MIMO OFDM Capacity Maximizing Beamforming for Large Doppler Scenarios

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***Abstract*—Performance of OFDM systems is limited by inter carrier interference (ICI) under high Doppler scenarios such as that encountered in high speed trains such as TGV. Several publications have addressed the receiver design for SISO (single input single output) and SIMO (single input multiple output) to combat ICI. Notably, the use of multiple receive antennas is a very effective way to combat ICI. In this paper, we consider a MIMO (multiple input multiple output) scenario and iteratively design a transmit beamformer to maximize the sum capacity across all the subcarriers in the presence of ICI. Our interest lies in analyzing the impact of excess of transmit (Tx) antennas in combating ICI. A time varying frequency selective channel is considered. In particular, we consider a linear model for the channel variation across the OFDM symbol as we focus on LTE(long term evolution)-like systems and train velocities up to 450kmph. As the original cost function is non-convex, we first follow the difference of convex functions (DC) approach to obtain a convex cost function. However, we later reinterpret the DC approach as a majorization technique. To solve the joint problem of power allocation across subcarriers and precoder design for all the useful subcarriers, we employ the cyclic minimization approach to alternately optimize the precoder design and transmit power allocation across subcarriers. The convergence of the iterative approach is proved and the theory is validated via numerical simulations.**

I. INTRODUCTION

As is well known, high Doppler encountered in HST (high speed train) environments violates the orthogonality requirement for OFDM (Orthogonal Frequency Division Multiplexing), resulting in ICI. The SINR (signal to interference plus noise ratio) analysis due to ICI can be found in ([1],[2]). Several prior publications have focused on receiver techniques to mitigate ICI. It is known that multiple receive antennas in a SIMO scenario is very effective in cancelling out the ICI (for example, see [3]). In this paper, we focus on a MIMO scenario and derive the channel capacity in the presence of ICI caused by channel variation. Our interest lies in analyzing the impact of excess of antennas in combating ICI. Specifically, for LTE systems, due to the large subcarrier spacing, we consider a linear channel variation as was done in ([4], [5], [3]).We consider a frequency selective scenario and iteratively arrive at the optimal beamformer for every subcarrier. The algorithm can also easily account for the presence of the guard and DC subcarriers to model a realistic transmission scenario. To solve the joint problem of power allocation across subcarriers and precoder design for all the useful subcarriers, we employ the cyclic minimization([6]) approach to alternately optimize the precoder design and transmit power allocation across subcarriers. As a first step, the DC convex approach ([7]) is used to get a convex cost function. Alternatively, we also interpret this as a minorization([6]) of the original cost function. Using this approach, the beamformer directions for every subcarrier is readily obtained as generalized eigen vectors. Once the beamformer directions are obtained, the power allocations across the MIMO streams of all the subcarriers are jointly derived. The convergence of the proposed methodology is also shown. The main contributions of this paper are as follows

- We design an ICI-aware beamformer for a MIMO OFDM system under FIR (finite impulse response) multipath channel that can also take into account guard subcarriers and DC subcarrier.
- To solve the joint problem of power allocation across subcarriers and precoder design for all the useful subcarriers, we employ the cyclic minimization approach to alternately optimize the precoder design and transmit power allocation across subcarriers.
- We reinterpret the difference of convex approach as a majorization technique
- The convergence of the entire beamformer design including the power allocation is proved and then shown numerically via simulations.

The rest of the paper is organized as follows. We first present the system model in Section II. This is followed by the design of the beamformer in section III. Simulation results are presented for different scenarios in section IV. Finally, conclusions are given in section V. In the following discussions, a bold notation in small letters indicates a vector and bold notation with capital letters indicates a matrix. We use the abbreviation DC to refer to both the DC subcarrier as well as the difference of convex approach.

II. SYSTEM MODEL

Consider a multiple input multiple output (MIMO) system with N_t transmit antennas and N_r receive antennas. An OFDM framework is chosen with N subcarriers and sampling rate f_s. Out of the total N subcarriers, let N_u be the number of utilized subcarriers. For instance, this would account for the guard subcarriers and DC subcarrier in an OFDM system.Let P be the maximum sum power requirement across all the subcarriers and let P_i be the individual power at any subcarrier i such that $\sum_{i=0}^{N-1} P_i = P$. We consider a time-varying Rician fading FIR channel. Thus for every combination of Tx(transmit) and Rx(receive) antenna, the time domain channel at sample n of an OFDM symbol may be represented as

$$\mathbf{h}(n) = \mathbf{h}_0 + \mathbf{h}^{'}(n) \quad (1)$$

where $\mathbf{h}_0$ represents the average channel across the OFDM symbol and $\mathbf{h}^{'}$ captures the time variation,has average value zero, and is orthogonal to $\mathbf{h}_0$. It is easy to see that with this formulation, the ICI contribution comes entirely from $\mathbf{h}^{'}(n)$ (see [3] for example). To continue the analysis, we can approximate $\mathbf{h}^{'}(n)$ by a polynomial function. For instance, a linear model is considered in [4], [5]. For an LTE-like OFDM system, we also choose a linear model due to the significant subcarrier spacing compared to the Doppler frequency being considered. Thus, for the duration of an OFDM symbol, (1) may be rewritten as

$$\mathbf{h}(n) = \mathbf{h}_0 + (n - \frac{N-1}{2})\mathbf{h}_1 \tag{2}$$

where $\mathbf{h}_1$ is a constant across the OFDM symbol and captures the time variation per sample.

Upon taking the N-point FFT, the frequency domain representation of carrier index k across the N_r receive antennas would become

$$\mathbf{y}_k = \mathbf{H}_{0k}\mathbf{d}_k + \sum_{l=0, l\neq k}^{N-1} \mathbf{H}_{1l}\mathbf{d}_l \Xi_{k,l} + \hat{\nu}_{\mathbf{k}} \tag{3}$$

$\mathbf{H}_{0k}$ (dimension $N_r \times N_t$) is the mean frequency domain channel observed at subcarrier k. The second term in equation (3) represents the ICI (inter carrier interference) caused by time variance due to Doppler. $\mathbf{H}_{1k}$ is the frequency domain channel component corresponding to $\mathbf{h}_1$ at subcarrier k, $\mathbf{d}_k = [d_k(1) \cdots d_k(N_t)]^T$ is the $N_t \times 1$ vector of transmitted data symbols on the carrier k. $\hat{\nu}_{\mathbf{k}}$ is the $N_r \times 1$ vector of AWGN (additive white Gaussian noise) noise observed at carrier index k. The variance of $\hat{\nu}_{\mathbf{k}}$ is normalized to be unity.

$$\Xi_{k,l} = \frac{1}{N}\sum_{n=0}^{N-1}\left(n - \frac{N-1}{2}\right)e^{j2\pi(k-l)\frac{n}{N}} \tag{4}$$

Let the transmit covariance matrix of subcarrier k be $\mathbf{Q}_k = \mathtt{E}(\mathbf{d}_k\mathbf{d}_k^H)$ where $\mathtt{E}(\cdot)$ is the expectation operator. Thus, the capacity of this MIMO system across all the subcarriers in the presence of both ICI and AWGN noise would be given as follows.

$$\mathbf{C} = \sum_{k=0}^{N-1}\log|\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}| \tag{5}$$

where $\bar{\mathbf{R}}_k = \mathbf{I} + \sum_{l=0,l\neq k}^{N-1}|\Xi_{k,l}|^2\mathbf{H}_{1l}\mathbf{Q}_l\mathbf{H}_{1l}^H$. Note that this formulation can include guard subcarriers and DC subcarrier by simply forcing their respective transmit covariances to zero.

We are interested in determining the optimal $\mathbf{Q}_k$ such that the capacity of the link is maximised under a power constraint

$$\begin{aligned} \mathtt{f}_0 : \max_{\mathbf{Q}_k}\mathbf{C} &= \max_{\mathbf{Q}_k}\sum_{k=0}^{N-1}\log|\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}| \\ \mathtt{subject\ \ to} \quad &\sum_{k=0}^{N-1}\mathtt{tr}\{\mathbf{Q}_k\} \leq P. \end{aligned} \tag{6}$$

III. Beamformer Design

The objective function $\mathtt{f}_0$ in (6) is non-convex in the covariance matrix $\mathbf{Q}_i$ and hence we follow an iterative approach. In addition, to solve the joint problem of power allocation across subcarriers and precoder design for all the useful subcarriers, we employ the alternating (cyclic) minimization approach to alternately optimize the precoder design and transmit power allocation across subcarriers. At the beginning of the iteration for the subcarrier i, let P_i be the power constraint and let $\bar{\mathbf{Q}}_i$ be the current values of the precoder. The objective is now two-fold,

- Update the value of $\mathbf{Q}_i$ for every used subcarrier i. This is done similar to ([7], [8]), but we later interpret this as a majorization technique ([6]). This is given in subsection III-A.
- Updation of power allocation across all the subcarriers. This is given in subsection III-B.

A. Covariance matrix update

Our iterative optimization algorithm operates one subcarrier at a time. With focus on subcarrier i, on the same lines as [7], the objective function $\mathtt{f}_0$ may be rewritten as

$$\begin{aligned} &\max_{\mathbf{Q}_i}\sum_{k=0}^{N-1}\log|\mathbf{I} + \mathbf{H}_{0k}\mathbf{Q}_k\mathbf{H}_{0k}^H\bar{\mathbf{R}}_k^{-1}| \\ &\quad = \max_{\mathbf{Q}_i}\{\log|\mathbf{I} + \mathbf{H}_{0i}\mathbf{Q}_i\mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}| + f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})\} \end{aligned} \tag{7}$$

where $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i}) = \sum_{l\neq i}\log|\mathbf{I} + \mathbf{H}_{0l}\mathbf{Q}_l\mathbf{H}_{0l}^H\bar{\mathbf{R}}_l^{-1}|$. $\mathbf{Q}_{-i}$ refers to the transmit covariances of all the subcarriers except the i^{th}. It is shown in [7] ($Lemma \quad 1$) that $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})$ is convex in $\mathbf{Q}_i$.Thus, equation (7) is the sum of a concave and convex function and hence the overall capacity is a non-convex function.

As in [7], we replace the convex function $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})$ by the linear term in it's Taylor series expansion evaluated at $\bar{\mathbf{Q}}_i$ and $\bar{\mathbf{Q}}_{-i}$ (current value of the covariance matrices $\mathbf{Q}_i$ and $\mathbf{Q}_{-i}$). However, we also add an additional constant term $f_i(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i})$ which is the value of $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})$ evaluated at $\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}$. Thus effectively, we replace a convex function $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})$ by it's tangent at $\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}$. This insight is used to give an alternative interpretation of this step in Proposition 1. We thus construct an alternative convex sub-problem to $\mathtt{f}_0$ at each subcarrier i.

$$\begin{aligned} \mathtt{f}_1 : \log|\mathbf{I} + \mathbf{H}_{0i}\mathbf{Q}_i\mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}| - \mathtt{tr}\left\{\mathbf{B}_i(\mathbf{Q}_i - \bar{\mathbf{Q}}_i)\right\} + \\ f_i(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}) \\ \mathtt{subject\ \ to} \quad \mathtt{tr}\{\mathbf{Q}_i\} \leq \bar{P}_i \end{aligned} \tag{8}$$

where $\mathbf{B}_i$ is the negative Hermitian of the derivative of $f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})$ with respect to $\mathbf{Q}_i$ evaluated at $\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}$. $\bar{P}_i$ indicates the current value of P_i at any give stage of the algorithm. $\mathbf{B}_i$ is given in equation (9) below (see also [9]).

$$\mathbf{B}_i = -\left[\frac{\partial f_i(\mathbf{Q}_i, \mathbf{Q}_{-i})}{\partial \mathbf{Q}_i}\right]^H = \sum_{l \neq i} |\Xi_{l,i}|^2 \mathbf{H}_{1i} \left\{\bar{\mathbf{R}}_l^{-1} - (\bar{\mathbf{R}}_l + \mathbf{H}_{0l}\mathbf{Q}_l\mathbf{H}_{0l}^H)^{-1}\right\} \mathbf{H}_{1i}^H \quad (9)$$

The Lagrangian for $\mathtt{f}_1$ may now be written as

$$\mathbf{L}(\mathbf{Q}_i, \mu_i) = \log|\mathbf{I} + \mathbf{H}_{0i}\mathbf{Q}_i\mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}| - \mathtt{tr}\left\{\mathbf{B}_i(\mathbf{Q}_i - \bar{\mathbf{Q}}_i)\right\} + f_i(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}) - \mu_i(\mathtt{tr}\{\mathbf{Q}_i\} - \bar{P}_i) \quad (10)$$

where $\mu_i \geq 0$.

The term $\log|\mathbf{I}+\mathbf{H}_{0i}\mathbf{Q}_i\mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}|$ is concave in $\mathbf{Q}_i$. As $\mathbf{B}_i$ is a constant, $\mathtt{tr}\left\{\mathbf{B}_i(\mathbf{Q}_i - \bar{\mathbf{Q}}_i)\right\}$ is an affine function. Thus $-\mathtt{tr}\left\{\mathbf{B}_i(\mathbf{Q}_i - \bar{\mathbf{Q}}_i)\right\}$ and $-\mu_i(\mathtt{tr}\{\mathbf{Q}_i\}-\bar{P}_i)$ are also concave. This makes $\mathbf{L}(\mathbf{Q}_i, \mu_i)$ a concave problem (see [10]). We now proceed to solve this convex optimization problem.

We derive the optimal transmit directions for subcarrier i along the same lines as [7],[11]. Let $\mathbf{A}_i = \mathbf{H}_{0i}^H\bar{\mathbf{R}}_i^{-1}\mathbf{H}_{0i}$. Taking $\mathbf{Q}_i = \mathbf{V}_i\mathbf{\Lambda}_i\mathbf{V}_i^H$,where $\mathbf{V}_i$ is a square matrix of dimension N_t with unit norm columns and $\mathbf{\Lambda}_i$ be a diagonal matrix with non-negative entries that represent the power allocation across the different transmit streams. Ignoring the constant terms, the maximization may be written as

$$\max_{\mathbf{V}_i} \log|\mathbf{I}+\mathbf{V}_i^H\mathbf{A}_i\mathbf{V}_i\mathbf{\Lambda}_i| - \mathtt{tr}\left\{\mathbf{V}_i^H(\mathbf{B}_i + \mu_i\mathbf{I})\mathbf{V}_i\mathbf{\Lambda_i}\right\} \quad (11)$$

Note that $\mathbf{B}_i$ is symmetric positive semi-definite and hence, so is $\mathbf{B}_i+\mu_i\mathbf{I}$. We can then define the Cholesky decomposition for $\mathbf{B}_i + \mu_i\mathbf{I}$ as $\mathbf{W}\mathbf{W}^H$ where $\mathbf{W}$ is a lower-triangular Cholesky factor. Define $\tilde{\mathbf{V}}_i = \mathbf{W}^H\mathbf{V}_i$. Equation (11) may be rewritten as

$$\max_{\tilde{\mathbf{V}}_i} \log|\mathbf{I}+\tilde{\mathbf{V}}_i^H\mathbf{W}^{-1}\mathbf{A}_i\mathbf{W}^{-H}\tilde{\mathbf{V}}_i\mathbf{\Lambda}_i| - \mathtt{tr}\left\{\tilde{\mathbf{V}}_i\tilde{\mathbf{V}}_i^H\mathbf{\Lambda}_i\right\} \quad (12)$$

Let the eigen decomposition of $\mathbf{W}^{-1}\mathbf{A}_i\mathbf{W}^{-H}$ be $\mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$, where $\mathbf{U}$ is a unitary matrix. Then if $\tilde{\mathbf{Q}}_i = \mathbf{U}^H\tilde{\mathbf{V}}_i\mathbf{\Lambda}_i\tilde{\mathbf{V}}_i^H\mathbf{U}$, equation (12) may be rewritten as

$$\max_{\tilde{\mathbf{Q}}_i \geq \mathbf{0}} \log|\mathbf{I} + \mathbf{\Sigma}\tilde{\mathbf{Q}}_i| - \mathtt{tr}\left\{\tilde{\mathbf{Q}}_i\right\} \quad (13)$$

By Hadamard inequality ([12, p.279]), the optimal $\tilde{\mathbf{Q}}_i$ has to be diagonal. Hence, $\tilde{\mathbf{U}}^H\tilde{\mathbf{V}}_i\mathbf{\Lambda}_i^{\frac{1}{2}} = \tilde{\mathbf{Q}}_i^{\frac{1}{2}} = \mathbf{U}^H\mathbf{W}^H\mathbf{V}_i\mathbf{\Lambda}_i^{\frac{1}{2}}$. By direct substitution, it is easy to see that $\mathbf{V}_i^H(\mathbf{B}_i+\mu_i\mathbf{I})\mathbf{V}_i\mathbf{\Lambda}_i = \tilde{\mathbf{Q}}_i$. Now, $\mathbf{V}_i^H\mathbf{A}_i\mathbf{V}_i\mathbf{\Lambda}_i = \tilde{\mathbf{Q}}_i\mathbf{\Sigma}$.

Thus the optimal $\mathbf{V}_i$ diagonalizes both $\mathbf{B}_i+\mu_i\mathbf{I}$ and $\mathbf{A}_i$ and can be interpreted as solution for the generalized eigenmatrix condition ([13])

$$\mathbf{A}_i\mathbf{V}_i = (\mathbf{B}_i + \mu_i\mathbf{I})\,\mathbf{V}_i\mathbf{\Sigma} \quad (14)$$

While (14) provides the directions for transmission, the optimal power allocation $\mathbf{\Lambda}_i$ has to be determined. This can be done as follows. The Lagrangian in equation (10) may be rewritten as

$$\mathbf{L}(\mathbf{Q}_i, \mu_i) = \log|\mathbf{I} + \mathbf{\Lambda}_i\mathbf{V}_i^H\mathbf{A}_i\mathbf{V}_i| - \mathtt{tr}\left\{\mathbf{V}_i^H(\mathbf{B}_i + \mu_i\mathbf{I})\mathbf{V}_i\mathbf{\Lambda}_i\right\} + \mu_i\bar{P}_i + f_i(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}) \quad (15)$$

Let $\mathbf{V}_i^H\mathbf{A}_i\mathbf{V}_i = \mathbf{D}_{1i}$, where $\mathbf{D}_{1i}$ is a diagonal matrix as $\mathbf{V}_i$ is generalized eigenmatrix of $\mathbf{A}_i$, $\mathbf{B}_i + \mu_i\mathbf{I}$. Let $\mathbf{D}_{2i}$ be a diagonal matrix containing the diagonal elements of the matrix $\mathbf{V}_i^H\mathbf{B}_i\mathbf{V}_i$. Equation (15) may be rewritten as

$$\mathbf{L}(\mathbf{Q}_i, \mu_i) = \log|\mathbf{I} + \mathbf{\Lambda}_i\mathbf{D}_{1i}| - \mathtt{tr}\left\{(\mathbf{D}_{2i} + \mu_i\mathbf{I})\mathbf{\Lambda}_i\right\} + \mu_i\bar{P}_i + f_i(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}) \quad (16)$$

Differentiating this with respect to λ_{ij} (the j^{th} diagonal entry of $\mathbf{\Lambda}_i$) yields the water filling equations,

$$\frac{\mathbf{D}_{1i}(j,j)}{1 + \mathbf{D}_{1i}(j,j)\lambda_{ij}} - (\mathbf{D}_{2i}(j,j) + \mu_i) = 0 \quad (17)$$

$$\lambda_{ij} = \left[\frac{1}{\mathbf{D}_{2i}(j,j) + \mu_i} - \frac{1}{\mathbf{D}_{1i}(j,j)}\right]^+ \quad \forall j \text{ such that } \mathbf{D}_{1i}(j,j) > 0 \quad (18)$$

where $[x]^+$ indicates $\max(x, 0)$.

The optimal μ_i can now be determined using a bisection search (see Table I) as λ_{ij} is monotonic in μ_i. Thus, the convex objective function $\mathtt{f}_1$ can be solved iteratively till $\mathbf{Q}_i$ converges.

B. Power allocation across the subcarriers

After obtaining one set of updated $\mathbf{Q}_i$ for all the subcarriers, one can now update the power allocation across the various subcarriers. Note that in this step, the optimal transmit directions across all the used subcarriers remain unchanged, and only the power allocation across the various transmit streams of all the used subcarriers is optimized. The Lagrangian for this scenario may be written as follows.

$$\mathbf{L}(\mathbf{\Lambda}, \eta) = \sum_{k=0}^{N-1} \log|\mathbf{I} + \mathbf{\Lambda}_k\mathbf{D}_{1k}| - \mathtt{tr}\left\{\mathbf{D}_{2k}\mathbf{\Lambda}_k\right\} - \eta\left(\sum_{k=0}^{N-1}\mathtt{tr}(\mathbf{\Lambda}_k) - P\right) \quad (19)$$

where η is the Lagrangian multiplier. Differentiating this with respect to λ_{ij} (the j^{th} diagonal entry of $\mathbf{\Lambda}_i$) yields the water filling equations,

$$\frac{\mathbf{D}_{1i}(j,j)}{1 + \mathbf{D}_{1i}(j,j)\lambda_{ij}} - (\mathbf{D}_{2i}(j,j) + \eta) = 0 \quad (20)$$

$$\lambda_{ij} = \left[\frac{1}{\mathbf{D}_{2i}(j,j) + \eta} - \frac{1}{\mathbf{D}_{1i}(j,j)}\right]^+ \quad \forall i \text{ such that } \mathbf{D}_{1i}(j,j) > 0 \quad (21)$$

The optimal η can now be determined using a bisection search as λ_{ij} is monotonic in η. Once all the λ_{ij} across all the subcarriers and their transmit streams are obtained, this is in turn used to update the transmit covariance matrix $\mathbf{Q}_i$ and the power allocation P_i of each used subcarrier i.

TABLE I. OVERALL ALGORITHM TO SOLVE OBJECTIVE FUNCTION $\mathtt{f}_0$

for i = 0,1 $\ldots$ $N-1$
  Initialize $P_i = \frac{P}{N_u}\mathbf{I}$ and $\mathbf{Q}_i = \frac{P_i}{N_t}\mathbf{I}$
  Initialize $\bar{\mathbf{R}}_i = \mathbf{I} + \sum_{l=0, l\neq k}^{N-1} |\Xi_{i,l}|^2 \mathbf{H}_{1i}\mathbf{Q}_l\mathbf{H}_{1i}^H$
Repeat until convergence
  Update Tx covariance matrix
  for i = 0,1 $\ldots$ $N-1$
  Repeat until convergence
    $\bar{\mathbf{R}}_i = \mathbf{I} + \sum_{l=0, l\neq k}^{N-1} |\Xi_{i,l}|^2 \mathbf{H}_{1i}\mathbf{Q}_l\mathbf{H}_{1i}^H$
    Compute $\mathbf{A}_i = \mathbf{H}_{0i}^H\bar{\mathbf{R}}^{-1}\mathbf{H}_{0i}$
    Compute $\mathbf{B}_i = \sum_{l\neq i} |\Xi_{l,i}|^2 \mathbf{H}_{1l}\left\{\bar{\mathbf{R}}_l^{-1} - (\bar{\mathbf{R}}_l + \mathbf{H}_{0l}\mathbf{Q}_l\mathbf{H}_{0l}^H)^{-1}\right\}\mathbf{H}_{1l}^H$
    Set $\underline{\mu_i} = 0$, $\bar{\mu_i} = \mu_{max}$
    Repeat until convergence
      $\mu_i = \frac{\underline{\mu}_i + \bar{\mu}_i}{2}$
      Compute the generalized eigenmatrix of $\mathbf{A}_i$ and $\mathbf{B}_i + \mu_i\mathbf{I}$
      Normalize the generalized eigenmatrix to have unit norm; denote it as $\mathbf{V}_i$.
      Set $\mathbf{D}_{1i} = \mathbf{V}_i\mathbf{A}_i\mathbf{V}_i^H$
      Set $\mathbf{D}_{2i}$ as the diagonal elements of $\mathbf{V}_i\mathbf{B}_i\mathbf{V}_i^H$
      Compute the transmit powers, $\lambda_{ij} = \left[\frac{1}{\mathbf{D}_{2i}(j,j)+\mu_i} - \frac{1}{\mathbf{D}_{1i}(j,j)}\right]^+$
      If any diagonal entries of $\mathbf{D}_{1i}$ are zero, corresponding λ_{ij} is set to zero.
      if $\mathtt{tr}(\mathbf{\Lambda}_i) > P_i$, set $\underline{\mu}_i = \mu_i$,else set $\bar{\mu}_i = \mu_i$
    Set $\mathbf{Q}_i = \mathbf{V}_i\mathbf{\Lambda}_i\mathbf{V}_i^H$
  Perform power allocation update
  Set $\underline{\eta} = 0$, $\bar{\eta} = \eta_{max}$
  Repeat until convergence
    $\eta = \frac{\underline{\eta}+\bar{\eta}}{2}$
    for l = 0,1 $\ldots$ $N-1$
      Set $\mathbf{D}_{1l} = \mathbf{V}_l\mathbf{A}_l\mathbf{V}_l^H$
      Set $\mathbf{D}_{2l}$ contain the diagonal elements of $\mathbf{V}_l\mathbf{B}_l\mathbf{V}_l^H$
      Compute the transmit powers, $\lambda_{lj} = \left[\frac{1}{\mathbf{D}_{2l}(j,j)+\eta} - \frac{1}{\mathbf{D}_{1l}(j,j)}\right]^+$
      If any diagonal entries of $\mathbf{D}_{1l}$ are zero, corresponding λ_{ij} is set to zero.
      if $\sum_{l=0}^{N-1}\mathtt{tr}(\mathbf{\Lambda}_l) > P$, set $\underline{\eta} = \eta$,else set $\bar{\eta} = \eta$
    for l = 0,1 $\ldots$ $N-1$
      Set $\mathbf{Q}_l = \mathbf{V}_l\mathbf{\Lambda}_l\mathbf{V}_l^H$
      Set $P_l = \mathtt{tr}(\mathbf{\Lambda}_l)$

C. Overall Algorithm and Convergence

The overall algorithm that solves $\mathtt{f}_0$ is given in Table I. At every iteration, a convex sub-problem $\mathtt{f}_1$ is created and optimized based on the updated value of $\mathbf{Q}_i, \mathbf{Q}_{-i}$ from the last iteration. A power allocation across all the subcarriers is performed at the end of one round of transmit covariance update for all subcarriers.

We now show that this algorithm is monotonically non-decreasing at each step of the iteration and hence attains convergence.

Proposition 1 : $\mathtt{f}_1$ is a minorization ([6]) function for $\mathtt{f}_0$ at any $\mathbf{Q}_i, \bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}$.

Proof: $\mathtt{f}_1$ was constructed by replacing a convex function with it's tangent. For a convex function, it is well known that the tangent is always a minorizer.

The non-decreasing behaviour of the algorithm in Table I is now shown below on the same lines as in [6]. Let $\bar{\mathbf{Q}}_i$ be the current value of $\mathbf{Q}_i$ at the beginning of an iteration, and let $\mathbf{Q}_i^*$ be the updated value. Then,

$$\begin{aligned}\mathtt{f}_0(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}) &= \mathtt{f}_1(\bar{\mathbf{Q}}_i, \bar{\mathbf{Q}}_{-i}) \\ &\leq \mathtt{f}_1(\mathbf{Q}_i^*, \bar{\mathbf{Q}}_{-i}) \\ &\leq \mathtt{f}_0(\mathbf{Q}_i^*, \bar{\mathbf{Q}}_{-i})\end{aligned} \tag{22}$$

where the first equality can be observed to be true by direct inspection whenever $\mathbf{Q}_i = \bar{\mathbf{Q}}_i$. The first inequality is because $\mathbf{Q}_i^*$ is the result of optimization in III-A, and the second inequality is due to Proposition 1. This shows that the transmit covariance update is non-decreasing.

The iterations for optimization of $\mathbf{Q}_i$ and power allocations are steps in cyclic minimization (actually maximization in this problem, also see [6]). Thus the overall algorithm in Table I results in a non-decreasing updated value of $\mathtt{f}_0$ at each step of the iteration. This ensures convergence to a maximum value.

IV. NUMERICAL RESULTS

We consider a MIMO fading channel based on equation (2). A single user MIMO scenario with signal to AWGN noise ratio of 25dB is considered. For every Tx-Rx pair,FIR Rayleigh fading channels are generated independently with the power delay profile (PDP) as [0 -5 -5] in dB for $\mathbf{h}_0$ and $\mathbf{h}_1$. An LTE OFDM system operating at unlicensed 2.4GHz band is considered with 15KHz of channel spacing. For simplicity, we consider 16 subcarriers though the proposed algorithm can support any FFT size. A Doppler frequency corresponding to 450kmph is assumed. The entries of $\mathbf{h}_1$ are scaled such that the overall ICI power experienced at any receive antenna corresponds to a Doppler frequency shift of 450kmph. The capacity of the iterative scheme under different scenarios is considered. In the simulation results presented, all subcarriers are assumed to be used.

Figure 1 shows a scenario with $N_t = 6$ transmit antennas and $N_r = 3$ receive antennas. As a reference, the capacity of a transmitter that implements the water filling algorithm without taking into account the ICI observed at the receiver is also shown. As expected, with the knowledge of ICI at the transmitter, the iterative algorithm is able to significantly improve the performance of the link. Also shown in the same figure is the performance with a scaled identity matrix used for transmit covariance on all the subcarriers. Figure 2 shows the performance of the the above schemes in the absence of ICI, as expected, the iterative method gives the same result as that of the standard MIMO water filling ([14]) in the absence of ICI.

Figure 3 considers a scenario with more number of receive antennas ($N_r = 5$) compared to transmit antennas ($N_t = 3$). Here, we find that the knowledge of ICI at the transmitter only results in marginal improvement of the overall capacity as the excess of receive antennas can be used effectively to suppress the ICI.

In all the simulation scenarios considered, we see that the iterations always exhibit a non-decreasing behaviour in the capacity as is predicted by the theory (section III-C).

V. CONCLUSION

In this paper, we present an iterative algorithm for the joint design of the optimal MIMO transmit precoder and the transmit power allocations for an OFDM system, in the presence

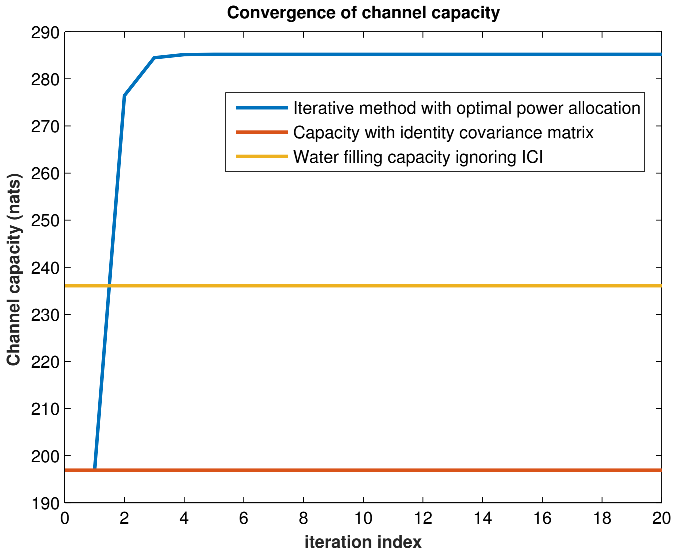

Fig. 1. Simulation Results with $N_t = 6$,$Nr = 3$ and Doppler of 450Kmph

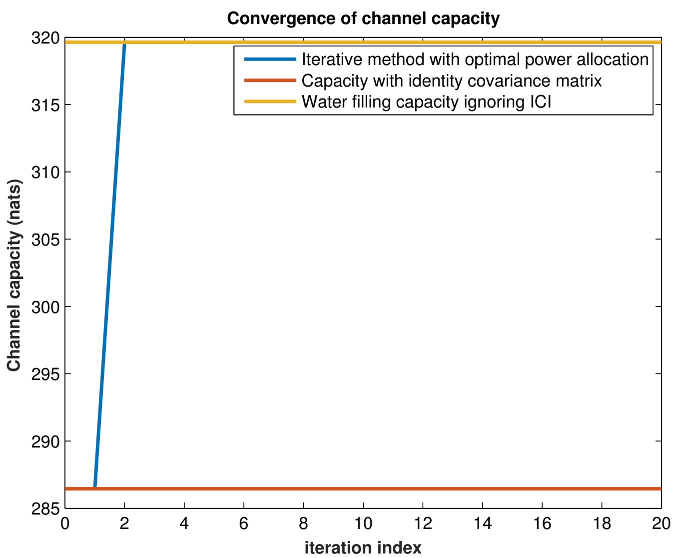

Fig. 2. Simulation Results with $N_t = 6$,$Nr = 3$ and zero Doppler

of ICI. The channel variations due to Doppler are modelled to be linear across the OFDM symbol. To solve the joint problem of power allocation across subcarriers and precoder design for all the useful subcarriers, we employ the cyclic minimization approach to alternately optimize the precoder design and transmit power allocation across subcarriers. As the problem of transmit covariance matrix design is not convex, we first construct an approximate convex problem following the difference of convex functions (DC) approach as in [7].We reinterpret the DC approach as a majorization technique([6]). The optimal $\mathbf{Q}$ is then derived iteratively for each subcarrier for a given power allocation. The updated covariance matrices are then used to update the power allocation across subcarriers. We are able to show analytically, the convergence of the iterative method and then give numerical results that show the convergence behaviour. Our iterative algorithm can also easily accommodate any unused subcarriers, like for instance,

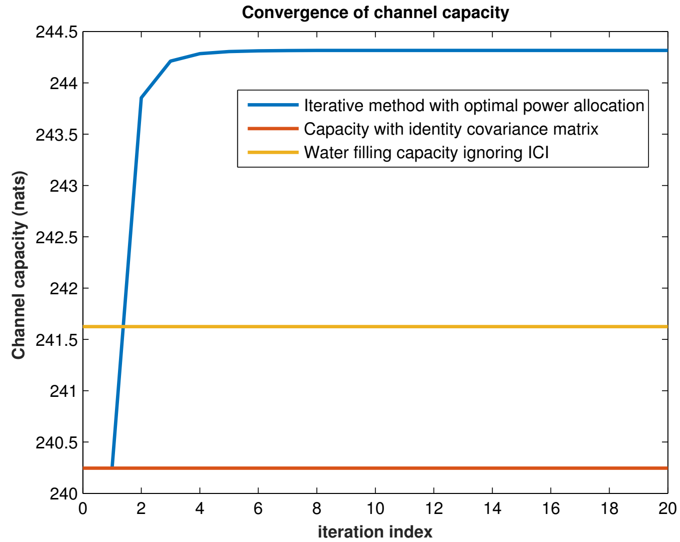

Fig. 3. Simulation Results with $N_t = 3$,$Nr = 5$ and Doppler of 450Kmph

the guard subcarriers or the DC subcarrier.

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