dagger A^\dagger Ad$ and variance $2N_0 d^\dagger A^\dagger Ad$, we can write

$$
P(x \to x') = E\left[Q\left(\sqrt{\frac{d^\dagger A^\dagger Ad}{2N_0}} \right) \right]
\tag{6}
$$

where expectation is with respect to the fading sequence a. Several general methods for evaluating or upper bounding the above expressions have been proposed in the literature (see for example [10, 11, 12, 13]).

For independent Rayleigh fading (i.e., when the $a[i]$ are complex circularly-symmetric Gaussian with mean 0 and variance 1), the Chernoff upper bound to the PEP is given by [7, Chap.13]

$$
P(x \to x') \leq \prod_{k \in \mathcal{K}} \frac{1}{1 + |d[k]|^2/4N_0}
\tag{7}
$$

where $\mathcal{K}$ is the set of indices k such that $d[k] \neq 0$. By letting $d_H(x, x')$ the Hamming distance between x and x' and E_s the average energy per symbol, for large E_s/N_0 we can write

$$
P(x \to x') \leq \left[\delta^2(x, x') \frac{E_s}{4N_0} \right]^{d_H(x, x')}
\tag{8}
$$

where

$$
\delta^2(x, x') = \left[\prod_{k \in \mathcal{K}} |x[k] - x'[k]|^2 / E_s \right]^{1/d_H(x, x')}
$$

is the geometric mean of the non-zero squared Euclidean distances between the components of x, x'. The latter result shows the important fact that the error probability is (approximately) inversely proportional to the *product* of the squared Euclidean distances between the components of x and x' that differ, and, to a more relevant extent, to a power of the signal-to-noise ratio whose exponent is the Hamming distance between x and x'.

By using a union bound, we see that the (bit, symbol or frame) error probability is dominated by the pairwise errors with the smallest d_H. The minimum Hamming distance $d_{H,\text{min}}$ of the code is sometimes referred to as "code diversity". The above discussion leads to the following code optimization criteria:

- Maximize code diversity $d_{H,\text{min}}$.

- Among the codes with maximum diversity, maximize the product distance $\delta^2(x, x')$ of pairs for which $d_H(x, x') = d_{H,\text{min}}$.

Codes optimized according to the above criteria (see [14, 10], do not necessarily yield a large minimum squared Euclidean distance, which is the classical criterion for good codes for the AWGN channel. On the other hand, optimal AWGN codes, such as Ungerboeck TCM schemes [15], perform poorly over the interleaved Rayleigh channel.

A way out from this *impasse* is obtained by giving up Ungerboeck's paradigm of combining coding and modulation in a single entity. In [16], schemes were designed in which coded modulation is generated by pairing an M-ary signal set with a binary convolutional code with the largest minimum free Hamming distance. Decoding was achieved by designing a metric aimed at keeping as its basic engine an off-the-shelf Viterbi decoder for the *de facto* standard, 64-state rate-1/2 convolutional code. Based on the latter concept, Zehavi [17] first recognized that the code diversity, and hence the reliability of coded modulation over a Rayleigh fading channel, could be further improved. Zehavi's idea was to make the code diversity equal to the smallest number of distinct *bits* (rather than *channel symbols*) along any error event. This is achieved by bit-wise interleaving at the encoder output, and by using an appropriate soft-decision bit metric as an input to the Viterbi decoder. One of Zehavi's findings, rather surprising *a priori*, was that on some channels there is a downside to combining demodulation and decoding. This prompted the investigation whose results are presented in a comprehensive fashion in [18] (see also [19]).

An advantage of this solution is its robustness, since changes in the physical channel affect the reception very little. Thus, it provides good performance with a Rayleigh fading channel as well as with an AWGN channel (and, consequently, with a Rice fading channel, which can be seen as intermediate between the latter two).

Recently, a very promising combination of bit-interleaved coded modulation with the concept of turbo-decoding [20, 21] has been discussed and analyzed.

3 Coding for delay-constrained channels

For delay-constrained systems, infinite interleaving is impossible. Code design for correlated fading is rather unpractical, since optimality criteria depend on the fading Doppler bandwidth, which in turns depends on the mobile speed. However, in many practical TDMA systems with slow frequency hopping (e.g., GSM), a code word is interleaved over a finite number M of TDMA bursts, transmitted over subcarriers separated by more than B_c. Then, even with very small Doppler, each block of symbols is affected by a different (independent) fading gain, constant over

the whole block. This fading channel model is called *block-fading* [6, 22].

In principle, we can imagine that a code word x of length $n = MN$ is partitioned into M blocks of length N. Let X be the $M \times N$ array obtained by writing the blocks of x by rows. The m-th block is sent over a constant fading channel with gain $a[m]$. Now, the fading vector affecting the transmission of x has length M, namely, $a = (a[1], \ldots, a[M])$. The output array corresponding to X is given by

$$Y = AX + Z \qquad (9)$$

where again $A = \text{diag}(a)$ and Z is a $M \times N$ array of comples Gaussian noise samples.

It is clear by comparing (3) and (9) that the i.i.d. channel and the block-fading channel are very similar, if we interpret X as a code word of length M, over a multidimensional signal set of dimensionality N. In particular, suppose that $x \in S^n$, where S is (say) a 1-dimensional complex signal set, such as QAM or PSK. Then, each row of the array X can be seen as a signal of the N-dimensional signal set S^N, obtained as the N-fold Cartesian product of S. The PEP analysis carried out for the i.i.d. fading case can be repeated here for the new N-dimensional signal set. In particular, for Rayleigh fading we find that the *code diversity* is the minimum Hamming block-distance, i.e., the minimum number of different rows between arrays X and X' corresponding to code words x and x' [23, 24]). Code constructions optimized for the block-fading channel are presented in [23].

An application of Singleton Bound [7] shows that the maximum block-Hamming distance achievable on an M-block fading channel is limited by

$$D \le 1 + \left\lfloor M \left(1 - \frac{R}{\log_2 |S|} \right) \right\rfloor$$

where $|S|$ is the cardinality of the signal set S and R is the code rate, expressed in bit/symbol. Note that binary signal sets ($|S| = 2$) are not effective in this case, so that codes constructed over high-level alphabets should be considered [23, 24]. As a matter of fact, bit-interleaved coded modulation behaves as a code constructed over a binary signal set. Then, it is not effective for the block-fading channel. For a deeper analysis of the relationship between code diversity and code rate, see [25, 26].

4 Diversity and power allocation techniques

The design procedure described in the sections above, and consisting of adapting the coding scheme to the channel, may suffer from a basic weakness. If the channel model is not stationary, as it is, for example, in a mobile-radio environment, then a code designed to be optimum for a fixed channel model might perform poorly when the channel varies. An alternative solution consists of doing the opposite, i.e., *matching*
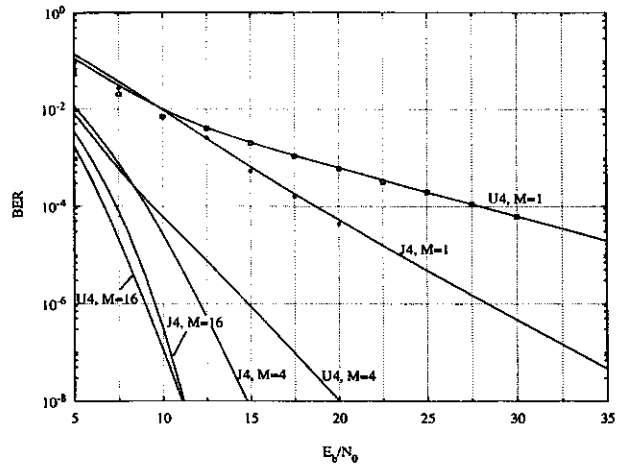

Figure 1: *Effect of antenna diversity on the performance of 4-state TCM schemes over the flat, independent Rayleigh-fading channel. J4 is optimum for the Rayleigh channel, while U4 is optimum for the Gaussian channel.*

the channel to the coding scheme: the latter is still designed for a Gaussian channel, while the former is transformed from fading channel (say) into an AWGN one. Here we shall examine two such robust solutions, the first based on antenna diversity and the second on feedback and dynamic power allocation.

4.1 Antenna diversity

In [8, 12, 27] the authors examined the synergy of coded modulation and antenna diversity reception with different detection techniques and interference conditions. It was shown that antenna diversity turns asymptotically a fading channel into an AWGN one, irrespectively of the fading statistics and correlation. Hence, codes optimized for the AWGN channel shall perform well also on fading channels with a sufficient amount of diversity.

In order to provide an example, consider the following PSK coded-modulation schemes:

J4: 4-state, rate-2/3 TCM scheme based on 8-PSK and optimized for Rayleigh-fading channels [10].

U4: 4-state rate-2/3 TCM scheme based on 8-PSK and optimized for the Gaussian channel.

Both schemes are coherently detected and maximal-ratio combined with perfect CSI. Fig. 1 compares the performance of U4 and J4 (two TCM schemes with the same complexity) over a Rayleigh-fading channel with M-branch diversity.

It is seen that, as M increases, the performance of U4 comes closer and closer to that of J4, and eventually U4 outperforms J4.

While antenna diversity reception is suited for the uplink of a wireless system, antenna diversity transmission is suited for the downlink. With transmission diversity, K antennas transmit at the same time and create interference at the receiver. The general channel model with block-fading, K transmitting antennas

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and M receiving antennas is still given by (9), where now $\mathbf{A}$ is a $M \times K$ matrix of fading gains, whose (i, j)-th element is the gain from transmitting antenna j to receiving antenna i, and where $\mathbf{X}$ is a $K \times N$ array formed by K blocks of length N of the transmitted code word written by rows.

Early schemes of transmission diversity were based on *repetition diversity*, i.e., where the same symbol is repeated by each antenna or delay-diversity, where copies of the same symbol were transmitted by the different antennas with some delay (see [28, 29]). More recently, code design criteria for transmission diversity have been proposed in [28] and information-theoretic analysis of transmission diversity schemes has been provided in [29, 30].

4.2 Dynamic power allocation

In some applications, the transmitter has information on the channel fading state. For example, CSI at the transmitter can be provided either by a dedicated feedback channel (some existing systems already implement a fast power control feedback channel) or by time-division duplex (TDD), where the uplink and the downlink time-share the same M subchannels and the fading gains can be estimated from the incoming signal. Then, the transmitter can allocate the transmitted power in order to compensate for the fading and keep the received SNR as constant as possible.

Consider the simplest such strategy over the flat block-fading channel. Assume that $\alpha[m] = |a[m]|^2$ is known at the transmitter. Then, an energy per symbol $E_s(\alpha[m]) = E_s/\alpha[m]$ can be allocated to the m-th block, in order to keep the received SNR constant. This way, the channel is transformed into an equivalent additive white Gaussian noise channel. The error probability is the same as if we had transmitted over a channel whose only effect is the addition of Gaussian noise. The average transmitted energy per symbol is then

$$E[|x|^2] = E_s E[1/\alpha[m]], \qquad (10)$$

Unfortunately, the above quantity might be infinite (e.g., in the case of Rayleigh fading). This means that this technique ("channel inversion") is simple, but not practical.

To avoid divergence of the average power (or an inordinately large value thereof) a possible strategy is the following. Choose

$$E_s(\alpha) = \begin{cases} E_s/\alpha & \text{if } \alpha > \alpha_0 \\ 0 & \text{otherwise.} \end{cases} \qquad (11)$$

By choosing appropriately the value of the threshold α_0 we trade a decrease of the average power value for an increase of error probability. Notice that if $\alpha < \alpha_0$, then transmission would cost too much in terms of energy and it is better to declare an "outage event" and save energy for more favorable channel conditions. Also, in a multiuser system this approach decreases the overall level of interference, since there will be a

fraction $P(\alpha < \alpha_0)$ of users not transmitting. A comprehensive information-theoretical analysis of power-control techniques for the block-fading channel is provided in [31].

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