Two-layer Precoding for Dimensionality Reduction in Massive MIMO

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***Abstract*—Massive MIMO (multiple-input multiple-output) is a promising technology for the upcoming 5G as it provides significant beamforming gains and interference reduction capabilities due to the large number of antennas. However, massive MIMO is computationally demanding, as the high antenna count results in high-dimensional matrix operations when conventional MIMO processing is applied. In this paper, we focus on two-stage digital beamforming, where the beamformer is split into a slow-varying statistics-based outer beamformer and an inner beamformer accounting for fast channel variations. We formulate two two-stage precoding optimization problems: weighted sum-rate maximization and minimum user rate maximization for a single-cell downlink system. We also provide different heuristic methods of forming the outer precoder matrix via user channel covariance, while the inner precoder is obtained as a result for the optimization problem. Unlike most previous work, which consider the outer precoder design based on energy maximization and user group location, our aim is to design it to offer a tradeoff between energy maximization and interference reduction, and also take into account the fairness between users. We evaluate the performance of the different heuristic methods as a function of the number of statistical pre-beams and a fixed user angular spread to see the overall effect of complexity reduction on the system sum-rate and minimum user rate. We also evaluate the advantages of different methods in terms of user fairness.**

I. Introduction

Massive MIMO has been regarded as one of the most promising technologies for the upcoming fifth generation (5G) cellular systems [1], [2], [3]. While traditional MIMO systems can house few antennas in the transmitter or receiver, massive MIMO can have tens, even hundreds of antennas. The increase in antenna count results in greatly increased degrees of freedom (DoF) and beamforming gains, opening up possibilities for increased data rates, diversity and reliability with reduced interference. At very high antenna numbers, the simpler signal processing methods such as maximal ratio transmission/combining (MRT/MRC) and zero-forcing also become near optimal [1].

Massive MIMO can provide significant gains and benefits in terms of throughput, energy efficiency and interference management. However, due to the greatly increased antenna count, conventional MIMO processing quickly becomes too computationally prohibitive due to the high-dimensional matrix operations in the precoders and decoders. In order to utilize conventional receiver and transmitter processing, such as minimum mean square error (MMSE), zero-forcing and regularized zero-forcing (RZF), the computation of matrix inverses based on the high dimensional channel matrix are necessary. The conventional methods with high dimensional channel also require accurate channel-state information (CSI) and thus, the CSI acquisition can get cumbersome with higher number of antennas.

The complexity reduction of massive MIMO systems has gained significant interest in the research community, both in terms of hybrid analog/digital beamforming and fully digital beamforming. The hybrid beamformer is a concatenation of an analog outer beamformer implemented with analog radio frequency (RF) components and a digital inner beamformer. The analog part forms pre-beams (analog beams for different spatially separated users/streams) and reduces the effective channel dimension, since there are less RF-chains than there are antennas. The digital beamformer can then multiplex the transmitted data streams on the effective channel with reduced dimension, i.e., reduced complexity. This structure can support as many data streams as there are RF-chains in the transceiver.

The focus of this paper is on fully digital two-layer beamforming in massive MIMO setting. Generally, the idea is the same as with hybrid analog/digital beamforming. In digital two-layer beamforming, an outer beamformer forms pre-beams for different users or propagation paths, effectively reducing the channel dimension by accounting only for the strongest paths. Then, an inner beamformer applies spatial multiplexing on the effective channel of reduced dimension. The effective channel also reduces the amount of coefficients in channel estimation and due to the statistical beams, the pilot SNR is increased, resulting in a more accurate channel estimate. The statistics-based outer precoder varies over long time scales compared to the inner precoder and thus requires less frequent updates.

One of the most notable current methods of fully digital two-stage (or two-layer) beamforming is the joint spatial division and multiplexing (JSDM), coined by Nam et al. in [4]. JSDM exploits the similar channel covariance eigenspaces of co-located users when forming the outer beamformer, resulting in slow-varying pre-beams based on the channel statistics. The method also exploits the fact that if the user groups are sufficiently well separated in the angle of arrival (AoA)

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domain, the inner beamformer can be made block diagonal reducing the complexity even further. The JSDM method is described in detail in [5] with various ways of forming the outer beamformer, for example using the eigenvectors of the per-group covariance matrices as outer beamformers. The article also provides performance analysis using the techniques of deterministic equivalents for different types of group processing, namely joint group and per-group processing (JGP and PGP). The concept of JSDM is extended further in [6], in which the authors consider the finite antennas regime and user grouping via minimum chordal distance. The article also considers opportunistic user selection and user scheduling. In [7] and [8] the user grouping problem is researched further, utilizing Fubini-Study distance in the former and weighted likelihood in the latter to enhance the grouping performance.

In addition to JSDM, other two-stage or multi-stage beamforming techniques have also been suggested. In [9], the authors consider interference mitigation with two-stage precoding, where the outer and inner beamformers are used to control the inter- and intra-cell interferences, respectively. The precoder design is formulated as a joint optimization of the outer precoders, user selection and power control. Furthermore, in [10] the precoder design is approached with a three-layer design in 3D channels, exploiting the low rank of the elevation covariance matrix. The three layers are utilized to mitigate inter-cell interference, provide the best possible signal level and spatially multiplex the transmit data for different users.

In this paper, we formulate weighted sum-rate maximization (WSRM) and minimum user rate maximization problems for a single-cell system utilizing two-stage beamforming. Following the approaches of [11] and [12] for WSRM and [13] for the minimum user rate, the problems are formulated as successive second order cone programs (SSOCP), which aim to optimize the inner precoder according to the fast channel variations with a given fixed outer precoder, i.e., the effective channel. The channel model used in this paper is the classical uniform linear array (ULA) model of [14], but extension to uniform planar arrays (UPA) is also straightforward. We propose different heuristic methods of constructing the outer precoder based on the total channel covariance and individual user covariance matrices, offering tradeoffs between sum-rate and user fairness. The different methods include:

- Choosing the eigenvectors corresponding to the strongest eigenvalues of the total channel covariance matrix.
- Choosing the total channel covariance eigenvectors providing the best match with user covariance matrices, one user at a time.
- Choosing the total channel covariance eigenvectors providing the best match with user covariance matrices among all users.

Unlike previous work in outer precoder design, which mostly focus on energy maximization based on user location and interference mitigation among different groups, our focus is to design the outer precoder based on long-term channel statistics to offer a tradeoff between energy maximization and interference mitigation between users. We also provide a design that aims to allocate statistical beams to all users, providing fairness between users by increasing individual user SNR. Using the different heuristic tactics, we construct the outer precoder and simulate the rate performance of the system as a function of the outer precoder dimension, i.e., the number of statistical pre-beams. This provides us insight into the tradeoff between dimensionality (complexity) reduction and both system sum-rate and minimum user rate. We also compare the results achieved with zero-forcing precoding and precoder optimization when formulating the inner precoder. The simulation results show that the outer precoder dimension can be reduced significantly without greatly impacting the achieved rates. Differences between various outer precoder formulation techniques are visible in the low-dimensional case, especially in the case of maximizing the minimum user rate with the fair matching mehtod.

II. System Model

We consider a downlink single-cell multi-user massive MIMO system, where a single base station (BS) with M transmit antennas serves K single-antenna user terminals (UT) with $M > K$. The received signal at user k can be expressed as

$$y_k = \mathbf{h}_k^{\mathrm{H}} \mathbf{v}_k x_k + \sum_{i \neq k} \mathbf{h}_k^{\mathrm{H}} \mathbf{v}_i x_i + n_k, \tag{1}$$

where the first term is the desired signal and the second term represents intra-cell interference. The channel between the base station and user k is denoted by $\mathbf{h}_k \in \mathbb{C}^{M \times 1}$, while $\mathbf{v}_k \in \mathbb{C}^{M \times 1}$ denotes the precoding vector of user k. The transmitted data symbol for user k is denoted by x_k with $\mathbb{E}[|x_k|^2] = 1$, $\forall k$. n_k represents the zero-mean white Gaussian noise at the receiver with variance N_0. The precoding is applied in two stages as $\mathbf{V} = \mathbf{BW}$, where $\mathbf{V} \in \mathbb{C}^{M \times K}$ is the total precoding matrix of all users, $\mathbf{B} \in \mathbb{C}^{M \times S}$ is the outer precoder based on slow-varying channel statistics and $\mathbf{W} \in \mathbb{C}^{S \times K}$ is the inner precoder applying multi-user processing based on the effective channel $\bar{\mathbf{H}} = \mathbf{H}^{\mathrm{H}}\mathbf{B}$ of dimensions $K \times S$. Here S is a design parameter describing the amount of statistical pre-beams used in the transmission.

The channel vector $\mathbf{h}_k$ is modeled as the classical multipath model for uniform linear arrays [14]:

$$\mathbf{h}_k = \frac{\beta_k}{\sqrt{L}} \sum_{l=1}^{L} \mathbf{a}(\theta_{k,l}) e^{j\phi_{k,l}}, \tag{2}$$

where β_k denotes the path loss between the base station and user k, L denotes the number of independent (and identically distributed, i.i.d) paths, $\phi_{k,l}$ is a random phase caused by the channel for path l, i.i.d. between different paths, and $\mathbf{a}(\theta)$ is the array signature vector given by

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi \frac{D}{\lambda} \cos(\theta)} \\ \vdots \\ e^{-j2\pi \frac{(M-1)D}{\lambda} \cos(\theta)} \end{bmatrix}, \tag{3}$$

where D is the BS antenna spacing, λ is the carrier wavelength and θ is the angle of departure (AoD). The user-specific channel correlation matrix can be defined as $\mathbf{R}_k = \mathbb{E}[\mathbf{h}_k\mathbf{h}_k^{\mathrm{H}}]$, and the sum of these determine the total channel correlation matrix $\mathbf{R} = \sum_k \mathbf{R}_k$.

Accounting for the two-stage precoding, the signal-to-interference-noise ratio (SINR) of user k can be expressed as

$$\gamma_k = \frac{\left|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}\mathbf{w}_k\right|^2}{\sum\limits_{i\neq k}\left|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}\mathbf{w}_i\right|^2 + N_0}. \quad (4)$$

Utilizing the SINR expression above, we can determine the weighted sum-rate of the system as

$$R = \sum_k \alpha_k \log_2(1+\gamma_k), \quad (5)$$

where $\alpha_k \geq 0$ is a user specific weight coefficient that can be determined with user scheduling.

III. Precoder Design

The joint optimization of $\mathbf{B}$ and $\mathbf{W}$ is highly complex due to their different variation time-scales. In this paper, we split the precoder design by considering the outer and inner precoders separately, starting with various heuristic methods of constructing the outer precoder $\mathbf{B}$. Furthermore, when optimizing the inner precoder, $\mathbf{B}$ is assumed to be fixed as it is based on the slow-varying channel statistics.

A. Outer precoder design

Let us first consider the outer precoding matrix $\mathbf{B}$. The outer precoder is based on slow-varying channel statistics, i.e., the user covariance matrices. Therefore, one obvious solution for the formulation of $\mathbf{B}$ is to decompose the total channel covariance matrix via eigenvalue decomposition (EVD) as $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathrm{H}}$ and choose S channel covariance eigenvectors corresponding to the S largest eigenvalues. This results in the outer precoder $\mathbf{B} = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_S] \in \mathbb{C}^{M\times S}$, which effectively forms pre-beams towards the strongest signal paths. We denote this selection method *eigen selection* (ES). The downside of this method is that it neglects user fairness and can be expected to prioritize users closer to the base station that have stronger signal paths and larger angular spreads which overlap with more statistical beams.

We can also adopt a more greedy approach by constructing the outer precoder one vector at a time from the total channel covariance eigenvectors $\mathbf{U} = [\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_M]$. We select the beamformers in $\mathbf{B}$ that maximize the per-user matching metric

$$\mathbf{u}_i = \underset{i,k}{\operatorname{argmax}}(\mathbf{u}_i{}^{\mathrm{H}}\mathbf{R}_k\mathbf{u}_i), \quad (6)$$

and exclude any selected eigenvector from future selections. This results in an outer precoder matrix $\mathbf{B}$ with orthogonal columns that covers the best per-user signal paths. This method, which we denote as *greedy matching* (GM), is very similar to ES, which collects the globally best signal paths. The fairness of the aforementioned method can be improved by also excluding the covariance matrix of user k that resulted in the best beamformer in (6) from future selections. After all the covariance matrices have been used once in the selection process, they can be included again in future selections. As a result, the outer precoder $\mathbf{B}$ will have orthogonal columns that cover all users, provided $S \geq K$. This *fair matching* (FM) method is not necessarily sum-rate optimal but improves the overall fairness between users.

B. Inner precoder design

The inner precoder design is formulated as a successive second-order cone program (SSOCP). For the WSRM, we mimic the precoder design of [11], which considered a multi-cell system, and for minimum user rate maximization we use a similar approach to [13]. We follow similar reasoning as in [11] in linearizing the non-convex constraints present in this design and we also account for the outer precoding matrix $\mathbf{B}$, which has an effect on the SINR expression and the power constraint. Overall, the precoder design can be cast as the following WSRM problem:

$$\begin{aligned} \underset{\mathbf{w}_k}{\text{maximize}} \quad & \prod_k (1+\gamma_k)^{\alpha_k} \\ \text{subject to} \quad & \sum_k \|\mathbf{B}\mathbf{w}_k\|_2^2 \leq P_{\text{tot}}, \end{aligned} \quad (7)$$

or as the following SINR-rate balancing problem:

$$\begin{aligned} \underset{\mathbf{w}_k}{\text{maximize}} \quad & \min_k(\gamma_k) \\ \text{subject to} \quad & \sum_k \|\mathbf{B}\mathbf{w}_k\|_2^2 \leq P_{\text{tot}}, \end{aligned} \quad (8)$$

where the total power constraint is denoted with P_{tot}. We consider problem (7) first and make a simple modification so it can solve (8). Alternatively, the SINR-rate balancing problem can be solved optimally via combination of power minimization and bisection as in [13], which essentially gives the same result as our proposed method.

Starting from (7), we can omit the logarithm as a monotonically non-decreasing function. Denoting the objective as $t_k = (1+\gamma_k)^{\alpha_k}$ and introducing a slack variable β_k for the denominator of the SINR expression, we can reformulate the weighted sum-rate maximization problem as

$$\begin{aligned} \underset{t_k,\beta_k,\mathbf{w}_k}{\text{maximize}} \quad & \prod_k t_k \\ \text{subject to} \quad & \frac{|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}\mathbf{w}_k|^2}{\beta_k} \geq t_k^{1/\alpha_k} - 1, \qquad \forall k, \\ & \sum_{i\neq k} |\mathbf{h}_k^{\mathrm{H}}\mathbf{B}\mathbf{w}_i|^2 + N_0 \leq \beta_k \quad \forall k, \\ & \sum_k \|\mathbf{B}\mathbf{w}_k\|_2^2 \leq P_{\text{tot}}. \end{aligned} \quad (9)$$

The left-hand side (LHS) of the first constraint in (9) is convex (quadratic over linear) and on the right-hand side (RHS), t_k^{1/α_k} is convex for $0 < \alpha_k \leq 1$ and concave when $\alpha_k > 1$. Therefore, linearization is required for both sides, or with proper scaling of α_k, only for the LHS. The last constraint for the total power is convex, and the second constraint can be expressed in an SOC form as

$$\left(\sum_{i\neq k}\left|\mathbf{h}_k^{\mathrm{H}}\mathbf{B}\mathbf{w}_i\right|^2 + (\sqrt{N_0})^2 + \frac{1}{4}(\beta_k-1)^2\right)^{\frac{1}{2}} \leq \frac{1}{2}(\beta_k+1), \forall k, \tag{10}$$

exploiting the fact that a hyperbolic constraint of the form $z^2 \leq xy$ can be expressed as $\|2z,(x-y)^{\mathrm{T}}\|_2 \leq (x+y)$, where $x, y \in \mathbb{R}_+$ [15].

To linearize the LHS and RHS of the first constraint, we follow the steps taken in [11], [12] by first dividing the LHS numerator into its real and imaginary parts as

$$p_k \triangleq \mathrm{Re}(\mathbf{h}_k^{\mathrm{H}}\mathbf{B}\mathbf{w}_k) \text{ and } q_k \triangleq \mathrm{Im}(\mathbf{h}_k^{\mathrm{H}}\mathbf{B}\mathbf{w}_k), \tag{11}$$

and taking the first order Taylor expansion of $(p_k^2+q_k^2)/\beta_k$ around the local point $\{\tilde{p}_k, \tilde{q}_k, \tilde{\beta}_k\}$, $\forall k$. As a result, the first constraint can be expressed as

$$1+\frac{2\tilde{p}_k}{\tilde{\beta}_k}(p_k-\tilde{p}_k)+\frac{2\tilde{q}_k}{\tilde{\beta}_k}(q_k-\tilde{q}_k) + \frac{\tilde{p}_k^2+\tilde{q}_k^2}{\tilde{\beta}_k}\left(1-\left(\frac{\beta_k-\tilde{\beta}_k}{\tilde{\beta}_k}\right)\right) \geq t_k^{1/\alpha_k}. \tag{12}$$

We also take the first order approximation for the RHS around the local point $\tilde{t}_k$, and obtain

$$t_k^{1/\alpha_k} \leq \tilde{t}_k^{1/\alpha_k} + \frac{1}{\alpha_k}\tilde{t}_k^{(1/\alpha_k)-1}(t_k-\tilde{t}_k), \tag{13}$$

and thus, the first constraint in (9) can be written as

$$1+\frac{2\tilde{p}_k}{\tilde{\beta}_k}(p_k-\tilde{p}_k)+\frac{2\tilde{q}_k}{\tilde{\beta}_k}(q_k-\tilde{q}_k)+\frac{\tilde{p}_k^2+\tilde{q}_k^2}{\tilde{\beta}_k}\times \left(1-\left(\frac{\beta_k-\tilde{\beta}_k}{\tilde{\beta}_k}\right)\right) \geq \tilde{t}_k^{1/\alpha_k} + \frac{1}{\alpha_k}\tilde{t}_k^{(1/\alpha_k)-1}(t_k-\tilde{t}_k). \tag{14}$$

After linearizing the first constraint of (9), fixing the outer beamformer $\mathbf{B}$, and transforming the second constraint into an SOC form, we can summarize the sum-rate maximization problem as a geometric mean maximization which can be formulated as an SOCP:

$$\begin{aligned} \underset{t_k,\beta_k,\mathbf{w}_k}{\text{maximize}} \quad & \left(\prod_{k=1}^{K} t_k\right)^{1/K} \\ \text{subject to} \quad & (14), (10) \\ & \sum_k \|\mathbf{B}\mathbf{w}_k\|_2^2 \leq P_{\text{tot}}, \end{aligned} \tag{15}$$

which can be solved iteratively by updating the local point $\{\tilde{p}_k, \tilde{q}_k, \tilde{\beta}_k\}$ after each optimization of (15) until convergence. The formulation of the objective into geometric mean has no effect on the optimal value. Also, as the precoder design optimization problem is non-convex, global optimality is not guaranteed.

The modification into minimum user rate maximization is straightforward. We change the objective in (15) to maximize a minimum SINR level r and add extra per-user constraints $t_k \geq r, \forall k$. This formulation aims to always maximize the weighted SINR of the weakest user. The rest of the constraints are same and can be linearized in a similar manner as the WSRM, and the formulation provides the same optimal performance as the combination of power minimization and bisection in [13].

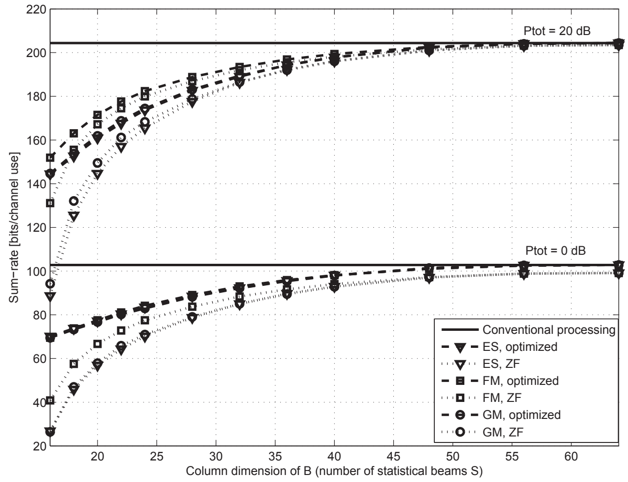

Fig. 1. System sum-rate as a function of the outer precoder dimension for different methods of choosing the outer precoder.

IV. Numerical Results

The simulation setup is as follows: We consider a single-cell case with 16 single-antenna users served by a base station with 64 antennas. The angular spread of the users is 15 degrees with 20 independent paths per user. The user weights are set to $\alpha_k = 1, \forall k$. The simulations are performed as a function of the outer precoder dimension S (i.e., number of statistical beams) and the results are averaged over 100 channel iterations where both precoders are updated for each iteration.

For our first numerical example, we assume that all users are randomly distributed around the base station with various path gains and investigate the tradeoff between sum-rate and outer precoder dimension S for three different transmit power cases, $P_{\text{tot}} = \{0, 20\}$ dB. The user path gains are uniformly distributed in range [0,20] dB to account for stronger users and still ensure a sufficient SNR for all users. We also plot the results where the inner precoder is obtained via ZF for comparison. The results of this simulation are presented in Fig. 1. We can immediately see that ZF has worse performance than our optimized case for all different methods of constructing the outer precoder. It is also apparent that the outer precoder dimension can be greatly reduced without a significant impact on the system sum-rate for all different cases. In terms of different outer precoder construction methods, the FM method performs slightly better than the others on high SNR but has similar performance on low SNR. The users close to the base station are served by a higher number of statistical beams in the ES and GM cases at the expense of the distant users. At high SNR, serving all users results in a higher total sum-rate due to the diminishing returns in the rates when only serving the near users.

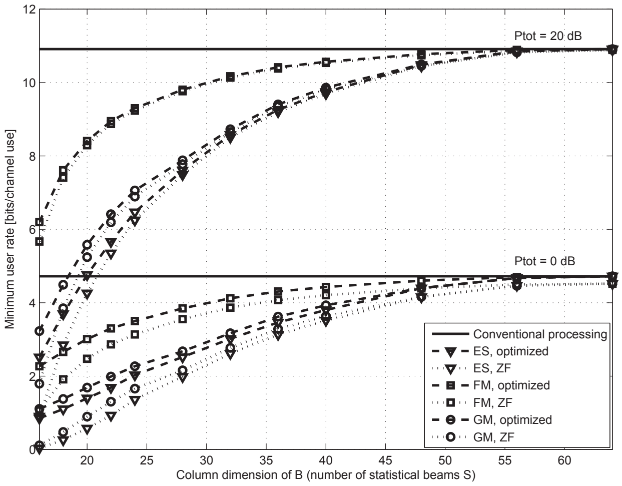

Fig. 2. Minimum user rate as a function of the outer precoder dimension for different methods of choosing the outer precoder.

Our second example considers the minimum user rate maximization, using the same transmit powers and path gains as in the first simulation, as a function of the outer precoder dimension S. The aim is to evaluate the performance of the different heuristic methods in a case that maximizes the per-user quality-of-service. The simulation results for minimum per-user rates are presented in Fig. 2. We can see that the method of constructing $\mathbf{B}$ has a significant effect with lower column dimensions, where utilizing the FM method results in the best minimum user rates. This is caused by selecting at least one statistical beam for all users, resulting in increased user SNR. Greedy matching also has a slight performance gain compared to eigen selection.

V. Conclusions

In this paper, we formulated two optimization problems for a single-cell system utilizing two-stage precoding. The optimization problems considered were weighted sum-rate maximization and minimum user rate maximization, focusing on optimizing the inner precoder with a fixed outer precoder. We also proposed different heuristic methods of constructing the outer precoder to provide either a good coverage of the strongest signal paths or fairness between users. The performance of these different heuristic methods was evaluated for both optimization cases as a function of the outer precoder column dimension to provide insight into the tradeoff between sum-rate or minimum user rate and the dimension reduction of the effective channel. The numerical results show that the outer precoder dimension can be greatly reduced without a significant impact on the data rates. The results also show that at low effective channel dimensions, the fair matching method results in the same or slightly better sum-rates, while the minimum user rate is significantly improved. Also at low dimensions, optimizing the inner precoder results in better data rates than using zero-forcing. When the column dimension of the outer precoder is high, however, all the different methods provide essentially the same sum-rate and minimum user rate. In this regime, optimizing the inner precoder is computationally intensive, so zero-forcing is preferred to reduce complexity.

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