WEIGHTED SUM RATE MAXIMIZATION OF MISO INTERFERENCE BROADCAST CHANNELS VIA DIFFERENCE OF CONVEX FUNCTIONS PROGRAMMING: A LARGE SYSTEM ANALYSIS

Wassim Tabikh¶‡ *Dirk Slock*¶ *Yi Yuan-Wu*‡

¶EURECOM, Sophia-Antipolis, France, Email:tabikh@eurecom.fr, slock@eurecom.fr

‡Orange Labs, Issy-les-Moulineaux, France, Email:wassim.tabikh,yi.yuan@orange.com

ABSTRACT

The weighted sum rate (WSR) maximizing linear precoding algorithm is studied in large correlated multiple-input single-output (MISO) interference broadcast channels (IBC). We consider an iterative WSR design via difference of convex functions (DC) programming as in [1], [2] and [3], focusing on the version in [3]. We propose an asymptotic approximation of the signal-to-interference plus noise ratio (SINR) at every iteration.

***Index Terms*—random matrix theory, beamforming, weighted sum rate maximization**

I. INTRODUCTION

In this paper, Tx may denote transmit/transmitter/ transmission and Rx may denote receive/receiver/reception. We consider the MISO IBC with linear precoding at the transmitter. In this case, we have C base stations (BS), each one of them is endowed with M antennas, whereas the K users of each cell c $\in 1, 2, ...C$ have single-antenna receivers. The precoding matrix that maximizes (local optimum) the WSR for IBC via DC programming for perfect channel state information (CSIT) is obtained from an iterative algorithm proposed in [1], [2] and [3].

In this contribution, we carry out a large system analysis of this latter optimal beamformer which we will denote by WSR-DC. Large system analysis which appeared in [4] and [5] allows to obtain deterministic (instead of fast fading channel dependent) expressions for various scalar quantities, facilitating the analysis of wireless systems. E.g. it may allow to evaluate beamforming performance without computing explicit beamformers. The analysis in [4] and [5] allowed e.g. the determination of the optimal regularization factor for the regularized zero-forcing precoders. A little known extension appeared in [6], [7] and [8] for optimal beamformers. However, in [6] and [7] the precoders are designed using the connection between WSR and the mean squared error (MSE), and in [8] the precoder aims to minimize the transmit power instead of maximizing the sum rate. These approaches are different than ours. Furthermore, the deterministic limits of the SINRs corresponding to the iterative IBC WSR-DC process leading to the optimal WSR are presented, which makes it possible to evaluate its performance more easily and compare with other algorithms and precoders.

Notation: The operators $()^H, tr(.) \; and \; E[.]$ denote conjugate transpose, trace and expectation, respectively. The $M \times M$ identity matrix is denoted I_M, $\ln(.)$ is the natural logarithm and diag is the diagonal matrix. The contribution of this paper is: a large system analysis of the WSR-DC iterative maxizimization problem.

II. STREAMWISE IBC SIGNAL MODEL

In the rest of this paper we shall consider a per stream approach (which in the perfect CSI case would be equivalent to per user). In an IBC formulation, one stream per user can be expected to be the usual scenario. In the development below, in the case of more than one stream per user, one can treat each stream as an individual user. So, consider an IBC with C cells with a total of K users. In this section we consider a system-wide numbering of the users. User k is served by BS b_k. The $N \times 1$ received signal at user k in cell b_k is

$$\mathbf{y}_k = \underbrace{\mathbf{H}_{k,b_k}\,\mathbf{g}_k\,x_k}_{\text{signal}} + \underbrace{\sum_{\substack{i\neq k\\ b_i=b_k}} \mathbf{H}_{k,b_k}\,\mathbf{g}_i\,x_i}_{\text{intracell interf.}} + \underbrace{\sum_{j\neq b_k}\sum_{i:b_i=j} \mathbf{H}_{k,j}\,\mathbf{g}_i\,x_i}_{\text{intercell interf.}} + \mathbf{v}_k \quad (1)$$

where x_k is the intended (white, unit variance) scalar signal stream, $\mathbf{H}_{k,b_k}$ is the $N \times M$ channel from BS b_k to user k. BS b_k serves $K_{b_k} = \sum_{i:b_i=b_k} 1$ users. We are considering a noise whitened signal representation so that we get for the noise $\mathbf{v}_k \sim \mathcal{CN}(0, I_N)$. The $M \times 1$ spatial Tx filter or beamformer (BF) is $\mathbf{g}_k$. Treating interference as noise, user k will apply a linear Rx filter $\mathbf{f}_k$ to maximize the signal power (diversity) while reducing any residual interference that would not have been (sufficiently) suppressed by the BS Tx. The Rx filter output is $\widehat{x}_k = \mathbf{f}_k^H \mathbf{y}_k$

$$\begin{aligned}\widehat{x}_k &= \mathbf{f}_k^H \mathbf{H}_{k,b_k}\,\mathbf{g}_k\,x_k + \sum_{i=1,\neq k}^{K} \mathbf{f}_k^H \mathbf{H}_{k,b_i}\,\mathbf{g}_i\,x_i + \mathbf{f}_k^H \mathbf{v}_k \\ &= \mathbf{f}_k^H \mathbf{h}_{k,k}\,x_k + \sum_{i\neq k} \mathbf{f}_k^H \mathbf{h}_{k,i}\,x_i + \mathbf{f}_k^H \mathbf{v}_k\end{aligned} \quad (2)$$

where $\mathbf{h}_{k,i} = \mathbf{H}_{k,b_i}\,\mathbf{g}_i$ is the channel-Tx cascade vector.

III. MAX WSR VIA DC

In this section we consider a system-wide numbering of the users. Consider as a starting point for the optimization the weighted sum rate (WSR)

$$WSR = WSR(\mathbf{g}) = \sum_{k=1}^{K} u_k \ \ln \frac{1}{e_k} \tag{3}$$

where $\mathbf{g}$ represents the collection of BFs $\mathbf{g}_k$, the u_k are rate weights, the $e_k = e_k(\mathbf{g})$ are the Minimum Mean Squared Errors (MMSEs) for estimating the x_k:

$$\begin{aligned}&\frac{1}{e_k}=1+\mathbf{g}_k^H\mathbf{H}_{k,b_k}^H\mathbf{R}_{\overline{k}}^{-1}\mathbf{H}_{k,b_k}\mathbf{g}_k=(1-\mathbf{g}_k^H\mathbf{H}_{k,b_k}^H\mathbf{R}_k^{-1}\mathbf{H}_{k,b_k}\mathbf{g}_k)^{-1}\\&\mathbf{R}_k = \mathbf{H}_{k,b_k}\mathbf{Q}_k\,\mathbf{H}_{k,b_k}^H + \mathbf{R}_{\overline{k}}\,,\ \mathbf{Q}_i = \mathbf{g}_i\mathbf{g}_i^H\,,\\&\mathbf{R}_{\overline{k}} = \sum_{i\neq k}\mathbf{H}_{k,b_i}\mathbf{Q}_i\,\mathbf{H}_{k,b_i}^H + I_{N_k}\,.\end{aligned} \tag{4}$$

$\mathbf{R}_k$, $\mathbf{R}_{\overline{k}}$ are the total and interference plus noise Rx covariance matrices resp. and e_k is the MMSE obtained at the output $\widehat{x}_k = \mathbf{f}_k^H\mathbf{y}_k$ of the optimal (MMSE) linear Rx $\mathbf{f}_k$,

$$\mathbf{f}_k = \mathbf{R}_k^{-1}\mathbf{H}_{k,b_k}\mathbf{g}_k = \mathbf{R}_k^{-1}\mathbf{h}_{k,k}\ . \tag{5}$$

The WSR cost function needs to be augmented with the power constraints

$$\sum_{k:b_k=j}\text{tr}\{\mathbf{Q}_k\} \leq P_j\ . \tag{6}$$

In a classical difference of convex functions (DC programming) approach, Kim and Giannakis [2] propose to keep the concave signal terms and to replace the convex interference terms by the linear (and hence concave) tangent approximation. More specifically, consider the dependence of WSR on $\mathbf{Q}_k$ alone. Then

$$\begin{aligned}&WSR = u_k \ln\det(\mathbf{R}_{\overline{k}}^{-1}\mathbf{R}_k) + WSR_{\overline{k}}\,,\\&WSR_{\overline{k}} = \textstyle\sum_{i=1,\neq k}^{K} u_i \ln\det(\mathbf{R}_{\overline{i}}^{-1}\mathbf{R}_i)\end{aligned} \tag{7}$$

where $\ln\det(\mathbf{R}_{\overline{k}}^{-1}\mathbf{R}_k)$ is concave in $\mathbf{Q}_k$ and $WSR_{\overline{k}}$ is convex in $\mathbf{Q}_k$. Since a linear function is simultaneously convex and concave, consider the first order Taylor series expansion in $\mathbf{Q}_k$ around $\widehat{\mathbf{Q}}$ (i.e. all $\widehat{\mathbf{Q}}_i$) with e.g. $\widehat{\mathbf{R}}_i = \mathbf{R}_i(\widehat{\mathbf{Q}})$, then

$$\begin{aligned}&WSR_{\overline{k}}(\mathbf{Q}_k,\widehat{\mathbf{Q}}) \approx WSR_{\overline{k}}(\widehat{\mathbf{Q}}_k,\widehat{\mathbf{Q}}) - \text{tr}\{(\mathbf{Q}_k - \widehat{\mathbf{Q}}_k)\widehat{\mathbf{A}}_k\}\\&\widehat{\mathbf{A}}_k = -\left.\frac{\partial WSR_{\overline{k}}(\mathbf{Q}_k,\widehat{\mathbf{Q}})}{\partial \mathbf{Q}_k}\right|_{\widehat{\mathbf{Q}}_k,\widehat{\mathbf{Q}}} = \sum_{i\neq k}^{K} u_i\mathbf{H}_{i,b_k}^H(\widehat{\mathbf{R}}_{\overline{i}}^{-1}-\widehat{\mathbf{R}}_i^{-1})\mathbf{H}_{i,b_k}\end{aligned} \tag{8}$$

Note that the linearized (tangent) expression for $WSR_{\overline{k}}$ constitutes a lower bound for it. Now, dropping constant terms, reparameterizing the $Q_k = \mathbf{g}_k\mathbf{g}_k^H$, performing this linearization for all users, and augmenting the WSR cost function with the constraints, we get the Lagrangian

$$\begin{aligned}&WSR(\mathbf{g},\widehat{\mathbf{g}},\lambda) = \sum_{j=1}^{C}\lambda_j P_j +\\&\sum_{k=1}^{K} u_k \ln(1 + \mathbf{g}_k^H\widehat{\mathbf{B}}_k\mathbf{g}_k) - \mathbf{g}_k^H(\widehat{\mathbf{A}}_k + \lambda_{b_k} I)\mathbf{g}_k\end{aligned} \tag{9}$$

where $$\widehat{\mathbf{B}}_k = \mathbf{H}_{k,b_k}^H\widehat{\mathbf{R}}_{\overline{k}}^{-1}\mathbf{H}_{k,b_k}\ . \tag{10}$$

The gradient (w.r.t. $\mathbf{g}_k$) of this concave WSR lower bound is actually still the same as that of the original WSR criterion! And it allows an interpretation as a generalized eigenvector condition

$$\widehat{\mathbf{B}}_k\,\mathbf{g}_k = \frac{1+\mathbf{g}_k^H\widehat{\mathbf{B}}_k\mathbf{g}_k}{u_k}\,(\widehat{\mathbf{A}}_k + \lambda_{b_k}I)\mathbf{g}_k \tag{11}$$

or hence $\mathbf{g}_k^{'} = V_{max}(\widehat{\mathbf{B}}_k, \widehat{\mathbf{A}}_k + \lambda_{b_k}I)$ is the (normalized) "max" generalized eigenvector of the two indicated matrices, with max eigenvalue $\sigma_k = \sigma_{max}(\widehat{\mathbf{B}}_k, \widehat{\mathbf{A}}_k + \lambda_{b_k}I)$. Let $\sigma_k^{(1)} = \mathbf{g}_k^{'H}\widehat{\mathbf{B}}_k\mathbf{g}_k^{'}$, $\sigma_k^{(2)} = \mathbf{g}_k^{'H}\widehat{\mathbf{A}}_k\mathbf{g}_k^{'}$. The advantage of formulation (9) is that it allows straightforward power adaptation: introducing stream powers $p_k \geq 0$ and substituting $\mathbf{g}_k = \sqrt{p_k}\,\mathbf{g}_k^{'}$ in (9) yields

$$WSR=\sum_{j}^{C}\lambda_j P_j + \sum_{k=1}^{K}\{u_k\ln(1+p_k\sigma_k^{(1)}) - p_k(\sigma_k^{(2)}+\lambda_{b_k})\}$$

which leads to the following interference leakage aware water filling

$$p_k = \left(\frac{u_k}{\sigma_k^{(2)}+\lambda_{b_k}} - \frac{1}{\sigma_k^{(1)}}\right)^{+} \tag{12}$$

where the Lagrange multipliers are adjusted to satisfy the power constraints $\sum_{k:b_k=j} p_k = P_j$. This can be done by bisection and gets executed per BS. Note that some Lagrange multipliers could be zero. Note also that as with any alternating optimization procedure, there are many updating schedules possible, with different impact on convergence speed. The quantities to be updated are the $\mathbf{g}_k^{'}$, the p_k and the λ_l. Whereas we focused on the case of one stream/user, the advantage of the DC approach is that it works for any number of streams/user, by simply taking more eigenvectors. The waterfilling then automatically determines (at each iteration) how many streams can be sustained.

IV. THE MISO CASE

In the MISO case, we have $\mathbf{C}_r = 1$ and we shall denote the matrices $\mathbf{R}$, $\mathbf{H}^H$ as the scalar r and the vector $\mathbf{h}$. We get $\mathbf{g}_k^{'} = V_{max}(\mathbf{B}_k, \mathbf{A}_k + \lambda_{b_k}I)$ and associated generalized eigenvalue $1/a_k = \lambda_{max}(\mathbf{B}_k, \mathbf{A}_k + \lambda_{b_k}I)$. Note that $\mathbf{g}_k^{'}$ is proportional to $(\mathbf{A}_k+\lambda_{b_k}I)^{-1}\mathbf{h}_{k,b_k}$ and that any scale factor in $\mathbf{g}_k^{'}$ gets compensated by the stream power p_k.

The proposed solution is amenable to large system analysis as in [7]. From now on, we will no more consider a system-wide numbering of the users. The WSR-DC precoder for user k in cell c can be expressed [3] as the following:

$$\mathbf{g}_{c,k}^{'} = (\mathbf{A}_{c,k} + \alpha_c I)^{-1}\mathbf{h}_{c,c,k} \tag{13}$$

$$\mathbf{A}_{c,k} = \sum_{(i,j)\neq(c,k)}^{K} u_i\mathbf{h}_{c,i,j}^H(r_{\overline{i,j}}^{-1} - r_{i,j}^{-1})\mathbf{h}_{c,i,j} \tag{14}$$

where $h_{c,c,k}$ is the $M \times 1$ channel between cell c and the user k of cell c, with $k \in [1, K_c]$. The

Lagrangian term α_c can be fixed optimally to $\alpha_c = tr\frac{\overline{\mathbf{D}}_c}{\rho_c}$ where $\overline{\mathbf{D}}_c = diag(\overline{d}_{c,1}, ...\overline{d}_{c,K})$ as in [9] and with $d_{i,j} = (r_{i,j}^{-1} - r_{i,j}^{-1}) = -r_{i,j}^{-1}(\mathbf{h}_{i,i,j}\mathbf{Q}_{i,j}\mathbf{h}_{i,i,j}^H)r_{i,j}^{-1} = a_{i,j}^2 \times w_{i,j}$ [4, Lemma 2]; where $a_{i,j}$ and $w_{i,j} = 1+\gamma_{i,j}$ are the Rx filter and the precoding weight as in [6]. The large system analysis treats all the users' fast fading parameters as identical and differentiates the users only by their second-order statistics (channels' covariance matrices), thus there is no need for waterfilling and a simple normalization parameter will be enough as in [4], [5] and [6].

V. MASSIVE MISO LIMIT

In this section we will derive the deterministic equivalent of the SINR for correlated channles. The channel $\mathbf{h}_{i,c,k}^H$ is correlated as
$\mathbb{E}\,[\mathbf{h}_{i,c,k}\mathbf{h}_{i,c,k}^H] = \mathbf{\Theta}_{i,c,k}$ thus

$$\mathbf{h}_{i,c,k} = \sqrt{M}\mathbf{\Theta}_{i,c,k}^{1/2}\mathbf{z}_{i,c,k} \tag{15}$$

where $\mathbf{z}_{i,c,k}$ has i.i.d. complex entries of zero mean and variance $\frac{1}{M}$ and the $\mathbf{\Theta}_{i,c,k}^{1/2}$ is the Hermitian square-root of $\mathbf{\Theta}_{i,c,k}$. The correlation matrix $\mathbf{\Theta}_{i,c,k}$ is positive semi-definite and of uniformly bounded spectral norm w.r.t. to M. For notational convenience, we denote $\mathbf{\Theta}_{c,c,k}$ as $\mathbf{\Theta}_{c,k}$. If the number of Tx antennas M becomes very large, the WSR (SINR) can be approximated using tools from Random Matrix Theory. In this section, performance analysis is conducted for the proposed precoder. The large-system limit is considered, where M and K go to infinity while keeping the ratio K/M finite such that $\limsup_M K/M < \infty$ and $\liminf_M K/M > 0$. The procedure should be understood in the way that, for each set of system dimension parameters M and K we provide an approximate expression for the SINR $\gamma_{c,k}$. Our precoder is initialized using a Matched Filter (MF) Precoder. Let $\gamma_{c,k}^{MF}$ be the SINR of user k of cell c under MF precoding then, $\gamma_{c,k}^{MF} - \overline{\gamma}_{c,k}^{MF} \xrightarrow{M\to\infty} 0$ [6], almost surely

$$\overline{\gamma}_{c,k}^{MF} = \frac{1}{\frac{1}{\beta_c\rho_c} + \frac{1}{M^2}\sum\limits_{(l,i)\neq(c,k)} tr\mathbf{\Theta}_{l,c,k}\mathbf{\Theta}_{l,i}} \tag{16}$$

where ρ_c is the signal-to-noise ratio and $\beta_c = \frac{K}{M}$. For the WSR-DC precoder, a deterministic equivalent of the SINR is provided in the following theorem

Theorem 1: Let $\gamma_{c,k}$ be the SINR of the kth user of cell c with the precoder defined in (13). Then, a deterministic equivalent $\overline{\gamma}_{c,k}^{(j)}$ at iteration $j > 0$ and under MF initialization, is given by $\overline{\gamma}_{c,k}^{(j)}$ is given by

$$\overline{\gamma}_{c,k}^{(j)} = \frac{(\overline{m}_{c,k}^{(j)})^2 \times (1+\overline{m}_{c,k}^{(j)})^2}{(\overline{\Upsilon}_{c,k}^{(j)} + \overline{\hat{\Upsilon}}_{c,k}^{(j)})\overline{d}_{c,k} + \overline{d}_{c,k}^{(j),2}\frac{\overline{\Psi}_c^{(j)}}{\rho_c}(1+\overline{m}_{c,k}^{(j)})^2} \tag{17}$$

where

$$\overline{m}_{c,k}^{(j)} = \frac{1}{M}tr\overline{\mathbf{\Theta}}_{c,k}^{(j)}V_c \tag{18}$$

$$\overline{\Psi}_c^{(j)} = \frac{1}{M}\sum_{i=1}^{K}\frac{1}{\overline{d}_{c,i}}e_{c,i}^{'} \tag{19}$$

$$\overline{\Upsilon}_{c,k}^{(j)} = \frac{1}{M}\sum_{l=1,l\neq k}^{K}\frac{1}{\overline{d}_{c,l}}e_{c,c,k,c,l}^{\prime} \tag{20}$$

$$\overline{\hat{\Upsilon}}_{c,k}^{(j)} = \frac{1}{M}\sum_{m=1,m\neq c}^{C}\frac{(1+\overline{m}_{c,k}^{(j)})^2}{(1+\overline{m}_{m,c,k}^{(j)})^2}\sum_{l=1}^{K}\frac{1}{\overline{d}_{m,l}}e_{m,c,k,m,l}^{'} \tag{21}$$

with $\overline{\mathbf{\Theta}}_{m,c,k} = \overline{d}_{c,k}\mathbf{\Theta}_{m,c,k}$, $\overline{m}_{m,c,k}^{(j)} = \frac{1}{M}tr\overline{\mathbf{\Theta}}_{m,c,k}^{(j)}\mathbf{V}_m$ and $\overline{a}_{c,k}^{(j)}, \overline{w}_{c,k}^{(j)}$ and $\overline{d}_{c,k}^{(j)}$ are given by

$$\overline{a}_{c,k}^{(j)} = \frac{1}{\sqrt{\overline{P}_{c,k}^{(j-1)}}}\frac{\overline{\gamma}_{c,k}^{(j-1)}}{1+\overline{\gamma}_{c,k}^{(j-1)}} \tag{22}$$

$$\sqrt{\overline{P}_{c,k}^{(j-1)}} = \frac{1}{\overline{d}_{c,k}^{(j-1)}}\sqrt{\frac{P_c}{\overline{\Psi}_c^{(j-1)}}}\overline{m}_{c,k}^{(j-1)} \tag{23}$$

$$\overline{w}_{c,k}^{(j)} = (1+\overline{\gamma}_{c,k}^{(j-1)}); \overline{d}_{c,k}^{(j)} = \overline{w}_{c,k}^{(j)}\overline{a}_{c,k}^{2,(j)}. \tag{24}$$

Denoting

$$\mathbf{V}_c = (\mathbf{F}_c + \overline{\alpha}_c\mathbf{I}_M)^{-1} \tag{25}$$

with $\overline{\alpha}_c^{(j)} = \frac{tr\overline{\mathbf{D}}_c^{(j)}}{M\rho_c}$, three systems of coupled equations have to be solved. First, we need to introduce $e_{m,c,k}\forall\{m,c,k\} \in \{\mathcal{C},\mathcal{C},\mathcal{K}_c\}$ which form the unique positive solutions of

$$e_{m,c,k} = \frac{1}{M}tr\overline{\mathbf{\Theta}}_{m,c,k}\mathbf{V}_m, \tag{26}$$

$$\mathbf{F}_m = \frac{1}{M}\sum_{j=1}^{C}\sum_{i=1}^{K}\frac{\overline{\mathbf{\Theta}}_{m,j,i}}{1+e_{m,j,i}}. \tag{27}$$

$e_{c,c,k}$ and $m_{c,c,k}$ denote $e_{c,k}$ and $m_{c,k}$ respectively. Secondly,we give $e_{1,1}^{\prime}, ..., e_{1,K}^{\prime}, ...e_{C,1}^{\prime}, ..., e_{C,K}^{\prime}$ which form the unique positive solutions of

$$e_{c,k}^{\prime} = \frac{1}{M}tr\overline{\mathbf{\Theta}}_{c,k}\mathbf{V}_c(\mathbf{F}_c^{\prime} + \mathbf{I}_M)\mathbf{V}_c, \tag{28}$$

$$\mathbf{F}_c^{\prime} = \frac{1}{M}\sum_{j=1}^{C}\sum_{i=1}^{K}\frac{\overline{\mathbf{\Theta}}_{c,j,i}e_{j,i}^{\prime}}{(1+e_{c,j,i})^2}. \tag{29}$$

And finally, we provide $e_{m,c,k,m,l}^{\prime}\forall\{m,c,k\} \in \{\mathcal{C},\mathcal{C},\mathcal{K}_c\}$

$$e_{m,c,k,m,l}^{\prime} = \frac{1}{M}tr\overline{\mathbf{\Theta}}_{m,c,k}\mathbf{V}_m(\mathbf{F}_{m,m,l}^{\prime} + \overline{\mathbf{\Theta}}_{m,l})\mathbf{V}_m \tag{30}$$

$$\mathbf{F}_{m,m,l}^{\prime} = \frac{1}{M}\sum_{j=1}^{C}\sum_{i=1}^{K}\frac{\overline{\mathbf{\Theta}}_{m,j,i}e_{m,j,i,m,l}^{\prime}}{(1+e_{m,j,i})^2}. \tag{31}$$

For $j \geq 1$, define $\Gamma_c^{(j)} = \frac{1}{M}\mathbf{H}_c^H\overline{\mathbf{D}}^{(j)}\mathbf{H}_c + \overline{\alpha}_c^{(j)}\mathbf{I}_M$, with $\mathbf{H}_c = [\mathbf{h}_{c,1,1}, ..., \mathbf{h}_{c,i,j}, ..., \mathbf{h}_{c,C,K}]^H s.t\ (i,j) \neq (c,k)$ and

$\overline{\mathbf{D}} = diag(\overline{\mathbf{D}}_1, ...\overline{\mathbf{D}}_C)$. The precoder at the end of iteration j is given by

$$\overline{\mathbf{g}}_{c,k}^{(j)} = \frac{\xi_c^{(j)}}{M} (\mathbf{\Gamma}_c^{(j)})^{-1} \mathbf{h}_{c,c,k} \tag{32}$$

for each user k in the cell c, where $\xi_c^{(j)}$ is

$$\xi_c^{(j)} = \sqrt{\frac{P_c}{\frac{1}{M^2} tr(\mathbf{\Gamma}_c^{(j)})^{-2} \mathbf{H}_{\hat{c}}^H \overline{\mathbf{A}}_c^{H,(j)} \overline{\mathbf{W}}_c^{2,(j)} \overline{\mathbf{A}}_c^{(j)} \mathbf{H}_{\hat{c}}}} \tag{33}$$

$$= \sqrt{\frac{P_c}{\Psi_c^{(j)}}}. \tag{34}$$

where $H_{\hat{c}} = [h_{c,c,1}, ..., h_{c,c,K}]^H$. We derive the deterministic equivalents of the normalization term $\xi_c^{(j)}$, the signal power $|\overline{\mathbf{g}}_{c,k}^{H,(j)} \mathbf{h}_{c,c,k}|^2$ and the interference power $\sum_{m=1}^{C} \sum_{l \neq k \; if \; m=c}^{K} \mathbf{h}_{m,c,k}^H \overline{\mathbf{g}}_{m,l}^{(j)} \overline{\mathbf{g}}_{m,l}^{H,(j)} \mathbf{h}_{m,c,k}$ similarly to [4], [5] and [6], i.e, using the same logic and mathematical approach, but for a more complex problem. The proof is omitted due to lack in space.

VI. NUMERICAL RESULTS

We plot the IBC WSR-DC algorithm with MF initialization and compare it to the large system approximation in Theorem 1. The channel correlation matrix is modeled as [4]. Figure 1 and Figure 2 show the WSR-DC precoder and its approximation for $C = 2$ for i.i.d. channels ($\mathbf{\Theta}_{m,c,k} = I_M$) and correlated channels respectively. For the simulations of the IBC WSR-DC algorithm, we have used 200 channel realizations. It can be observed that for i.i.d channels the approximation is very accurate which validates our asymptotic approach. Although the sum rate expression for the approximation approach (17) seems to be complex, however we need to calculate it only once per a given SNR, while we need to run the IBC WSR-DC simulations as many times (200) as the number of channel realizations.

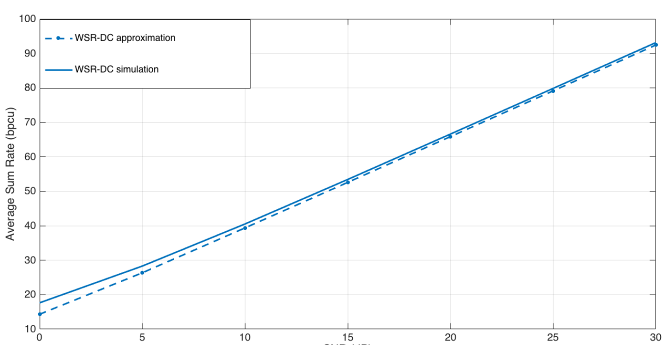

Fig. 1. Sum rate comparisons for C=2,K=4,M=20 with i.i.d channels

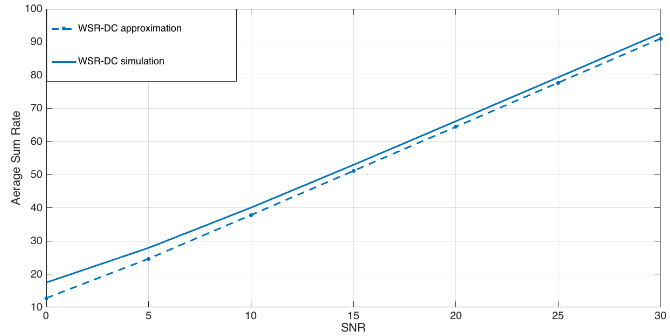

Fig. 2. Sum rate comparisons for C=2,K=4,M=20 with correlated channels

VII. CONCLUSION

In this work, we presented the large system approximate of the performance of the WSR-DC using tools from RMT.

ACKNOWLEDGMENTS

EURECOM's research is partially supported by its industrial members: ORANGE, BMW, SFR, ST Microelectronics, Symantec, SAP, Monaco Telecom, iABG, by the projects ADEL (EU FP7) and DIONISOS (French ANR). The research of Orange Labs is partially supported by the EU H2020 project Fantastic5G.

REFERENCES

[1] C.T.K. Ng and H. Huang"Linear Precoding in Cooperative MIMO Cellular Networks with Limited Coordination Clusters," IEEE Journal on Selected Areas in Communications, Vol. 28, No. 9, December 2010.

[2] S.-J. Kim and G.B. Giannakis, Optimal Resource Allocation for MIMO Ad Hoc Cognitive Radio Networks, IEEE Trans. Info. Theory, May 2011.

[3] W. Tabikh, D. Slock, and Y. Yuan-Wu,"MaMIMO IBC Beamforming Design with Combined Channel Estimate and Covariance CSIT," submitted to ISIT 2016.

[4] S. Wagner, R. Couillet, M. Debbah, and D.T.M. Slock, "Large System Analysis of Linear Precoding in MISO Broadcast Channels with Limited Feedback," IEEE Trans. on Information Theory, vol. 58, no. 7, pp. 4509 - 4537, 2012.

[5] S. Wagner, R. Couillet, M. Debbah, and D.T.M. Slock, "Deterministic Equivalent for the SINR of Regularized Zero-forcing Precoding in Correlated MISO Broadcast Channels with Imperfect CSIT," IEEE International Conference on Communications (ICC), Kyoto, Japan, 2011.

[6] S. Wagner and D. Slock, "Sum Rate Maximization of Correlated MISO Broadcast Channels under Linear

Precoding: A Large System Analysis," 12th IEEE International Workshop on Signal Processing Advances in Wireless Communications, Jun 2011, San Francisco, United States.

[7] W. Tabikh, D. Slock and Y. Yuan-Wu, "Weighted sum rate maximization of correlated MISO interference broadcast channels under linear precoding: A large system Analysis," VTC Spring Workshop, 2016, Nanjing, China.

[8] S. Lakshminarayana, M. Assaad and M. Debbah, "Coordinated Multicell Beamforming for Massive MIMO: A Random Matrix Approach," IEEE Trans. on Information Theory, vol. 61, no. 6, 2015.

[9] S. Christensen, R. Agarwal, E. de Carvalho and J. M. Cioffi, *Weighted Sum-Rate Maximization using Weighted MMSE for MIMO-BC Beamforming Design,* IEEE Trans. Wireless Commun., vol. 7, no. 12, pp. 47924799, Dec. 2008.